Transport near the chiral critical point

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Based on: 2005.02885, 2101.10847

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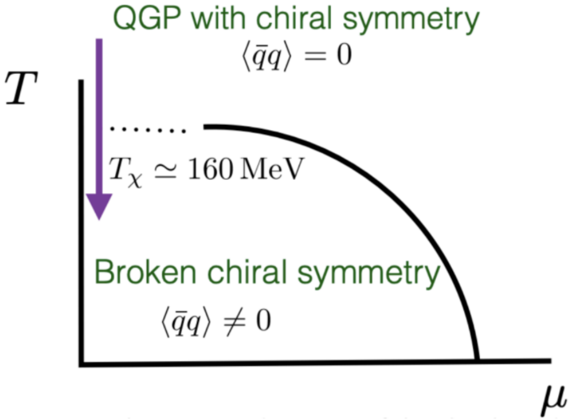
Stony Brook University

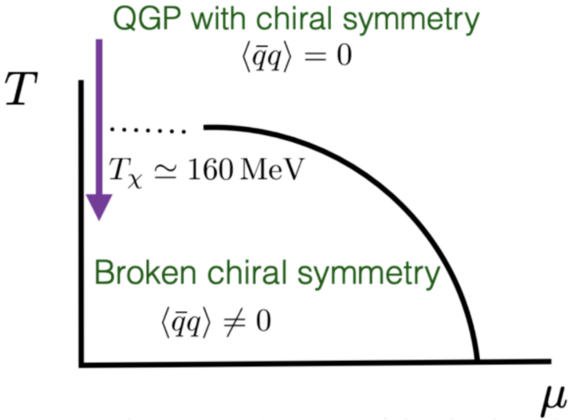
FWF Der Wissenschaftsfonds.

Physical motivation



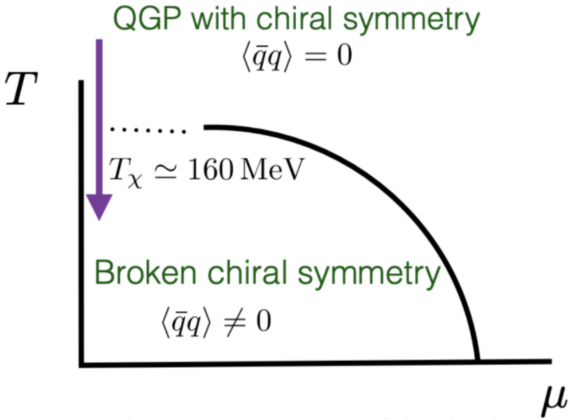
Approximate chiral symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

Physical motivation



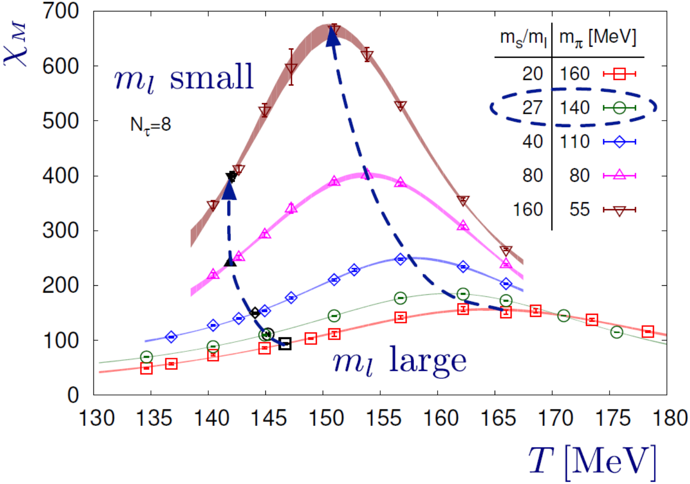
Approximate chiral symmetry $SU(2)_L \times SU(2)_R \sim O(4)$
Lattice points to transition in $O(4)$ universality class (1903.04801)

Physical motivation



What role does chiral symmetry play in heavy ion collisions?

