Bound states in a quark-gluon plasma from lattice QCD

Rasmus Larsen

University of Stavanger

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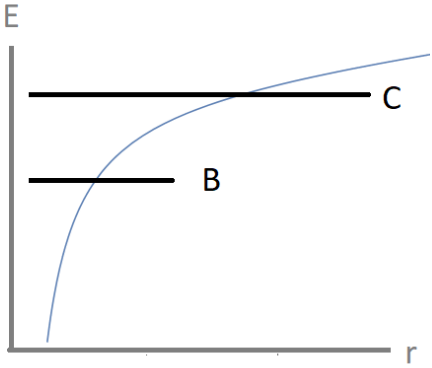
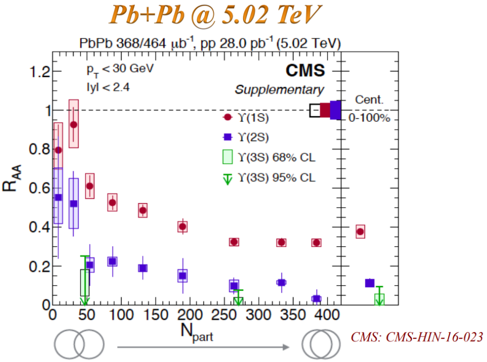
- Understand what happens to states of quarks and anti quarks at finite temperature:
  - Use Bottomonium states as probe for change in color screening
  - Experimental results show suppression of Bottomonium states at finite temperature





Approach

- 2 different measurements to understand bound states
  - 1: Measure the energy of state propagating through complex time
  - 2: Measure the energy of 2 infinitely heavy quarks, separated by distance r

1:

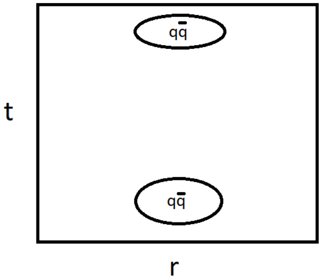


Figure: Illustration of a Upsilon correlation measurement.

2:

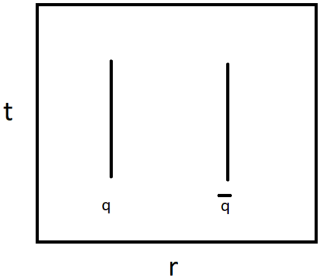


Figure: Illustration of a Wilson line correlation measurement.

Approach 1: Correlation of Bottomonia States

- Main observable: Correlation function $C(\tau)$
  - $C(\tau)$ is the zero momentum of the state of interest

$$\int d^3x < O(\tau, x)O^\dagger(0,0) > \; = \; C(\tau) \; = \; \int_0^\infty \rho(\omega)\exp(-\omega\tau)d\omega \qquad (1)$$

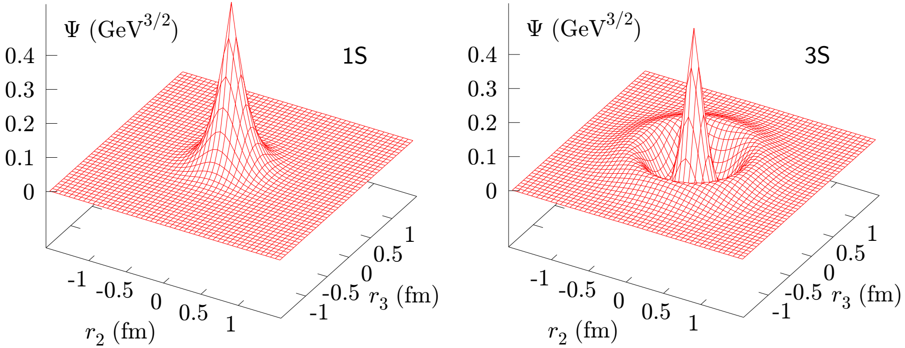
$$O_i(\tau, \mathbf{x}) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r}, \tau)\Gamma q(\mathbf{x}, \tau) \qquad (2)$$

- Invert equation to find spectral function $\rho(\omega)$

Extended Sources

- Source calculated from discretized schroedinger equation with confining potential that reproduces zero temperature spectrum

$$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r},t)\Gamma q(\mathbf{x},t) \quad (3)$$



