Quantum Simulation of Non-equilibrium Dynamics and Thermalization of the Schwinger Model

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Why Quantum Computing

- Usefulness of Euclidean lattice QCD at finite temperature: QCD matter crossover at zero chemical potential, some transport coefficients (e.g. heavy quark diffusion)

- Challenges of Euclidean lattice QCD, sign problem (signal-to-noise ratio): fermion at finite chemical potential, real-time observables

- Use quantum computers to simulate quantum systems

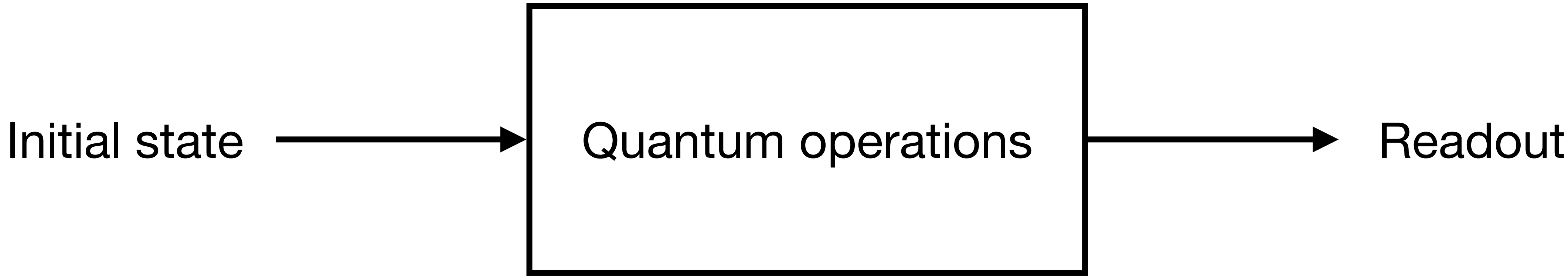
"Simulating physics with computers" R. P. Feynman, 1982

- Quantum computing devices are developing quickly: superconducting circuits (IBM Q, Google, rigetti), trapped ion (Ion Q)

"Quantum supremacy using a programmable superconducting processor" Google AI Quantum, 2019

Elements of Quantum Computing

- Three elements of quantum circuits: **state initialization, gate operation, readout**



- **State initialization**
  - Ground state: adiabatic state preparation
  - Thermal state (important to study quantum systems at finite temperature, e.g. calculate real time correlators): **couple with ancilla qubits**, quantum Metropolis, imaginary time evolution

Contents

- Prepare thermal state for toy model: U(1) gauge theory in 1+1D (Schwinger model)
- Embedded in thermal environment of scalar fields —> open quantum system
- Lindblad equation for Schwinger model in quantum Brownian motion limit —> thermalization
- Simulation on IBM Q simulators and real devices

Schwinger Model

- **U(1) gauge theory in 1+1D**

$$\mathcal{L} = \overline{\psi}(iD^\mu\gamma_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\gamma^0 = \sigma_z$$
$$\gamma^1 = -i\sigma_y$$

- **Hamiltonian formulation in axial gauge $A_0 = 0$**

$$\mathcal{H} = -i\overline{\psi}\gamma^1(\partial_1 + ieA)\psi + m\overline{\psi}\psi + \frac{1}{2}E^2$$

$$A = A_1$$
$$E = F^{10}$$

- **Discretization**

Staggered fermion $\chi(n) = \sqrt{a}\,(\gamma^1)^n\psi(n)$

J. B. Kogut and L. Susskind
Phys. Rev. D11(1975) 395–408

Jordan-Wigner transform $\chi(n) = \Big(\prod_{m<n} i\sigma_z(m)\Big)\sigma^-(n)$

