Jet momentum broadening in an anisotropic quark-gluon plasma

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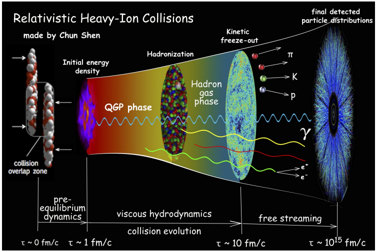
Introduction

- Heavy-ion collisions produce high T QCD matter.
- Medium affects jets, allowing for jet tomography:
  - Transverse momentum broadening.
  - Medium-induced emission.
- Medium is anisotropic, also in hydro stage.
- Need non-equilibrium calculations of e.g. jet momentum broadening.
- Relevant for kinetic theory stage and QGP hydrodynamic stage.

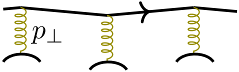


[Chun Shen, 2014]

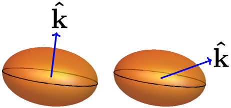
Anisotropic momentum broadening

- Total transverse momentum broadening:

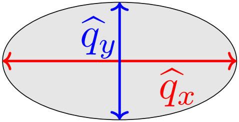
$$\widehat{q} = \frac{d\langle \mathbf{p}_\perp^2 \rangle}{dt}$$



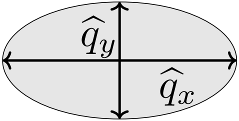
- Anisotropic plasma: Two partons at same place have different $\widehat{q}(\hat{\mathbf{k}})$.



- Momentum broadening itself is anisotropic:
  - $\widehat{q}_x \neq \widehat{q}_y$ with $\widehat{q}_x = \frac{d\langle p_x^2 \rangle}{dt}$
- Affects the rate of gluon emission.
  - Will depend on jet parton direction.



Anisotropic momentum broadening



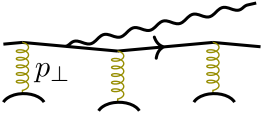
- Momentum broadening given by

$$\widehat{q}_i = \int d^2p_\perp \, p_i^2 \, \mathcal{C}(\mathbf{p}_\perp),$$

- In thermal equilibrium

$$\mathcal{C}(\mathbf{p}_\perp) \sim g^2 C_F \frac{m_D^2}{\mathbf{p}_\perp^2 \left(\mathbf{p}_\perp^2 + m_D^2\right)}$$

[Aurenche, Gelis, Zaraket (2002); Caron-Huot (2008)]



- E.g. determines rate of collinear emission $\sim \mathrm{Re}\, \mathbf{f}$ where

$$\mathbf{p}_\perp = i\delta E \, \mathbf{f}(\mathbf{p}_\perp) + \int_{\mathbf{q}_\perp} \mathcal{C}(\mathbf{q}_\perp) \left[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\right]$$

[Aurenche, Gelis, Kobes, Zaraket (1998); Arnold, Moore, Yaffe(2001);

Non-equilibrium: Hauksson, Jeon, Gale (2017)]

- Want **anisotropic** $\mathcal{C}(\mathbf{p}_\perp)$.

Some earlier work

Calculations of $\mathcal{C}(\mathbf{p}_\perp)$ in thermal equilibrium:

- LO: Arnold, Moore, Yaffe (2001); Aurenche, Gelis, Zaraket (2002).
- NLO: Caron-Huot (2008).
- On lattice for gT kicks: E.g. Panero, Rummukainen, Schaefer (2014); Moore, Schlichting, Schlusser, Soudi (2021).

Non-equilibrium calculations of momentum broadening (also for heavy quark):

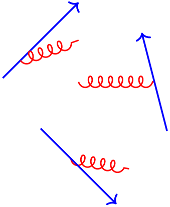
- Classical-statistical field theory: Boguslavski, Kurkela, Lappi, Peuron (2020).
- CGC: Carrington, Czajka, Mrowczynski (2020)
  Ipp, Muller, Schuh (2020)
- HTL-like simulation: Schenke, Strickland, Dumitru, Nara, Greiner (2009)
  Mrowczynski (2017)
- ...

Do a semi-analytic, non-equilibrium calculation of $\mathcal{C}(\mathbf{p}_\perp)$ in HTL regime.

Perturbative QGP

- Two scales in plasma:
    - Hard quarks and gluons at energy Λ.
      [Arnold, Moore, Yaffe, 2003]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \mathcal{C}[f, A]$$



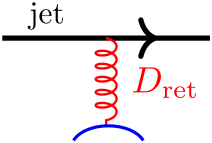
    - Soft gluon fields at energy $g\Lambda$

$$\mathcal{D}_\mu F^{\mu\nu} = j^\nu[f]$$

- Integrating out f gives HTL retarded correlator.
  [Blaizot, Iancu, 2001; Mrowczynski, Thoma, 2000;

  Anisotropic evaluation: Romatschke, Strickland (2003)]

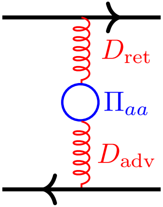
$$D^{\mu\nu}_{\text{ret}}(x, y) = \theta(t_x - t_y)\,\langle\, [A^\mu(x), A^\nu(y)]\,\rangle$$



Perturbative QGP

- Need

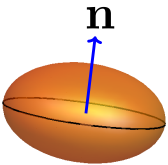
$$D^{\mu\nu}_{rr}(Q) = [D_{\rm ret}(Q)\,\Pi_{aa}(Q)\,D_{\rm adv}(Q)]^{\mu\nu}$$



- Determined by hard quasiparticle momentum distribution:

$$f(\mathbf{p}) = \sqrt{1+\xi}\, f_{\rm eq}\left(\sqrt{p^2+\xi(\mathbf{n}\cdot\mathbf{p})^2}\right)$$

[Romatschke, Strickland (2003)]



- Have evaluated $D^{\mu\nu}_{rr}$ fully.

