Heavy Quark Diffusion on Lattice from Gradient Flow and Varying Temperatures

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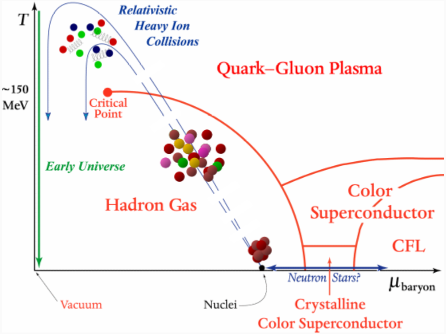
Introduction: QCD and QGP

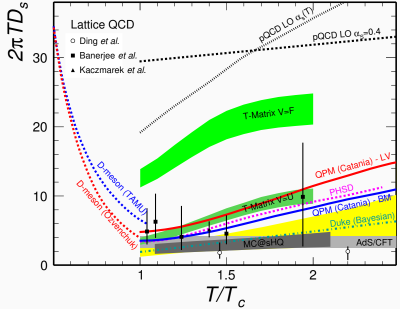
- We aim to understand the strongly coupled Quark Gluon Plasma (QGP)
- QGP generated at particle accelerators such as LHC/RHIC

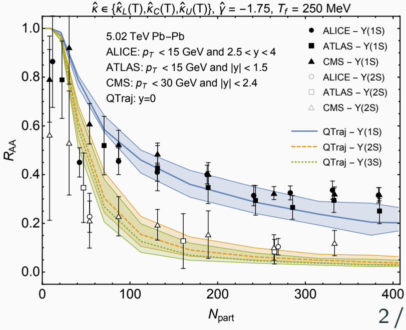


- The QGP can be described in terms of transport coefficients
- In this talk we focus on the heavy quark momentum diffusion coefficient κ
- κ related to experimental quantities nuclear modification factor R_{AA} and elliptic flow ν_2

Need for spatial diffusion coefficient

- R_{AA} and v_2 described by spatial diffusion coefficient D_x
- Observed v_2 is larger than expected from kinetic models but agrees more with hydrodynamic models
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}





UP: X. Dong CIPANP (2018)
DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, JHEP 05 (2021) 136

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  → Brownian motion; Follows Langevin dynamics

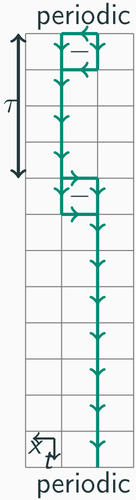
$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

- Heavy quark momentum diffusion coefficient κ related to many interesting phenomena

Such as: Spatial diffusion coefficient $D_\mathrm{s} = 2T^2/\kappa$,
Drag coefficient $\eta_\mathrm{D} = \kappa/(2MT)$,
Heavy quark relaxation time $\tau_\mathrm{Q} = \eta_\mathrm{D}^{-1}$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy quark diffusion from lattice: Euclidean Correlator



- Traditional approach using current correlators has a transport peak at zero
- HQEFT inspired Euclidean correlator peak free

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(\beta,0)]\rangle}$$

- Chromoelectric field E needs discretization
- On Lattice E has non-physical self-energy contribution
  $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$
  (Christensen and Laine PLB02 (2016))
- Traditionally solved with multilevel algorithm (quenched)
- Recently Gradient Flow measurement has been used
  (Altenkort *et.al.* PRD103 (2021))

$\Rightarrow$ Automatically renormalizes, could be expanded to un-quenched

Caron-Huot *et.al.*JHEP04 (2009) 053

Heavy quark diffusion from lattice: Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh \frac{\omega}{2T}}$$

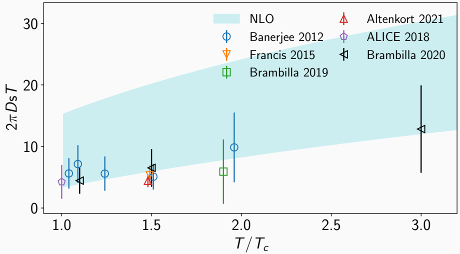
$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

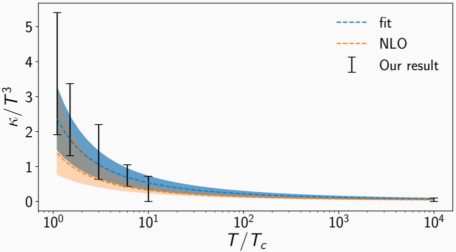
- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- Compare to ansatz: Trivial IR behavior ($\omega \ll T$)

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

- Perturbative behavior (N)LO in UV ($\omega \gg T$):
- Use 5-loop running for the coupling
- Systematics from varying the scale by factor of 2

