In medium Langevin dynamics of heavy particles

Strong and Electro-Weak Matter 2021

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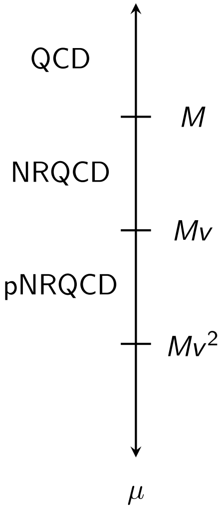
Introduction

Motivation

use heavy quarks and their bound states to probe the strongly coupled medium formed in heavy ion collisions

- high mass M of bottom (and charm) quarks and the short formation time of their bound states make them ideal probes of the quark gluon plasma (QGP)
- ideally suited for treatment using the formalism of open quantum systems (OQS) and effective field theory (EFT)
  - OQS: allows for the rigorous treatment of a quantum system of interest (heavy quark(onium)) coupled to an environment (QGP)
  - EFT: nonrelativistic QCD (NRQCD) and potential NRQCD (pNRQCD) are EFTs of the strong interaction taking advantage of the large mass of the heavy quark and the resultant nonrelativistic nature of the system

potential Non-Relativistic QCD (pNRQCD)



- effective theory of the strong interaction obtained from full QCD via non-relativistic QCD (NRQCD) by successive integrating out of the hard (M) and soft (Mv) scales where $v \ll 1$ is the relative velocity in a heavy-heavy bound state
- degrees of freedom are singlet and octet heavy-heavy bound states and ultrasoft gluons
- small bound state radius and large quark mass allow for double expansion in r and M^{-1}

Langevin Dynamics

Langevin Dynamics

- Langevin equations

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\eta_D p_i + \xi_i(t),\ \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t'),\ \eta_D = \frac{\kappa}{2MT},$$

  where p_i is the momentum of the particle (heavy quark), η_D is the drag coefficient, and ξ_i encodes the random, uncorrelated interactions of the particle with the medium
- κ is the heavy quark momentum diffusion coefficient
- as shown by Casalderrey-Solana and Teaney, for an in medium heavy quark, integration of force-force correlator along the Schwinger-Keldysh contour gives κ in terms of a chromo electric correlator[1]

[1] Phys. Rev. D 74, 085012 (2006).

Heavy Quarkonium Sector

Evolution Equations of in Medium Coulombic Quarkonium[2]

$$\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = -i\,[h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t)),$$

$$\frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} = -i\,[h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$

- $\rho_{s,o}(t)$: density matrix of color singlet, octet bound state
- $h_{s,o} = \frac{\mathrm{p}^2}{M} + V_{s,o}$: singlet, octet Hamiltonian
  - $V_s = -\frac{C_f \alpha_s(1/a0)}{r}$: singlet potential
  - $V_o = \frac{\alpha_s(1/a0)}{2N_c r}$: octet potential
- Σ, Ξ: encode medium interactions in correlators of the form

$$\Sigma,\, \Xi \sim \langle \tilde{E}^{a,j}(0,\mathbf{0})\tilde{E}^{a,j}(s,\mathbf{0})\rangle, \quad \tilde{E}^{a,i}(s,\mathbf{0}) = \Omega(s)E^{a,i}(s,\mathbf{0})\Omega(s)^\dagger,$$

$$\Omega(s) = \exp\left[-ig\int_{-\infty}^{s} \mathrm{d}s' A_0(s',\mathbf{0})\right]$$

[2] Phys. Rev. D 97, 074009 (2018).

Master Equation

evolution equations can be rewritten as master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H,\rho(t)] + \sum_{n,m} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger}L_i^n, \rho(t)\right\}\right),$$

where

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix}, \quad H = \begin{pmatrix} h_s + \mathrm{Im}(\Sigma_s) & 0 \\ 0 & h_o + \mathrm{Im}(\Sigma_o) \end{pmatrix},$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i, \quad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}, \quad L_i^2 = \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} \\ 1 & 0 \end{pmatrix} r^i,$$

$$L_i^3 = \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}, \quad h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$A_i^{uv} = \frac{g^2}{6N_c} \int_0^\infty \mathrm{d}s\, e^{-ih_u s} r^i e^{ih_v s} \langle \tilde{E}^{a,j}(0,\mathbf{0}) \tilde{E}^{a,j}(s,\mathbf{0}) \rangle$$

Lindblad Form

- for $(\pi)T \gg E$ (where E is the binding energy), $e^{-ih_{s,o}s} \approx 1$, and the medium interactions are encoded in the transport coefficients

$$\kappa = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}t \Big\langle \Big\{ \tilde{E}^{a,i}(t,0), \tilde{E}^{a,i}(0,0) \Big\} \Big\rangle,$$
$$\gamma = -\frac{ig^2}{6N_c}\int_0^\infty \mathrm{d}t \Big\langle \Big[\tilde{E}^{a,i}(t,0), \tilde{E}^{a,i}(0,0) \Big] \Big\rangle$$

- as shown by Casalderrey-Solana and Teaney, κ is the heavy quark momentum diffusion coefficient occurring in a Langevin equation[3]; γ is its dispersive counterpart
- evolution equation can be written as Lindblad equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H(t),\rho] + \sum_n \left(C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right)$$

[3] Phys. Rev. D 74, 085012 (2006).

