Shedding light on photon and dilepton spectral functions[1,2]

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– SEWM • via Zoom • July 2021 –

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[2] supported by the SNF under grant 200020B-188712

Theoretical tools

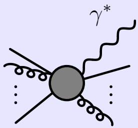
$$\mathcal{L} = -\tfrac{1}{4}F^2 + \sum_f \bar{\psi}\,(i\not{D} - m_f)\,\psi\ ; \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu}$$

perturbation theory

weak-coupling expansion:

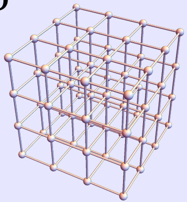
production rates

$$\omega \frac{d\Gamma}{d^3\boldsymbol{k}} = \int d\Phi \left| \quad \right|^2$$



$\times$ (thermal weight)

lattice QCD



$$G_{\mu\nu}(\tau, \boldsymbol{k}) = \int_{\mathbf{x}} e^{\,i\,\boldsymbol{kx}}\, \langle j_\mu(\tau, \mathbf{x}) j_\nu(0) \rangle$$

$\tau \equiv it$ 'imaginary time'

... photons are 'clean' probes b/c they do not re-interact!

(& closely related *dileptons pairs*, e.g. from $q\bar{q} \to \gamma^* \to e^+ e^-$)

Basic relations from pert. theory

[Weldon (1990)]

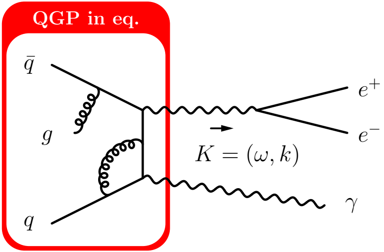
[Gale, Kapusta (1991)]

Dilepton rate:

$$\frac{d\Gamma_{e^- e^+}}{d\omega\, d^3\boldsymbol{k}} \simeq \frac{\alpha_{\rm em}^2 n_{\rm B}(\omega)}{3\pi^2 M^2} C_{\rm em}\, \rho_{\rm V}(\omega, k)$$

Photon rate:

$$\frac{d\Gamma_{\gamma}}{d^3\boldsymbol{k}} \simeq \frac{\alpha_{\rm em} n_{\rm B}(k)}{2\pi^2 k} C_{\rm em}\, \rho_{\rm V}(k, k)$$



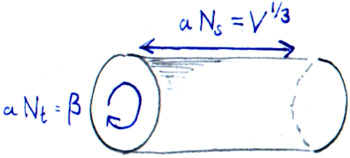
$M^2 \equiv K^2 = \omega^2 - k^2$ invariant mass, $n_{\rm B}$ is the Bose distribution

$$\rho_{\mu\nu}(\omega, k) = \mathrm{Im}\Big[\Pi_{\mu\nu}(\omega + i0^+, k)\Big]$$

Vector channel spectral function $\rho_{\rm V} \equiv \rho_\mu{}^\mu = 2\rho_{\rm T} + \rho_{\rm L}$

Lattice CHALLENGES

$\boxed{N_t \times N_s^3}$ grid



continuum limit: $a \to 0$

thermodynamic : $V \to \infty$

Can $\rho(\omega, k)$ be extracted, for *real* frequencies?

simple inversion is ill-posed : sensitive to input (overdetermined)

$$G(\tau, k) = \int_0^\infty \frac{d\omega}{2\pi} \, \rho(\omega, k) \, \frac{\cosh[(\frac{1}{2}\beta - \tau)\omega]}{\sinh[\frac{1}{2}\beta\omega]}$$

But: by hook or by crook ... [Aarts, Allton, Foley, Hand, Kim (2007)]