NRQCD

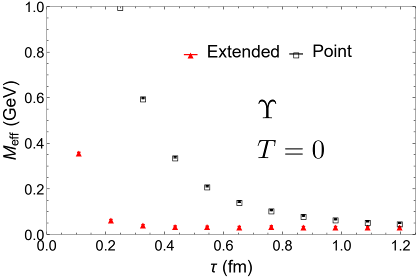
- Method: Non Relativistic QCD (NRQCD) on lattice
  - Bottom mass $\sim 4GeV$
  - Lattice spacing $a^{-1} \sim 3GeV$
    - with $O(v^4)$ corrections, plus $O(v^6)$ spin corrections
  - 2+1 flavor HotQCD configurations from $T = 151MeV$ to $T = 334MeV$
  - Pion mass 160MeV, Kaon mass physical
  - Explore Υ, χ_b

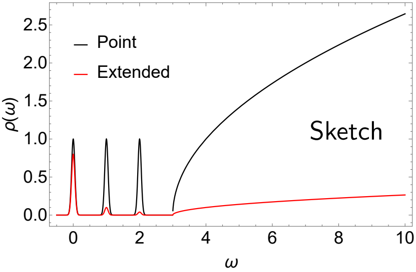
Effective Mass

- Plateaus of the effective mass M_{eff} $->$ Mass state exists in $\rho(\omega)$

$$M_{eff} = \frac{1}{a}\log[C(\tau)/C(\tau+a)] = -\frac{\partial}{\partial_\tau}\log(C(\tau)) \tag{4}$$

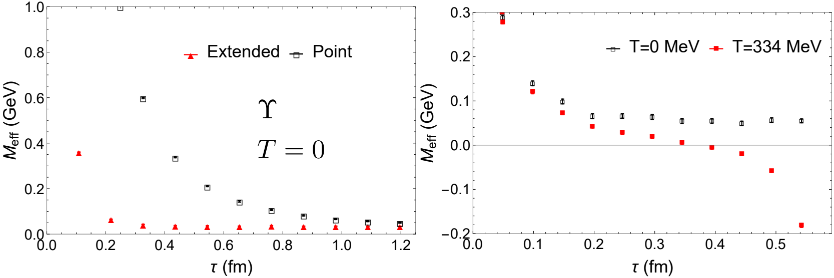
- Continuum in $\rho(\omega)$ with point sources dominates contribution to correlation function
- Solution:
  - Use sources with finite (extended) size $\rightarrow$ project onto specific region in ω





Continuum Subtracted S-wave

- Extended sources greatly reduces continuum contribution

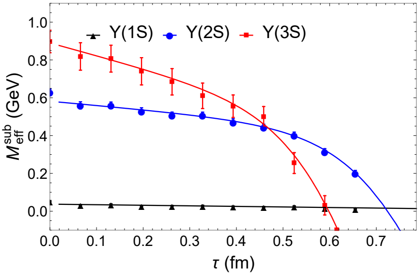


- Small τ behavior similar at $T = 0$ and $T \neq 0$
- Extract continuum $C_{high}(\tau)$ from $T = 0$ results
- 0 Corresponds to energy of η_b at $T = 0MeV$.

$$
\begin{aligned}
C(\tau) &= Ae^{-M\tau} + C_{high}(\tau) \\
C_{sub}(\tau, T) &= C(\tau, T) - C_{high}(\tau)
\end{aligned}
\qquad (5)
$$

Finite Temperature Subtracted Effective Mass

- Drop in effective mass as $\tau \to 1/T$
- Linear behavior at small to mid range τ



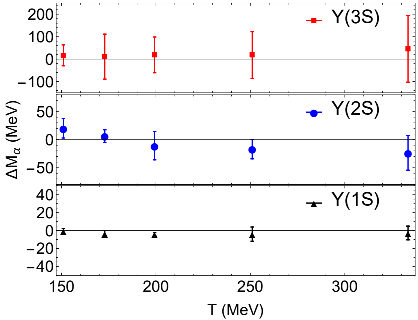
- Information in correlation function is thus

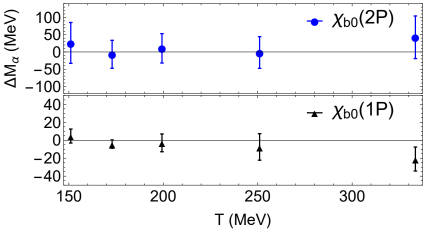
$$C_{sub}(\tau, T) \sim \exp(-M_\alpha \tau + \frac{1}{2}\Gamma_\alpha^2 \tau^2 + O(\tau^3)) \quad (6)$$

$$\rho_\alpha(\omega, T) = A_\alpha(T) \exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right) + A_\alpha^{\rm cut}(T)\,\delta\left(\omega - \omega_\alpha^{\rm cut}(T)\right)$$

