Imaginary part of the effective potential and momentum diffusion of heavy quarks from real-time Yang-Mills dynamics

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arxiv ePrint: 2102.12587

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What's this talk about?

- **In-medium Heavy quarkonia → imaginary part of the potential**
- **Considering the classical regime (effectively) of Yang-Mills theory/ static limit of the potential**
- **Perturbative and nonperturbative methods**
- **Larger lattices/ smaller spacing/ various temperatures (or β)**
- **Compact functional forms/ Relation to** κ

In-medium Heavy quarks

$$V_{\mathrm{KMS}}(r,T) = \left(\frac{1-e^{-m_D r}}{m_D r}\right)\sigma r - \frac{\alpha}{r}e^{-m_D r}$$

Karsch,Mehr, and Satz : Z. Phys. C37, 617 (1988)

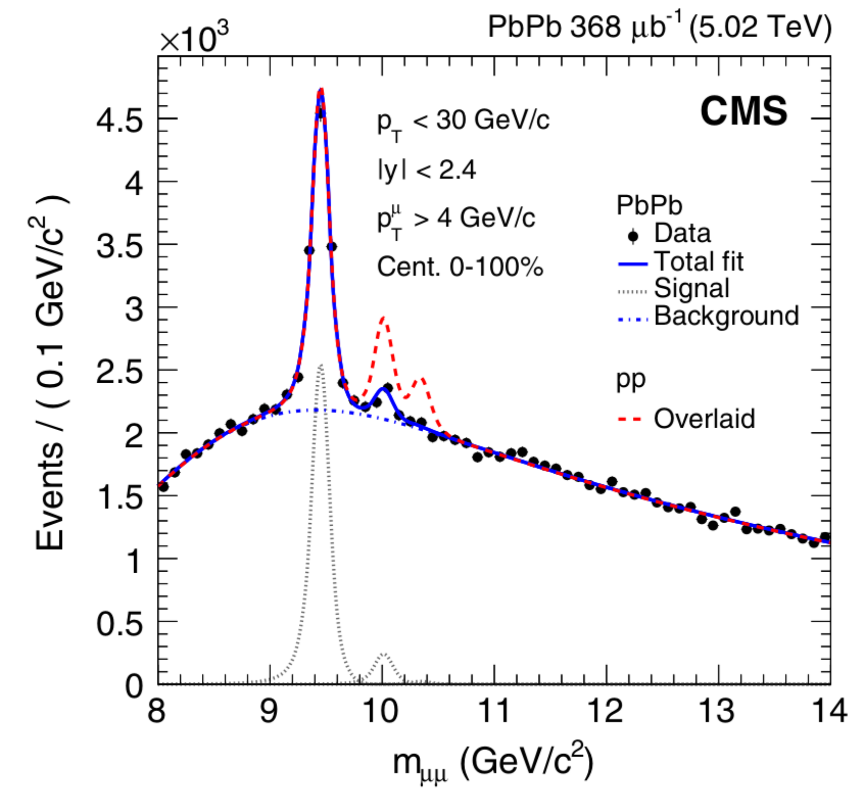
SEWM Talks by:

Peuron (Today)

Larsen (Tuesday)

Leino (Tuesday)

Vander Griend (Thursday)



Plot from: **Phys. Rev. Lett. 120, 142301 (2018)**

Heavy quarks Im[*V*]

Thermal medium → finite width of the quarkonium peak in the electromagnetic production rate. [**Laine et al JHEP 03 (2007) 054 (hep-ph/0611300)**]

Melting of quarkonium peaks

Similar studies, phenomenological applications:
Phys.Rev.D79:114003, (2009)
Phys. Lett. B., 811, 135949 (2020)
And for many other references: check arxiv: 2102.12587

Stochastic Hamiltonian

$$H(\mathbf{r},t) = -\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) + \tilde{\nu}\left(\mathbf{R}+\frac{\mathbf{r}}{2},t\right) - \tilde{\nu}\left(\mathbf{R}-\frac{\mathbf{r}}{2},t\right)$$

$$\langle\tilde{\nu}(\mathbf{r},t)\rangle = 0, \qquad \langle\tilde{\nu}(\mathbf{r},t)\tilde{\nu}(\mathbf{r}',t')\rangle = D(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

$$i\frac{d}{dt}\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\rangle = \left(-\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) - i\{D(\mathbf{0}) - D((\mathbf{r}))\}\right)\langle\psi_{Q\bar{Q}}(\mathbf{r},t)\rangle$$

Kajimoto et al (2018): Phys. Rev. D97, 014003, Rothkopf (2020) : Phys. Rept. 858, 1-117

Im[V]: 2nd order pt (HTL, HCL)

HTL result: [2nd order pt with dimensional regularization]

$$\mathrm{Im}[V^{(2)}(r)] = -\frac{C_F g^2 T}{4\pi}\phi(m_{D,\mathrm{HTL}} r)$$

Laine et al: JHEP 03 (2007) 054

$$\phi(x) \equiv 2\int_0^\infty \mathrm{d}z \frac{z}{(1+z^2)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

$$m^2_{D,\mathrm{HTL}} = 4g^2 N_c \int \frac{d^3p}{(2\pi)^3}\frac{f_{\mathrm{BE}}(p)}{p} = \frac{N_c g^2 T}{3}$$

HCL result: [2nd order pt with cubic lattice regularization]

