Sound predictions for cosmological phase transitions

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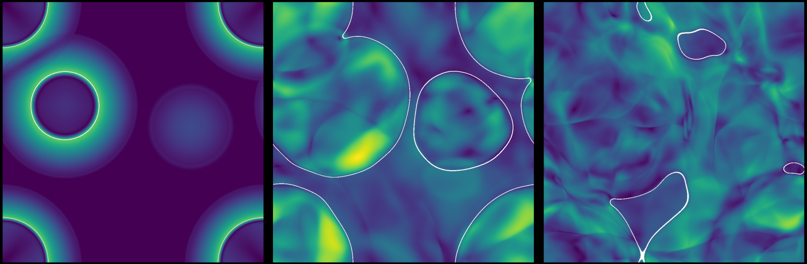


Strong and Electroweak Matter
30 June, 2021

Collaborators

Many important contributions for today's talk due to:

Djuna Croon
Andreas Ekstedt
Sinan Guyer
Joonas Hirvonen
Anna Kormu
Jonathan Kozaczuk
Johan Löfgren
Lauri Niemi
Michael Ramsey-Musolf
Kari Rummukainen
Philipp Schicho
Tuomas Tenkanen
David Weir
Graham White

Cosmological first-order phase transitions

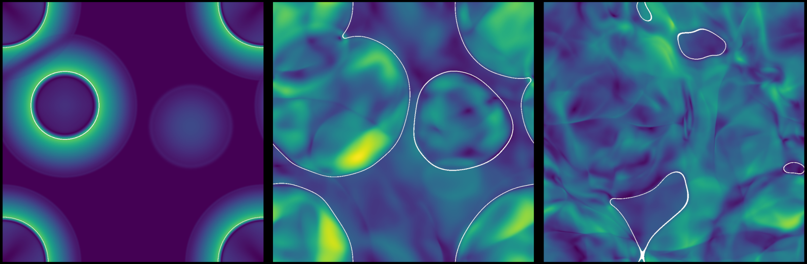


Figure: Cutting et al. arXiv:1906.00480.

- **Transition dynamics**
  - Bubbles nucleate, expand and collide
  - This creates long-lived fluid flows, and gravitational waves
- **Observable remnants**

  Such as $(n_B - n_{\bar{B}})/s$, stochastic gravitational wave backgrounds, topological defects, magnetic fields, . . .

$\Rightarrow$ new probe of particle physics

Gravitational waves from phase transitions: the pipeline

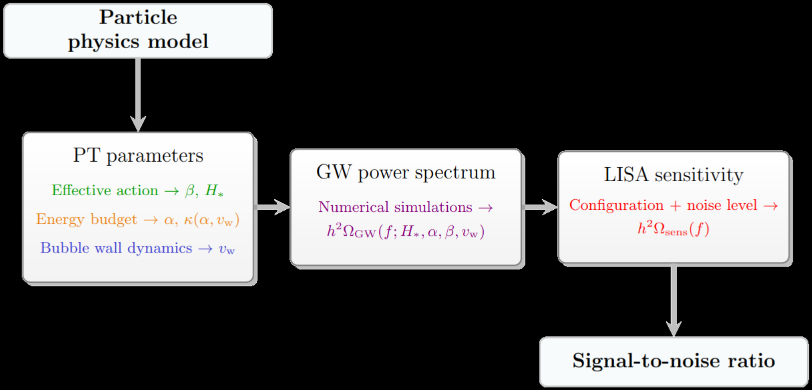


Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \mathrm{SNR}(f)$, Caprini et al. arXiv:1910.13125

Sound predictions

- How reliable are current predictions?
- Where do uncertainties come from?
- How to overcome them?

Perturbative sensitivity

- GW spectra of first-order phase transitions in any given specific model are very sensitive to details of calculation.

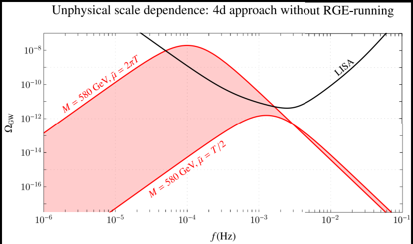


Figure: Renormalisation scale dependence of GW spectrum at one parameter point in SMEFT, Croon et al. arXiv:2009.10080.

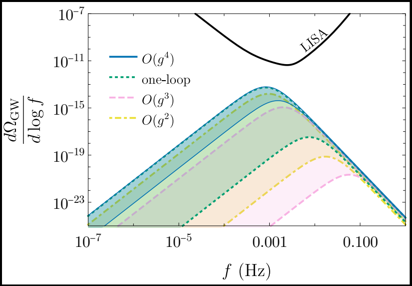


Figure: Renormalisation scale dependence of GW spectrum at one parameter point in Z_2-xSM, OG & Tenkanen arXiv:2104.04399.