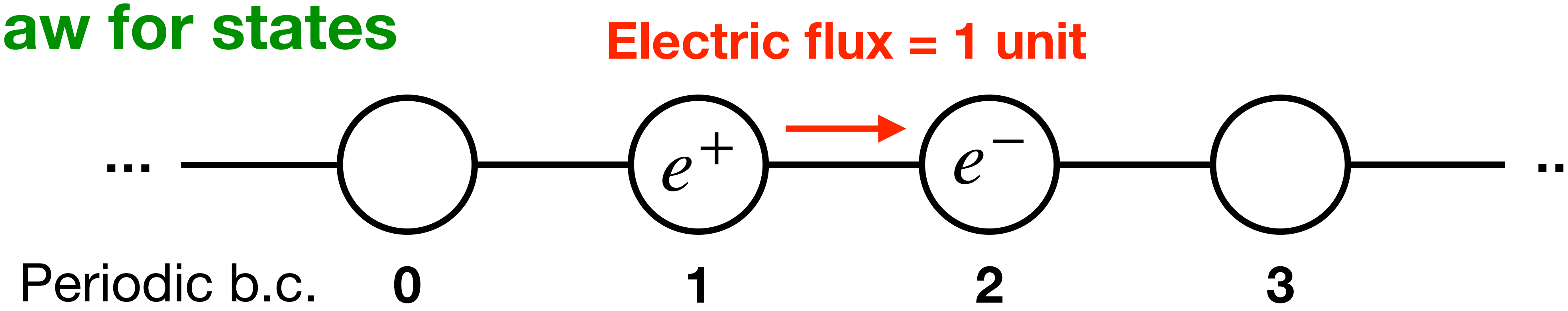
E field and gauge coupling solved by ladder $[E(n), U(n+1,n)] = eU(n+1,n)$

$$H_S = \frac{1}{2a}\sum_n \Big(\sigma^+(n)L_n^-\sigma^-(n+1) + \sigma^+(n)L_{n-1}^+\sigma^-(n-1)\Big) + \frac{m}{2}\sum_n(-1)^n\big(\sigma_z(n)+1\big) + \frac{ae^2}{2}\sum_n \ell_n^2$$

States in Schwinger Model

- **Impose Gauss law for states**

$$\partial_1 E = e\psi^\dagger\psi$$



Even sites: fermion, odd sites: anti-fermion

- **Focus on states with specific momentum and symmetry**

k=0 state: first find states that are equivalent under cyclic permutation
then symmetrized linear combination (trivial Fourier transform)

Positive parity: reflection w.r.t. a chosen site

- **Can write a code to generate the general states, specific momentum & parity states and corresponding H**

$$H_S^{\mathbf{k=0},+} =$$

$$\begin{pmatrix}
-4m & \frac{\sqrt{2}}{a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}}{a} & \frac{ae^2}{2}-2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{a} & ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & ae^2 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2a} & 0 & 0 & 0 & \frac{3}{2}ae^2-2m & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2a} & \frac{1}{\sqrt{2}a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 2ae^2 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 & 2ae^2+4m & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & \frac{5}{2}ae^2-2m & 0 & \frac{1}{2a} & & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & \frac{5}{2}ae^2+2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & 3ae^2 & 0 & \frac{1}{a} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 3ae^2 & \frac{1}{\sqrt{2}a} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{7}{2}ae^2-2m & \frac{1}{a} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 4ae^2-4m
\end{pmatrix}$$

8 fermion sites

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$\rho(t=0) = \rho_S \otimes \rho_E$

Subsystem & environment

Unitary evolution

Time reversible

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

Subsystem & environment

Trace out (integrate out) environment

$\rho_S(t=0)$

Subsystem

Non-unitary

Time irreversible

$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

From Open Quantum System to Lindblad Equation

Review: XY, 2102.01736

Subsystem: non-unitary, time-irreversible evolution

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$$

Markovian (weak-coupling)

Quantum Brownian motion (high T)

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$

Quantum optical limit (low T)

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S, \rho_S(t)\big] + L_i \rho_S(t) L_i^\dagger - \frac{1}{2}\big\{L_i^\dagger L_i, \rho_S(t)\big\}$$

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Quantum optical limit (low T)

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

τ_E: **environment correlation**

τ_S: **subsystem intrinsic time**

τ_R: **relaxation time**

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S, \rho_S(t)\right] + L_i \rho_S(t) L_i^\dagger - \frac{1}{2}\left\{L_i^\dagger L_i, \rho_S(t)\right\}$$

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$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

Markovian (weak-coupling)

Quantum Brownian motion (high T)

$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$

$\tau_E \sim \frac{1}{T},\ \tau_S \sim \frac{1}{\Delta E}$

For QFT in continuum expect $\Delta E \to 0$ so $\tau_S \to \infty$

Quantum optical limit (low T)

$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S, \rho_S(t)\right] + L_i \rho_S(t) L_i^\dagger - \frac{1}{2}\left\{L_i^\dagger L_i, \rho_S(t)\right\}$$

