QCD Plasma Out of Equilibrium

Strong and Electroweak Matter 2021

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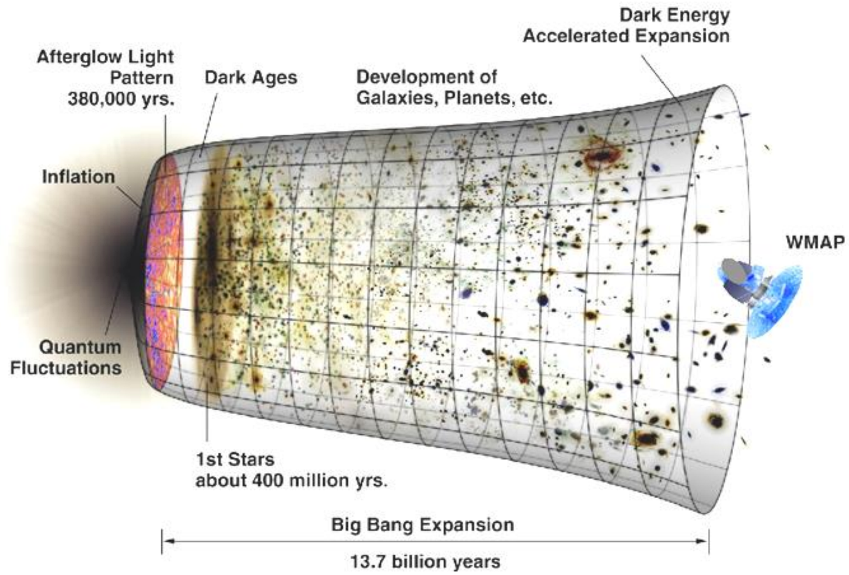
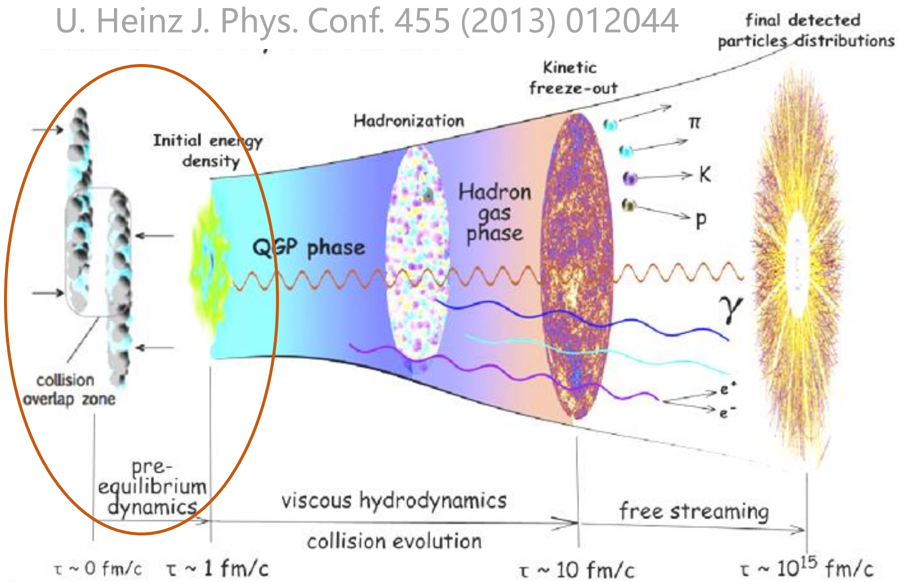
CRC–TR 211
Strong-interaction matter under extreme conditions

Equilibration after bangs



U. Heinz J. Phys. Conf. 455 (2013) 012044



Early universe (Big Bang)

Standard Model matter produced and equilibrated between inflation and Big Bang nucleosynthesis (BBN)

Heavy-ion collision (little bang)

Off-thermal plasma produced in initial collision and equilibrated into thermal hydrodynamic states:

Kinetic equilibration

Yang-Mills plasma (gluon saturated) equilibrated into quark-gluon plasma (quarks + gluon):