Mass

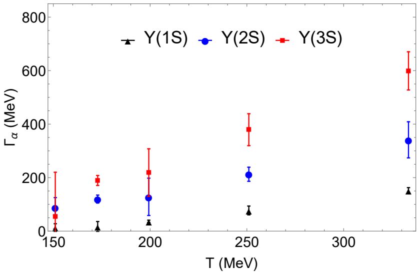
- The mass is found to be consistent with zero temperature results [R. Larsen et al., arXiv:1910.07374]
- $\Delta M_\alpha = M_\alpha(T) - M_\alpha(0)$

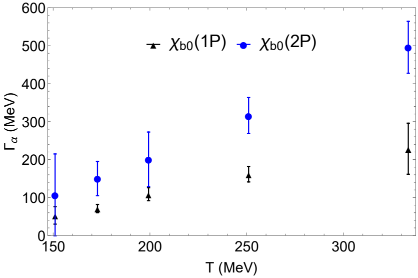




Spectral Width

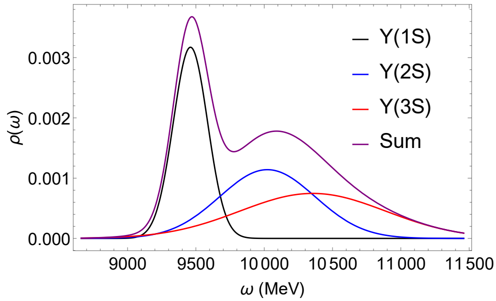
- Spectral width grows with temperature like T^2 [R. Larsen et al., arXiv:1910.07374]
- The higher the energy of the state, the wider the spectral function becomes





Picture at finite temperature

- Our results indicate the following picture
  - No significant change in energy/mass of states
  - Large spectral width, such that states start to overlap
  - Spectral function with equal weight at $T = 334MeV$ shown below



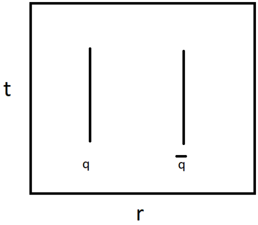
Approach 2: Wilson Lines Correlator

- Wilson line is the product of Links

$$W(t,x) = \prod_{i}^{t} U_4(i,x) \quad (7)$$

- Infinitely heavy quarks stay fixed at same position
- Propagating from $\tau = 0$ to $\tau = t$ will be done by a wilson line of length t
- A quark and anti-quark will interfere with each other, to create different states based on the possible energies

$$C(t,x) = Tr(W(t,0)W(t,x)^{\dagger}) \quad (8)$$



- Measurement is not gauge invariant
  - We gauge fix to Coulomb gauge

Measurements with Wilson Lines

- Measurement contain contribution from continuum, just like for Bottomonia
- We remove the continuum in the same procedure as for Bottomonia

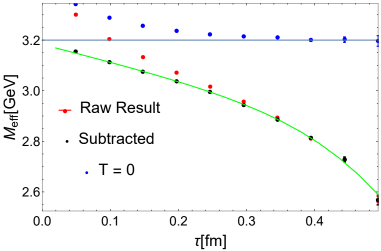


Figure: T=334MeV, r=0.44fm.

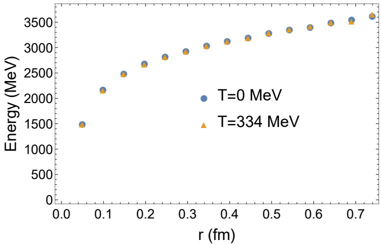
- Information in correlation function (Black points) is thus

$$C_{sub}(\tau,T) \sim \exp(-M_\alpha\tau + \frac{1}{2}\Gamma_\alpha^2\tau^2 + O(\tau^3)) \quad (9)$$

$$\rho_\alpha(\omega,T) = A_\alpha(T)\exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right) + A_\alpha^{\text{cut}}(T)\,\delta\left(\omega - \omega_\alpha^{\text{cut}}(T)\right)$$

Energy and Width of Wilson Lines Correlator

- Almost no change in energy, but increasing width



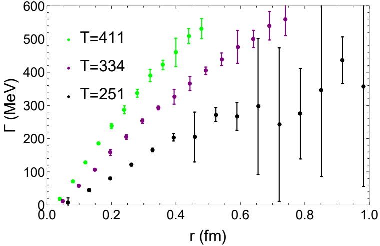


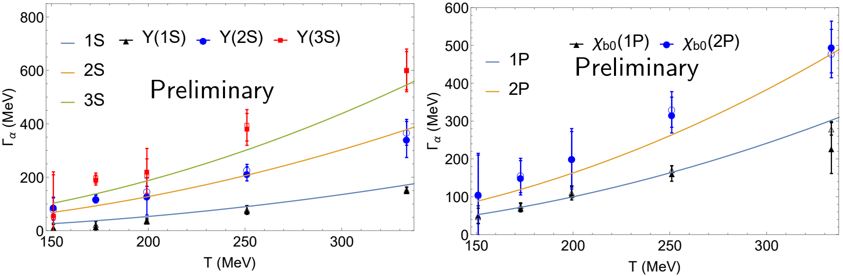
Figure: (Left) Energy obtained for $T = 334MeV$. (Right) Width at several temperatures around 300 MeV.

