Lattice study of the SU(3) gluodynamics phase transition in presence of rotation

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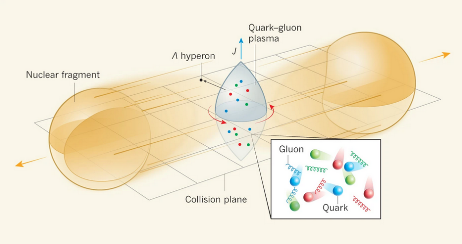
V.V. Braguta, A.Yu. Kotov, D.D. Kuznedelev, A.A. Roenko JETP Lett. 112 (2020) 1, 6-12

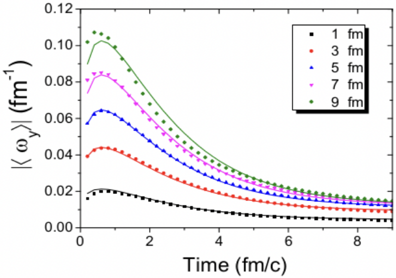
V.V. Braguta, A.Yu. Kotov, D.D. Kuznedelev, A.A. Roenko Phys.Rev.D (Vol. 103,No. 9)

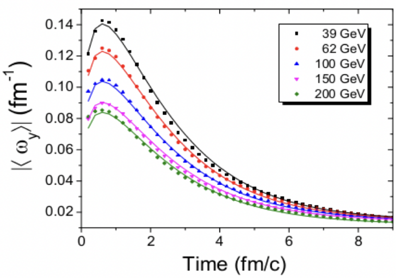
https://www.nature.com/articles/548034a



- QGP with nonzero angular momentum is created in heavy ion collisions.
- Cores of rotating compact stars
- Another type of extremal external conditions akin high temperature and density

Phys.Rev.C 94, 044910 (2016)

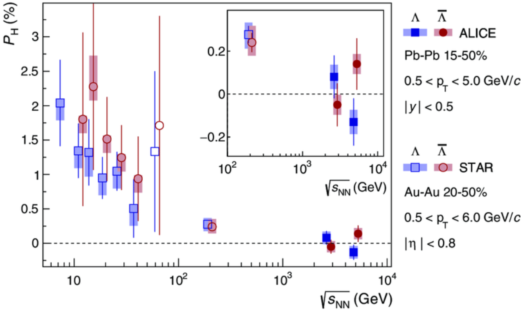




- Au-Au (left up to $\sqrt{s} = 200 GeV$, right $b = 7$ fm)
- $\Omega \sim (4-28)\mathrm{MeV}$ ($\Omega \sim 20\mathrm{MeV} \Rightarrow v \sim c$) at distances 7 fm
- Relativistic rotation of QGP

Angular velocity in experiments:

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$
- $\Omega \sim (4 - 18)\mathrm{MeV}$
- Relativistic rotation of QGP



Nature 548, 62–65 (2017)

Related research

- NJL framework
  - S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94–99
  - M.N. Chernodub, Shinya Gongyo, Phys.Rev.D 95 (2017) 9, 096006
  - M.N. Chernodub, Shinya Gongyo, JHEP 01 (2017) 136
  - Yin Jiang, Jinfeng Liao, Phys.Rev.Lett. 117 (2016) 19, 192302
  - Hui Zhang, Defu Hou, Jinfeng Liao, e-Print: 1812.11787 [hep-ph]
- Holography
  - Xun Chen, Lin Zhang, Danning Li, Defu Hou, Mei Huang, e-Print: 2010.14478
- HRG model
  - Y.Fujimoto, K.Fukushima, Y.Hidaka Phys.Lett.B 816 (2021) 136184
- Compact QED in 2+1
  - M.N.Chernodub Phys.Rev.D 103 (2021) 5, 054027
- Lattice QCD
  - Arata Yamamoto, Yuji Hirono, Phys.Rev.Lett. 111 (2013) 081601
  - Arata Yamamoto, e-Print: 2103.00237

Common features

- Impact of rotation on the temperature of chiral transition is the object of study in the NJL-model based works
- Drop of the chiral phase transition with angular velocity
- Possible explanation -polarization of the chiral condensate (Phys.Rev.Lett. 117 (2016) 19, 192302)
- Drop of the confinement/deconfinement transition with angular velocity

Rotating QGP on the Lattice

- QGP is in thermodynamic equilibrium
- The study is carried out in the co-rotating reference frame.
- Rotation acts as an external gravitational field

$$\text{Causality principle} : \Omega r < c$$

- Only bounded systems can be studied.
- Boundary conditions are important!

