Electroweak transitions due to magnetic field: lattice results

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Preliminary

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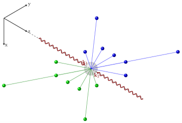
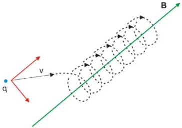
The instability

	Electric field	Magnetic field
QED	Not-stable	stable
Elweak	Not-stable	Not-stable





Landau levels:

$$\text{scalar:} \quad E_n^2 = k_z^2 + (2n+1)eH + m^2$$

$$\text{spinor:} \quad E_n^2 = k_z^2 + (2n+1)eH - 2eH \cdot s + m^2 \qquad s = \pm\frac{1}{2}$$

$$\text{vector:} \quad E_n^2 = k_z^2 + (2n+1)eH - 2eH \cdot s + m^2 \qquad s = \pm 1, 0$$

$\Rightarrow$ Exists instability: $eH_{crit}^{(1)} = m^2$

J. Ambjorn, P. Olesen: Phys. Let. B214 (1988) 565, Nucl. Phys. B315 (1989) 606, Nucl. Phys. B330 (1990) 193, arXiv:hep-ph/9304220 (1993).

EW theory (without fermions) + Unitary gauge $\begin{pmatrix} 0 \\ \phi \end{pmatrix}$

$$
\begin{aligned}
\mathcal{L} = \;& - \left\{ \frac{1}{2} |\tilde{D}_\mu W_\nu - \tilde{D}_\nu W_\mu|^2 + \frac{1}{4} f_{\mu\nu}^2 + \frac{1}{4} Z_{\mu\nu}^2 + (\partial_\mu \phi)^2 \right\} && \textit{(kinetic)} \\
& - \left\{ \frac{g^2 \phi^2}{2} W_\mu^\dagger W_\mu + \frac{1}{2} \frac{g^2 \phi^2}{\cos^2 \theta} \frac{1}{2} Z_\mu^2 - 2\lambda \phi_0^2 \phi^2 \right\} && \textit{(mass)} \\
& - \imath g \left(f_{\mu\nu} \sin\theta + Z_{\mu\nu} \cos\theta \right) W_\mu^\dagger W_\nu && \textit{(magnetic moment)} \\
& - \frac{1}{2} g^2 \left(\left(W_\mu^\dagger W_\mu \right)^2 - W_\mu^{\dagger 2} W_\mu^2 \right) + \lambda \left(\phi^4 + \phi_0^4 \right) && \textit{(4-order)}
\end{aligned}
$$

Mass term for W in the presence of a magnetic field ($f_{12} = H$):

$$
\begin{pmatrix} W_1^\dagger & W_2^\dagger \end{pmatrix} \begin{pmatrix} m_W^2 & \imath eH \\ -\imath eH & m_W^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}
$$

$\Rightarrow$ Eigenvalues are $m^2 = m_W^2 \pm eH$

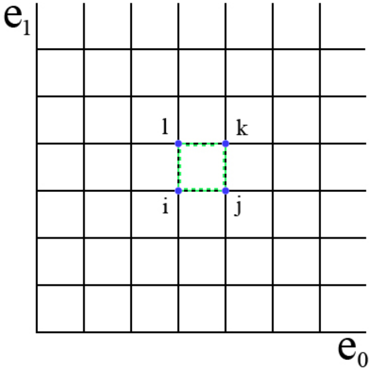
if $eH > m_W^2 \Rightarrow$ imaginary mass

$\Rightarrow$ condensation!