Chemical equilibration

QCD Effective Kinetic Theory

Simulation with QCD Effective Kinetic Theory (EKT)

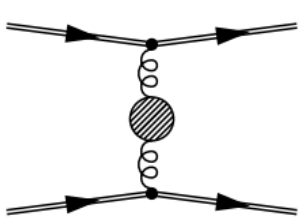
$$\left(\frac{\partial}{\partial\tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow2}[f](\tau, p_T, p_\parallel)$$

Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

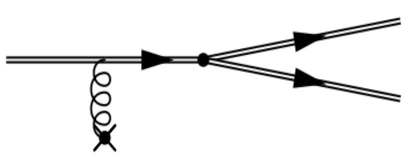
Solving a set of coupled Boltzmann equations

Including all light quarks/antiquarks and gluon $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

Including LO 2↔2 elastic scatterings and 1↔2 inelastic scatterings with back reaction



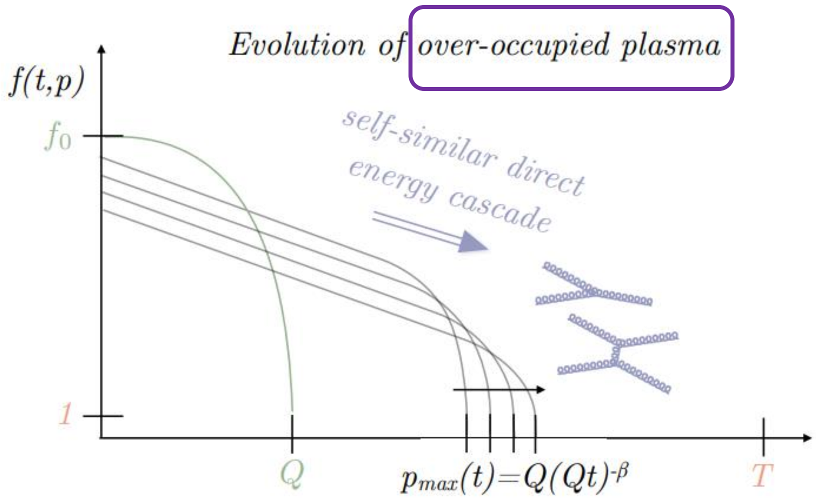
2↔2: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation

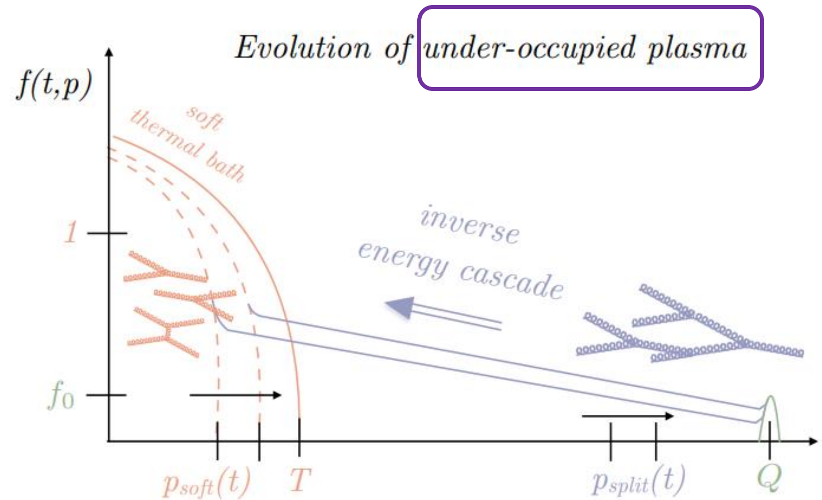


1↔2: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation

Turbulence in QCD equilibration

Two typical far-from-equilibrium systems





Over-occupied plasma

- Separation of scale

  $\langle p \rangle_0 \ll T$

- Direct energy cascade

  low → high momentum

- Initial state in HICs

Under-occupied plasma

- Separation of scale

  $\langle p \rangle_0 \gg T$

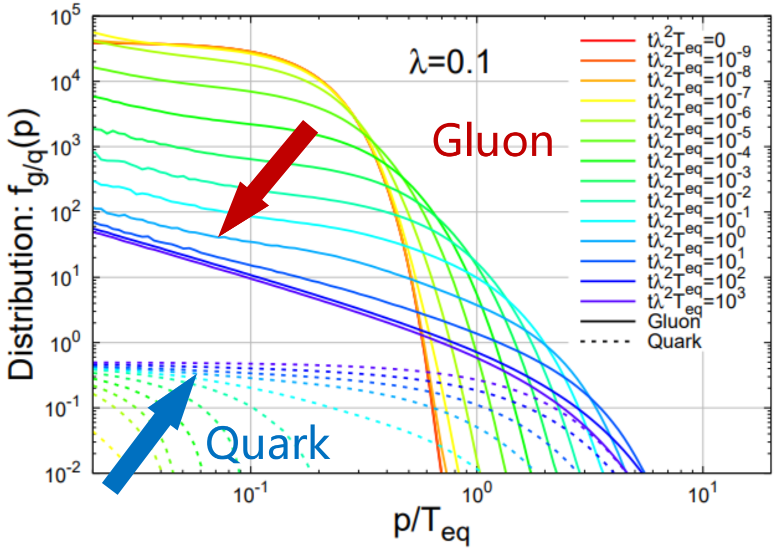
- Inverse energy cascade

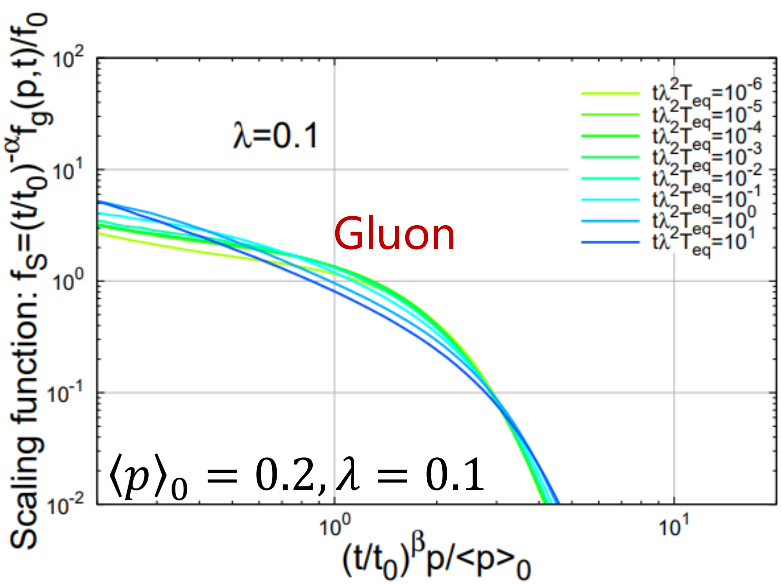
  high → low momentum

- Jets in HICs

Schlichting, Teaney, ARNPS 69 (2019) 447

Over-occupied plasma





Self-similar energy cascade

Self-similar scaling spectra

$$f_g(p,t) = (t/t_0)^{\alpha} f_0 f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Universal Scaling Function

$$f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Scaling Exponents from Yang-Mills plasma

