Matching glasma to hydro via kinetic theory

Aleksas Mazeliauskas

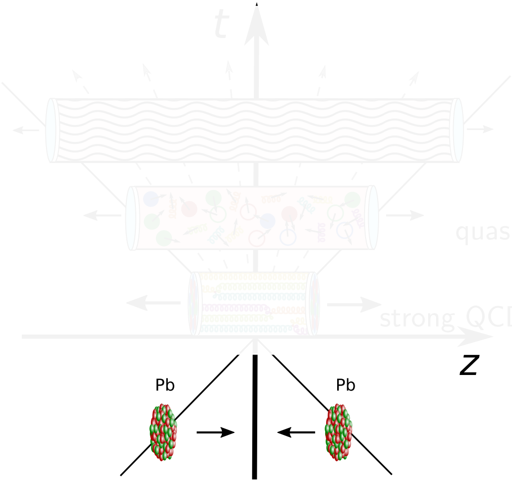
Theoretical Physics Department, CERN

June 28, 2020

Thermalisation in QCD at weak couplings $\alpha_s \ll 1$

Berges, Heller, AM, Venugopalan (2020) [1]

See also review by Teaney and Schlichting (2019) [2]



Fluid expansion
$t \sim 1 - 10\,\mathrm{fm}/c$

$T(t,x), u^\mu(t,x)$ $f_g \sim 1$

Equilibration
$t \sim 1\,\mathrm{fm}/c$

quasi-particles $1 \ll f_g \ll \frac{1}{\alpha_s}$

Initial state
$t \ll 1\,\mathrm{fm}/c$

strong QCD fields $f_g(p \sim Q_s) \sim \frac{1}{\alpha_s} \gg 1$

Incoming nuclei

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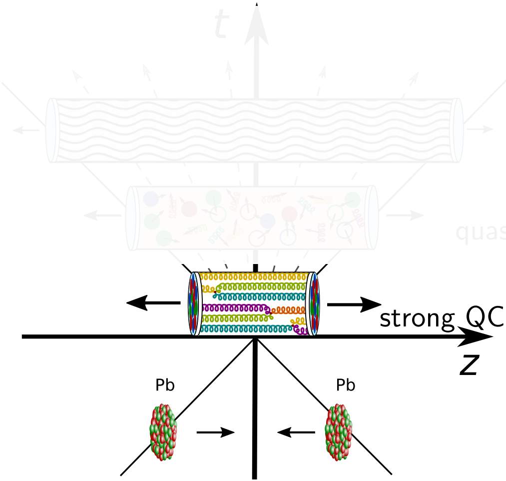

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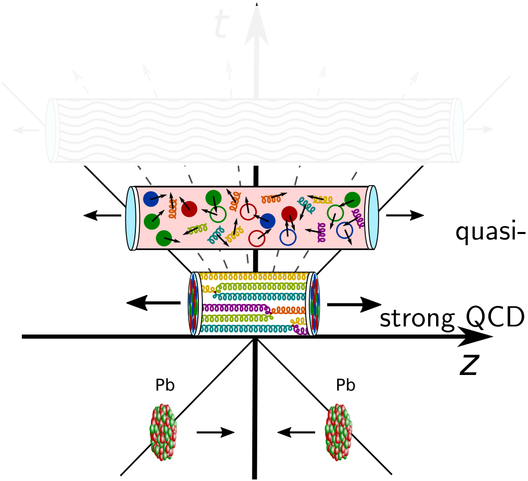

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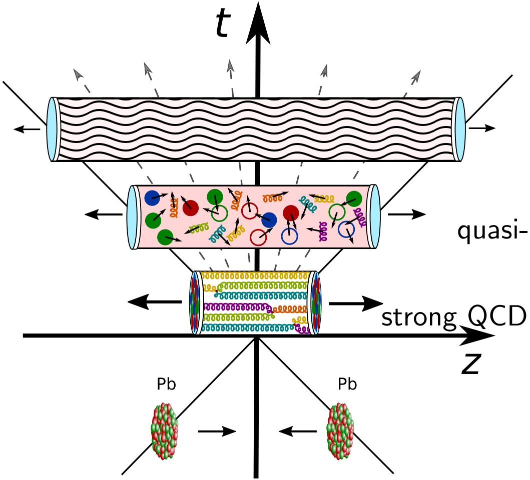

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Incoming nuclei

QCD kinetic theory—bridge between glasma initial state and late hydrodynamics.