- Preliminary [Rasmus Larsen,Peter Petreczky, Alexander Rothkopf, Johannes Heinrich Weber and more]
- Width obtained from Wilson lines correlator consistent with width of Υ when looking at distance of Υ 's average radius.

Wilson Lines Correlators and Bottomonia States

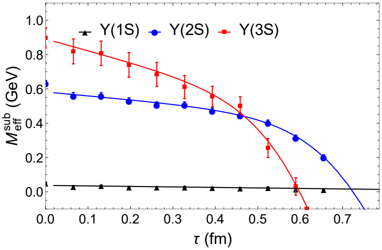
- Calculate spectral width of bottomonia from Wilson lines results
- Wave function from discretized Shroedinger equation

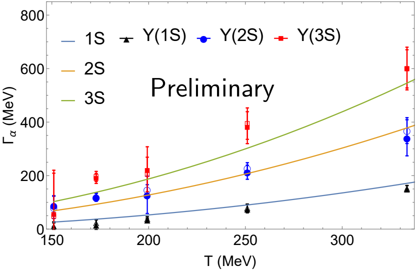
$$\Gamma_\alpha = \int dx^3 |\psi_\alpha(x)|^2 \Gamma_r(|x|) \quad (10)$$



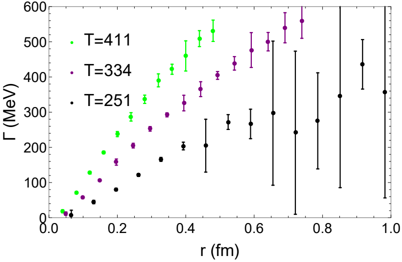
- Width obtained from linear rise in width of Wilson lines correlator
- Wavefunctions calculated with mass 5.5GeV.

Conclusion





- Linear behavior in effective mass observed
- Behavior explained by large spectral width of bottomonia $\sim 160 - 600 MeV$
- 2S and 3S, 1P and 2P spectral functions overlap strongly above $T = 200 MeV$
- Width from Wilson lines correlator shows same behavior as bottomonia results



Extra

Bethe-Salpeter Wavefunction

- Bethe-Salpeter amplitude for bottomonium

$$C^r_\alpha(\tau) = \langle O^r_{qq}(\tau)\tilde{O}^\dagger_\alpha(0)\rangle, \tag{11}$$

- Measure probability of quarks to be separated

$$O^r_{qq}(\tau) = \int d^3x\bar{q}(\mathbf{x},\tau)\Gamma q(\mathbf{x}+\mathbf{r},\tau). \tag{12}$$

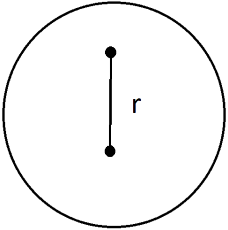
- Remove Energy dependence to get time independent result

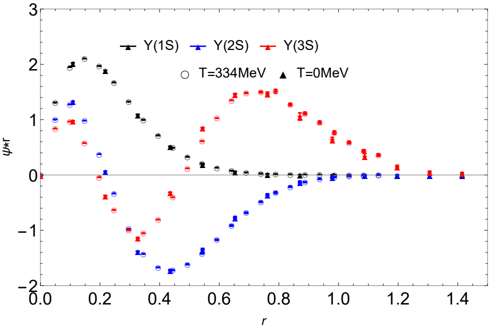
$$e^{E_\alpha\tau}\tilde{C}^r_\alpha(\tau) \tag{13}$$

- or normalize to 1

Spatial Localization of States

- Arxiv:2008.00100
- Look at the change in shape using the Bethe-Salpeter wavefunction
- Shows the distribution of the state as a function of distance r