$(N \to \infty,\ t \to \infty)$

$$\mathrm{Im}[V^{(2)}_{\mathrm{cl}}(r)]/g^2T = -\frac{\pi C_F N_c^2}{\beta}\int_0^1 \mathrm{d}^3\mathbf{x}\frac{1-\cos(\pi x_3 r/a)}{(\tilde{\mathbf{x}}^2 + N_c^2\Sigma/\pi\beta)^2}\int_{-1}^1 \mathrm{d}^3\mathbf{y}\frac{\delta(\tilde{\mathbf{x}}\cdot\mathring{\mathbf{y}})}{(\tilde{\mathbf{y}}^2)^{1/2}}$$

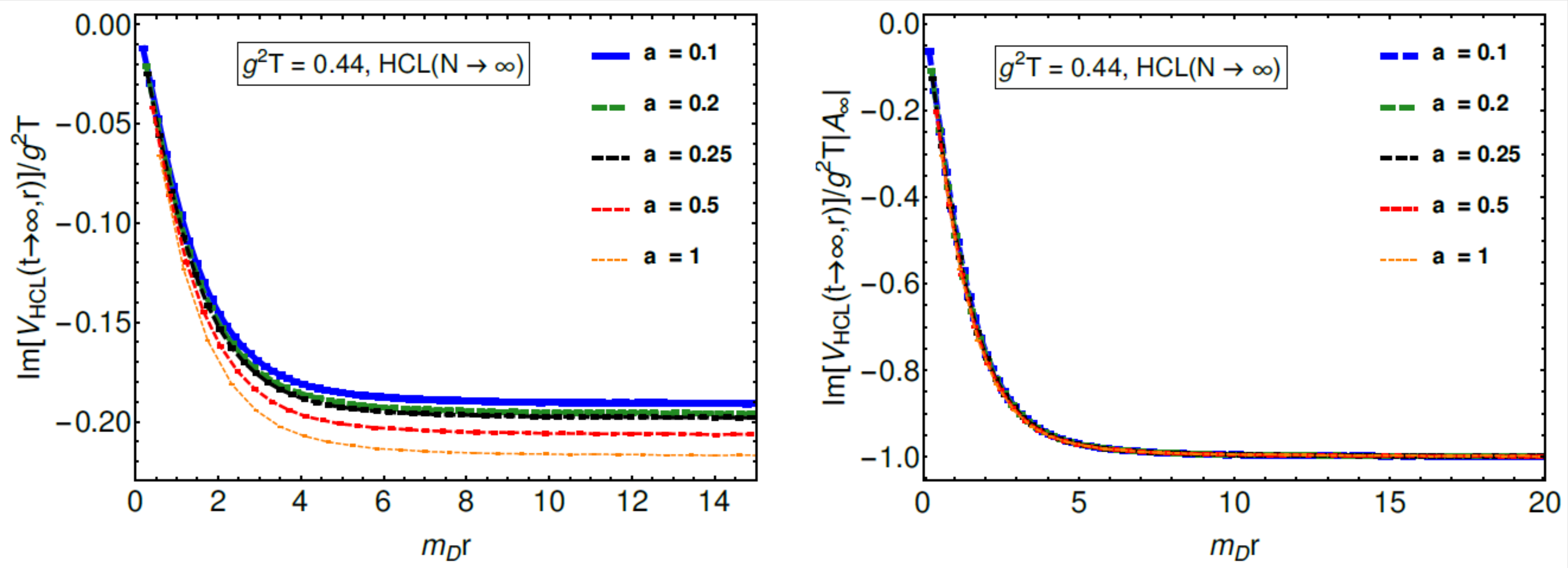
$$\tilde{x}_i \equiv 2\sin\left(\frac{\pi x_i}{2}\right), \qquad \mathring{x}_i \equiv \sin(\pi x_i), \qquad x_i \in (-1,1)$$

Laine, Philipsen, Tassler: JHEP 0709:066, (2007)

$$\beta = \frac{2N_c}{g^2 T a}$$

$$m^2_{D,\mathrm{HCL}} = \frac{g^2 T N_c}{2a}\left(\frac{3\Sigma}{2\pi} - 1\right)$$

HCL plots, *A*ϕ ansatz



$$\mathrm{Im}[V_{\mathrm{cl}}^{(2)}(r)] = g^2 T A_\infty \phi\left(B\, m_D r\right)$$

Classical-Statistical Lattice Yang-Mills

SU(3) gauge theory, YM action:

$$S[A] = -\frac{1}{2}\int d^4x \, \mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right)$$

In terms of link variables

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

Classical equations of motion:

$$U_j(t,\mathbf{x}) \approx \exp\left(igaA_j(t,\mathbf{x})\right)$$

$$U_j(t+a_t/2,\mathbf{x}) = \exp\left(ia_t agE^j(t,\mathbf{x})\right)U_j(t-a_t/2,\mathbf{x}),$$

$$gE^j(t+a_t,\mathbf{x}) = gE^j(t,\mathbf{x}) - \frac{a_t}{a^3}\sum_{j\neq i}\left[U_{ij}(t-a_t/2,\mathbf{x}) + U_{i(-j)}(t-a_t/2,\mathbf{x})\right]_{\mathrm{ah}}$$

Plaquette:

$$U_{ij}(\mathbf{x}) = U_i(\mathbf{x})U_j(\mathbf{x}+\mathbf{a}_i)U_i^\dagger(\mathbf{x}+\mathbf{a}_j)U_i^\dagger(\mathbf{x})$$