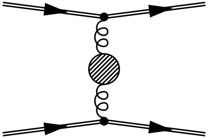
High temperature QCD kinetic theory

Arnold, Moore, Yaffe (2003)[5]

Weakly coupled $g \ll 1$ gas of quark and gluon quasi-particles

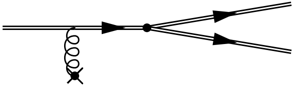
$$\text{Boltzmann eq.:} \quad \partial_t f + \underbrace{\frac{\mathbf{p}}{|p|} \cdot \nabla f}_{\text{expansion}} = - \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]}_{\text{in-medium QCD collisions}}$$

1. $2 \leftrightarrow 2$ elastic scatterings



with screening mass $m_D \sim gT$

2. $1 \leftrightarrow 2$ medium induced collinear radiation:



including interference effects (LPM)

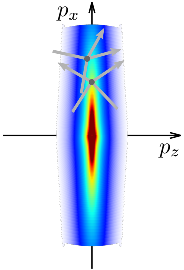
Contains necessary physics for thermalisation and in-medium parton energy loss.

Sequential emission by Arnold, Gorda, Iqbal (2020) [3], spectral fermion function on lattice by Boguslavski, Lappi, Mace, Schlichting (2021) [4]
For other developments see Peuron's talk on Monday; Haukkson's and Schlusser's talks on Thursday

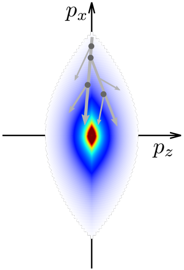
“Bottom-up” thermalisation scenario

Baier, Mueller, Schiff, and Son (2001)[10]

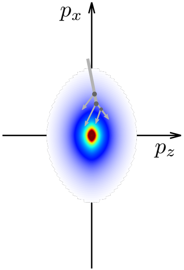
Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$



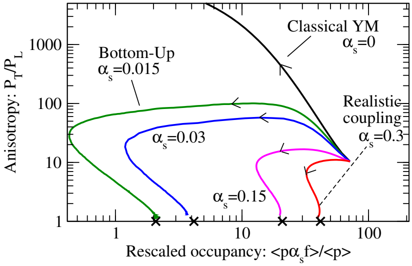
$2 \leftrightarrow 2$ broadening



collinear cascade



mini-jet quench



Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018) [6–9]

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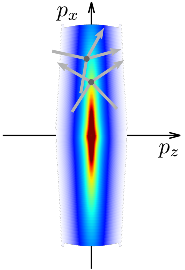

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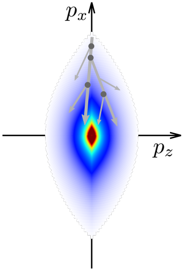

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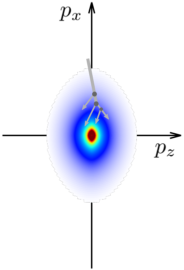

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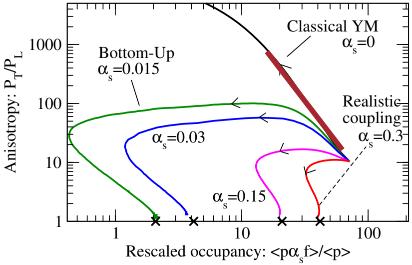

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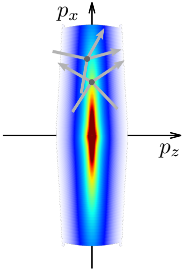

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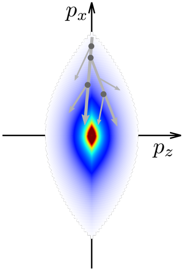

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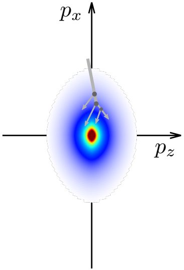

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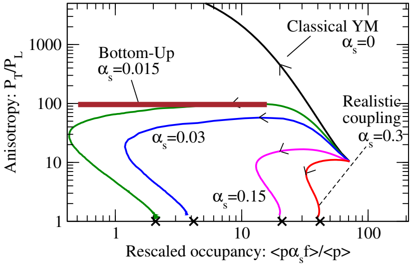