Schwinger Model Coupled w/ Thermal Scalars

- **Hamiltonians** $H = H_S + H_E + H_I$

$$H_E = \int dx \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{3!}g\phi^3\right]$$

$$H_I = \lambda \int dx\, \phi(x)\overline{\psi}(x)\psi(x) = \int dx\, O_E(x) O_S(x)$$

- **Lindblad equation in quantum Brownian motion limit**

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S, \rho_S(t)\big] + L\rho_S(t)L^\dagger - \frac{1}{2}\big\{L^\dagger L, \rho_S(t)\big\}$$

Only one Lindblad operator: $L = \sqrt{aN_f D(k_0 = 0, k = 0)}\left(O_S - \frac{1}{4T}\left[H_S, O_S\right]\right)$

$$O_S^{\alpha\beta} = \frac{1}{aN_f}\sum_n \left\langle k = 0, \alpha \left| \frac{(-1)^n(\sigma_z(n) + 1)}{2} \right| k = 0, \beta \right\rangle$$

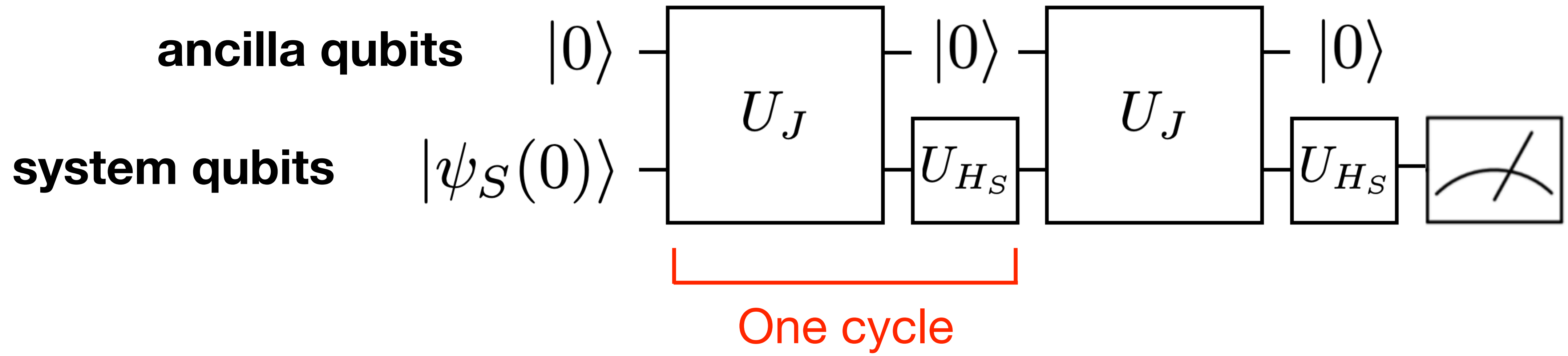
Relevant property of environment (thermal scalars) encoded by

$$D(k_0, k) = \int dt dx\, e^{ik_0 t - ikx}\mathrm{Tr}_E\big(\rho_E\, O_E^{(\mathrm{int})}(t,x) O_E^{(\mathrm{int})}(0,0)\big)$$

$$\rho_E = \frac{1}{Z}e^{-\beta H_E}$$

Simulation on Quantum Computers

- **Stinespring dilation theorem: simulate non-unitary from unitary**



No. of ancilla qubits depend on No. of Lindblad operators
After each cycle, ancilla qubits are reset

$$\rho(t) = |0\rangle_a\langle 0|_a \otimes \rho_S(t) = \begin{pmatrix} \rho_S(t) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \qquad J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