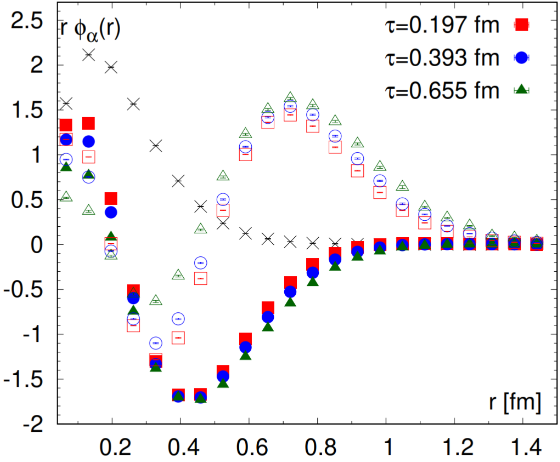




- No significant difference in shape is seen at $\tau \sim 0.4fm$

Bethe-Salpeter Wavefunction 2

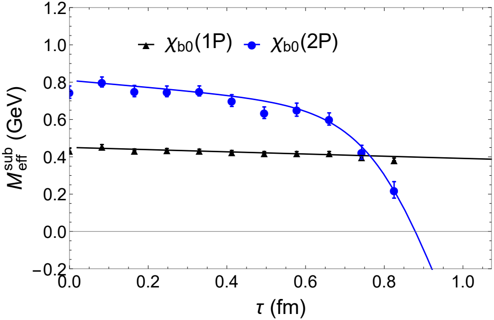
- For large temperature BS wavefunction is seen to move out slightly to larger distances as τ increases
- T=251MeV



Backup

Continuum Subtracted P-wave

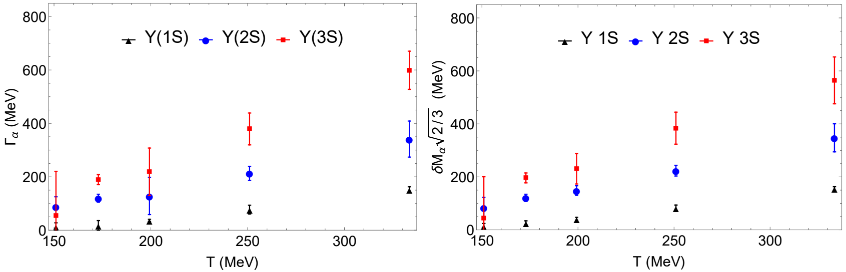
- Same procedure works for χ_b
- Figure below is for $T = 199MV$



- M_{eff}^{sub} goes to energy of the $T = 0$ result when extrapolated to $\tau = 0$

Ansatz Comparison

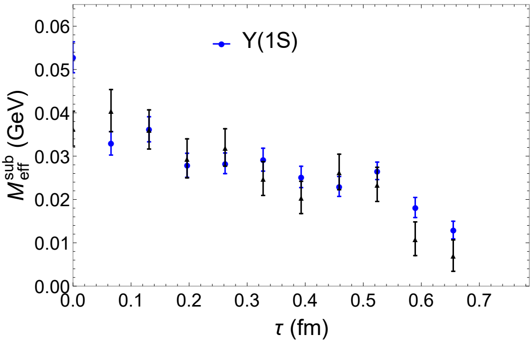
- Ansatz with Gaussian spectral function (left) and 3 delta functions representing a width (right) are consistent
- δM_α is difference between middle delta function and the two other delta functions



- Width $\sqrt{\langle\omega^2\rangle - \langle\omega\rangle^2}$ is equal to:
  - Γ_α for Gaussian ansatz
  - $\delta M_\alpha \sqrt{2/3}$ for 3 delta functions ansatz

Dependence on Source

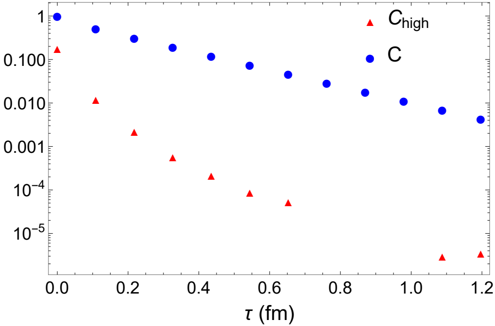
- Slope consistent between smeared sources and wavefunction sources
- Drop off at $\tau \sim 1/T$ smaller for wavefunctions



Continuum Subtracted

- Small τ behavior same at $T = 0$ and $T \neq 0$
- Extract difference from groundstate at $T = 0$

$$C(\tau) \quad = \quad Ae^{-M\tau} + C_{high}(\tau)$$



- Subtract result from finite temperature result

$$C_{sub}(\tau, T) = C(\tau, T) - C_{high}(\tau). \tag{14}$$