Gibbs entropy from entanglement in electric quenches

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with:

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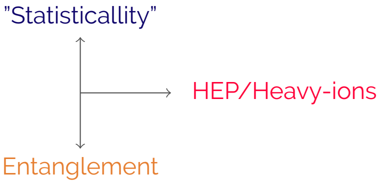
arXiv: 2106.00838

SEWM21, 28th of June 2021

Goal



Goal

"Statisticallity"

↕

Entanglement

→ HEP/Heavy-ions

Background

EPR pairs $|00\rangle + |11\rangle$

Information scrambling, chaos

Black hole physics

Quantum computing

$\dots$

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Model

$1+1$ QED, $e^2 \to 0$:

$$S = \int \mathrm{d}^2x\, \hat{\bar{\psi}}\, (i\gamma^\mu \partial_\mu + A_1(t)\gamma^1 - m)\, \hat{\psi}$$

$A(t)$ homogeneous *background*

$\hat{\psi} = \begin{pmatrix} \hat{\psi}_1 \\ \hat{\psi}_2 \end{pmatrix}$ dynamical fermion

$\to$ "Schwinger" pair creation

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Riemann ζ

C_{2n}: moments of multiplicity distribution

Like [Klich, Levitov, 2008] in quantum shot noise!

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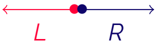

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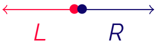

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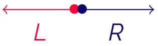

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$$H_{t*} = \int \mathrm{d}x_1 \hat{\bar{\psi}} \left(-\gamma^1 \left(i\partial_1 + A_1(t^*)\right) + m\right) \hat{\psi}$$

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Different vacuum at every t^*

Particle creation:

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See also [Z. Ebadi and B. Mirza, 2014]

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Microscopic origin

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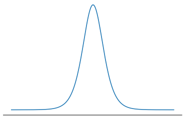

- Asymptot. α, β well-known
- Can also compute time-dep.
- $\gamma = \frac{\mathcal{E}\tau}{m}, m\tau$

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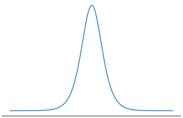

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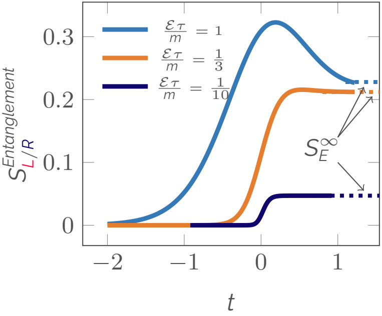
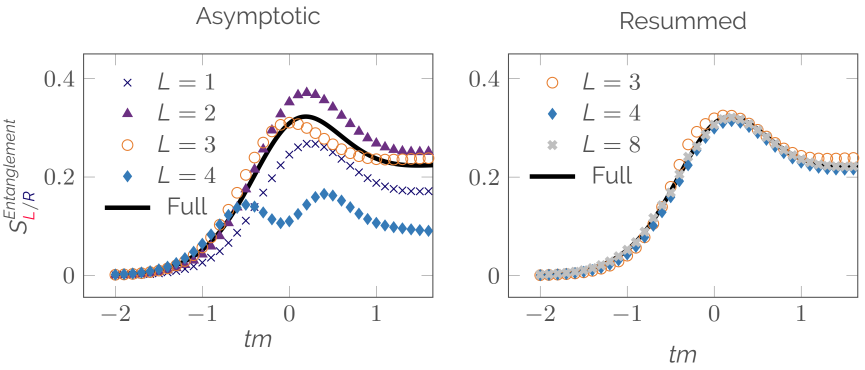


$S_{L/R}^{Entanglement}$ from observables

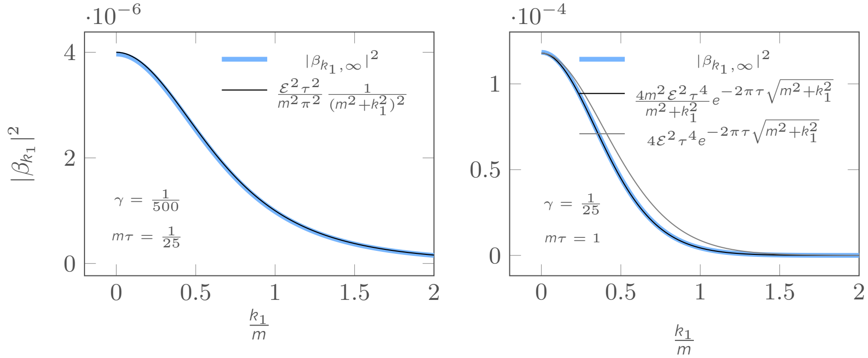


$$S_{L/R}^{Entanglement} = 2\sum_{l=1}^{\infty} \zeta(2l) C_{2l} \to 2\sum_{n=1}^{\infty} f_C(n), \qquad f_C(x) = \sum_{l=1}^{\infty} \frac{C_{2l}}{x^{2l}}$$

↑

Padé-Borel (resum. with finite # of C_{2n})

Bonus/Outlook



Understand better link to thermalization

Take home

- In this model $S_{L/R}^{Entanglement} = S^{Gibbs}$
- Efficient way to compute from multiplicity distrribution
- Many outlooks

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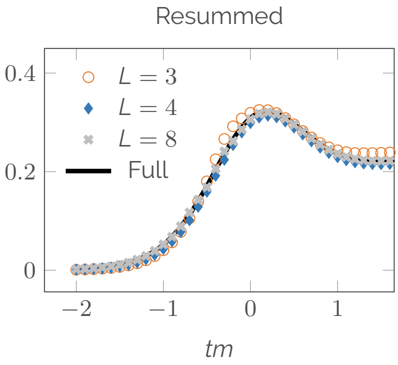

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Extension to $3 + 1D$

Physical set-up

Thermalization

Holography

Thank you!

Comparison to thermal