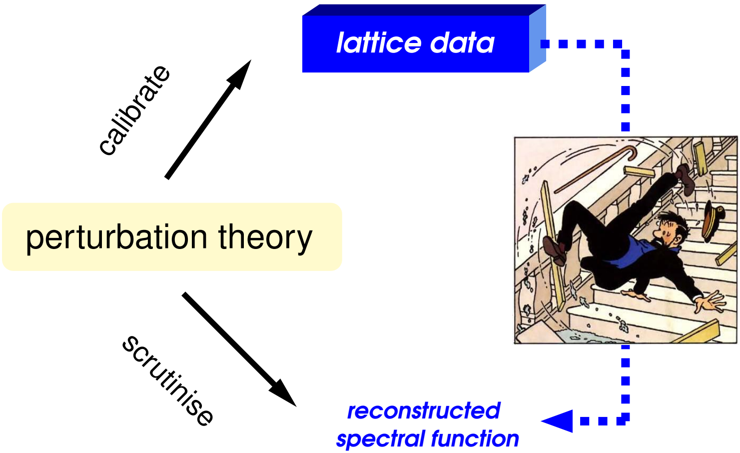
Lattice

UV-finite correlator $G_{\rm H} = 2(G_{\rm T} - G_{\rm L})$ [Brandt, *et al* (2018)]

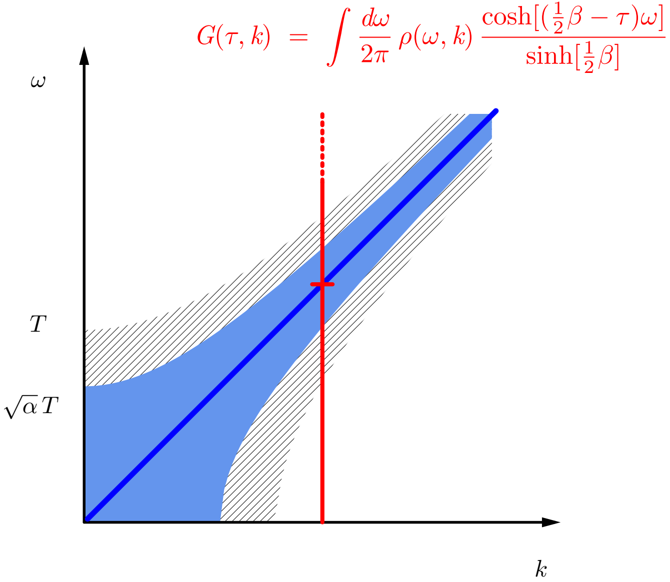
Properties:

- no vacuum part, $\lim_{T\to 0} \rho_{\rm H} = 0$
- expansion, $\rho_{\rm H} \;=\; \alpha_{\rm s}\, 64\pi\, k^2 \int_{\boldsymbol{p}} \frac{p}{\pi} \frac{4(4n_{\rm F} - n_{\rm B})}{9M^4} + O\left(\frac{T^6}{M^4}\right)$
- sum rule, $\int_0^\infty d\omega\, \omega\, \rho_{\rm H}(\omega, k) \;=\; 0$ [Caron-Huot (2009)]

$\Rightarrow$ Improved control over systematic uncertainties!


lattice data
calibrate
perturbation theory
scrutinise
reconstructed
spectral function

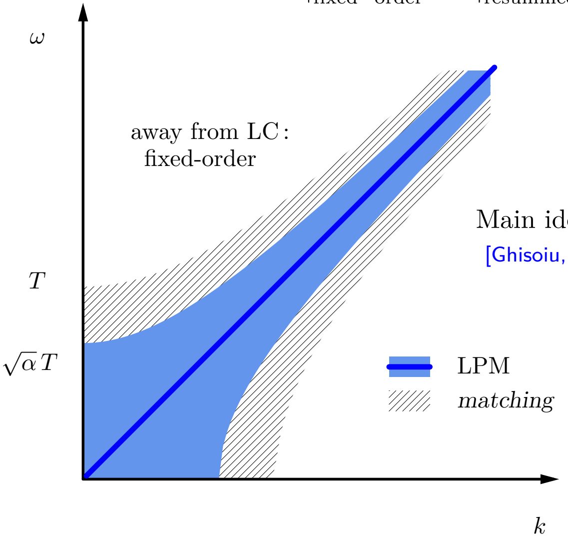
What we do



$G(\tau, k) \;=\; \int \frac{d\omega}{2\pi}\, \rho(\omega, k)\, \frac{\cosh[(\frac{1}{2}\beta - \tau)\omega]}{\sinh[\frac{1}{2}\beta]}$
ω
T
$\sqrt{\alpha}\, T$
k