Grigoriev and Rubakov (1988): Nucl. Phys. B 299/ **Ambjorn and Krasnitz (1995)**: Phys. Lett. B 362 / **Bödeker, McLerran, and Smilga (1995)**: Phys. Rev. D 52, 4675/ **Arnold (1997)**: Phys. Rev. D 55, 7781/ **Iancu (1998)**: hep-ph/9809535/ **Moore and Rummukainen (2000)** Phys. Rev. D 61, 105008/ **Kurkela and Moore (2012)**: Phys. Rev. D 86, 056008

Setup

Solve the EOM with small time-steps: $a_t/a = 0.01$

Observables: $$\langle O\rangle(t) \equiv \frac{1}{n}\sum_{j=1}^{n} O_j(t)$$

Berges et al 2014: Phys. Rev. D 89 (2014) 114007 [1311.3005]

Boguslavski et al 2018: Phys. Rev. D 98 (2018) 014006 [1804.01966]

See also the talk by **Peuron (Today)**

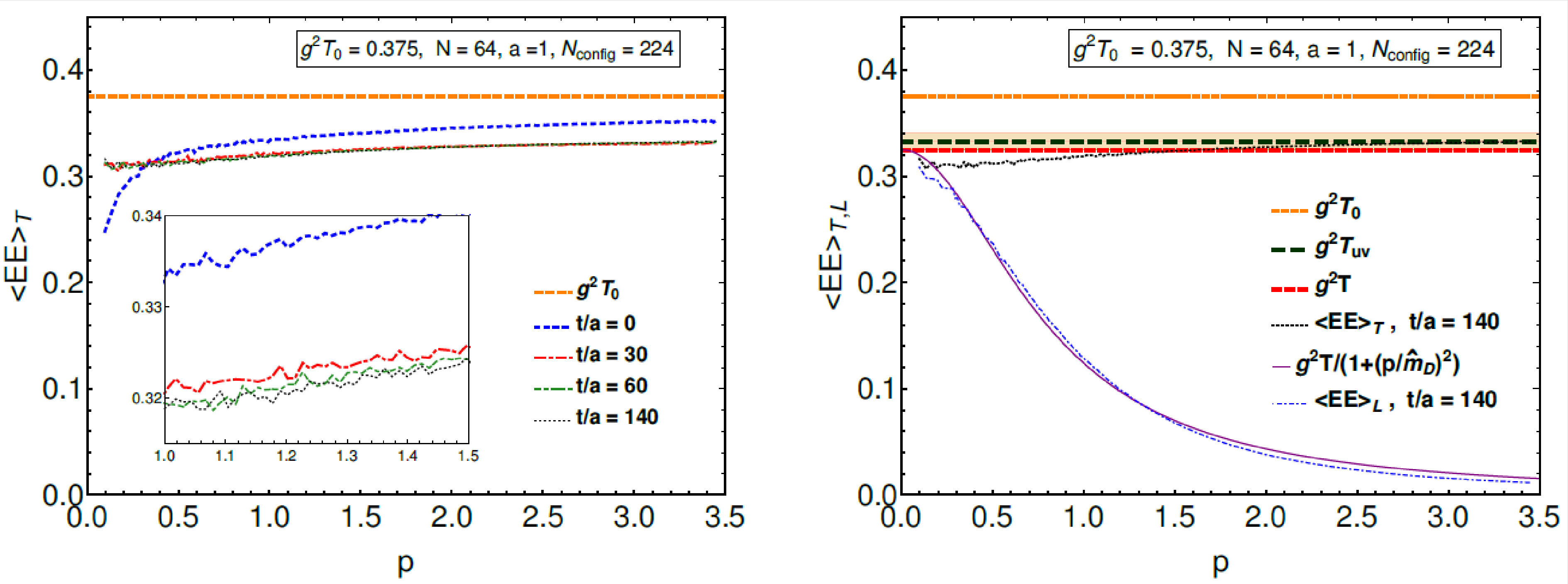
Initialize such that:

$$g^2\langle EE\rangle_T = g^2 T_0\,, \qquad g^2\langle EE\rangle_L = 0$$
$$g^2\langle AA\rangle_T = \frac{g^2 T_0}{p^2}\,, \qquad g^2\langle AA\rangle_L = 0$$

Temperature and Debye mass from chromo-electric field correlations:

$$g^2\langle EE\rangle_T \approx g^2 T\,, \qquad g^2\langle EE\rangle_L \approx g^2 T\,\frac{\hat{m}_D^2}{p^2+\hat{m}_D^2}$$

Thermalization, extraction of *T* and *mD*



$$g^2\langle EE\rangle_T \approx g^2 T\,, \qquad g^2\langle EE\rangle_L \approx g^2 T\,\frac{\hat{m}_D^2}{p^2+\hat{m}_D^2}$$

Im[V]: nonperturbative

Following **Laine et al 2007** [JHEP 03 (2007) 054 (hep-ph/0611300)]:

$$i\partial_t C_{\rm cl}(t,r) = V_{\rm cl}(t,r) C_{\rm cl}(t,r)$$

With classical Wilson loop:

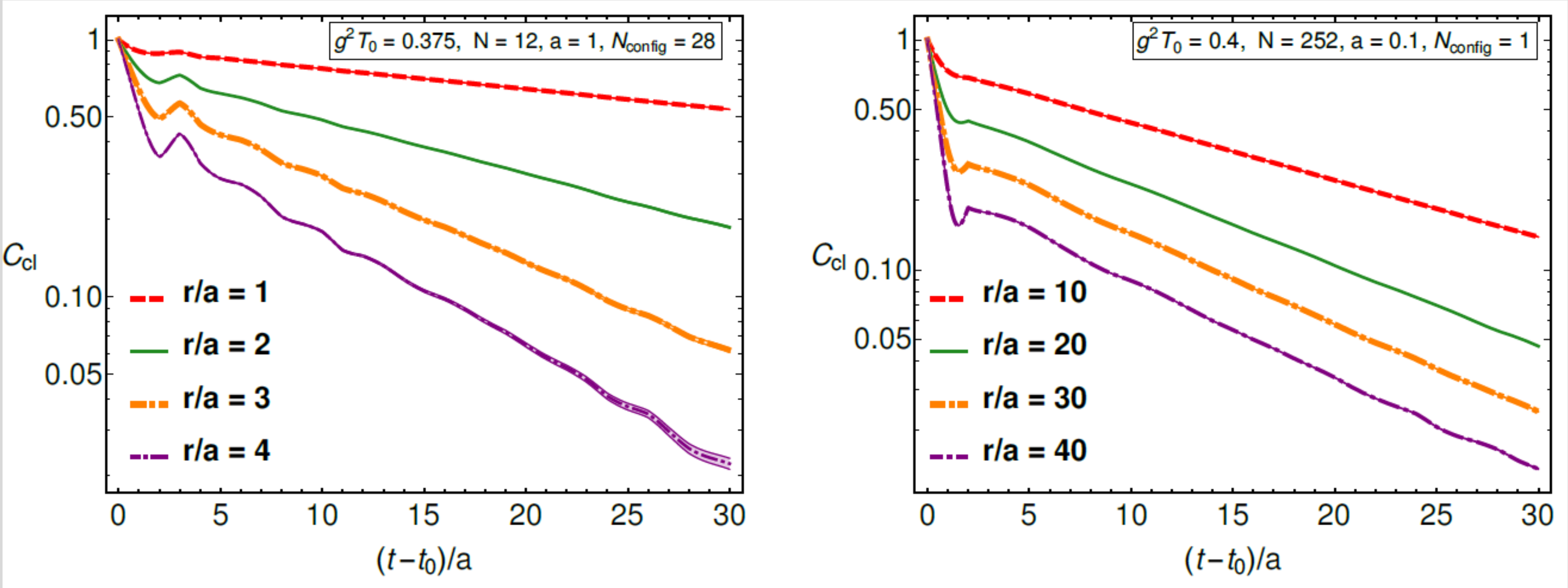
$$C_{\rm cl}(t,r) \equiv \frac{1}{N_c} \mathrm{Tr} \left\langle W[(t_0,\mathbf{x});(t,\mathbf{x})]\, W[(t,\mathbf{x});(t,\mathbf{0})]\, W[(t,\mathbf{0});(t_0,\mathbf{0})]\, W[(t_0,\mathbf{0});(t_0,\mathbf{x})] \right\rangle$$

Static limit:

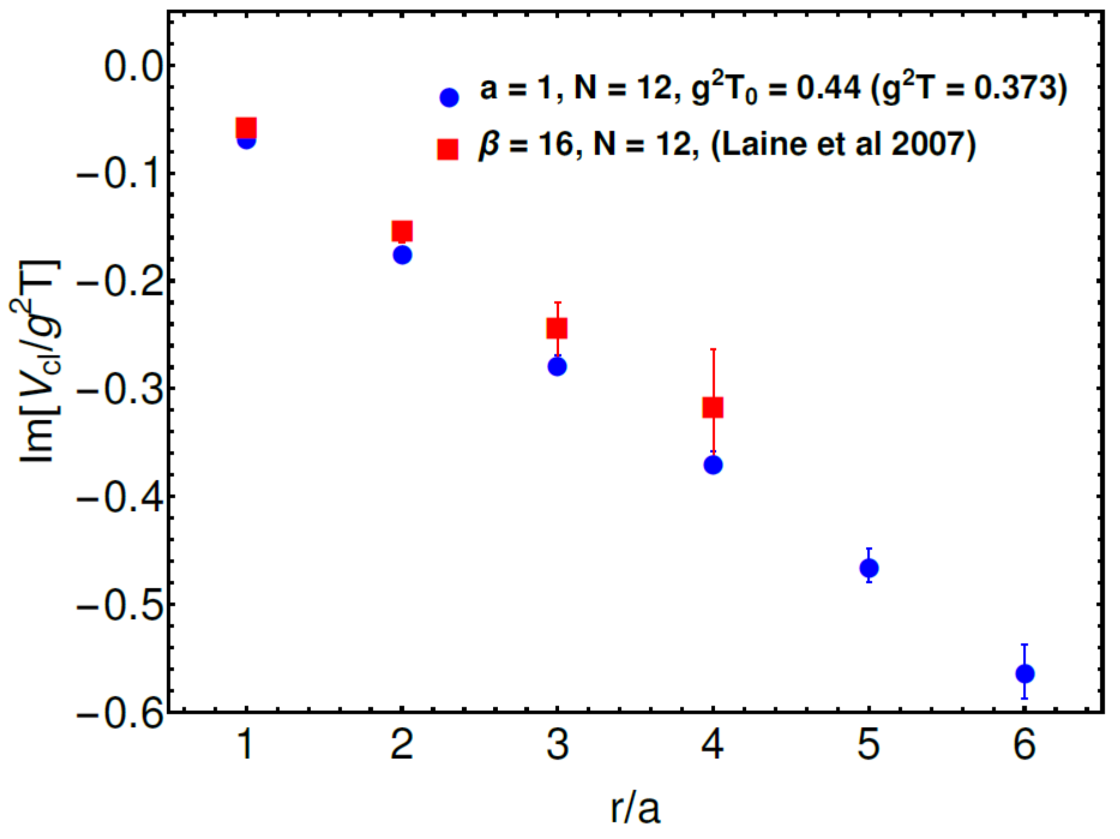
$$\mathrm{Im}[V_{\rm cl}(r)] \equiv \lim_{t\to\infty} \mathrm{Im}[V_{\rm cl}(t,r)]$$