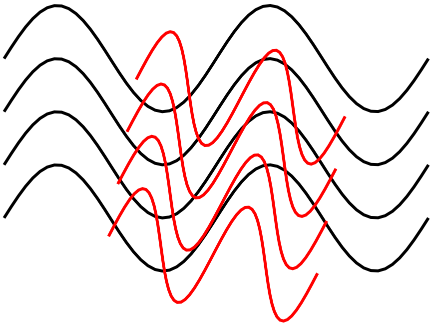
$O(4)$ scaling as seen on the lattice



Chiral susceptibility: $\chi_M \propto \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial m_l} \sim m_l^{1/\delta - 1} f_\chi(z)$, $z \equiv t m_l^{-1/\beta\delta}$

Plot from HotQCD collaboration: arXiv:1903.04801

Hydrodynamics in the chiral limit[1]



equilibrated hydro modes, $k \ll m_\pi$
superfluid modes, $k \sim m_\pi$

At long distances, effective theory of QCD is hydrodynamics
At finite quark mass, theory should be superfluid-like for $L \sim m_\pi^{-1}$

[1]Son hep-ph/9912267,Son and Stephanov hep-ph/020422

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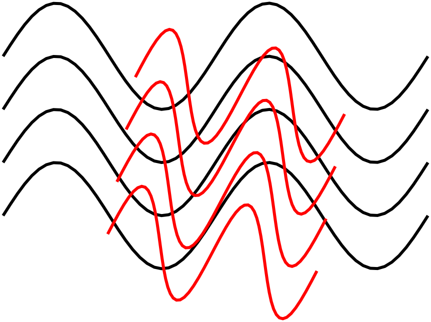

equilibrated hydro modes, $k \ll m_\pi$

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Question: How do these modes contribute to hydrodynamic variables and transport?

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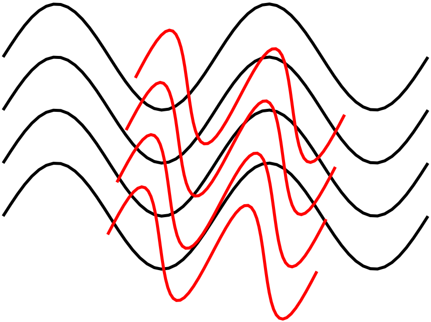

equilibrated hydro modes, $k \ll m_\pi$

superfluid modes, $k \sim m_\pi$

Hydrodynamic variables get correction due to pions, e.g.

$$\eta = \eta_{\rm hydro} + \Delta\eta$$

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Setup at a glance[2]

- Start from generating functional

$$W[g_{\mu\nu}, A_\mu, H] = \int_x (p(T) + S_{O(4)})$$

$$S_{O(4)} = \frac{\chi_0}{4}\mu_{ab}^2 - \frac{1}{2}\Delta^{\mu\nu} D_\mu \phi_a D_\nu \phi_a - V(\phi)$$

μ_{ab} is the $O(4)$ chemical potential (e.g. $\mu_{0i} = \mu_A$)

$O(4)$ vector: $\phi_a = (\sigma, -\pi_i)$

$V(\phi_\alpha \phi_\alpha) = m_0^2(t)\phi^2 + \lambda\phi^4 - H\phi$

$m_0^2(t) \sim (T - T_c)$

[2] Rajagopal/Wilczek arXiv:9210253, Son/Stephanov arXiv:0204226, Jensen et al 1203.3556

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- Compute ideal equations of motion (Josephson constraint from entropy conservation $u \cdot \partial\phi_a + \mu_{ab}\phi_b = 0$)

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- Compute ideal equations of motion
- Add dissipation in standard way:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu}$$

$$J^\mu = J^\mu_{\rm ideal} + q^\mu$$

$$u \cdot \partial\phi_a + \mu_{ab}\phi_b = \Xi_a$$

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  $\Rightarrow$ Compute change in transport coefficients