Lattice Quantum Field Theory

Problems of QFT:

- divergence (UV and IR)
- nonperturbative effects are playing a key role in the QCD



QCD on the lattice:

- vertex – fields
  $\psi(x) \to \psi(x_i)$
- edge (link) – gauge fields
  $A_\mu \to U(L) = e^{\imath g_0 \int_L A_\mu \, dx^\mu}$

gauge transformation:

$$U(L) \to g^{-1}(L_{end})\, U(L)\, g(L_{begin})$$

Wilson: $S_W = \sum_{plaquettes} S_P$, where $S_P = \beta\left(1 - \frac{1}{N} Re\, Tr\, U_P\right)$

Calculation of observables

1. The partition function has the form:

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S[U]}$$

2. The configurations U_i are generated with Boltzmann weight:

$$p(U)\,\mathcal{D}U \sim e^{-S(U)}\,\mathcal{D}U$$

3. The calculation of the average value:

$$\langle A \rangle = \mathcal{Z}^{-1} \int \mathcal{D}U \, A[U] \, e^{-S[U]} \quad \text{or} \quad \langle A \rangle = \sum_i A[U_i]/N_{conf}$$

EW on the lattice

Dynamical fields:

- $U_{x,\mu} = \exp\left(\imath \frac{\sigma_i}{2} W^i_{x,\mu}\right) \in \mathrm{SU}(2)$
- $\theta_{x,\mu} \in \mathcal{R}$
- $\phi_x = \begin{pmatrix} \phi_{1,x} \\ \phi_{2,x} \end{pmatrix}$

$$
\begin{aligned}
S \;=\; & \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\, U_{x,\mu\nu}\right) + \frac{\beta_Y}{2} \sum_{x,\mu<\nu} \theta^2_{x,\mu\nu} \quad \textit{(gauge)} \\
+ & \sum_{x} \left(-\kappa \phi^\dagger_x \phi_x + \lambda \left(\phi^\dagger_x \phi_x\right)^2\right) \quad \textit{(Higgs)} \\
+ & \sum_{x,\mu} \left| \phi_x - e^{\imath\left(\theta_{x,\mu} + \theta^B_{x,\mu}\right)} U_{x,\mu} \phi_{x+\hat{\mu}} \right|^2 \quad \textit{(interaction)}
\end{aligned}
$$

Boundary condition: periodic

Magnetic field : along Z direction

Lattice size: 64×48^3

Parameters: β, β_Y, κ, λ, $\theta^B_{x,\mu}$.

Where is physical point?

Finding a physical point

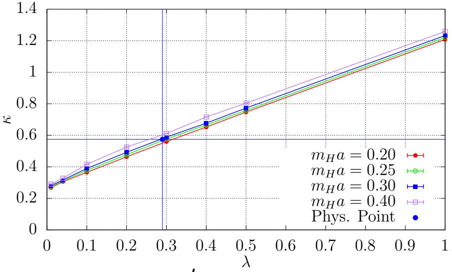
$e \approx 0.303 \quad m_H \approx 125.3$ GeV
$g \approx 0.642 \quad m_Z \approx 91.2$ GeV
$g' \approx 0.344 \quad m_W \approx 80.4$ GeV
$\sin^2\theta_W \approx 0.223$

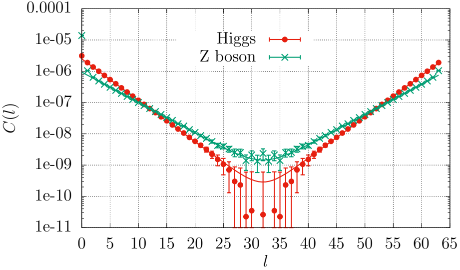
$$\beta = \frac{4}{g^2}\,, \quad \beta_Y = \frac{1}{g'^2} \equiv \frac{1}{g^2 \tan^2 \theta_W}$$

$$\Rightarrow \beta = 4\beta_Y \tan^2\theta_W$$

Our values: $\beta_Y = 7$, $\beta = 8$.