See also: **Lehmann and Rothkopf [arXiv:2012.10089, (2020)]**

Evolution of Classical Wilson loop



Im[*V*]/*g*^2*T*, comparison with previous results

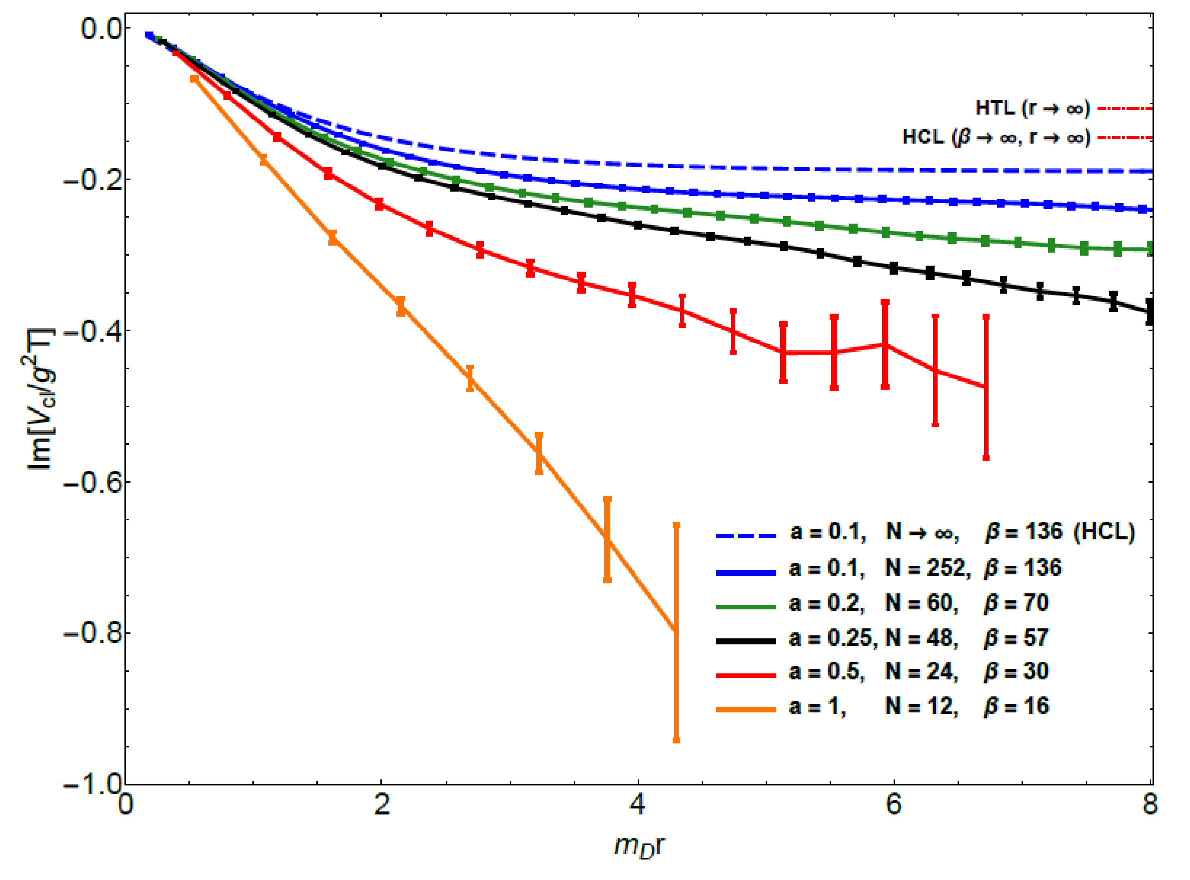


Compared to:

Laine, Philipsen, Tassler:

JHEP 0709:066, 2007

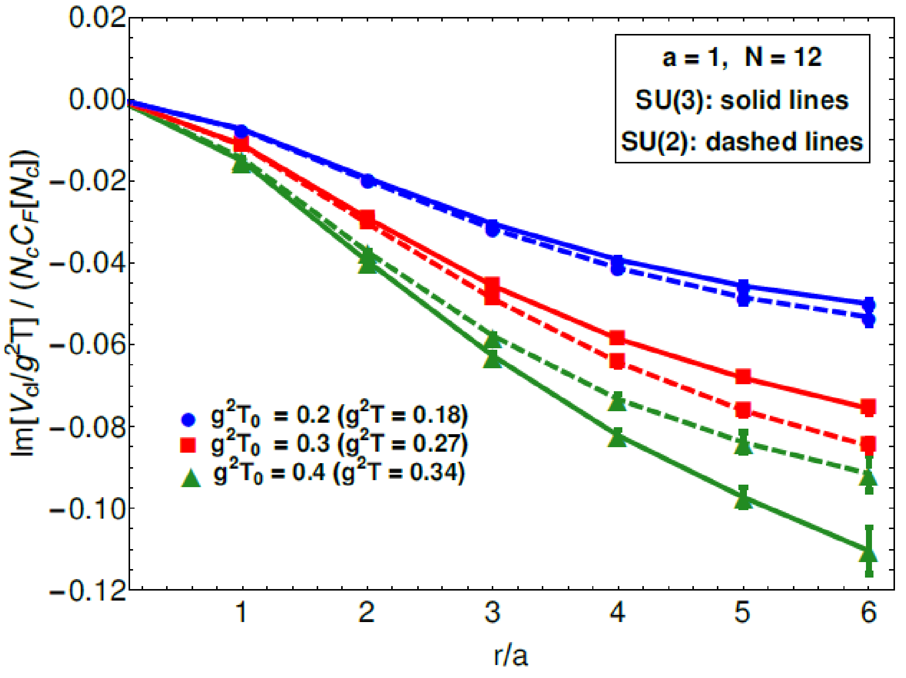
Im[V]: main results



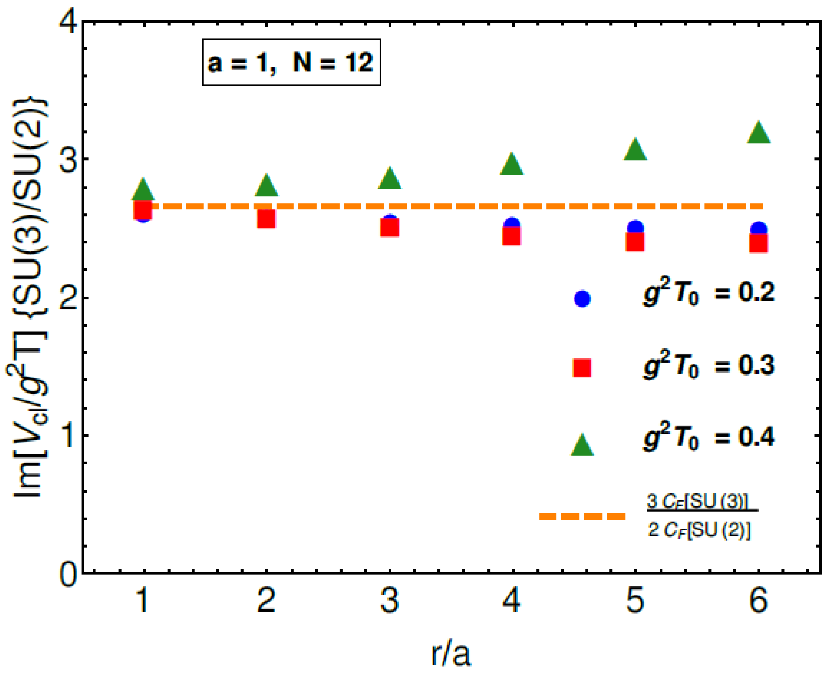
Classical divergence in Im[V] is moved into the Debye mass $m_D \sim 1/\sqrt{a}$

$$r_{\min} \ll 1/m_D \ll r_{\max}$$

SU(2) case



a = 1, N = 12
SU(3): solid lines
SU(2): dashed lines
g^2T_0 = 0.2 (g^2T = 0.18)
g^2T_0 = 0.3 (g^2T = 0.27)
g^2T_0 = 0.4 (g^2T = 0.34)
Im[V_{cl}/g^2T] / ($N_c C_F[N_c]$)
r/a



a = 1, N = 12
g^2T_0 = 0.2
g^2T_0 = 0.3
g^2T_0 = 0.4
$\frac{3\, C_F[SU(3)]}{2\, C_F[SU(2)]}$
Im[V_{cl}/g^2T] {SU(3)/SU(2)}
r/a

Momentum diffusion

Heavy-quark diffusion coefficient

$$\frac{d}{dt}\langle(\Delta p)^2\rangle = \kappa$$

Langevin equation for momentum of heavy-quarks of mass M in a medium of temperature T (M >> T)

$$\frac{dp_i}{dt} = -\eta_D\, p_i + \xi_i(t), \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$