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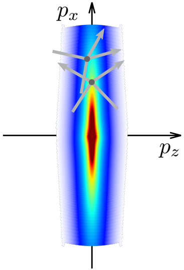

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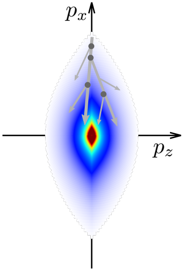

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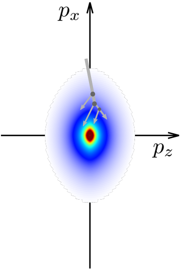

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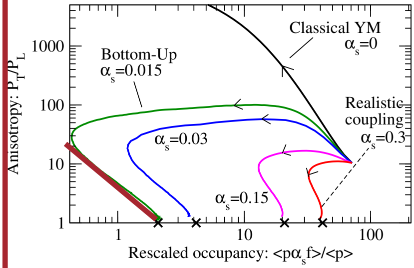

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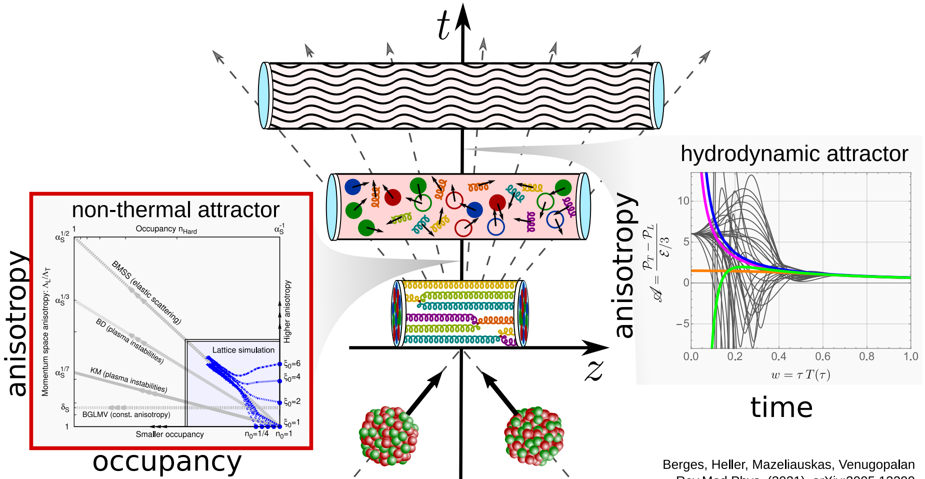
Non-thermal and hydrodynamic attractors in QCD thermalisation

Simplification of non-equilibrium dynamics thanks to attractor behaviour.

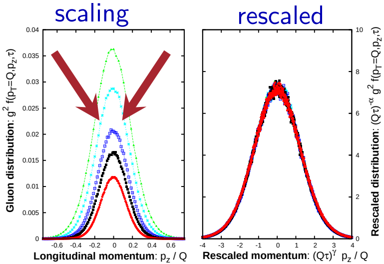


Berges, Heller, Mazeliauskas, Venugopalan
Rev.Mod.Phys. (2021), arXiv:2005.12299

Non-thermal fixed point for gauge theories

Highly occupied gluon evolution described by classical-statistical Yang-Mills

Aarts, Berges (2002) [11], Mueller, Son (2004) [12], Jeon (2005) [13]



Self-similar scaling $\Longrightarrow$ loss of information

Berges, Schenke, Schlichting, Venugopalan (2014) [14]

$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$

Scaling phenomena also seen in scalar theories, cold atom experiments

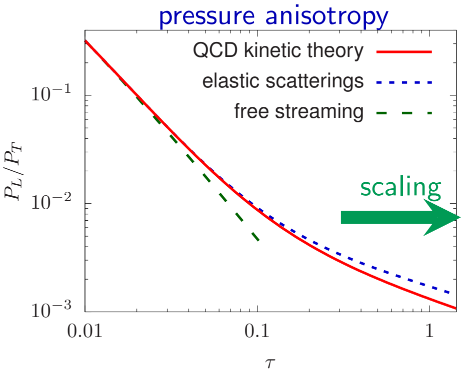
Orioli et al. (2015) [15], Mikheev et al. (2018) [16], Prüfer et al. (2018) [17], Erne et al. (2018) [18]