$$U_{H_S} = e^{-iH_S\Delta t}$$

$$U_J = e^{-iJ\Delta t}$$

Results from Simulators

- **Observables** $\langle A \rangle = \mathrm{Tr}(\rho_S \mathrm{A})$

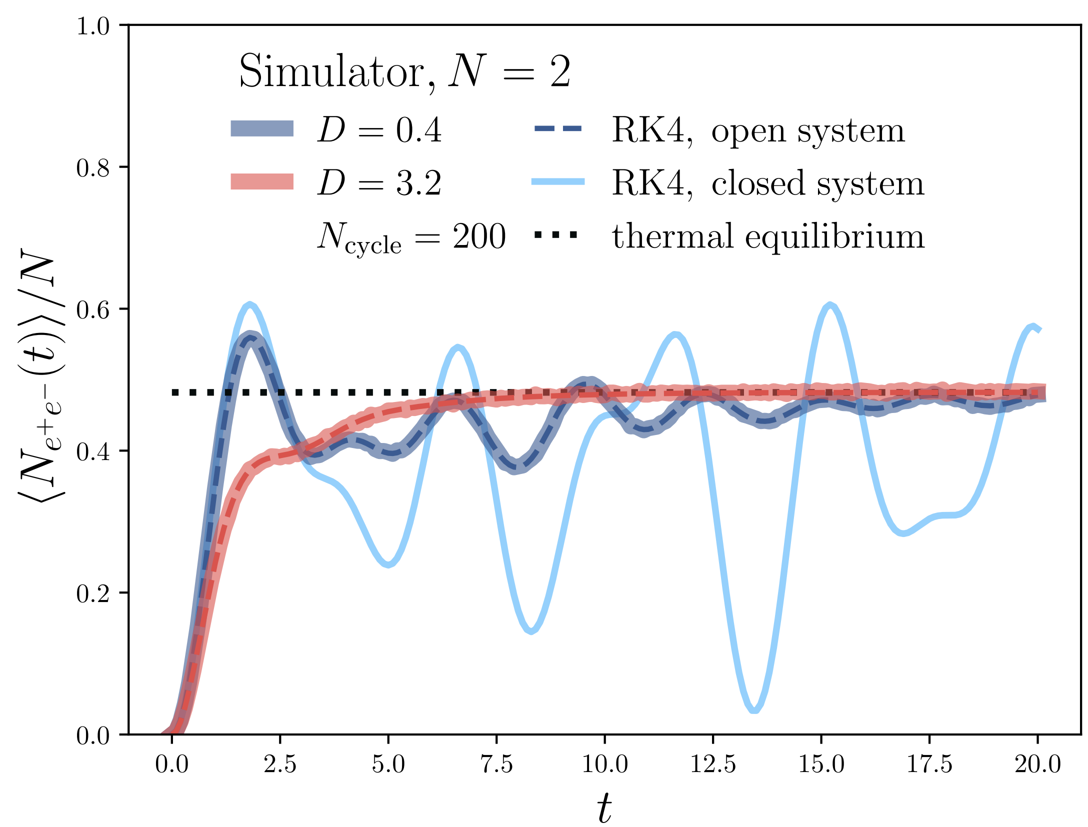
$$e = \frac{1}{a},\ m = \frac{0.1}{a},\ \beta = 0.1a,\ a = 1$$

