A nonperturbative look into two-step electroweak phase transition

Lauri Niemi

Based on: "Thermodynamics of a Two-Step Electroweak Phase Transition"
LN, M. J. Ramsey-Musolf, T. V. I. Tenkanen, D. Weir
Phys.Rev.Lett. 126 (2021) 17 [arXiv:2005.11332]

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Two-step phase transitions: basic setup

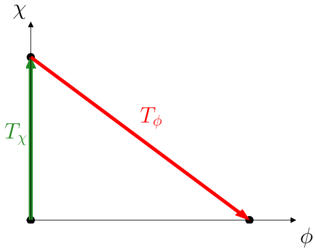
- Consider two-field potential:

$$V(\phi,\chi) = \frac{1}{2}m_1^2\phi^2 + \frac{1}{2}m_2^2\chi^2 + \lambda_1\phi^4 + \lambda_2\chi^4 + \lambda_{12}\phi^2\chi^2$$

so that the minimum is at $\langle\phi\rangle \neq 0$.

- At finite temperature, effectively $m^2 \to m^2(T)$. Symmetry restoration at $T > T_\phi$.

- **Two-step PT**: Phase with $\langle\phi\rangle \neq 0$ preceded by symmetry breaking in the χ direction.



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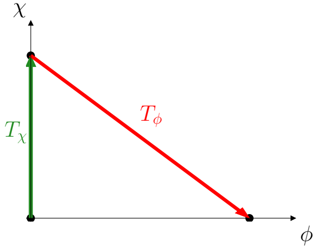
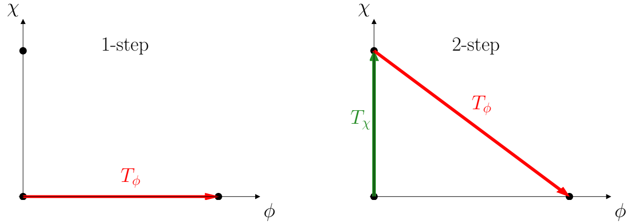
Realistic case: $\phi\to$ SM Higgs
$\chi\to$ new scalar



Two-step electroweak phase transition (EWPT)?

- Higgs condensation occurs smoothly in the SM: no electroweak phase transition.
- Possibly richer thermal history in beyond the SM settings:



radiatively generated crossover
or 1st order transition

1st order transition at T_ϕ
through tree-level effects

- Two-step EWPT are often very strong → gravitational waves, baryogenesis . . .

A minimal extension of the EW sector

$$V(\phi,\Sigma) = m_\phi^2\ \overset{\text{SM Higgs}}{\phi^\dagger\phi}\ + \lambda(\phi^\dagger\phi)^2 + m_\Sigma^2\ \mathrm{Tr}\,\overset{\text{SU(2) adjoint scalar}}{\Sigma^2}$$

$$+\, b_4(\mathrm{Tr}\,\Sigma^2)^2 + a_2\phi^\dagger\phi\,\mathrm{Tr}\,\Sigma^2$$

“Phases” at finite temperature:

- ϕ condenses → SM-like Higgs regime.
- Σ condenses → Georgi-Glashow-like regime with t’Hooft-Polyakov magnetic monopoles.
- No Higgsing → Gauge fields strongly coupled in the infrared (at high T).

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Two-step EWPT

arXiv:1212.5652

The perturbativity issue

- High-T perturbation theory is unreliable in the infrared:

$$\text{expansion parameter} \sim g^2 n_B(m) \xrightarrow{m \ll T} \frac{g^2 T}{m}$$

- Source of serious uncertainty for gravitational wave predictions.

See talk by O. Gould earlier today!

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Our goals:

1. Verify the two-step structure non-perturbatively with lattice simulations.
2. Benchmark perturbation theory by computing transition strength and critical temperature (1-loop vs. 2-loop vs. lattice).