[Phys. Lett. B284 (1992) 371; Nucl.Phys. B544 (1999) 357]





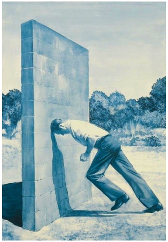
$$\frac{m_Z^{ph.}}{m_H^{ph.}} = 0.7280$$

$m_H a = 0.3049(2) \quad m_Z a = 0.2237(3)$

$m_Z = (91.88 \pm 0.12)$ GeV (err. $< 1\%$)

The most interesting: W boson

Note: W boson – not diagonal part of U matrix! $\begin{pmatrix} U_{11} & U_{12} \\ -U_{12}^* & U_{11}^* \end{pmatrix}$



- No gauge invariant observable for W in our theory:((

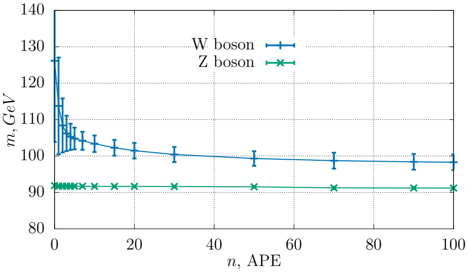
[We have not found]

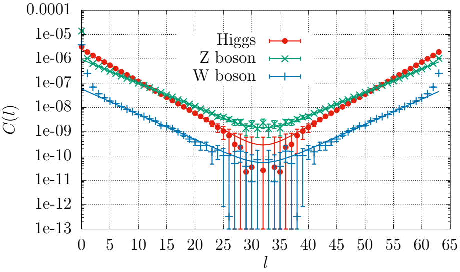
Our solution: $\langle\, U_{12}^*(0) \cdot U_{12}(t)\,\rangle$

- Maximal tree gauge for U(1) + Unitary gauge for SU(2)
- Spatial APE smearing

[arXiv:hep-lat/0409141]

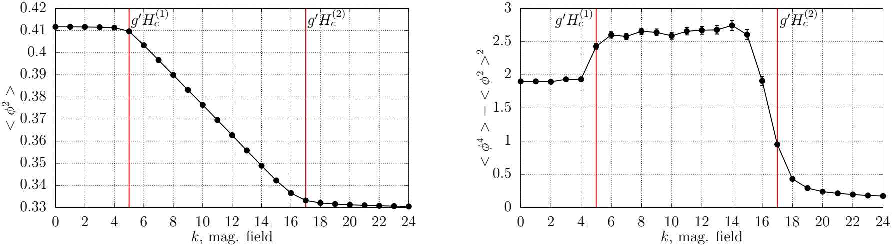
$$m_W = 98.3 \pm 1.1\,\mathrm{GeV}$$





Added a magnetic field

$$m_H a = 0.3049(2) \quad \Rightarrow \quad a = 0.4804(3) \times 10^{-18}\, m$$



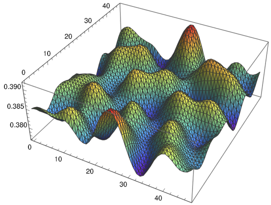
1. We see two transitions: $3.3(4) \times 10^{19}$ Tesla and $11.4(4) \times 10^{19}$ Tesla.
2. Transitions are smooth.

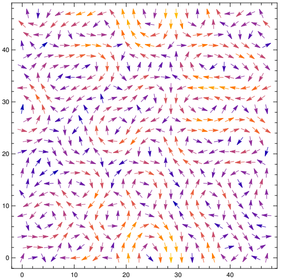
$$\sqrt{g' H_c^{(1)}} = 48.0 \pm 2.4 \text{ GeV} \quad \sim (48.8 \pm 2.5)\% \cdot m_W^{(our)}$$

$$\sqrt{g' H_c^{(2)}} = 88.4 \pm 1.3 \text{ GeV} \quad \sim (70.6 \pm 1.0)\% \cdot m_H$$

Higgs condensate and vortices on the lattice ($k = 9$)