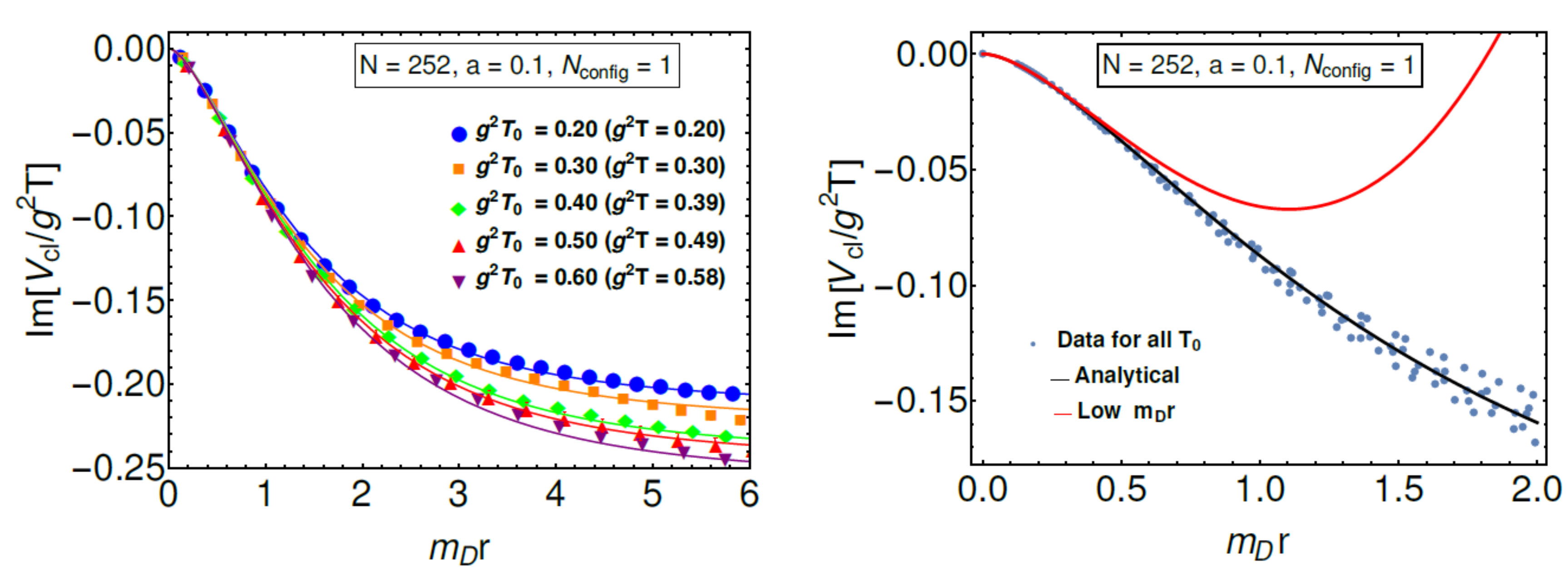
$$\kappa = \frac{1}{3}\int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t)\xi_i(0)\rangle$$

See for example: Moore and Teaney (2005): Phys. Rev. C71, 064904

Momentum diffusion from Im[*V*]

Low distance: $\mathrm{Im}[V_{\mathrm{cl}}^{(2)}(r)] \simeq -\frac{1}{2}\kappa r^2$

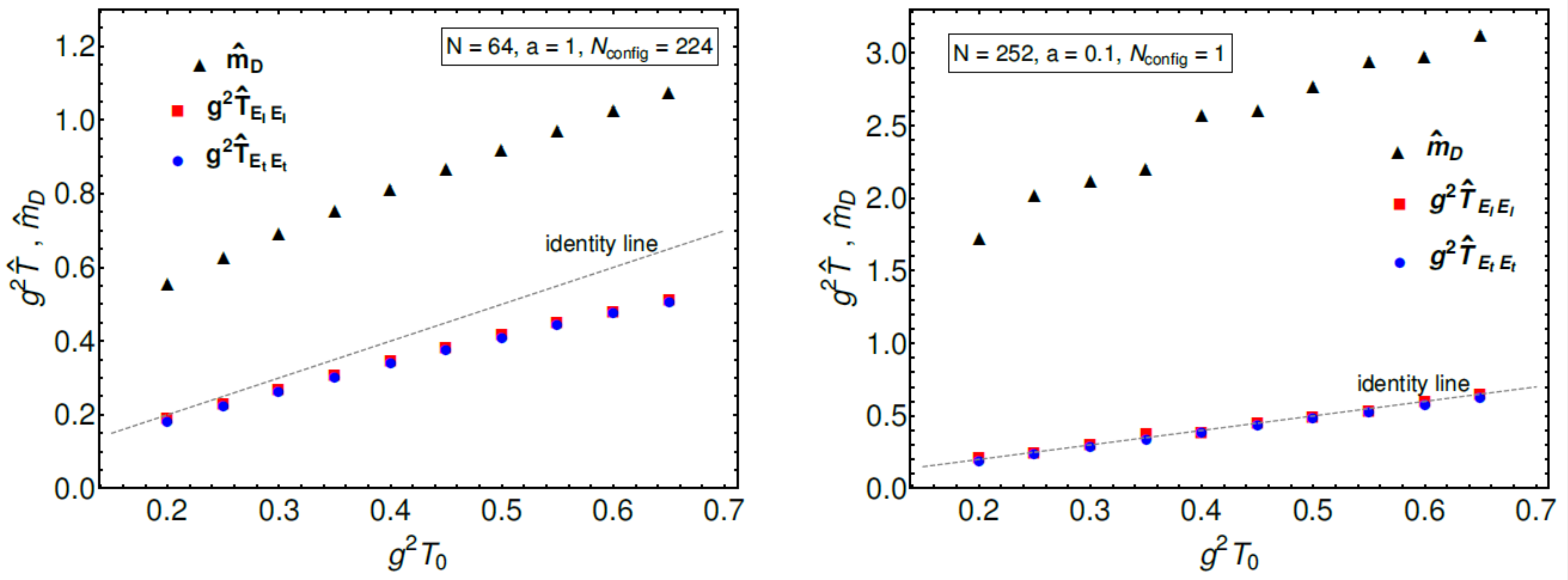
$$\kappa_V^{\mathrm{latt}} \approx C_F\, g^2 T \hat{m}_D^2 \left(0.086 \log\left(\frac{T}{\hat{m}_D}\right) + 0.018\right)$$