Recent prior results





- Recent multilevel results

  Brambilla *et.al.* PRD102 (2020)

- Quenched multilevel simulations, very wide temperature range $1.1\,T_c - 10^4\,T_c$
- Can fit temperature dependence:

$$\frac{\kappa^{\mathrm{NLO}}}{T^3} = \frac{g^4 C_{\mathrm{F}} N_{\mathrm{c}}}{18\pi}\left[\ln\frac{2T}{m_{\mathrm{E}}} + \xi + C\frac{m_{\mathrm{E}}}{T}\right].$$

- Other lattice studies

  Meyer NJP13 (2011),

  Ding *et.al.*JPG38 (2011),

  Banerjee *et.al.* PRD85 (2012),

  Francis *et.al.* PRD92 (2015)

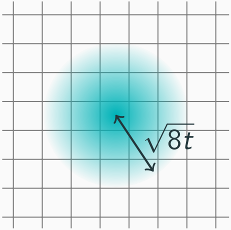
  Altenkort *et.al.* PRD103 (2021)

Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$

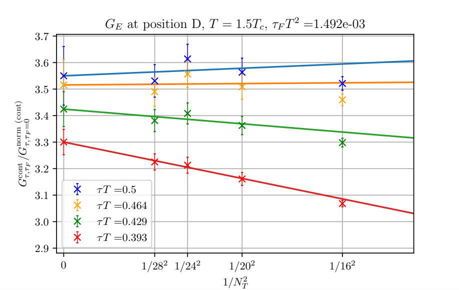
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$

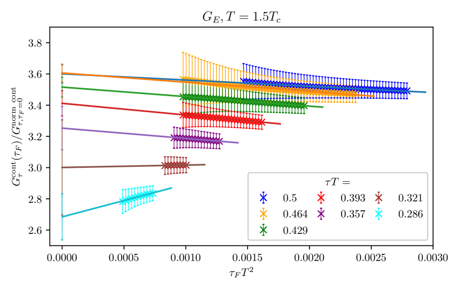
$$B_{0,\mu} = A_\mu \;\leftarrow \text{the original gauge field}$$



- Evolve gauge along fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}
- Automatically renormalizes gauge invariant observables
- Zero flowtime limit $\mathcal{O}(x,t) \xrightarrow{t\to 0} \sum_j c_j(t)\mathcal{O}_j^R(x)$

Procedure





- Measure $G_\mathrm{E}(\tau_F, \tau)$, τ_F: flowtime, τ: E-field separation
- Take continuum limit
- Take zero flowtime limit.

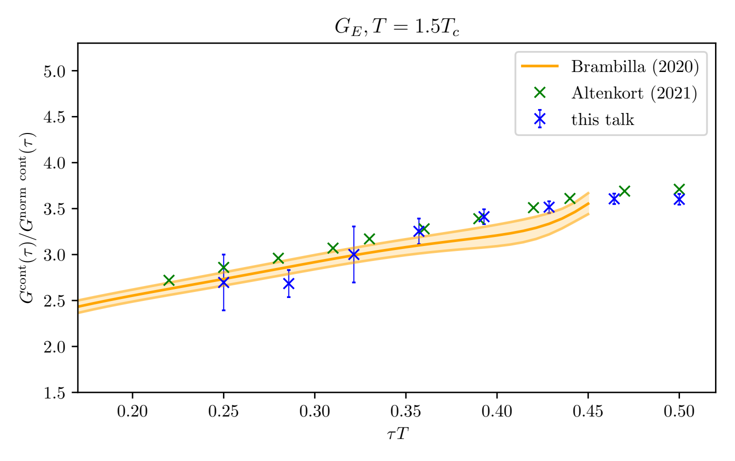
  Must be taken before solving $\rho(\omega)$: (Altenkort *et.al.* PRD103 (2021))

  Limit flow regime:

$$a \lesssim \sqrt{8\tau_F} \lesssim \frac{\tau - a}{3}$$

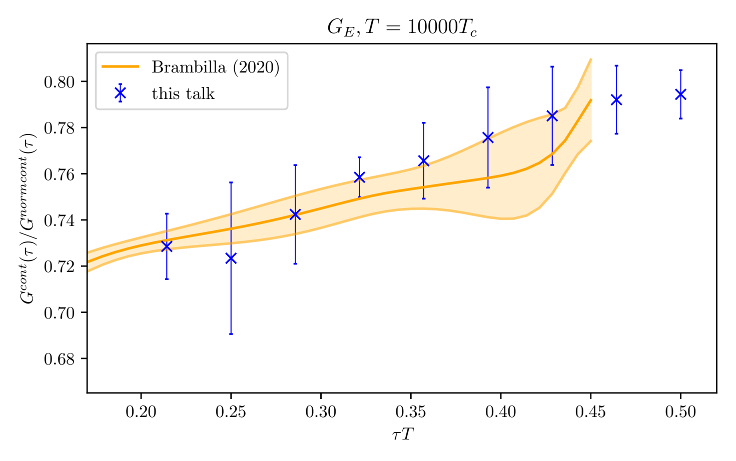
- Find κ trough $\rho(\omega)$. (In this talk we focus on Euclidean correlators)