High-T effective description and the lattice

- Dimensional reduction at high T → super-renormalizable 3D effective theory valid at scales $\ll \pi T$. arXiv:1802.10500

$$S_{\text{3D}} = \int d^3x \Big\{ \frac{1}{2}\,\mathrm{Tr}\,F_{ij}F_{ij} + |D_i\phi|^2 + \mathrm{Tr}[D_i,\Sigma]^2 + \bar{m}_\phi^2(T)\phi^\dagger\phi + \bar{\lambda}(T)(\phi^\dagger\phi)^2 + \bar{m}_\Sigma^2(T)\,\mathrm{Tr}\,\Sigma^2 + \bar{b}_4(T)(\mathrm{Tr}\,\Sigma^2)^2 + \bar{a}_2(T)\phi^\dagger\phi\,\mathrm{Tr}\,\Sigma^2 \Big\}$$

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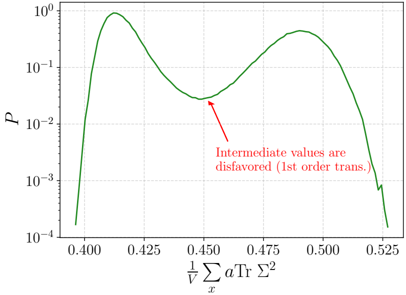
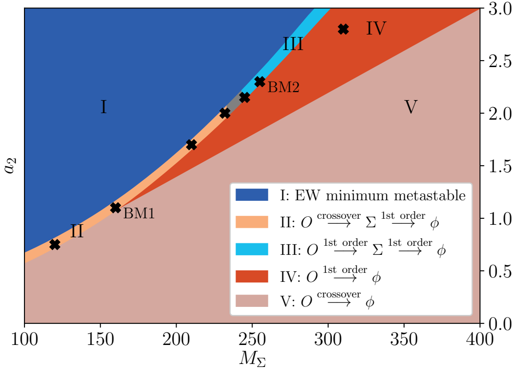
- Put on 3D lattice and simulate (standard)
  …but much easier than simulations in 4D!
- For strong transitions we apply multicanonical sampling.



Mapping out the phase structure

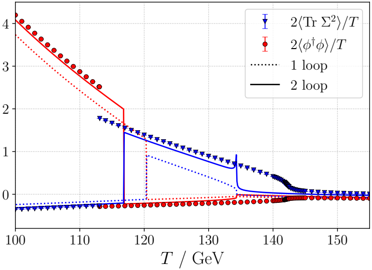
Combine simulations and perturbative scans (fixed $b_4 = 0.25$):



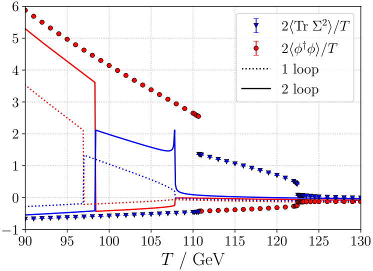
- Two-step EWPT only possible in a narrow region (II & III). Second step always 1st order.
- Crossover in region V found by integrating out Σ and using existing lattice results.

arXiv:1802.10500

Comparison with perturbation theory



$(M_\Sigma, a_2, b_4) = (160\ \mathrm{GeV}, 1.1, 0.25)$



$(M_\Sigma, a_2, b_4) = (255\ \mathrm{GeV}, 2.3, 0.25)$

- Gauge-invariant effective potential is IR divergent at the first transition (effective mass $=0$ at tree level).
- For $\Sigma \to \phi$ transition, discrepancy in T_c is $\mathcal{O}(10\%)$ and $\gtrsim 30\%$ in strength (L/T_c^4).
- **2-loop corrections are very significant, but not sufficient for high precision calculations.**

Conclusions and outlook

Findings for SM + adjoint Higgs theory.

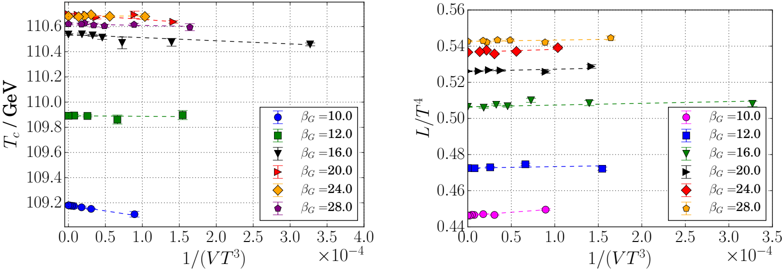
- Two-step phase cosmological transitions exist, but restricted to a small region of free parameter space.
- Loop corrections have large impact on transition strength even when potential barrier is present at tree level – scalar interactions are relatively strong.

Future directions.

- Extend analysis to other motivated extensions of the SM.
- Study bubble nucleation rate in realistic models with lattice methods.

Thank you for your attention!

Finite-size effects (backup)



- Mild dependence on lattice spacing and volume. Here $\beta_G = 4/(ag^2T)$.
- Data is for a strong 1st order $\Sigma \to \phi$ transition.