[For 00 component, see Nopoush, Guo, Strickland (2017)]

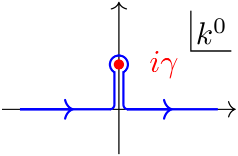
- Collision kernel is then

$$\mathcal{C}(\mathbf{q}_\perp) = g^2 C_F \int \frac{dq^0 dq^z}{(2\pi)^2}\, D^{\mu\nu}_{rr}(Q) v_\mu v_\nu\, \delta(v\cdot Q)$$

Instabilities in anisotropic QGP

- Naively get divergent $\mathcal{C}(\mathbf{p}_\perp)$ in anisotropic medium.
    - There are unstable modes that grow exponentially.

      [See e.g. Mrowczynski, Schenke, Strickland (2016)]
    - Starting at $t_0 = -\infty$ gives unstable modes infinite time to grow.



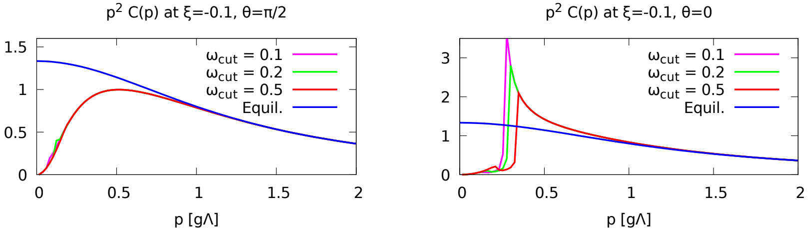
- Theoretically consistent: Start at $t_0 = 0$ and for $\xi \ll 1$ get [Hauksson, Jeon, Gale (2020)]

$$D_{\text{ret}}(t_x, t_y; \mathbf{k}) = \underbrace{\int \frac{dk^0}{2\pi}\, e^{-ik^0(t_x - t_y)}\, \widehat{D}_{\text{ret}}(K)}_{\substack{K \sim g\Lambda \\ \text{fluctuating modes}}} + \underbrace{\theta(t_x - t_y) \sum_i A_i e^{\gamma_i (t_x - t_y)}}_{\substack{\gamma \sim \xi g\Lambda \\ \text{instabilities}}}$$

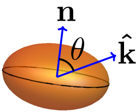
$$D_{rr}(t_x, t_y; \mathbf{k}) \approx \overbrace{\int \frac{dk^0}{2\pi}\, e^{-ik^0(t_x - t_y)}\, \widehat{D}_{\text{ret}} \Pi_{aa} \widehat{D}_{\text{adv}}} + \overbrace{\sum_{i,j} \# \frac{e^{\gamma_i t_x} e^{\gamma_j t_y} - 1}{\gamma_i + \gamma_j}}$$

Results

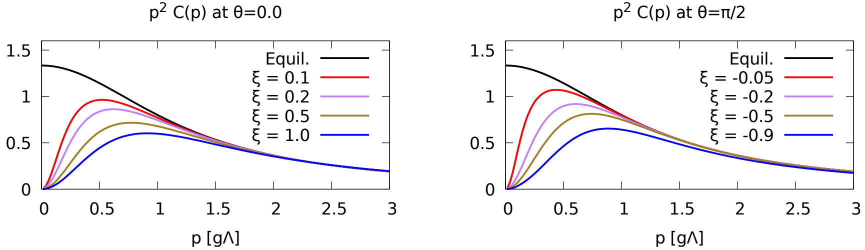
- Separate fluctuating modes and instabilities by ω_{cut}.
- Modes below ω_{cut} beyond this study: Subtract poles off.
    - E.g. classical-statistical simulations suggest that no instability modes during hydro stage. [E.g. Berges, Boguslavski, Schlichting, Venugopalan (2014)]



- For certain θ, more sensitive to ω_{cut}:
    - $\widehat{q} \sim \xi^{3/2} \log \omega_{\mathrm{cut}}$
    - Complementing with description of deep IR cancels ω_{cut}.

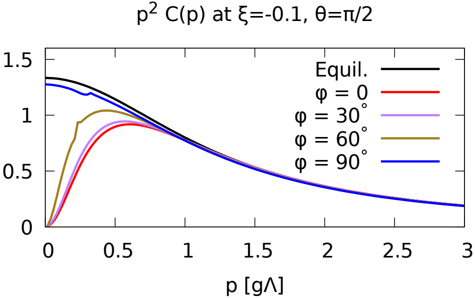


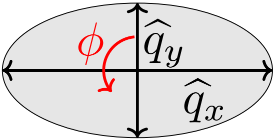
Results



- Less momentum broadening in more anisotropic plasma.
- Leads to less gluon radiation.
- Especially important for photon radiation and kinetic theory.
- Part of why medium has limited effect on jets in small systems?

Results





- Anisotropic momentum broadening.
- Radiated gluon emitted in a preferred direction.

Conclusions

- QGP in heavy-ion collisions is anisotropic.
- Momentum broadening and gluon radiation depends on direction of jet.
- Calculated $\mathcal{C}(\mathbf{p}_\perp)$ microscopically.
    - Reduced momentum broadening.
- At certain values of ξ and θ it is sensitive to the fate of instabilities:
    - Need to complement with deep IR physics.