Scaling in leading order QCD kinetic theory

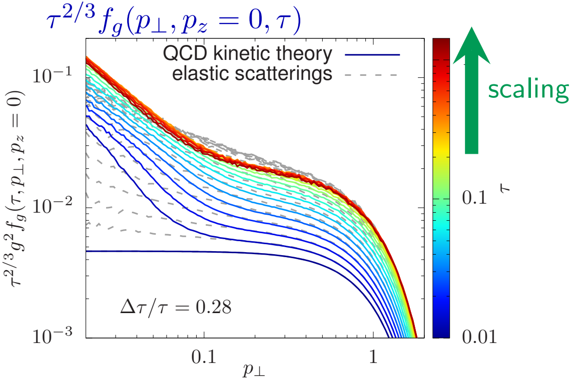
In kinetic theory scaling regime is reached at late times

AM and Berges (2019) [19]

$$f_g(p_\perp, p_z, \tau) = \tau^{-2/3} f_S(p_\perp, \tau^{1/3} p_z),$$



pressure anisotropy



$\tau^{2/3} f_g(p_\perp, p_z = 0, \tau)$

Early time QCD kinetic theory evolution differs from free streaming!

Pre-scaling regime in QCD kinetic theory

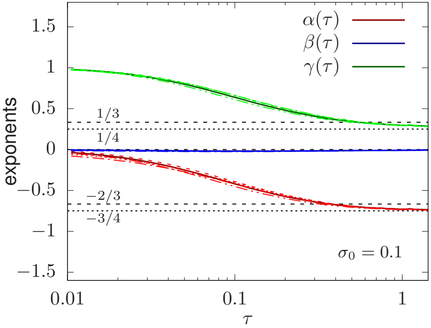
Non-equilibrium dynamics undone by self-similar renormalization

$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

Scaling exponents $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$ can be time dependent!

AM and Berges (2019) [19]

scaling exponents



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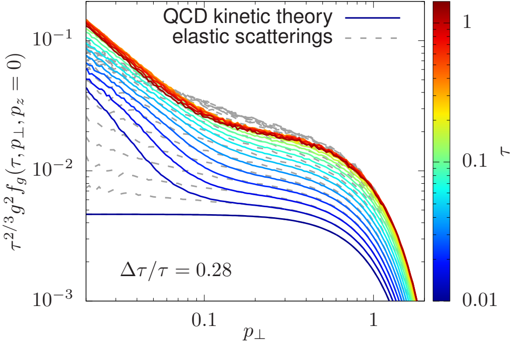

Much earlier collapse to scaling solution f_S — pre-scaling regime.

Pre-scaling regime in QCD kinetic theory

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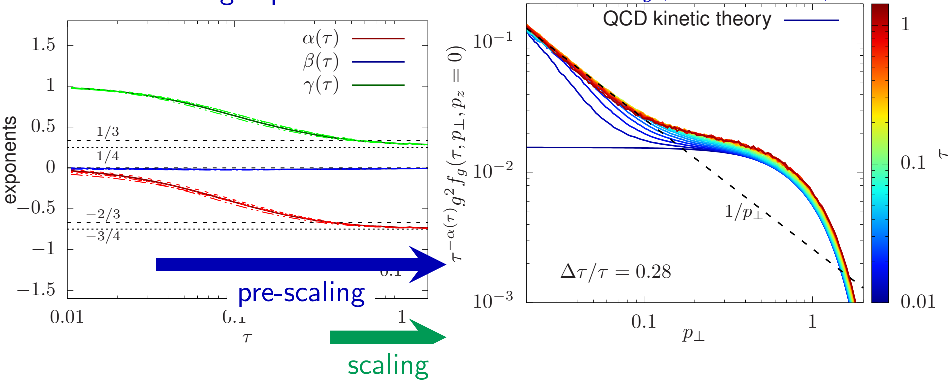