Unwrapping perturbative sensitivity

- Ω_{GW} depends very strongly on the temperature of the transition,
$$\Omega_{\mathsf{GW}} \propto \frac{(\Delta V_*)^2}{T_*^8},$$
so an apparently innocuous uncertainty in T_* can still result in a huge uncertainty in Ω_{GW}.
- Uncertainties in thermodynamic parameters are themselves quite large

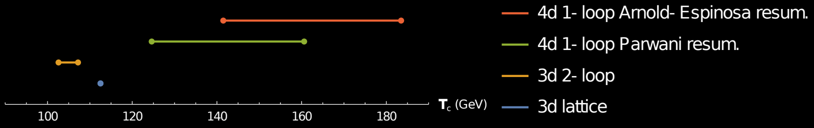


Figure: Theoretical uncertainties for T_c at one benchmark point in the 2HDM, Niemi et al. arXiv:1904.01329.

Origins of theoretical uncertainties

- **Infrared enhancements at high-T**

  Due to the high occupancy of infrared bosons, the effective expansion parameter α_{eff} grows

$$\alpha_{\text{eff}} \sim \lambda \frac{1}{1 - e^{p/T}} \approx \lambda \frac{T}{p},$$

  lighter modes are more strongly coupled:

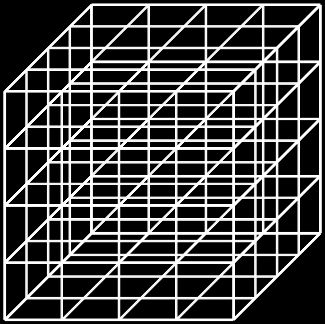
$$\begin{aligned} \text{hard}: &\quad p \sim \pi T \Rightarrow \alpha_{\text{eff}} \sim \lambda, \\ \text{soft}: &\quad p \sim \sqrt{\lambda} T \Rightarrow \alpha_{\text{eff}} \sim \sqrt{\lambda}, \\ \text{softer}: &\quad p \sim \lambda^{3/4} T \Rightarrow \alpha_{\text{eff}} \sim \lambda^{1/4}, \\ \text{ultrasoft}: &\quad p \sim \lambda T \Rightarrow \alpha_{\text{eff}} \sim 1. \end{aligned}$$

- **Effective field theory**

  3d EFTs provide a means to organise calculations involving these different modes and their different couplings.

  Farakos et al. '94, Braaten & Nieto '95, Kajantie et al. '95

Lattice vs perturbation theory



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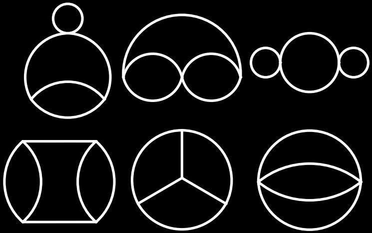


The theory

- Real, singlet scalar extension of the SM (xSM):

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \mathscr{L}_{\text{singlet}} + \mathscr{L}_{\text{portal}} \, ,$$
$$\mathscr{L}_{\text{singlet}} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \, ,$$
$$V(\phi) = \sigma\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4 \, .$$

  Focus on phase transition in the singlet direction.

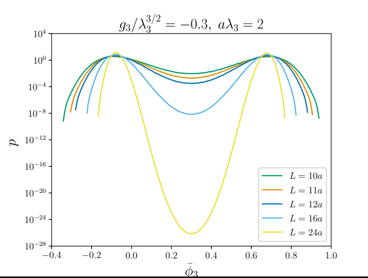
- The 3d EFT:

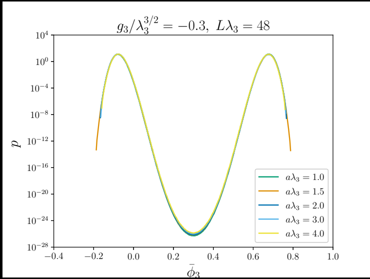
$$\mathscr{L}_3 = \frac{1}{2}(\partial_i \phi_3)^2 + V_3(\phi_3) \, ,$$
$$V_3(\phi_3) = \sigma_3\phi_3 + \frac{1}{2}m_3^2\phi_3^2 + \frac{1}{3!}g_3\phi_3^3 + \frac{1}{4!}\lambda_3\phi_3^4 \, .$$

  Can think of ϕ_3 as the zero Matsuara mode.

Lattice simulations

- Monte-Carlo simulations of 3d EFT sample the thermal distribution of field configurations, $p \propto e^{-H[\phi]/T}$.





- Efficient update algorithms known. Kajantie et al. '95
- Superrenormalisability $\Rightarrow$ exact lattice-continuum relations. Laine '95

Perturbative expansion in 3d EFT

- In general loops within the 3d EFT are suppressed by

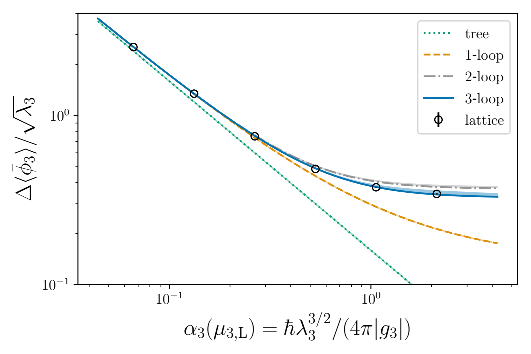
$$\frac{\lambda_3}{m_3}, \quad \frac{g_3^2}{m_3^3}.$$

- Near T_c, the effective mass is $m_3 \sim |g_3|/\sqrt{\lambda_3}$, and hence the 3d loop expansion parameter is

$$\alpha_3 = \frac{\hbar}{(4\pi)} \frac{\lambda_3^{3/2}}{|g_3|}.$$

- This diverges as one approaches the Z_2-symmetric second-order transition $\Rightarrow$ perturbation theory breaks down completely.