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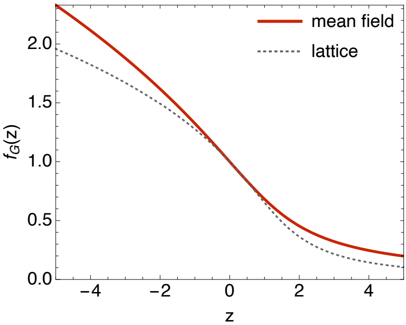
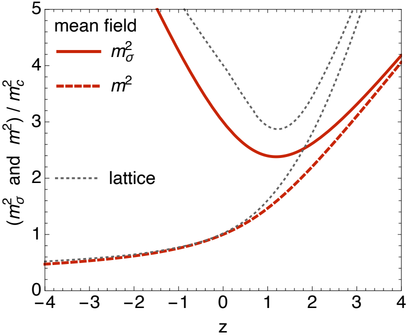
Mean field compared to lattice[3]

$$0 = \frac{dV}{d\phi} = m_0^2(t)\,\bar{\sigma} + \frac{\lambda}{3!}\bar{\sigma}^3 - H.$$



O(4) scaling function $f_G \propto \bar{\sigma}$



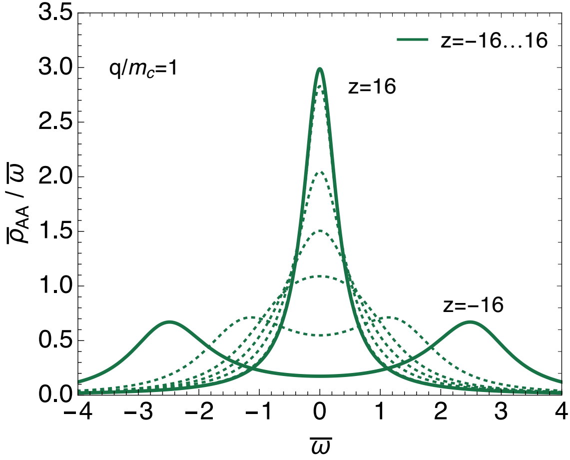
Pion and sigma masses

Note: $z \propto \frac{T - T_c}{T_c}$

[3]Engels Vogt arXiv:0911.1939, Engels Karsch arXiv:1105.0584

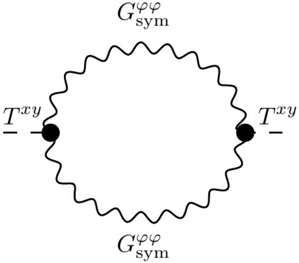
Spectral density for axial charge density-density correlator

$(\chi_0 \omega_k)^{-2} G^{\varphi\varphi}_{sym} = \frac{T}{\omega} \rho_{AA}$



For $z \gg 0$, $\rho_{AA}/\omega \propto Dk^2/(\omega^2 + (Dk^2)^2)$, diffusion of quarks.
For $z \ll 0$, pair of peaks interpret as propagating pions
$\Rightarrow$ Transition from QGP to pion propagation from EOM!

Modification of shear viscosity



$$2T\eta = \int d^4x \left\langle \tfrac{1}{2}\{T^{xy}(t,\boldsymbol{x}), T^{xy}(0,\mathbf{0})\}\right\rangle$$

In linearized regime, relevant components for shear viscosity:

$$\Delta T^{xy} = \underbrace{\partial^x \delta\sigma \partial^y \delta\sigma}_{\text{condensate contribution}} + \underbrace{\bar{\sigma}^2 \partial^x \varphi_a \partial^y \varphi_a}_{\text{pion contribution}}$$

Break up computation into two pieces:

$$\Delta\eta \equiv I^{xy}_{\sigma\sigma} + I^{xy}_{\varphi\varphi}$$

Modification of shear viscosity

Break up computation into two pieces:

$$\Delta\eta \equiv I^{xy}_{\sigma\sigma} + I^{xy}_{\varphi\varphi}$$

Sigma correlator:

$$G^{\sigma\sigma}_{sym}(\omega, k) = 2\frac{T}{\omega}\mathrm{Im}\, G^{\sigma\sigma}_R(\omega, k) = \frac{2T\Gamma}{\omega^2 + \Gamma^2(k^2 + m_\sigma^2)^2}$$

which leads to

$$I^{xy}_{\sigma\sigma} = 2\int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{(2\pi)}(k^x k^y G^{\sigma\sigma}_{sym})^2 = \frac{2T^2}{(30\pi^2\Gamma)}\int_0^\Lambda \frac{k^6 dk}{(k^2 + m_\sigma^2)^3}$$
$$= \underbrace{\frac{2T^2}{30\pi^2\Gamma}\Lambda}_{\text{divergent}} - \underbrace{\frac{2T^2 m_\sigma}{32\pi\Gamma}}_{\text{finite piece}} + \dots$$

