Dissipative spin hydrodynamics from the method of moments

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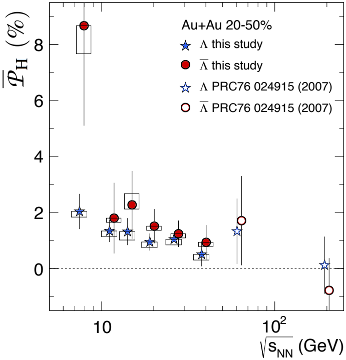
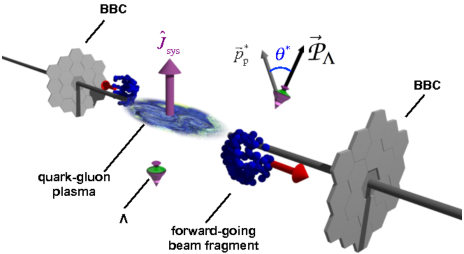
CRC-TR 211 Strong-interaction matter under extreme conditions

DFG

- **Weak decay of Λ hyperons:** daughter particles emitted preferably along polarization direction $\boldsymbol{\mathcal{P}}$.
- **Angular distribution of emitted momenta in hyperon rest frame**
  B.I. Abelev, I. Selyuzhenkov. et al. (STAR), PRC76, 024915 (2007)

$$\frac{dN}{d\cos\theta_\star} = \frac{1}{2}\left(1+\alpha|\boldsymbol{\mathcal{P}}|\cos\theta_\star\right)$$





L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

$\Longrightarrow$ Polarization of Λ hyperons along global angular momentum!

How do particles get polarized?

- Non-central heavy-ion collisions: large orbital angular momentum
- Conversion of orbital angular momentum into spin
  $\Longrightarrow$ Global rotation leads to polarization

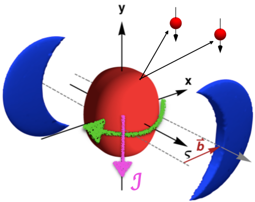


Figure from W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- Estimate vorticity from thermal approach in global equilibrium
  F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, PRC 95 (2017) 5, 054902

$$\omega \approx T(\mathcal{P}_\Lambda + \mathcal{P}_{\bar{\Lambda}})$$

Quark-gluon plasma is the "most vortical fluid ever observed"

L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

$$\omega \approx (9 \pm 1) \times 10^{21}\mathrm{s}^{-1}$$

Great Red Spot of Jupiter $10^{-4}\,\mathrm{s}^{-1}$

- Want: polarization dynamics from microscopic theory $\Longrightarrow$ spin hydrodynamics

- Spin is quantum property
  $\Longrightarrow$ starting point: quantum field theory

- Derived kinetic theory for massive spin-1/2 particles from quantum field theory using Wigner-function formalism
  NW, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, PRD100, 056018 (2019)
  J.-H. Gao and Z.-T. Liang, PRD100, 056021(2019)
  K. Hattori, Y. Hidaka, and D.-L. Yang, PRD100, 096011 (2019)
  Y.-C. Liu, K. Mameda, and X.-G. Huang, Chin.Phys.C 44 (2020) 9, 094101
  NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL, 2005.01506, PRD ,2103.04896

- Now: **Derive hydrodynamic equations of motion from kinetic theory,**
  use method of moments
  G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047

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NW, E. Speranza, X.-l. Sheng, Q. Wang, and D.H. Rischke, PRL, 2005.01506, PRD, 2103.04896

- Phase-space distribution function $f(x, p, \mathfrak{s})$, depends on spin variable $\mathfrak{s}^\mu$
- **Boltzmann equation**

$$p \cdot \partial f(x, p, \mathfrak{s}) = \mathfrak{C}[f]$$

- Nonlocal collision term

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \, [f(x + \Delta_1, p_1, \mathfrak{s}_1) \times f(x + \Delta_2, p_2, \mathfrak{s}_2) - f(x + \Delta, p, \mathfrak{s}) f(x + \Delta', p', \mathfrak{s}')]$$