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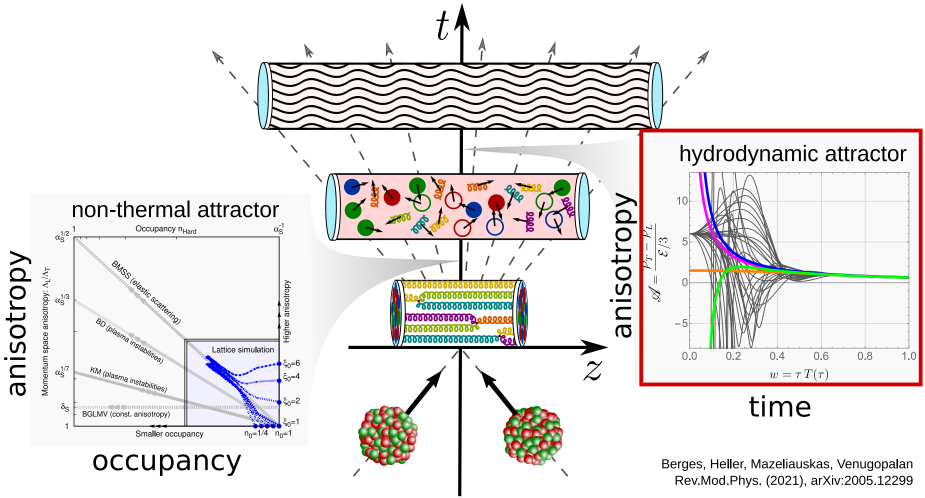

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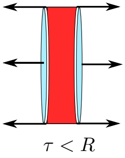
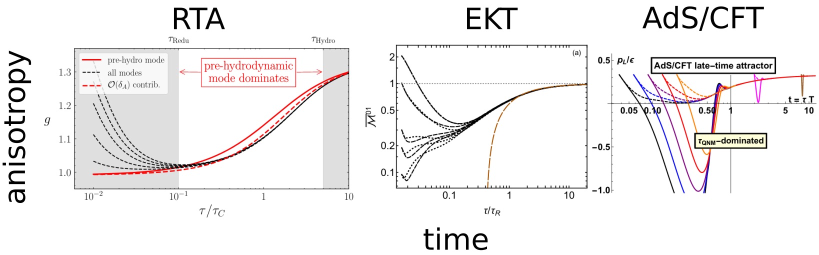
Beyond conventional gradient expansion: hydrodynamic attractors

Pressure anisotropy collapse to a hydrodynamic attractor

Heller, Janik, Witaszczyk (2011)[20]

$$\frac{\tau\partial_\tau e}{e} = -1 - \frac{P_L}{e} = -\frac{4}{3} + \underbrace{\frac{16}{9}\frac{\eta/s}{\tau T}}_{\text{1st gradient}} \;\ldots$$





Brewer, Yan, Yi (2019) [21], Almaalol, Kurkela, Strickland (2020) [22], Kurkela, van der Schee, Widemann, Wu (2019) [23]

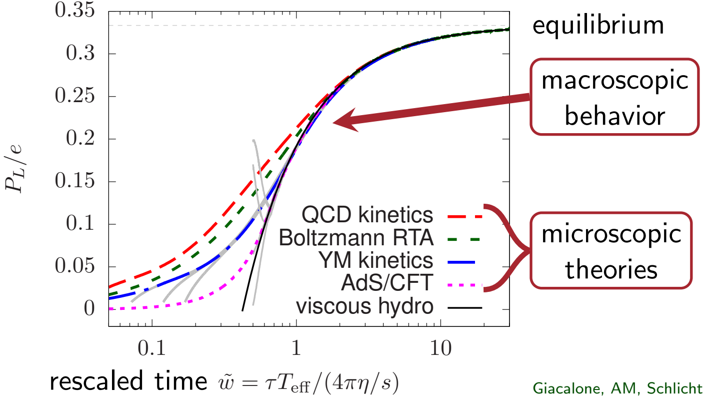
see reviews: Florkowski, Heller and Spalinksi (2017)[24], Romatschke and Romatschke (2017) [25]

What can we learn from far-from-equilibrium dynamics?