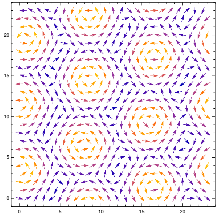
$\beta = 8$





VS

AHM

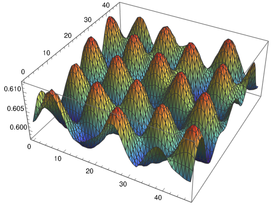


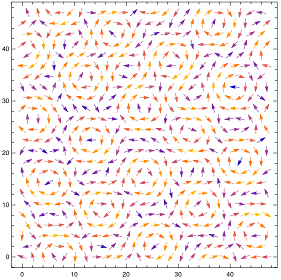
$\Leftarrow$ Do not form a lattice

Number of vortices: $\neq k$ $\searrow$ $2k$

Hill VS pit

$\beta = 20$





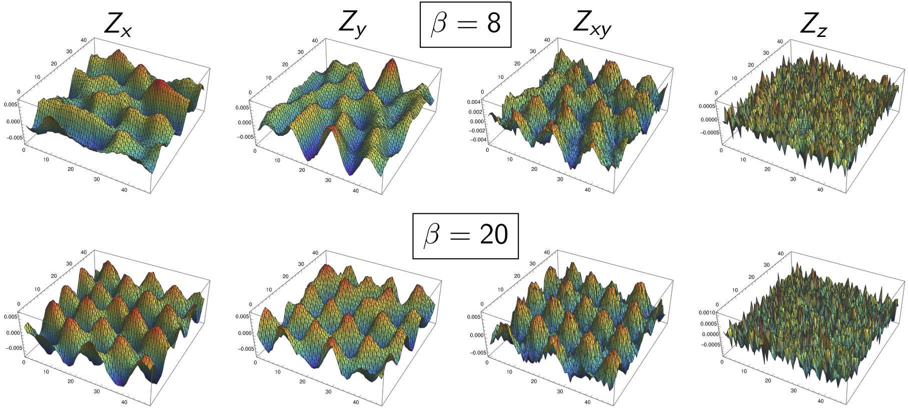
Deeper in phase with condensation exist lattice. $\nearrow$ $2k$

$$\beta : 8 \quad \rightarrow \quad 20$$

$$\sin^2\theta_W : 0.223 \quad \rightarrow \quad 0.417$$

$$\frac{m_Z}{m_H} : 0.7335(9) \quad \rightarrow \quad 0.609(3)$$

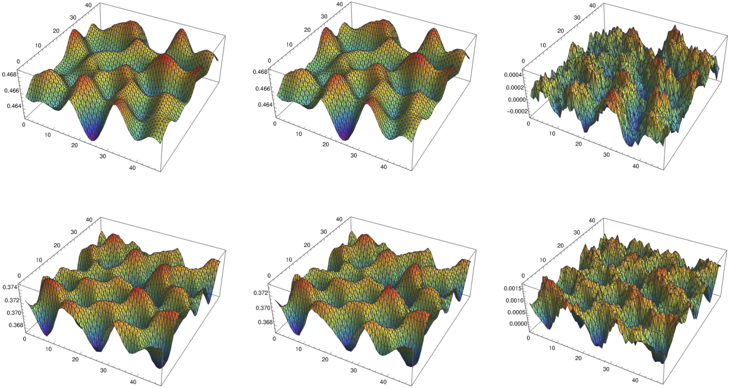
Z-induced fields ($k = 9$)



- x and y components sense magnetic field.
- Z flux built from Z_x and Z_y represent position of vortex.