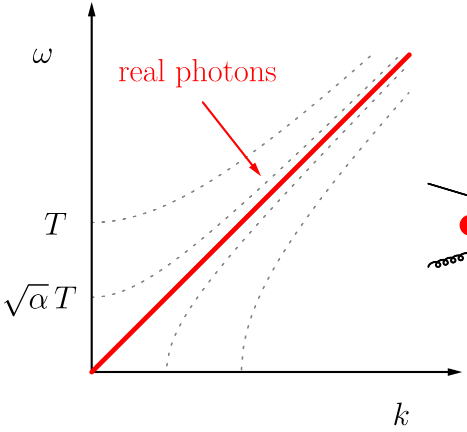
What we do

$$\rho(\omega,k) \;\simeq\; \rho\Big|^{\rm NLO}_{\rm fixed-order} + \rho\Big|^{\rm LPM}_{\rm resummed} - \rho\Big|_{\rm overcounting}$$



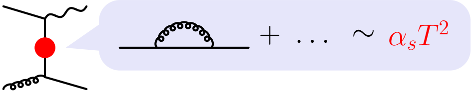
Main idea:

[Ghisoiu, Laine (2014)]



Thermal Screening

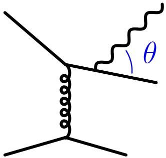
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{-\pi\,\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}}{3s^2}\,\frac{t^2+s^2}{t\,s}$$



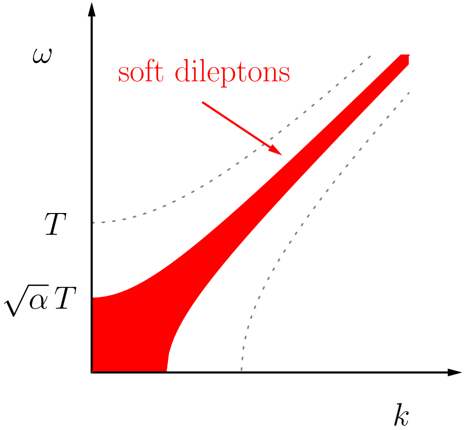
[Kapusta, Lichard, Seibert (1991)]

Landau-Pomeranchuk-Migdal (LPM)

$$\int \frac{\mathrm{d}\cos\theta}{E(1-\cos\theta)} = \infty$$



LO: [Arnold, Moore, Yaffe (2001)] , NLO: [Ghiglieri, *et al* (2013)]



LO: [Aurenche, *et al* (2002)]

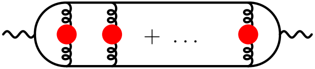
NLO: [Ghiglieri, Moore (2014)]

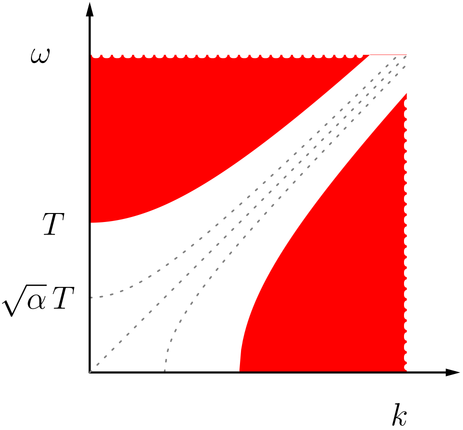
light-like correlator

$\Downarrow$ [Caron-Huot (2009)]

Effective Field Theory

'ladder diagrams' for $M^2 \ll T^2 \rightarrow$ LPM effect + *Hard Thermal Loops*

$\rho_{\mu\nu}(\omega, \boldsymbol{k}) \;=\; \mathrm{Im}\Big[\mu$  $\nu\Big]$



For $M \gtrsim T$, no resummation.

fixed-order NLO...

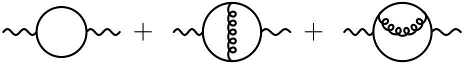
$\omega > k$: Laine (2013) [1310.0164]