  Particle positions displaced by Δ^μ

- Nonlocal collision term: conversion of orbital angular momentum into spin $\Longrightarrow$ spin alignment with vorticity

- **Spin hydrodynamics is based on conserved quantities:**

  W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, PRC 97, no. 4, 041901 (2018)
  W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, PRD 97, no. 11, 116017 (2018)
  W. Florkowski, F. Becattini, and E. Speranza, APB 49, 1409 (2018)

  charge current

  $$\partial \cdot N = 0$$

  energy-momentum tensor

  $$\partial_\mu T^{\mu\nu} = 0$$

  \+ **total** angular-momentum tensor

  $$J^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + \hbar S^{\lambda,\mu\nu}$$

  orbital part　　　spin tensor

  $$\partial_\lambda J^{\lambda,\mu\nu} = 0 \qquad \Longrightarrow \qquad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

  $$a^{[\mu} b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$$

- Nonlocal collision term $\Longleftrightarrow T^{[\nu\mu]} \neq 0$

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- Decompose currents with respect to fluid velocity u^μ

$$N^\mu = n u^\mu + n^\mu \,,$$

$$T^{\mu\nu}_{\mathsf{sym}} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0 + \Pi) + u^{(\mu} h^{\nu)} + \pi^{\mu\nu} \,,$$

$$S^{\lambda,\mu\nu} = u^\lambda \tilde{\mathfrak{N}}^{\mu\nu} + \Delta^\lambda_\alpha \tilde{\mathfrak{P}}^{\alpha\mu\nu} + 2 u_{(\alpha} \tilde{\mathfrak{H}}^{\lambda)\mu\nu\alpha} + \tilde{\mathfrak{Q}}^{\lambda\mu\nu} + \frac{\hbar}{2m} \partial^{[\nu} \left[\epsilon_0 u^{\mu]} u^\lambda - \Delta^{\mu]\lambda}(P_0 + \Pi) + \pi^{\mu]\lambda} \right]$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu, \;\; \tilde{A}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} A_{\alpha\beta}$$

- 4+10+24 degrees of freedom ↔ 1+4+6 equations of motion
  $\Longrightarrow$ need additional equations of motion to close system of equations
- All components can be related to moments of distribution function, e.g., spin-energy tensor

$$\tilde{\mathfrak{N}}^{\mu\nu} \equiv -\frac{1}{2m} \epsilon^{\mu\nu\alpha\beta} u_\alpha \langle E_p^2 \, \mathfrak{s}_\beta \rangle$$

$\Longrightarrow$ derive additional equations of motion from kinetic theory

use method of moments

G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, PRD 85 (2012) 114047

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- Expand distribution function around equilibrium up to first order in gradients

$$f = f_{\text{eq}} + \delta f = f_{0p}\left[1 + \frac{\hbar}{4}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu} + \phi_p + \mathfrak{s}\cdot\zeta_p\right],$$

equilibrium, deviations from equilibrium

Zeroth-order distribution function

$$f_{0p} \equiv \frac{1}{(2\pi\hbar)^2} e^{-\beta_0 u\cdot p + \alpha_0}$$

inverse temperature β_0, chemical potential α_0,
spin potential: thermal vorticity + dissipative corrections

$$\Omega_{\mu\nu} = \varpi_{\mu\nu} + \mathcal{O}(\partial)$$

- Define dissipative moments

$$\rho_n^{\mu_1\cdots\mu_l} \equiv \langle E_p^n\, p^{\langle\mu_1}\ldots p^{\mu_l\rangle}\rangle_\delta$$

$$\tau_n^{\mu,\mu_1\cdots\mu_l} \equiv \langle E_p^n\, \mathfrak{s}^\mu p^{\langle\mu_1}\ldots p^{\mu_l\rangle}\rangle_\delta$$

$A^{\langle\mu_1\cdots\mu_n\rangle}$: traceless, symmetric projection orthogonal to u^μ