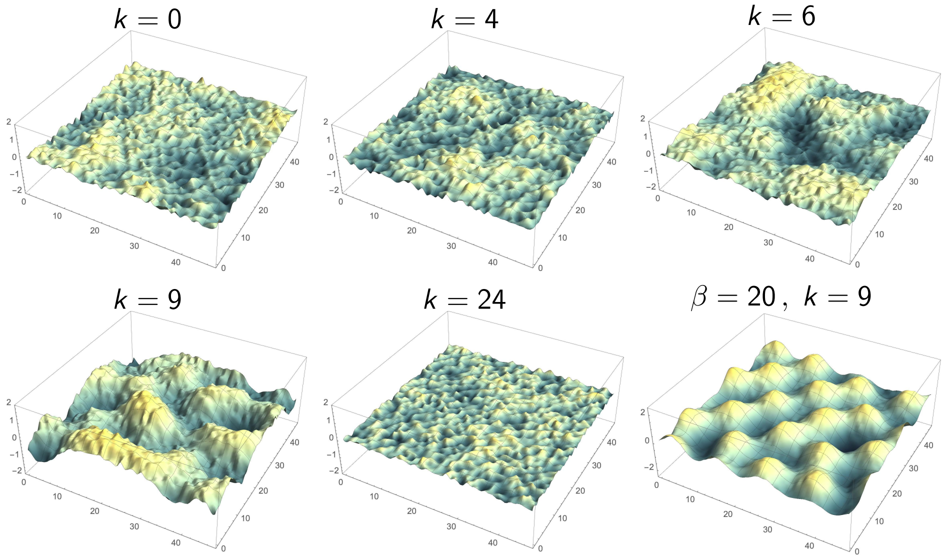
W fluctuations ($k = 9$)

	$\frac{1}{2}(W_x + W_y)$	W_z	$\frac{1}{2}(W_x + W_y) - W_z$
$\beta = 8$			
$\beta = 20$			

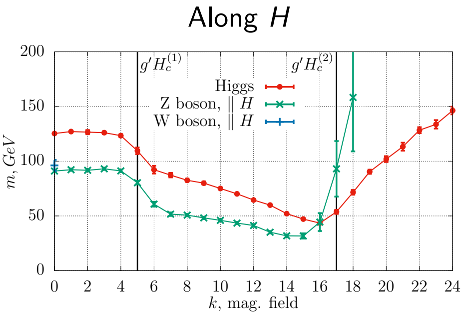


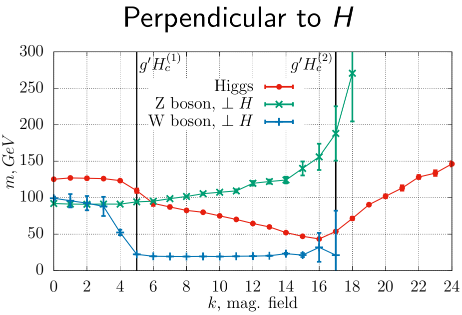
- x and y fluctuations a little bit more than in z component.
- fluctuations in $z \Rightarrow$ not of Ambjorn-Olesen type.

Some videos of Higgs field



Mass dependence





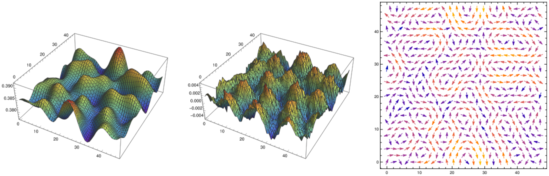
$$k < k_c^{(1)} \quad - \quad m_H,\ m_Z = \text{const}$$

$$k_c^{(1)} < k < k_c^{(2)} \quad - \quad m_H,\ m_{Z\parallel} \searrow \quad \text{and} \quad m_{Z\perp} \nearrow$$

$$k > k_c^{(2)} \quad - \quad m_H \nearrow \quad \text{and} \quad \text{[no bosons anymore]}$$

Conclusion

1. We observed magnetic field-induced phase transition in Electroweak theory at zero temperature.
2. Transitions occur at weaker magnetic fields than predicted.
3. We observed vortices (not of Ambjorn-Olesen type).
4. Smooth (crossover) transition.
5. At the physical point, we see no vortex lattice.



Questions remain: gas or liquid? Finite-T diagram.

[Thank you]