G_{E} at $1.5\,T_{\mathrm{c}}$



- After continuum and zero flowtime limits, we replicate the previous studies:
  - Brambilla *et.al.* PRD102 (2020) Previous multilevel
  - Altenkort *et.al.* PRD103 (2021) Previous Gradient flow

G_{E} at $10^4\,T_{\mathrm{c}}$



- We see agreement with our previous multilevel result at large τ
- Small separations, larger lattices needed for continuum limit (in progress)

Mass-suppressed effects to HQ diffusion

- Considering full Lorentz force:

$$F(t) = \dot{p} = q\,(E + v \times B))\,(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_{\rm tot} \simeq \kappa_{\rm E} + \frac{2}{3}\langle \mathrm{v}^2 \rangle \kappa_{\rm B}$$

- $\kappa_{\rm B}$ related to correlation of chromo magnetic fields:

$$G_{\rm B}(\tau) = \sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

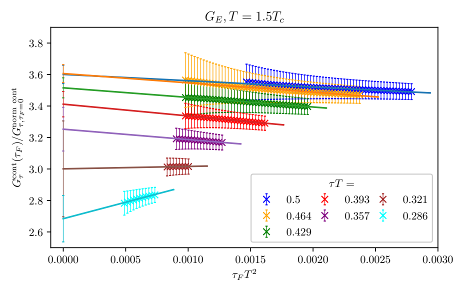
$$G_{\rm B}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\rm B}(\omega, T)K(\omega, \tau T)\;, \qquad \kappa_{\rm B} = \lim_{\omega \to 0} \frac{2T\rho_{\rm B}(\omega)}{\omega}$$

- Same tree level expansion as $G_{\rm E}$, NLO has divergence:

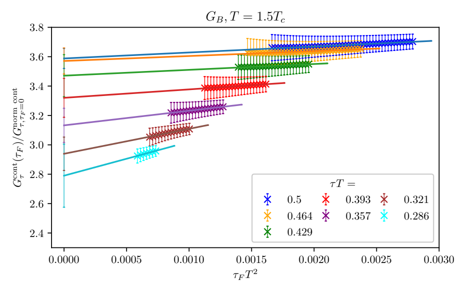
$$\rho_{\rm B} = \frac{g^2 C_f \omega^3}{6\pi}\left[1 - \frac{g^2 C_A}{(4\pi)^2}\frac{2}{\varepsilon} + (\mathrm{finite})\right] + \mathcal{O}(g^6)$$

A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Flowtime dependence of G_{E} and G_{B} at $1.5\,T_{\mathrm{c}}$

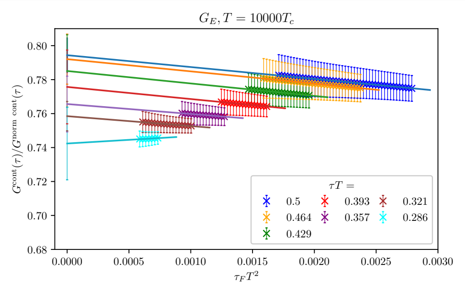


G_{E}

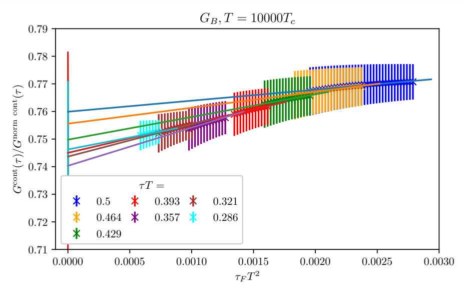


G_{B}

- We observe different small flow time scaling between G_{E} and G_{B}
- Possible indication of divergence or $\log(\tau_F)$ contributions



G_E



G_B

- Similar story at higher temperatures

Conclusions

- Prior Study: Measured κ_E at wide range of temperatures with multilevel
- Now: Measured G_E with gradient flow.
- At $1.5\,T_\mathrm{c}$ we replicate the existing results, promising results at large temperatures
- Preliminary results on G_B.
- Possible indication of divergent contribution to zero flowtime limit

Thank You