- δf can be expressed in basis of dissipative moments

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- Conservation laws
  $\Longrightarrow$ equations of motion for thermodynamic potentials α_0, β_0, u^μ, $\Omega^{\mu\nu}$
- Boltzmann equation $\Longrightarrow$ exact equations of motion for spin moments $\tau_n^{\mu,\mu_1\cdots\mu_l}$, e.g.,

$$\dot{\tau}_r^{\langle\mu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\rangle} =$$

$$\left[\xi_r^{(0)}\theta + \frac{G_{2(r+1)}}{D_{20}}\Pi\theta - \frac{G_{2(r+1)}}{D_{20}}\pi^{\lambda\nu}\sigma_{\lambda\nu} - \frac{G_{3r}}{D_{20}}\partial\cdot n\right]\omega_0^\mu - \frac{\hbar}{2m}I_{(r+1)1}\Delta_\lambda^\mu\nabla_\nu\tilde{\Omega}^{\lambda\nu}$$

$$-\frac{\hbar}{2m}\tilde{\Omega}^{\langle\mu\rangle\nu}\left[I_{(r+1)1}I_\nu - I_{(r+2)1}\frac{\beta_0}{\epsilon_0+P_0}\left(-\Pi\dot{u}_\nu + \nabla_\nu\Pi - \Delta_{\nu\lambda}\partial_\rho\pi^{\lambda\rho}\right)\right]$$

$$+r\,\dot{u}_\nu\tau_{r-1}^{\langle\mu\rangle,\nu} + (r-1)\sigma_{\alpha\beta}\tau_{r-2}^{\langle\mu\rangle,\alpha\beta} - \Delta_\lambda^\mu\nabla_\nu\tau_{r-1}^{\lambda,\nu} - \frac{1}{3}\left[(r+2)\tau_r^{\langle\mu\rangle} - (r-1)m^2\tau_{r-2}^{\langle\mu\rangle}\right]\theta$$

$$-\frac{\hbar}{2m}I_{(r+1)0}\epsilon^{\mu\nu\alpha\beta}u_\nu\dot{\Omega}_{\alpha\beta}$$

$\nabla^\mu \equiv \Delta_\nu^\mu\partial^\nu$, $\theta \equiv \nabla\cdot u$, $\sigma^{\mu\nu} \equiv \nabla^{\langle\mu}u^{\nu\rangle}$, $\omega_0^\mu \equiv -(1/2)\epsilon^{\mu\nu\alpha\beta}u_\nu\Omega_{\alpha\beta}$, $I^\mu \equiv \nabla^\mu\alpha_0$

- Collision term

$$\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_n\rangle} \equiv \int d\Gamma\, E_p^{r-1}\mathfrak{s}^\mu p^{\langle\mu_1}\cdots p^{\mu_n\rangle}\mathfrak{C}[f]$$

- Infinite number of coupled equations $\Longrightarrow$ need truncation procedure
- **Idea:** Approximate moments which do not appear in conservation laws by those which do appear
- **Conventional hydrodynamics:**

$$n^\mu \equiv \rho_0^\mu, \qquad h^\mu \equiv \rho_1^\mu, \qquad \Pi \equiv -\frac{m^2}{3}\rho_0, \qquad \pi^{\mu\nu} \equiv \rho_0^{\mu\nu}$$

particle diffusion | heat flux | bulk viscous pressure | shear stress

$\Longrightarrow$ 14-moment approximation

- **Spin hydrodynamics:** additional dynamical moments from spin tensor

$$\mathfrak{n}^\nu \equiv \tau_2^\nu, \qquad \mathfrak{p}^\mu \equiv \tau_0^\mu, \qquad \mathfrak{z}^{\lambda\mu} \equiv \tau_1^{(\langle\mu\rangle,\lambda)}, \qquad \mathfrak{q}^{\lambda\mu\nu} \equiv \tau_0^{\nu,\mu\lambda}$$

spin energy | spin pressure | spin diffusion | spin stress

$\Longrightarrow$ 14+24-moment approximation

- Matching conditions remove certain dynamical degrees of freedom

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- General form of collision terms