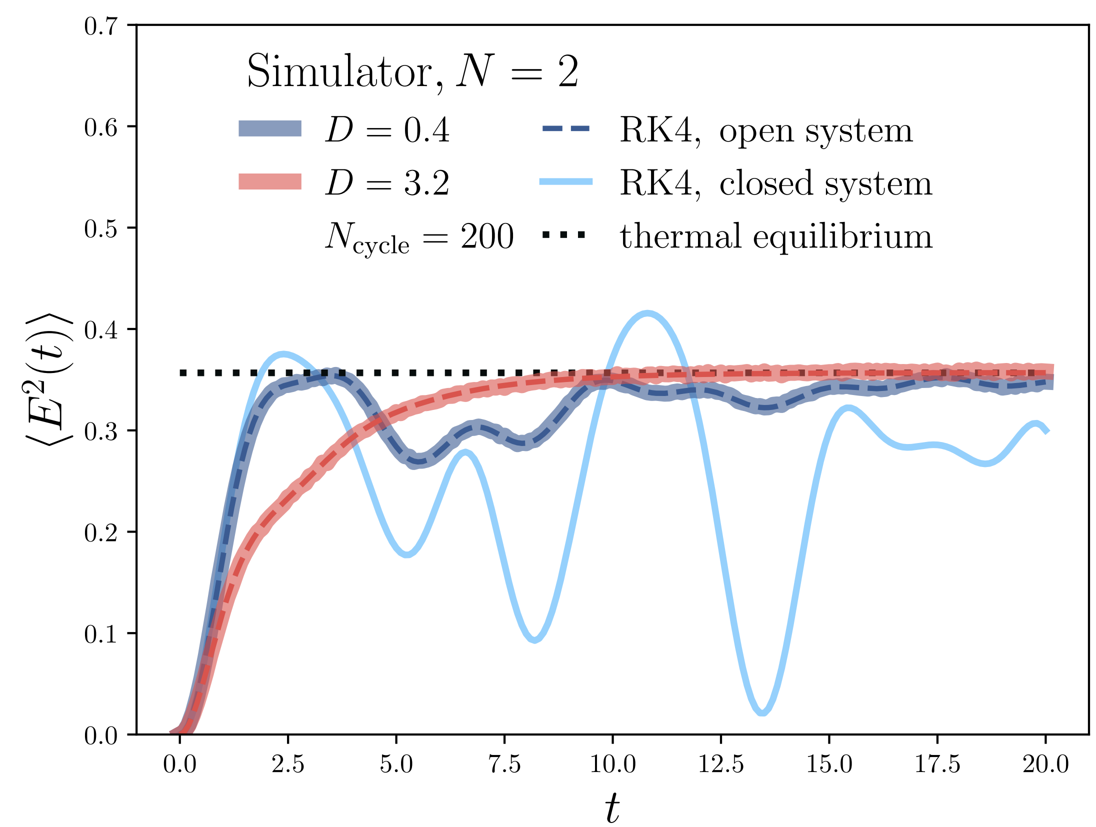
Average of E flux squared

$$\hat{A}_{E^2} = \frac{1}{2Na} \int dx\, E^2(x) = \frac{e^2}{2N} \sum_n \ell_n^2\,,$$