Integrating equations of motion on an attractor can relate early and late times

Giacalone, AM, Schlichting, (2019) [26]

$$e\left(\tau_{\text{therm}}\right) = e_0 \exp\left(-\int_{\tilde{w}_0}^{\tilde{w}_{\text{therm}}} \frac{4d\tilde{w}}{\tilde{w}} \frac{1+P_L/e}{3-P_L/e}\right) \Longrightarrow \left(\tau^{\frac{4}{3}} e\right)_{\text{therm}} \propto C_\infty \left(e\tau\right)_0^{8/9}$$



Giacalone, AM, Schlichting, (2019) [26]

Entropy production from hydrodynamic attractor

- In equilibrium entropy per rapidity $\frac{dS}{dy} = A_\perp (s\tau)_{\text{therm}}$ given by

$$(s\tau)_{\text{therm}} \propto \left(e\tau^{\frac{4}{3}}\right)_{\text{therm}}^{\frac{3}{4}}$$

- Hydrodynamic attractor relates $\left(e\tau^{4/3}\right)_{\text{therm}}$ to energy at early times

$$(s\tau)_{\text{therm}} = \frac{4}{3} C_\infty^{3/4} \left(4\pi\frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3} (e\tau)_0^{2/3}.$$

- Pocket formula for particle production from energy deposition

Giacalone, AM, Schlichting, (2019) [26]

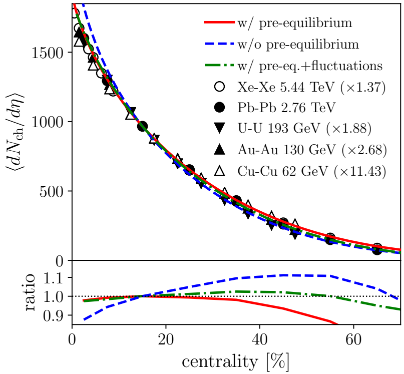
$$\underbrace{\left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle}_{\text{final-state}} \approx A_\perp \; \frac{N_{\text{ch}}}{S} \underbrace{\frac{4}{3} C_\infty^{3/4} \left(4\pi\frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3}}_{\text{medium properties}} \underbrace{\left(\frac{1}{A_\perp} \left\langle \frac{dE_\perp}{d\eta} \right\rangle_0 \right)^{2/3}}_{\text{glasma initial-state}}$$

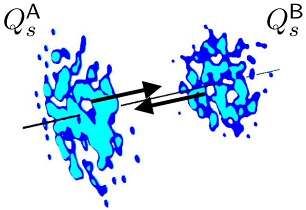
All relevant-prefactors and powers included!

Universal centrality dependence of particle multiplicity

Three predictors for particle multiplicity:

$$\left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle \propto \underbrace{\frac{dS_{\text{therm}}}{d\eta}}_{\text{equilibration}}, \quad \underbrace{\frac{dN_{\text{gluons}}}{d\eta}}_{\text{no equilibration}}, \quad \underbrace{\left\langle \frac{dS_{\text{therm}}}{d\eta} \right\rangle}_{\text{e-by-e fluctuations}}.$$





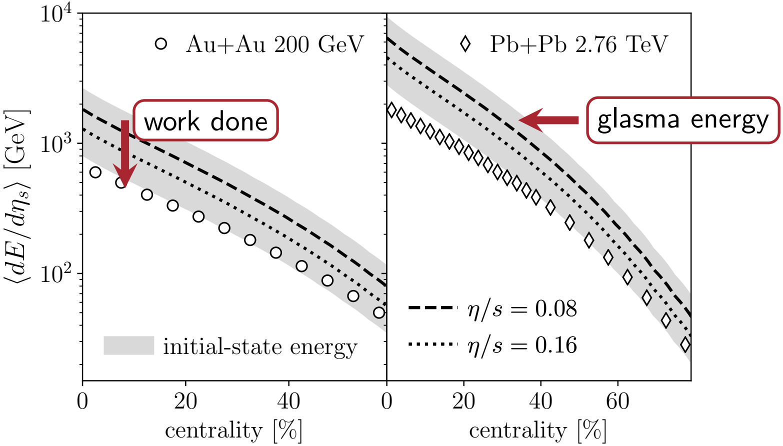
glasma energy density

$$(e\tau)_0 \propto Q_s^{2<}(\mathbf{x}_\perp)\sqrt{Q_s^{2>}(\mathbf{x}_\perp)}$$