$$\mathfrak{C}_{r-1}^{\mu,\langle\mu_1\cdots\mu_l\rangle} = \sum_{n=0}^{N_l} B_{rn}^{(l)} \tau_n^{\mu,\langle\mu_1\cdots\nu_l\rangle} + \int [d\Gamma]\, \mathcal{W}\, E_p^{r-1} p^{\langle\mu_1} \cdots p^{\mu_l\rangle} \mathfrak{s}^\mu f_{0p} f_{0p'} \times \left[-\frac{\hbar}{4}(\Omega_{\alpha\beta} - \varpi_{\alpha\beta})\Sigma_{\mathfrak{s}}^{\alpha\beta} + \frac{1}{2}\partial_{(\beta}\beta_{\alpha)}\Delta^\beta p^\alpha \right]$$

dissipative spin moments
difference between spin potential and thermal vorticity
from nonlocal collision term, thermal shear

- Invert matrix $B_{rn}^{(l)}$ to obtain final form of equations of motion

Obtain equations of motion of all dynamical spin moments, e.g.,

$$
\begin{aligned}
&\tau_{\mathfrak{p}0}\dot{\mathfrak{p}}^{\langle\mu\rangle} - 6\tau_{\mathfrak{n}0}\dot{\mathfrak{q}}^{\nu\mu}{}_{\nu} - \mathfrak{p}^{\langle\mu\rangle} = \\
&\mathfrak{e}^{(0)}\left(\tilde{\Omega}^{\mu\nu} - \tilde{\varpi}^{\mu\nu}\right)u_\nu + \left(\mathfrak{K}^{(0)}_{\theta\omega}\theta + \mathfrak{K}^{(0)}_{\theta\omega\Pi}\Pi\theta + \mathfrak{K}^{(0)}_{\pi\sigma\omega}\pi^{\lambda\nu}\sigma_{\lambda\nu} + \mathfrak{K}^{(0)}_{n\omega}\partial\cdot n\right)\omega_0^\mu \\
&+ \left[\mathfrak{K}^{(0)}_{I\Omega}I_\nu + \mathfrak{K}^{(0)}_{\Pi\Omega}\left(-\Pi\dot{u}_\nu + \nabla_\nu\Pi - \Delta_{\nu\lambda}\partial_\rho\pi^{\lambda\rho}\right)\right]\tilde{\Omega}^{\langle\mu\rangle\nu} + \mathfrak{K}^{(0)}_{\nabla\Omega}\Delta^\mu_\lambda\nabla_\nu\tilde{\Omega}^{\lambda\nu} \\
&+ \mathfrak{g}_1^{(0)}\mathfrak{z}^{\mu\nu}F_\nu + \mathfrak{g}_2^{(0)}\sigma_{\alpha\beta}\mathfrak{q}^{\langle\mu\rangle\alpha\beta} - \mathfrak{g}_3^{(0)}\Delta^\mu_\lambda\nabla_\nu\mathfrak{z}^{\lambda\nu} + \mathfrak{g}_4^{(0)}\theta\mathfrak{p}^{\langle\mu\rangle} \\
&- \mathfrak{g}_5^{(0)}\theta\mathfrak{q}^{\nu\mu}{}_{\nu} + \mathfrak{g}_6^{(0)}\mathfrak{z}^{\mu\nu}I_\nu + \left(\mathfrak{g}_7^{(0)}\mathfrak{p}_\nu + \mathfrak{g}_8^{(0)}\mathfrak{q}^{\lambda}{}_{\nu\lambda}\right)\left(\sigma^{\nu\mu} + \omega^{\nu\mu}\right) + \mathfrak{K}^{(0)}_{\dot{\Omega}}\left(\dot{\omega}_0^{\langle\mu\rangle} - \tilde{\Omega}^{\langle\mu\rangle\nu}\dot{u}_\nu\right)
\end{aligned}
$$

relaxation times
transport coefficients

and similar equations for $\mathfrak{z}^{\lambda\mu}$ and $\mathfrak{q}^{\lambda\mu\nu}$