Modification of shear

Combining the ingredients, we find

$$\eta = \eta_{\Sigma}^{\rm phys} \underbrace{- \frac{T}{32\pi\Gamma}(md_A + m_\sigma + md_A u^2(1-r^2)f(r,u))}_{\Delta\eta},$$

where the physical part captures the cut-off dependent piece

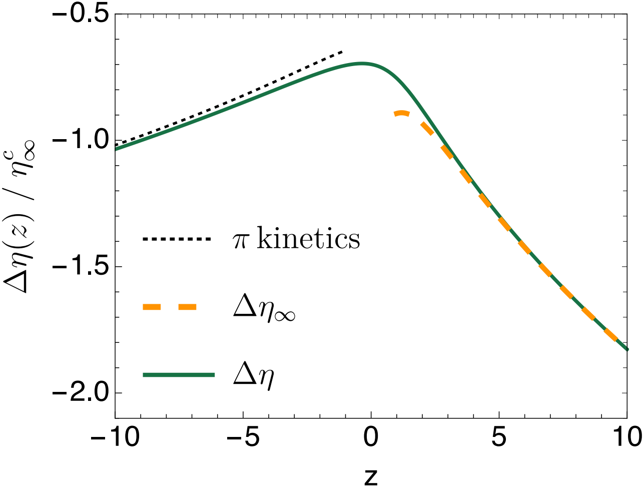
$$\eta_{\Sigma}^{\rm phys} = \eta_{\Sigma}(\Lambda) + \delta_{\alpha\alpha}\frac{T\Lambda}{30\pi^2\Gamma},$$

where

$$\begin{aligned} f(r,u) &= \frac{32\pi}{30\pi^2}\int_0^\infty \frac{dk}{m}\frac{k^6}{(k^2+m^2)^3}\frac{m^2k^2}{(k^2+r^2m^2)(k^2+u^2m^2)}, \\ &= \frac{8u^4+24u^3+48u^2+45u+15}{15(u+1)^3(u^2-r^2)} + (r \leftrightarrow u). \end{aligned}$$