See recent work by Jakub, Kamata, Martinez, Spaliński (2020) [27] for related early results in AdS/CFT by van der Schee (2014) [28]

Centrality dependence of initial state energy

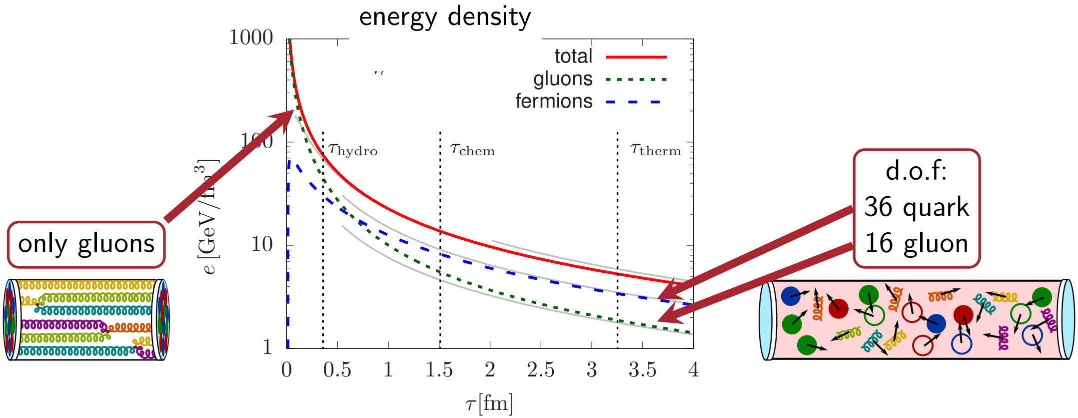
Bands are variations of $C_\infty = [0.8\text{-}1.15]$, $\eta/s = [0.08\text{-}0.24]$



Inferring initial state energy densities from measured multiplicity.

Fermion production in QCD kinetic theory

Fermions are produced through fusion $gg \to q\bar{q}$ and splitting $g \to q\bar{q}$.



only gluons

d.o.f:
36 quark
16 gluon

Kurkela, AM (2018) [29, 30]

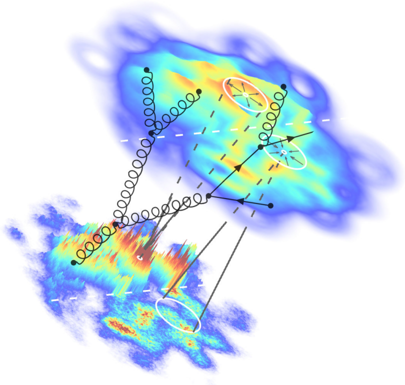
We found the timescale of chemical equilibration in the Quark Gluon Plasma.

Recent extensions to finite baryon chemical potential (see Du's talk) Du and Schlichting (2020) [31, 32]

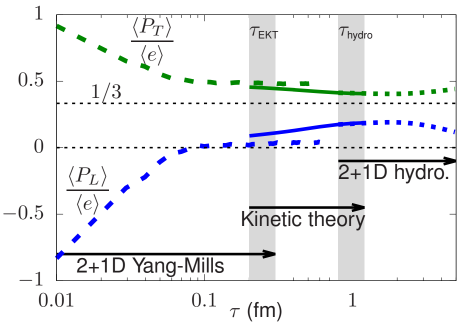
Transverse pre-equilibrium evolution

KøMPøST— event-by-event kinetic pre-equilibrium for heavy ion collisions.

$$\underbrace{\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}},\mathbf{x}')}_{\text{goes into hydro}} = \int d^2\mathbf{x}'\ \underbrace{G^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}-\mathbf{x}',\tau_{\text{hydro}},\tau_{\text{EKT}}\right)}_{\text{linear response function}}\ \underbrace{\delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}},\mathbf{x}')}_{\text{initial conditions}}.$$



`https://github.com/KMPST/KoMPoST` [33]



Kurkela, AM, Paquet, Schlichting and Teaney (2018)[8, 9]