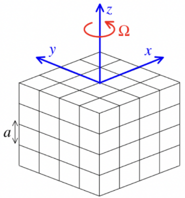
Lattice setup

- Rotation is accounted via metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system

$$N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$$



Phys.Rev.Lett. 111 (2013) 081601

- Partition function ($\hat{H}$ is conserved)

$$Z = \operatorname{Tr} \exp\left[-\beta \hat{H}\right]$$

- Euclidean action

$$S_G = \frac{1}{2g^2} \int d^4x \, \big[(1 - r^2\Omega^2) F^a_{xy} F^a_{xy} + (1 - y^2\Omega^2) F^a_{xz} F^a_{xz} + \\ + (1 - x^2\Omega^2) F^a_{yz} F^a_{yz} + F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} - \\ - 2iy\Omega(F^a_{xy} F^a_{y\tau} + F^a_{xz} F^a_{z\tau}) + 2ix\Omega(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau}) - 2xy\Omega^2 F^a_{xz} F^a_{zy}\big]$$

- Ehrenfest–Tolman effect:
  Temperature is not constant in space at thermal equilibrium in the presence of gravitational field

$$T(r)\sqrt{g_{00}} = 1/\beta = const$$

$$T(r)\sqrt{(}1 - r^2\Omega^2) = 1/\beta$$

- Temperature on the boundaries is higher, that in the center

$$T(r > 0) > T(r = 0)$$

- One would expect that rotation decreases the critical temperature
- We introduce the notation $T = T(r = 0) = 1/\beta$

Boundary conditions

- `Periodic b.c`
  - $U_\mu(x) = U_\mu(x + N_i \vec{i})$
  - Not appropriate for the field of velocities of rotating body
- `Dirichlet b.c`
  - $A_\mu|_{x\in\partial D} = 0 \Rightarrow U_\mu(x)|_{x\in\partial D} = 1$
  - Explicit breaking of $\mathbb{Z}_3$ symmetry
- `Open b.c`
  - $F_{\mu\nu}|_{x\in\partial D} = 0 \Rightarrow U_\square|_{\square\in\partial D} = 0$

Sign problem

$$S_G = \frac{1}{2g^2}\int d^4x \, \Big[(1 - r^2\Omega^2)F^a_{xy}F^a_{xy} + (1 - y^2\Omega^2)F^a_{xz}F^a_{xz} + \\ + (1 - x^2\Omega^2)F^a_{yz}F^a_{yz} + F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} - \\ - 2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2F^a_{xz}F^a_{zy}\Big]$$

- The Euclidean action has imaginary part (sign problem)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach is valid for the studied range of Ω ($\Omega \leqslant 50$ MeV)

Lattice setup

Determination of the critical temperature

- Polyakov line

$$\mathcal{L} = \left\langle \mathrm{Tr}\, \mathcal{T} \exp\left[ig \oint_{[0,1/T]} A_4\, dx^4 \right] \right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right)$$