Total number of fermion pairs

$$\hat{A}_{N_{e^+e^-}} = \sum_{n,\,\text{even}} \sigma^+(n)\sigma^-(n)$$

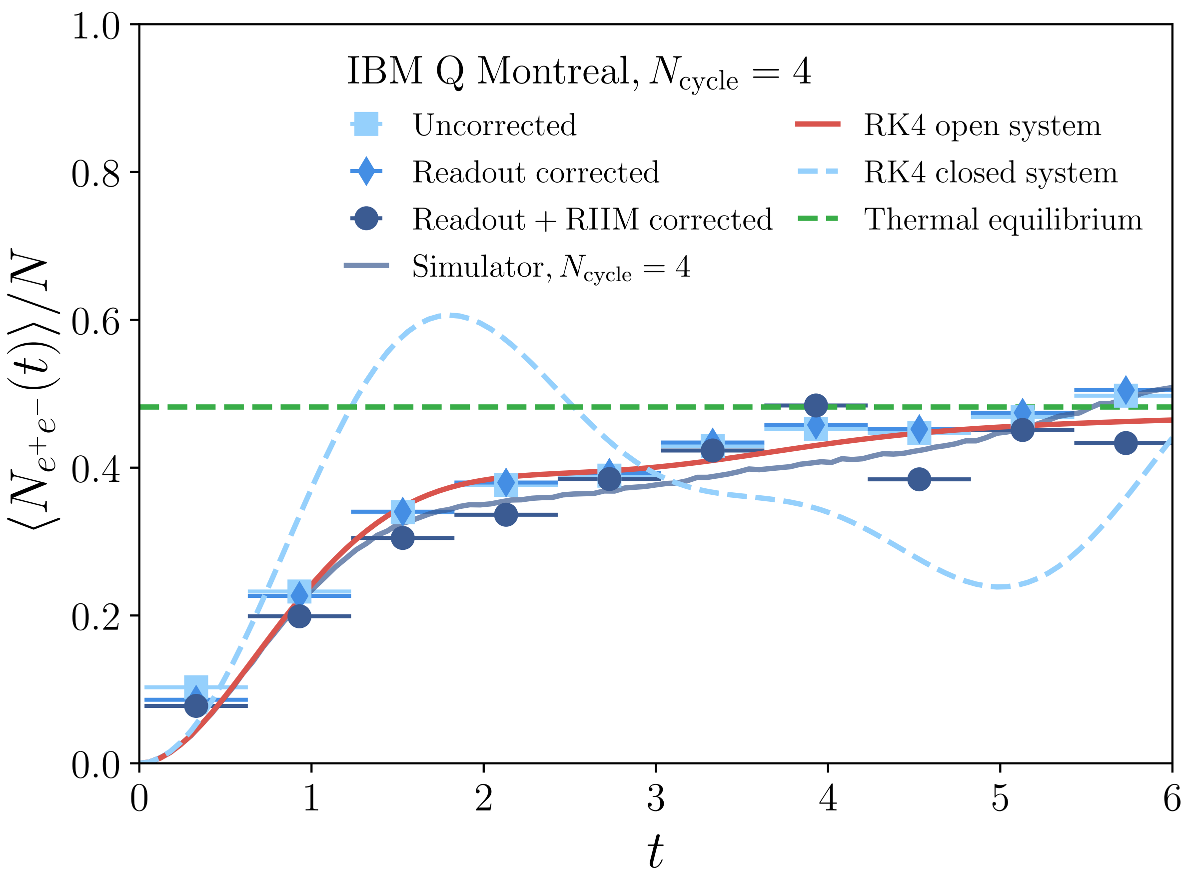


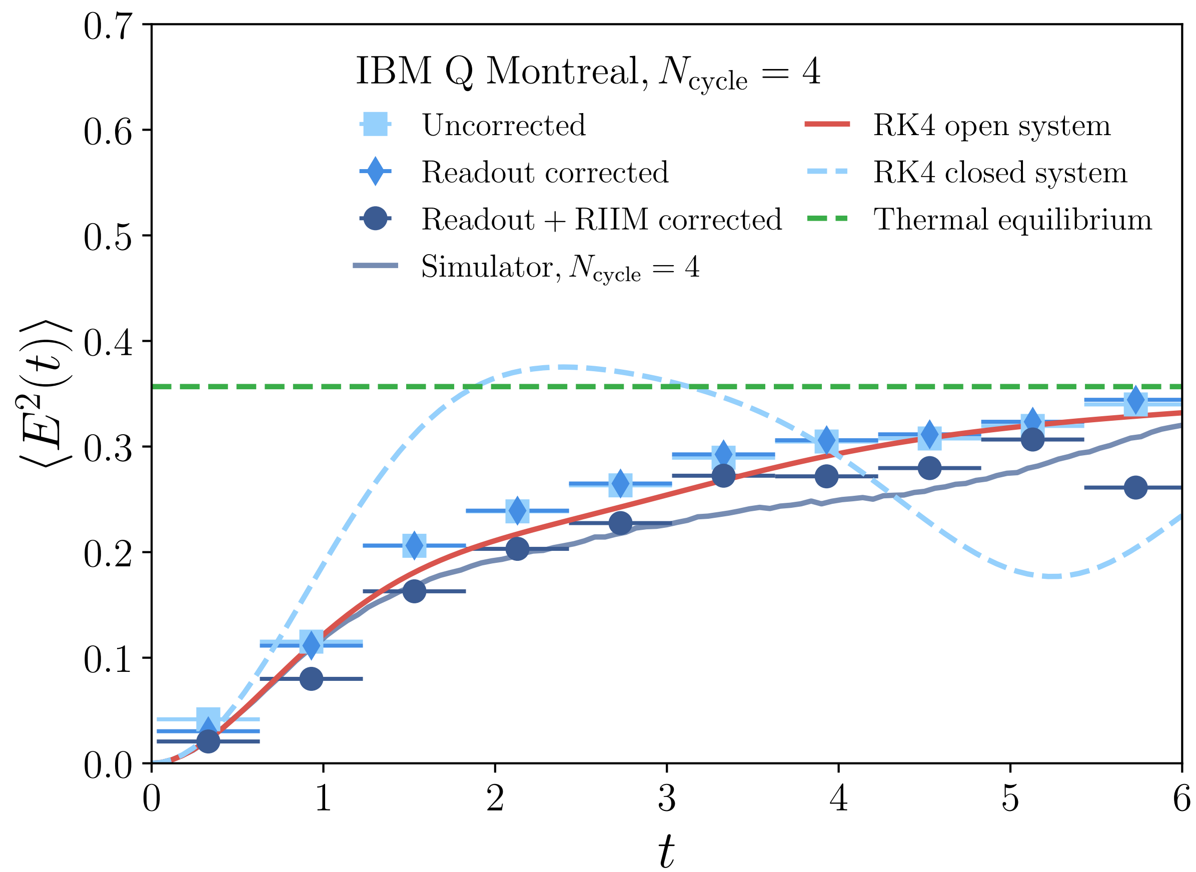


Interaction with environment leads to thermalization

Thermalization rate depends on D

Results from Real Quantum Devices





- **Error mitigation**

Readout error correction: small effect

RIIM (random identity insertion method): correct CNOT gate error, big effect

Possible to extend to higher number of cycles
& prepare states close to thermal equilibrium

Summary

- Quantum simulation of Schwinger model as open quantum system (non-equilibrium dynamics) —> prepare thermal state as initial state

- Couple with thermal scalar fields and trace out environment

- Lindblad equation in quantum Brownian motion limit

- Simulation results from IBM Q simulators and real devices: possible to achieve thermalization by using current devices