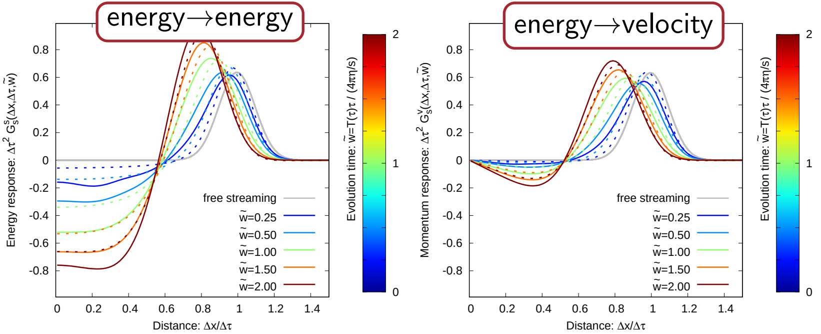
Kinetic theory response functions

All components of energy-momentum tensor generated by kinetic response

Kurkela, AM, Paquet, Schlichting and Teaney (2018) [9]

$$G^{\mu\nu}\left(\tau,\tau_0,|\mathbf{x}-\mathbf{x}_0|,e(\tau_0),\lambda\right)\Rightarrow G^{\mu\nu,\mathrm{univ}}\left(\tilde{w},\Delta x/\Delta\tau\right)$$

Comparison between Yang-Mills kinetic theory (dotted) and RTA (solid)



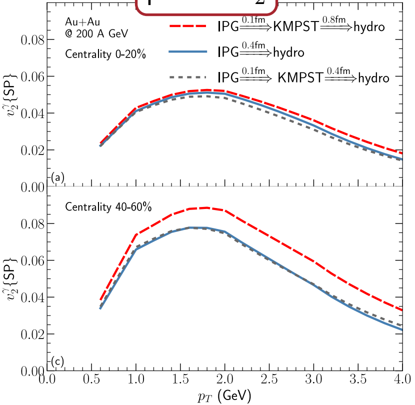
Kamata, Martinez, Praschke, Ochsenfeld, Schlichting (2020) [34]

Kinetic response functions evolve from free-streaming to hydrodynamic-like.

KøMPøST in action

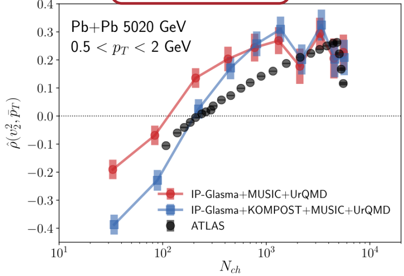
Effects of pre-equilibrium evolution on different observables

photon v_2



Gale, Paquet, Schenke, Shen (2021) [35]

$p_T - v_2$ corr.



Schenke, Shen, Teaney (2020) [36]

Noticeable pre-equilibrium effects even in large systems.

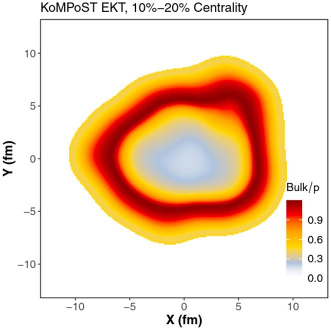
KøMPøST improves causality in hydro, see Plumberg, Almaalol, Dore, Noronha, Noronha-Hostler (2021) [37]

Towards complete pre-equilibrium evolution in QCD kinetic theory

- Non-conformal pre-equilibrium $\Longrightarrow$ *better matching to hydro*
- Chemical equilibration $\Longrightarrow$ *evolution of charges and photon emission*
- Real 2D/3D+1 space-time evolution $\Longrightarrow$ *crucial for small systems*

for first steps see Törnkvist's talk

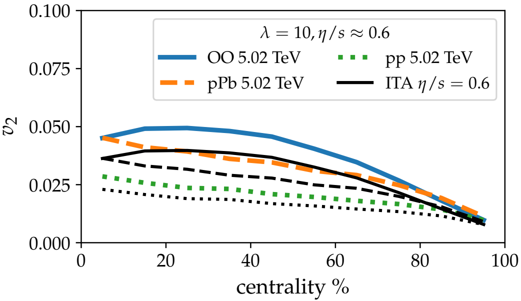
conformality violations



Nunes da Silva, Chinellato, Hippert, Serenone,

Takahashi, Denicol, Luzum, Noronha (2020) [38]

flow in 2D+1 single hit kinetic theory



Kurkela, AM, Törnkvist (2021) [39]

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