$\Longrightarrow$ closed set of relaxation equations

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- Observable in heavy-ion collisions: Pauli-Lubanski vector
  F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, AP338, 32 (2013)
  F. Becattini, arXiv:2004.04050
  E. Speranza, NW, EPJA 57 (2021) 5, 155
  L. Tinti, W. Florkowski, arXiv:2007.04029

$$\Pi^\mu(p) = \frac{1}{2\mathcal{N}} \int d\Sigma_\lambda p^\lambda dS\, \mathfrak{s}^\mu f(x,p,\mathfrak{s})$$

- Equilibrium:

$$\Pi^\mu_{\mathbf{eq}}(p) = -\frac{\hbar}{4m\mathcal{N}} \int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\mu\nu}(E_p u_\nu + p_{\langle\nu\rangle})$$

- Express dissipative corrections through dynamical spin moments:

$$\delta\Pi^\mu(p) = \frac{1}{2\mathcal{N}}\left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2}\right)\int d\Sigma_\lambda p^\lambda \left\{\chi_\mathfrak{p}\mathfrak{p}^{\langle\nu\rangle} - 6\chi_\mathfrak{n}\mathfrak{q}^{\rho\nu}{}_\rho + \mathfrak{x}_\mathfrak{p} u^\nu \mathfrak{z}^\lambda{}_\lambda \right.$$
$$\left. + \left[\chi_\mathfrak{z}\mathfrak{z}^{\nu\alpha} + \left(\mathfrak{x}_\mathfrak{q}\mathfrak{q}^{\lambda\alpha}{}_\lambda + \mathfrak{x}_\mathfrak{p}\mathfrak{p}^{\langle\alpha\rangle}\right)u^\nu\right]p_{\langle\alpha\rangle} + \left(\chi_\mathfrak{q}\mathfrak{q}^{\langle\nu\rangle\alpha\beta} + \mathfrak{x}_\mathfrak{z} u^\nu \mathfrak{z}^{\langle\alpha\beta\rangle}\right)p_{\langle\alpha}p_{\beta\rangle}\right\}$$

- So far: kept terms up to second order in equations of motion
  $\Longrightarrow$ transient hydrodynamics
- Now: keep only first-order terms $\Longrightarrow$ Navier-Stokes limit
- No additional dynamical quantities,
  everything can be expressed in terms of α_0, β_0, u^μ, $\Omega^{\mu\nu}$ and their derivatives
- Full expression of Pauli-Lubanski vector lengthy
- From nonlocal collision term: contributions independent of spin potential

$$\delta\Pi^\mu(p) \simeq \frac{1}{2\mathcal{N}} \left(g^\mu_\nu - \frac{p^\mu p_\nu}{p^2} \right) \int d\Sigma_\lambda p^\lambda \chi_\sigma \sigma_\rho{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\tau\rho} u_\tau p_{\langle\alpha} p_{\beta\rangle} + \dots$$

$\Longrightarrow$ contribution from shear to local polarization, vanishes after momentum integration

Effects of shear important for description of local Λ polarization

B. Fu, S. Y.F. Liu, L. Pang, H- Song, Y. Yin, arXiv:2103.10403
F. Becattini, M. Buzzegoli, A. Palermo, arXiv:2103.10917
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, arXiv:2103.14621

Conclusions and outlook

- Derived second-order spin hydrodynamics from kinetic theory using method of moments
- Dissipative corrections to Pauli-Lubanski vector
  $\Longrightarrow$ depend on all dynamical spin moments
  $\Longrightarrow$ Navier-Stokes limit: contributions from spin potential and shear

Outlook:

- Stability analysis of equations of motion, numerical implementation
  $\Longrightarrow$ dissipative effects of polarization in heavy-ion collisions
- Kinetic theory and hydrodynamics for spin-1 particles
  Huang, Mitkin, Sadofyev, Speranza, JHEP 10 (2020) 117
  Hattori, Hidaka, Yamamoto, Yang, JHEP 02 (2021) 001
  D. Wagner, NW, E. Speranza, D. H. Rischke, to appear
- Include electromagnetic fields
  $\Longrightarrow$ dissipative spin magnetohydrodynamics