$\omega < k$: GJ (2019) [1910.07552]

explicit singularity for $M^2 \to 0$

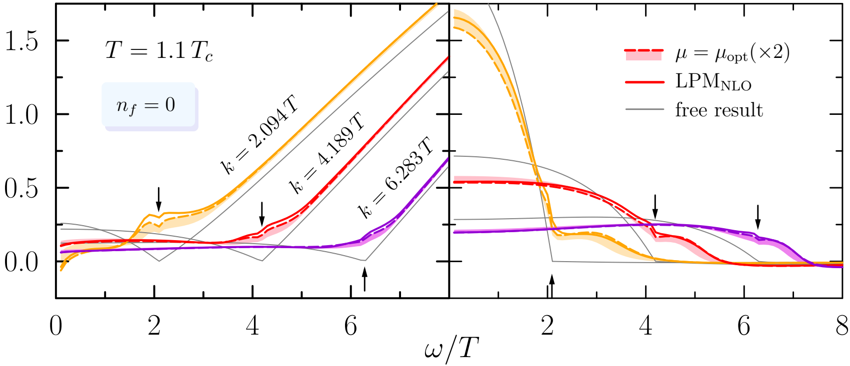
$$\rho_{\rm V} \;\simeq\; \alpha_{\rm s}\, T^2\, \frac{N_c\, C_{\rm F}}{4} \log \frac{T^2}{M^2}$$

$$\Pi^{\mu\nu} \;=\; e^2 \left[\, \textstyle\sum_{l=0}^{\infty} g^{2l}\, \Pi^{\mu\nu}_{(l)} \,\right] \;+\; O(e^4)$$

$=$  $+ \ldots$

What we find

$G(\tau)$ requires knowing $\rho(\omega, k)$, for *ALL* frequencies:



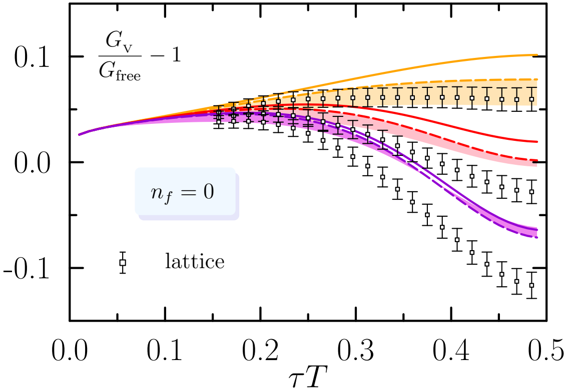
Spectral functions,

Left: $\rho_V/(\omega T) = (2\rho_T + \rho_L)/(\omega T)$

Right: $\rho_H/(\omega T) = 2(\rho_T - \rho_L)/(\omega T)$

What we find

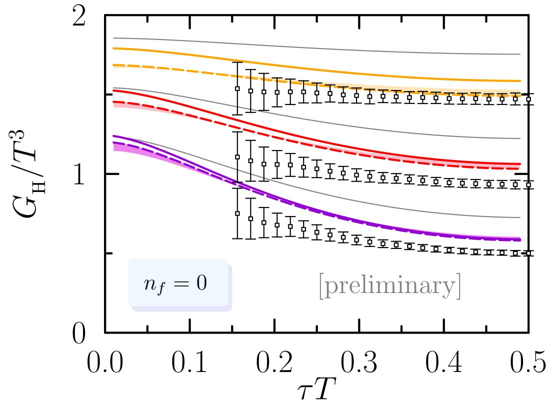
Comparing with the lattice [GJ, Laine (2019)] (also $n_f = 2$!)



(G_{free} includes no QCD corrections, $\alpha_s \to 0$)

What we find

Comparing with the lattice Bala, Kaczmarek, Jackson (work in progress)



(G_{free} includes no QCD corrections, $\alpha_s \to 0$)

Summary

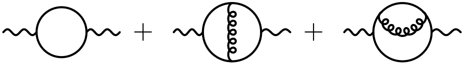
Arxiv: 1910.09567
1910.07552

- spacelike virtualities *complete* the pQCD calculation
- considered UV-finite **difference** $\rho_{\mathrm{T}}(\omega) - \rho_{\mathrm{L}}(\omega)$
- provide **cross-check** for reconstructed spectral fncs.