Langevin Form

- taking $e^{-ih_{s,o}s} \approx 1 - ih_{s,o}s$, medium interactions take more complicated form as Hamiltonian term gives rise to terms suppressed by E/T, i.e.,

$$A_i^{uv} = \frac{r_i}{2}(\kappa - i\gamma) + \left(-\frac{ip_i}{2MT} + \frac{\Delta V_{uv} r_i}{4T}\right)\kappa$$

- evolution equation can no longer be written as a Lindblad equation without introducing subleading corrections
- following the procedure of Blaizot and Escobedo[4], we project the evolution equations onto eigenstates of the bound state radius $\langle r|$ and $|r'\rangle$ corresponding to the radius pre- and post-, respectively, interaction with the medium; we work in the system of coordinates

$$\mathrm{r}_+ = \frac{\mathrm{r} + \mathrm{r}'}{2}, \quad \mathrm{r}_- = \mathrm{r} - \mathrm{r}'$$

[4] JHEP 06 (2018) 034.

Scaling

the projected evolution parameters depend on the operators/quantities r_+, r_-, ∇_+, ∇_-, $V_{s,o}$, κ, and γ; we assign a scaling to extract leading order evolution

- bound state is Coulombic

$$r_+ \sim 1/\sqrt{EM}, \quad \nabla_+ \sim \sqrt{EM}$$

- potential scales as the binding energy

$$V_{s,o} \sim E$$

- κ, γ are thermal quantities of dimension 3

$$\kappa,\ \gamma \sim (\pi T)^3$$

- interaction with medium thermalizes bound state

$$r_- \sim 1/\sqrt{\pi TM}, \quad \nabla_- \sim \sqrt{\pi TM}$$

Leading Order Evolution

- as $M \gg \pi T \gg E$, there are two small parameters in which to expand; for $(\pi T)/M \sim E/(\pi T)$, the leading order evolution operators are of order πT

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\rho_s^{\mathbf{rr}'}\\ \rho_o^{\mathbf{rr}'}\end{pmatrix} = \begin{pmatrix}-r_+^2\kappa & \frac{1}{N_c^2-1}r_+^2\kappa \\ r_+^2\kappa & -\frac{1}{N_c^2-1}r_+^2\kappa\end{pmatrix}\begin{pmatrix}\rho_s^{\mathbf{rr}'}\\ \rho_o^{\mathbf{rr}'}\end{pmatrix} + \cdots\,,$$

  where $\rho_{s,o}^{\mathbf{rr}'} = \langle \mathrm{r}|\rho_{s,o}(t)|\mathrm{r}'\rangle$ and the ellipsis indicates terms suppressed by addition powers of $(\pi T)/M \sim E/(\pi T)$
- evolution matrix has eigenvalues

$$\{\lambda_0,\,\lambda_8\} = \left\{0, -r_+^2\kappa\frac{N_c^2}{N_c^2-1}\right\}$$

Corrections to Leading Order Evolution I

- à la Blaizot and Escobedo, move to basis in which LO evolution is diagonal

$$\rho_0 = \frac{\rho_s + \rho_o}{N_c^2}, \qquad \rho_8 = \frac{(N_c^2 - 1)\rho_s - \rho_o}{N_c^2},$$

- include terms suppressed by powers of $(\pi T)/M \sim E/(\pi T)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\rho_0^{\mathbf{rr}'}\\ \rho_8^{\mathbf{rr}'}\end{pmatrix} = \begin{pmatrix}\ell_{00}^{(1)} + \ell_{00}^{(2)} & \ell_{08}^{(1)} + \ell_{08}^{(2)}\\ \ell_{80}^{(1)} + \ell_{80}^{(2)} & \ell_{88}^{(0)} + \ell_{88}^{(1)} + \ell_{88}^{(2)}\end{pmatrix}\begin{pmatrix}\rho_0^{\mathbf{rr}'}\\ \rho_8^{\mathbf{rr}'}\end{pmatrix} + \cdots\,,$$

where superscripts in parenthesis indicate degree of suppression in $\sqrt{(\pi T)/M} \sim \sqrt{E/(\pi T)}$ with respect to LO evolution and the ellipsis indicates further suppressed terms

Corrections to Leading Order Evolution II

- evolution matrix has eigenvalues $\{\lambda_0', \lambda_8'\}$ which reduce to $\{\lambda_0, \lambda_8\}$ in limit $(\pi T)/M \sim E/(\pi T) \to 0$
- λ_0' given by

$$\lambda_0' = \ell_{00}^{(1)} + \ell_{00}^{(2)} - \frac{\ell_{08}^{(1)}\,\ell_{80}^{(1)}}{\ell_{88}^{(0)}} + \cdots$$