Summary and outlook

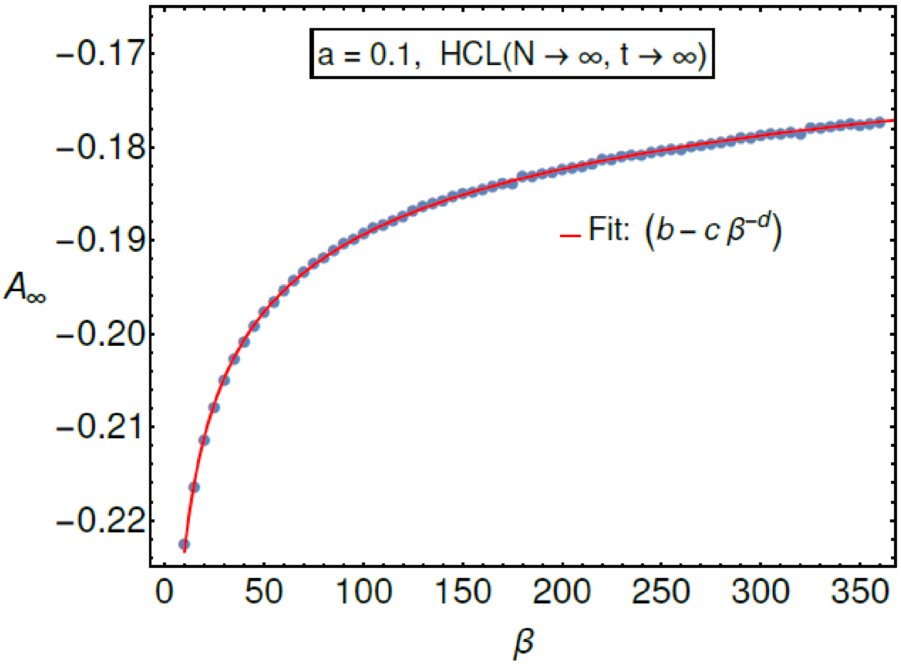
- **Nonperturbative Im[*V*], larger *N*, smaller *a***
- ***A* ϕ ansatz**
- **κ extraction**
- **Outlook: expanding and anisotropic cases**

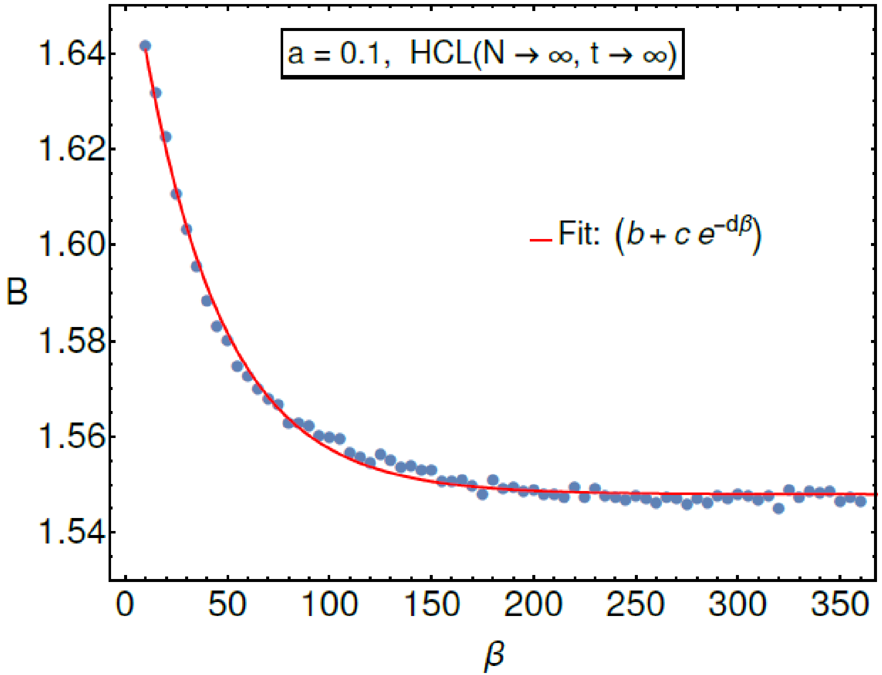
Backup: *T(T0)*



N = 64, a = 1, N_{config} = 224
$\hat{m}_D$
$g^2\hat{T}_{E_l E_l}$
$g^2\hat{T}_{E_t E_t}$
identity line
$g^2\hat{T}$, $\hat{m}_D$
$g^2 T_0$
N = 252, a = 0.1, N_{config} = 1
$\hat{m}_D$
$g^2\hat{T}_{E_l E_l}$
$g^2\hat{T}_{E_t E_t}$
identity line
$g^2\hat{T}$, $\hat{m}_D$
$g^2 T_0$

Backup: HCL large-β





$$A_{\infty}^{\mathrm{HCL}}(\beta) \approx -0.144 - 0.139\,\beta^{-0.24}$$
$$B^{\mathrm{HCL}}(\beta) \approx 1.548 + 0.12\,e^{-0.025\beta}\,,$$

Backup: Low distance Im[*V*]

$$\mathrm{Im}[V_{\mathrm{cl}}(r)] \simeq -r^2\,\frac{1}{9}\,|A_\infty|B^2\,g^2T\,m_D^2\,(4-3\gamma-3\log(Bm_Dr))$$

$$\simeq -\frac{1}{2}\,r^2\,\left[\frac{2}{9}\,|A_\infty|B^2\,g^2Tm_D^2\left(3\log\left(\frac{T}{m_D}\right)+4-3\gamma-3\log\left(\frac{2B}{D}\right)-3\log\left(\frac{DrT}{2}\right)\right)\right]$$

$D \sim \mathcal{O}(1)$

$r \approx 2/T$

$$\Rightarrow \quad \mathrm{Im}[V_{\mathrm{cl}}^{(2)}(r)] \simeq -\frac{1}{2}\,\kappa_V\,r^2$$