Overfull \hbox, (badness 10000)

Fixed-order calculation

$$\Pi^{\mu\nu} \;=\; e^2 \left[\textstyle\sum_{l=0}^{\infty} g^{2l}\, \Pi^{\mu\nu}_{(l)} \right] \;+\; O(e^4)$$

$=$  $+ \dots$

The leading-order ('free') part is simple: (Here no HTL approx!)

$$\mathrm{Im}\big[\, g_{\mu\nu}\Pi^{\mu\nu}_{(0)} \,\big] \;=\; -K^2\,\frac{N}{4\pi}\left\{\; \Theta(k_-) + 2\frac{T}{k}\log\frac{1+e^{-k_+}}{1+e^{-|k_-|}} \;\right\},$$

$$\begin{aligned}\mathrm{Im}\big[\, \Pi^{00}_{(0)} \,\big] \;&=\; k^2\,\frac{N}{4\pi}\left\{\; \frac{1}{3}\Theta(k_-) \right.\\ &+\; 4\frac{T^2}{k^2}\Big(\mathrm{Li}_2\big(-e^{-\beta k_+}\big) + \mathrm{sgn}(k_-)\mathrm{Li}_2\big(-e^{-\beta|k_-|}\big)\Big)\\ &+\; \left. 8\frac{T^3}{k^3}\Big(\mathrm{Li}_3(-e^{-\beta k_+}) - \mathrm{Li}_3(-e^{-\beta|k_-|})\Big) \right\}.\end{aligned}$$

To finally give ... $\Pi_{\mathrm{L}} = \dfrac{K^2}{\boldsymbol{k}^2}\,\Pi^{00}\;, \quad \Pi_{\mathrm{T}} = -\,\dfrac{1}{2}\Big(\Pi_\mu^{\;\mu} + \dfrac{K^2}{\boldsymbol{k}^2}\Pi^{00}\Big)\,.$

$$\rho^{(m,n)}_{abcde}(K) \equiv \mathrm{Im} \sumint_{P,Q} \frac{p_0^m q_0^n}{P^{2a} Q^{2b} (K-P-Q)^{2c} (K-P)^{2d} (K-Q)^{2e}}$$

$$\begin{aligned}
\mathrm{Im}\left[\, g_{\mu\nu}\Pi^{\mu\nu}_{(1)}\,\right] &= 8(1-\epsilon)Nc_F \Big\{ 2(1-\epsilon)K^2\big(\rho^{(0,0)}_{11020} - \rho^{(0,0)}_{10120}\big) \\
&+ 2\rho^{(0,0)}_{11010} + 2\epsilon\,\rho^{(0,0)}_{11100} - \tfrac{1}{2}(3+2\epsilon)K^2\rho^{(0,0)}_{11011} \\
&- 2(1-\epsilon)\rho^{(0,0)}_{1111(-1)} + 4K^2\rho^{(0,0)}_{11110} - K^4\rho^{(0,0)}_{11111} \Big\} ,
\end{aligned}$$

$$\begin{aligned}
\mathrm{Im}\left[\,\Pi^{00}_{(1)}\,\right] = 4Nc_F \Big\{ & \; 2(1-\epsilon)\rho^{(0,0)}_{10110} + 2\epsilon\rho^{(0,0)}_{11100} + (1+\epsilon)k^2\,\rho^{(0,0)}_{11011} \\
& - 2(1-\epsilon)\rho^{(0,0)}_{1111(-1)} + 4\big[\,(1-2\epsilon)k_0^2 - k^2\,\big]\,\rho^{(0,0)}_{11110} \\
& + 8\epsilon\, k_0\,\rho^{(1,0)}_{11110} - 8(1-\epsilon)k_0\,\rho^{(0,1)}_{11110} + \big[\,(1-2\epsilon)k_0^2 \\
& + k^2\,\big]\,K^2\rho^{(0,0)}_{11111} + 4\epsilon\,K^2\,\rho^{(1,1)}_{11111} - 4(1-\epsilon)K^2\,\rho^{(2,0)}_{11111} \Big\} .
\end{aligned}$$

Apply general ‘cutting’ rules to each **master diagram** ... [Jeon (1993)]

What we do

$G(\tau)$ requires knowing $\rho(\omega, k)$, for *ALL* frequencies:

5-loop $\alpha_s(\mu)$ at 'optimal' scale

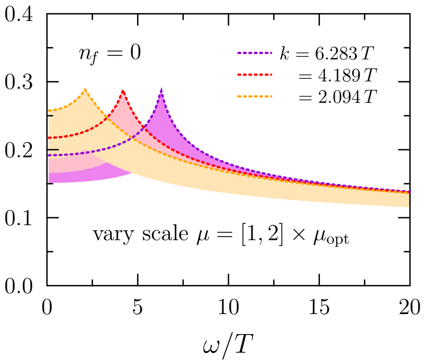
$$\mu_{\rm opt} = \sqrt{|M^2| + (\xi \cdot \pi T)^2}$$

where $\xi = 1\,(2)$ for $n_f = 0\,(2)$

Near the light cone, $\mu_{\rm opt} \sim T$

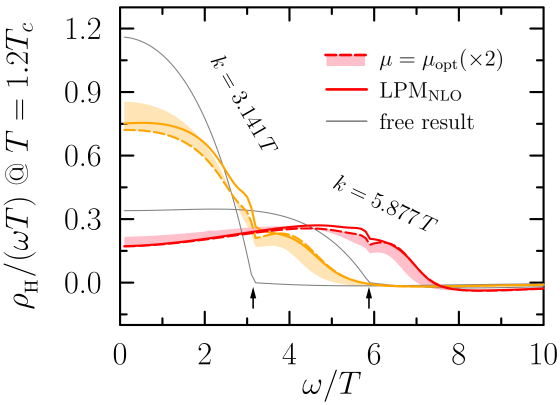
values of $\dfrac{k}{T}$ @ $T = 1.1\,T_c$:

$2\pi/3$,	$4\pi/3$,	2π
(2.09440)	(4.18879)	(6.28319)



What we find

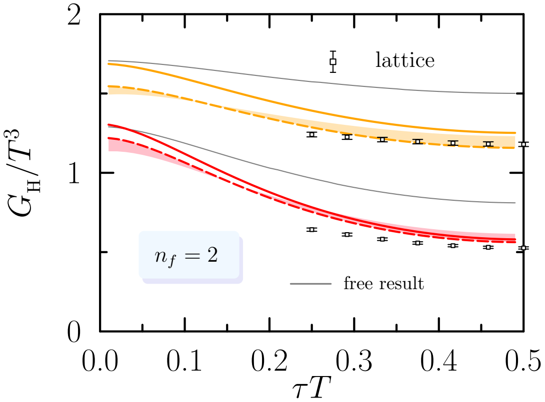
UV-finite correlator $G_{\rm H} = 2(G_{\rm T} - G_{\rm L})$



Spectral function depicted for 2-flavour QCD ($n_f = 2$)

What we find

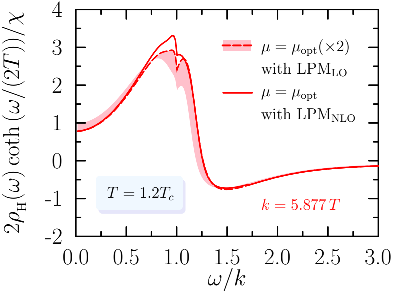
Comparing with the lattice [Cè, *et al* (2020)]



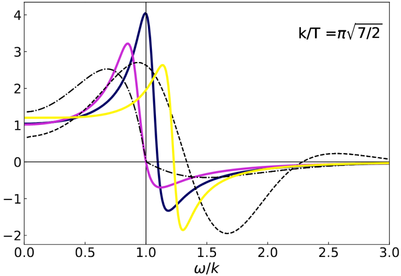
values of $\frac{k}{T}$ @ $T = 1.2\,T_c$: π, (3.14159) $\sqrt{7/2}\,\pi$ (5.87738)

Comparing with the lattice, $\rho_{\rm H}$ for $n_f = 2$ [Cè, *et al* (2020)]

perturbation theory



lattice reconstruction



Should ρ_{V} be negative in the very IR?

For $\omega, k \ll T$, the **hydrodynamic** prediction gives:

$$\frac{\rho_{\mathrm{T}}}{\omega} = -\chi_{\mathrm{q}} D$$

$$\frac{\rho_{\mathrm{L}}}{\omega} = -\chi_{\mathrm{q}} D \frac{K^2}{\omega^2 + D^2 k^4}$$

[Hong, Teaney (2010)]

$D =$ diffusion coefficient

$\chi_{\mathrm{q}} =$ charge susceptibility

Therefore $\lim_{\omega \to 0} \rho_{\mathrm{V}} / \omega$ crosses zero at $k = 1/(\sqrt{2} D)$