- Wigner transforming the evolution equation of the state evolved by λ_0' gives the Fokker Planck equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_+\right)\tilde{\rho}_0(t) = \left[\frac{\kappa}{4}\nabla_{\mathbf{p}}^2 + \frac{M}{2}\eta\nabla_{\mathbf{p}}\cdot\mathbf{v} + \frac{\gamma}{2}\mathbf{r}_+\cdot\nabla_{\mathbf{p}} + \left(\frac{\gamma}{\sqrt{\kappa}}\frac{\mathrm{r}_+\cdot\nabla_{\mathbf{p}}}{2N_c|\mathrm{r}_+|}\right)^2\right]\tilde{\rho}_0(t),$$

where $\tilde{\rho}_0(t)$ is the Wigner transform of the state evolved by λ_0', v is the relative velocity of the quark and antiquark and $\mathrm{p} = M\mathrm{v}/2$

Langevin Equation

the corresponding Langevin equations are

$$\frac{\mathrm{d}\mathrm{r}_i^+}{\mathrm{d}t} = \frac{2\mathrm{p}_i}{M}, \quad \frac{M}{2}\frac{\mathrm{d}^2 r_i^+}{\mathrm{d}t^2} = -F_i(\mathrm{r}^+) - \eta_{ij} p_j + \xi_i(t, \mathrm{r}^+) + \theta_i(t, \mathrm{r}^+).$$

where

- $\langle \xi_i(t, \mathrm{r}^+) \xi_j(t', \mathrm{r}^+) \rangle = \delta(t - t') \delta_{ij} \, \kappa$: ξ_i encodes random, uncorrelated interactions with medium; κ is heavy quark momentum diffusion coefficient
- $\eta_{ij}(\mathrm{r}^+) = \frac{\kappa}{2MT} \delta_{ij}$: Einstein relation between κ and drag coefficient η
- $\langle \theta_i(t, \mathrm{r}^+) \theta_j(t', \mathrm{r}^+) \rangle = \delta(t - t') \frac{r_i^+ r_j^+ \gamma^2}{4N_c^2 \kappa \, r_+^2}$: θ_i is second random force due to fluctuations in force between quark and antiquark which are, on average, 0
- $F_i(\mathrm{r}^+) = -\gamma \frac{r_i^+}{2}$: correction to quark-antiquark potential

Single Heavy Quark Sector

EFT for an in Medium Heavy Quark

- consider a single heavy quark of mass M described by nonrelativistic QCD (NRQCD)

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left(i\partial_0 - gA_0 + \frac{\nabla^2}{2M} \right) \psi$$

- consider interaction with medium gluons of temperature T such that $M \gg \sqrt{MT} \gg T$
- isolate gauge structure via field redefinitions

$$\psi(t,\mathbf{x}) \to \exp\left[ig \int_{\mathbf{0}}^{\mathbf{x}} \mathrm{d}\mathbf{x}' \cdot \mathbf{A}(t,\mathbf{x}') \right] \psi(t,\mathbf{x}),$$

$$\psi(t,\mathbf{x}) \to \exp\left[-ig \int_{-\infty}^{t} \mathrm{d}t' A_0(t',\mathbf{0}) \right] \psi(t,\mathbf{x})$$

- multipole expand to isolate contributions from gluons with momentum transfer T

$$\mathcal{L}'_{\mathrm{NRQCD}} = \psi^\dagger \left\{ i\partial_0 + x_i\, g\tilde{E}^{i,a}(t,\mathbf{0}) + \frac{\nabla^2}{2M} \right\} \psi$$

Single Quark Langevin

- analogously to heavy quarkonium case, evolution equations depend on

$$A_i = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}t\, e^{-iht} x^i e^{iht} \langle \tilde{E}^{a,j}(0,0)\tilde{E}^{a,j}(t,0)\rangle$$

- analogous analysis, i.e., expansion of exponentials to NLO, projection onto $\langle \mathbf{x}|$ and $|\mathbf{x}'\rangle$, and Wigner transform leads to Fokker-Planck equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla_+\right)\tilde{\rho}(t) = \left[\frac{\kappa}{2}\nabla_{\mathbf{p}}^2 + M\eta\nabla_{\mathbf{p}}\mathbf{v} + \gamma\,\mathbf{x}_+\cdot\nabla_{\mathbf{p}}\right]\tilde{\rho}(t)$$

with corresponding Langevin equations

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\mathbf{F} - \eta\,\mathbf{p} + \boldsymbol{\xi}(t),$$

where

$$\mathbf{F} = -\gamma\,\mathbf{x}_+, \quad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\,\delta_{ij}\delta(t-t'), \quad \eta = \frac{\kappa}{2MT},$$

Conclusions and Future Work

- heavy quarks and their bound states are excellent probes of QGP formed in HICs
- two main theoretical tools are EFTs and OQS
- scale hierarchy $M \gg \pi T$ makes Langevin equation natural candidate for description of dynamics
- evolution equations depend on chromo electric-electric correlators which reduce at lowest order to a linear combination of κ and γ [5]
- inclusion of higher order corrections à la Blaizot and Escobedo[6] allows for derivation of Langevin equation containing κ from first principles
- future work: rigorous integrating out of the scale $\sqrt{MT}$ from NRQCD and investigation of its affects

[5] Phys. Rev. D 97, 074009 (2018).
[6] JHEP 06 (2018) 034.

Thank you!