- T_c is determined from Gaussian fit of the $\chi(T)$

Open b.c

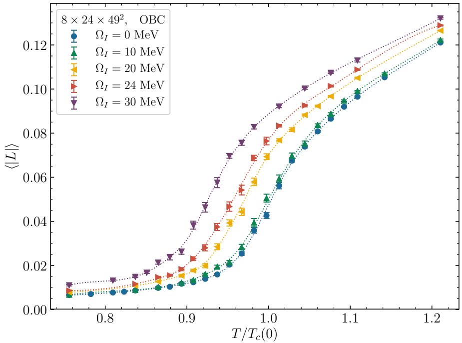


Figure: Dependence of the Polyakov loop susceptibility on the temperature

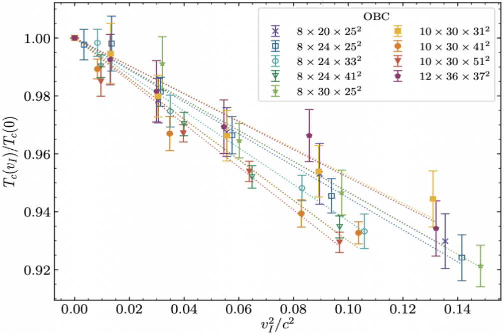


Figure: Dependence of the T_c on the linear velocity on the boundary

Periodic b.c

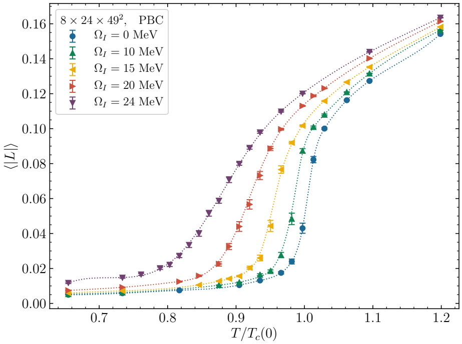


FIGURE: Dependence of the Polyakov loop susceptibility on the temperature

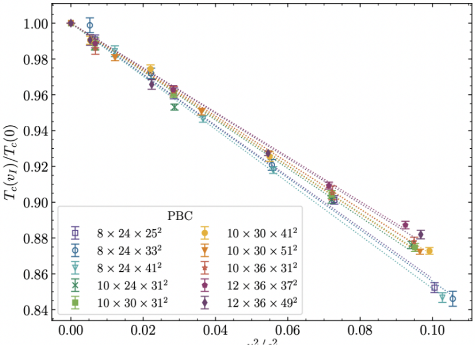


FIGURE: Dependence of the T_c on the linear velocity on the boundary

Dirichlet b.c

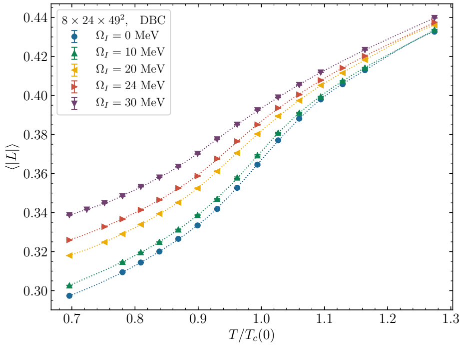


Figure: Dependence of the Polyakov loop susceptibility on the temperature

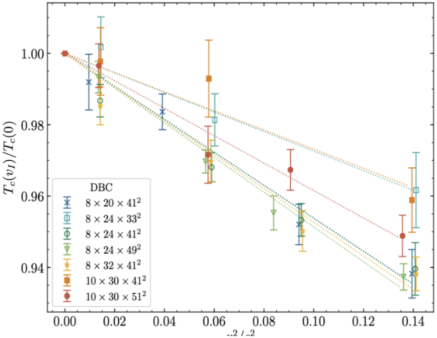


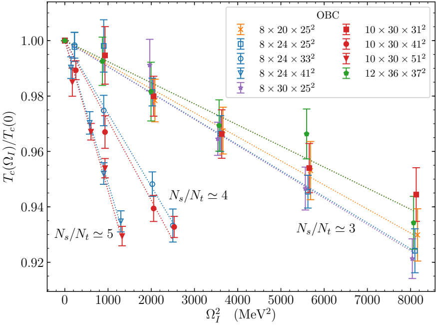
Figure: Dependence of the T_c on the linear velocity on the boundary

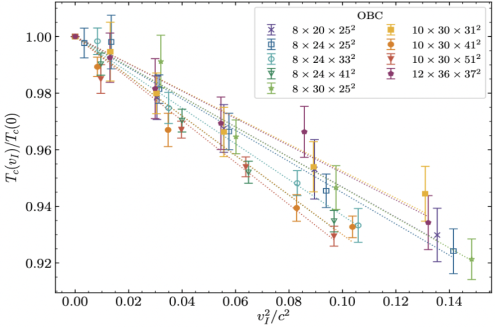
Results

- Dependence can be approximated by the formula:

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- Critical temperature increases with the angular velocity
- Results depend weakly on the lattice spacing and the volume

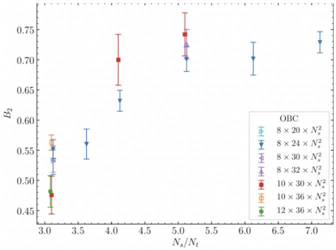




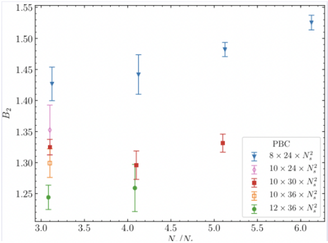
- Dependence on angular velocity is not universal (different for different $\frac{N_s}{N_t}$)
- Dependence on linear velocity v_I (at the boundary) is universal

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 v_I^2 \quad v_I = \Omega(N_s - 1)a/2 \quad C_2 = B_2(N_s - 1)^2 a^2/4$$

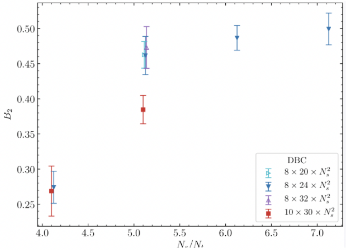
B_2 coefficient for different b.c.



Open b.c
$B_2 \sim 0.7$



Periodic b.c
$B_2 \sim 1.3$

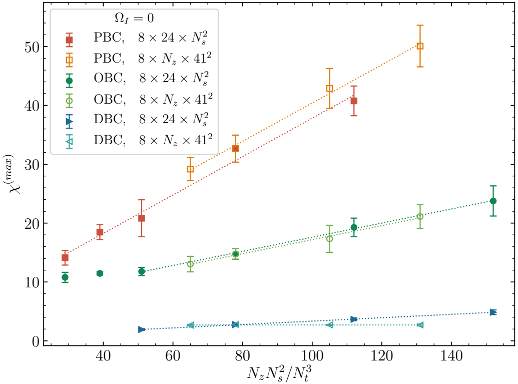


Dirichlet b.c
$B_2 \sim 0.5$

Volume dependence of the susceptibility

- Periodic b.c. : $\sim V$
- Dirichlet b.c. : $\sim const$
- Open b.c. : $\sim V$

Rotation doesn't change order of the phase transition!



Conclusions

- We have studied the influence of the relativistic rotation on the confinement/deconfinement phase transition
- Critical temperature of the confinement/deconfinement phase transition increases with Ω

- Rotation is known to decrease chiral transition critical temperature
- Is the impact of fermions larger than of gluons in the QCD?