Results: change in shear viscosity

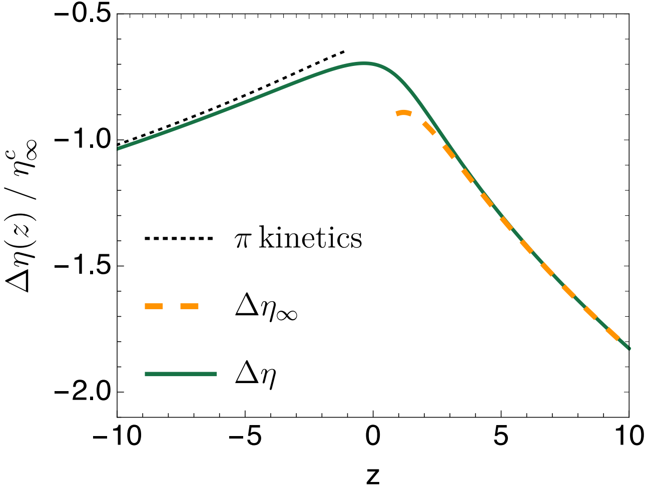
Note: $z \propto \frac{T - T_c}{T_c}$



Large z simple form $\Delta\eta_\infty = -\frac{T m_\sigma}{8\pi\Gamma}$

Results: change in shear viscosity

Note: $z \propto \frac{T - T_c}{T_c}$



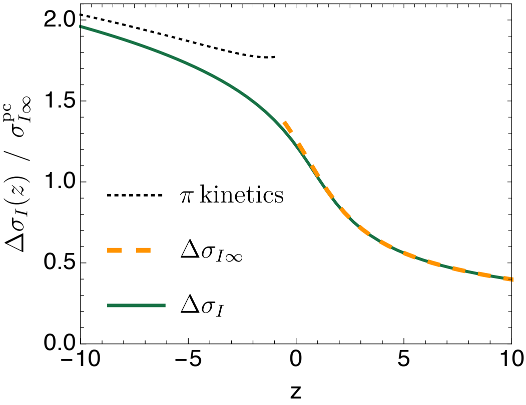
At $T_{pc} \simeq 155$ MeV, $\frac{\eta_{pc}^{\infty}}{sT} = \frac{0.3}{4\pi T}\left[\left(\frac{1.5}{\pi T \Gamma}\right)\left(\frac{5.4}{sT^{-3}}\right)\left(\frac{m_\sigma / T}{1.56}\right)\right]$

Results: change in isovector conductivity

Similar procedure for conductivity

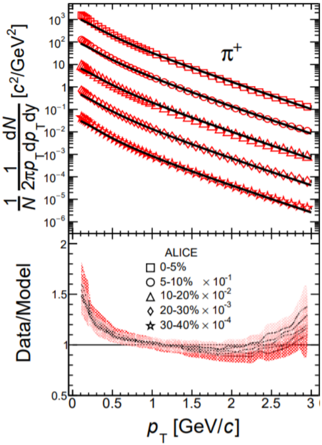
$$2T\sigma_I = \int d^4x \frac{1}{d_A} \left\langle \tfrac{1}{2}\{J^x_{V,a}(t,\boldsymbol{x}), J^x_{V,a}(0,\boldsymbol{0})\}\right\rangle ,$$

$$J^x_{V,a} = \sigma_0^2 f_{abc}\varphi_b \partial^x \varphi_c$$



At $T_{pc} \simeq 155$ MeV, $\frac{\sigma^{\infty}_{I,pc}}{\chi_Q} = \frac{0.5}{2\pi T}\left[\left(\frac{1.5}{\pi T\Gamma}\right)\left(\frac{1.27}{mT^{-1}}\right)\left(\frac{0.4}{\chi T^{-2}}\right)\right]$

Outlook: soft pions in experiment?



Plot from: D. Devetak *et al*, arXiv:1909.10485

Soft pions are harder to fit to hydrodynamic models

Estimating soft pion yield enhancement near critical point

Want to compare soft pion dispersion: $E_p^2 = v^2(p)p^2 + m^2(p)$
to vacuum dispersion: $E_{\mathrm{vac}}^2 = p^2 + m_\pi^2$

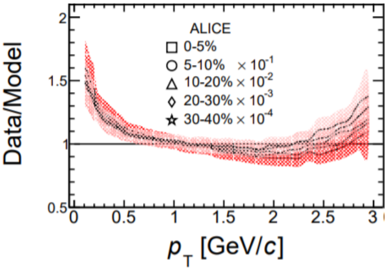
Quick model:

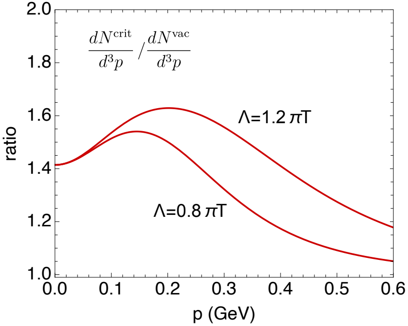
- Pion velocity to interpolate between $v_0^2 \approx 0.25$ for low momenta and $c = 1$ for high momenta
- With cut-off at $p \sim \Lambda$
- Similar interpolating form for $m^2(p)$

Compare to vacuum by computing ratio:

$$\text{ratio} = \frac{n(E_p)}{n(E_{\mathrm{vac}})} \approx \frac{E_{\mathrm{vac}}}{E_p}$$

Estimating soft pion yield enhancement near critical point





- Promising, enhancement is where it should be!
- Needs more robust treatment, e.g. constraining dispersion via lattice, second order hydro, including resonance decays, etc.
- Future: upgrade to Inner Tracking System at ALICE allows to see more low p_T particles, especially pions...