Results: lattice versus perturbation theory



$$\frac{1}{v_0}\Delta\langle\bar{\phi}_3\rangle = 2 + \sqrt{3}\,\alpha_3 + \frac{1}{2}\left(1 + 2\log\tilde{\mu}_3\right)\alpha_3^2$$
$$+\sqrt{3}\left[-\frac{3}{8\sqrt{2}}\xi + \frac{21}{32}\mathrm{Li}_2\frac{1}{4} - \frac{7\pi^2}{128} - \frac{1}{2} + \frac{21}{64}\log^2\frac{4}{3} + \frac{5}{8}\log\frac{4}{3}\right]\alpha_3^3$$
$$+O\left(\alpha_3^4\right)$$

OG arXiv:2101.05528

Implications

What does this teach us about this theory?

- The effective expansion parameter really is α_3
  - For strong transitions $\alpha_3 \sim \sqrt{\lambda}$
  - But as the transition gets weaker $\alpha_3 \to \infty$
- (Renormalisation group improved) perturbation theory is very accurate for $\alpha_3 \lesssim 1$

What about other theories?

- EFT results can be applied to a wide range of 4d theories with transitions in the ϕ-direction.
- Will perturbation theory be as effective for other 3d EFTs?
  - theories with two scale hierarchies? $\leftarrow$ typically $\alpha_3 \sim \lambda^{1/4}$
  - non-Abelian gauge theories? $\leftarrow$ high orders not computable

  Neither of these are deal-breakers, so there is promise.

Conclusions

- Phase transitions may be observable by GW detectors
- Calculational developments necessary for reliable Ω_{GW} predictions
- Effective field theory provides suitable tools
- For real scalar theory, perturbation theory agrees very well with lattice up to $\alpha_3 \lesssim 1$
- Promising for more difficult theories (see Niemi's talk)

Conclusions

- Phase transitions may be observable by GW detectors
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Thanks for listening!

Backup slides

QFT at high temperatures

- Equilibrium thermodynamics can be formulated in $\mathbb{R}^3 \times S^1$.

- Fields are expanded into Fourier (Matsubara) modes:

$$\Phi(\mathbf{x},\tau) = \sum_{n \text{ even}} \phi_n(\mathbf{x}) e^{i\pi T n\tau} \leftarrow \text{boson}$$

$$\Psi(\mathbf{x},\tau) = \sum_{n \text{ odd}} \psi_n(\mathbf{x}) e^{i\pi T n\tau} \leftarrow \text{fermion}$$

- Effective masses of Matsubara modes are

$$m_n^2 = m^2 + (n\pi T)^2$$

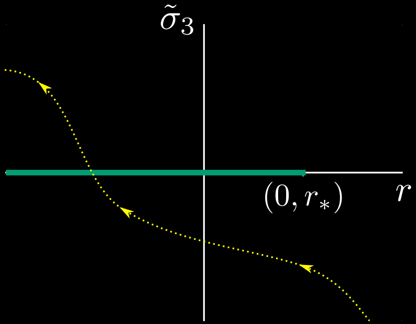
Phase diagram of EFT

By making the following shift

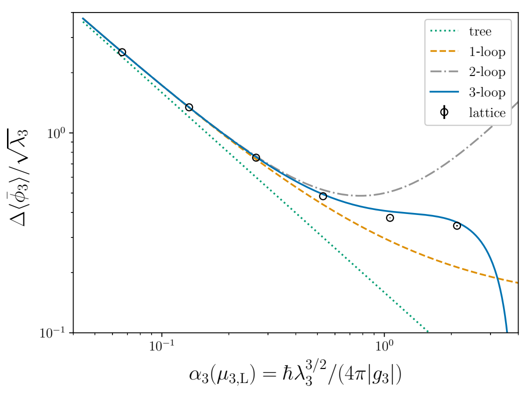
$$\phi_3 \to -\frac{g_3}{\lambda_3} + \phi_3 \ ,$$

the bare potential takes the form,

$$V_3 = \underbrace{\left(\sigma_3 + \frac{g_3^3}{3\lambda_3^2} - \frac{g_3 m_3^2}{\lambda_3}\right)}_{\tilde{\sigma}_3(T)} \phi_3 + \frac{1}{2}\left(\underbrace{m_3^2 - \frac{g_3^2}{2\lambda_3}}_{r(T)} + \delta m_3^2\right)\phi_3^2 + \frac{1}{4!}\lambda_3\phi_3^4 \ .$$



Results: lattice vs (unimproved) perturbation theory



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