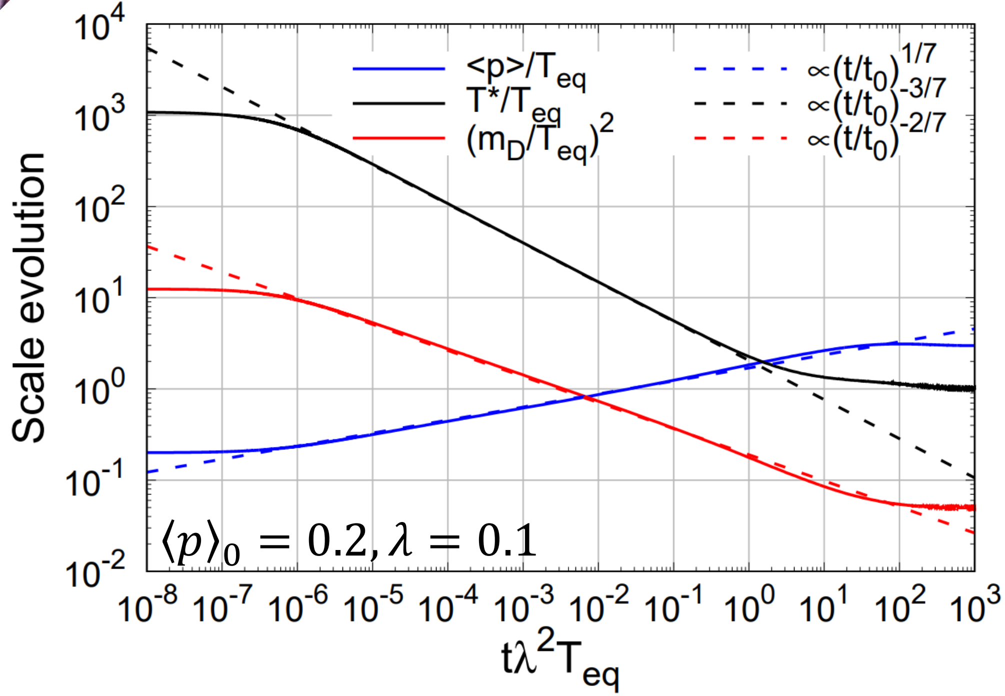
$$\alpha = -\frac{4}{7}, \beta = -\frac{1}{7}$$

Also work for quark-gluon plasma
gluon dominated

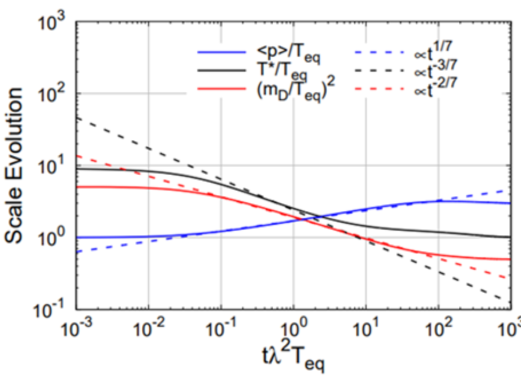
Quark spectra following gluon spectrum

XD, Schlichting, arXiv: 2012.09079
Berges, Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036

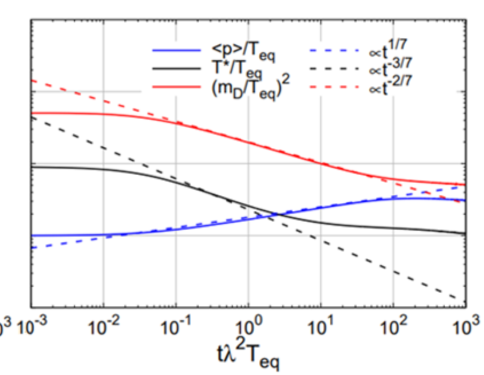
Over-occupied plasma



$\langle p\rangle_0 = 0.2, \lambda = 0.1$



$\langle p\rangle_0 = 1, \lambda = 1$



$\langle p\rangle_0 = 1, \lambda = 10$

Self-similar scaling

$$f \sim f_0 \left(\frac{t}{t_0}\right)^{-\frac{4}{7}}$$

Power-law evolution

$$p \sim \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{\frac{1}{7}} \quad T \sim g^2 f_0 \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{-\frac{3}{7}}$$

$$m_D^2 \sim g^2 f_0 \langle p\rangle_0^2 \left(\frac{t}{t_0}\right)^{-\frac{2}{7}}$$

Not limited to Yang-Mills plasma
But also for quark-gluon plasma

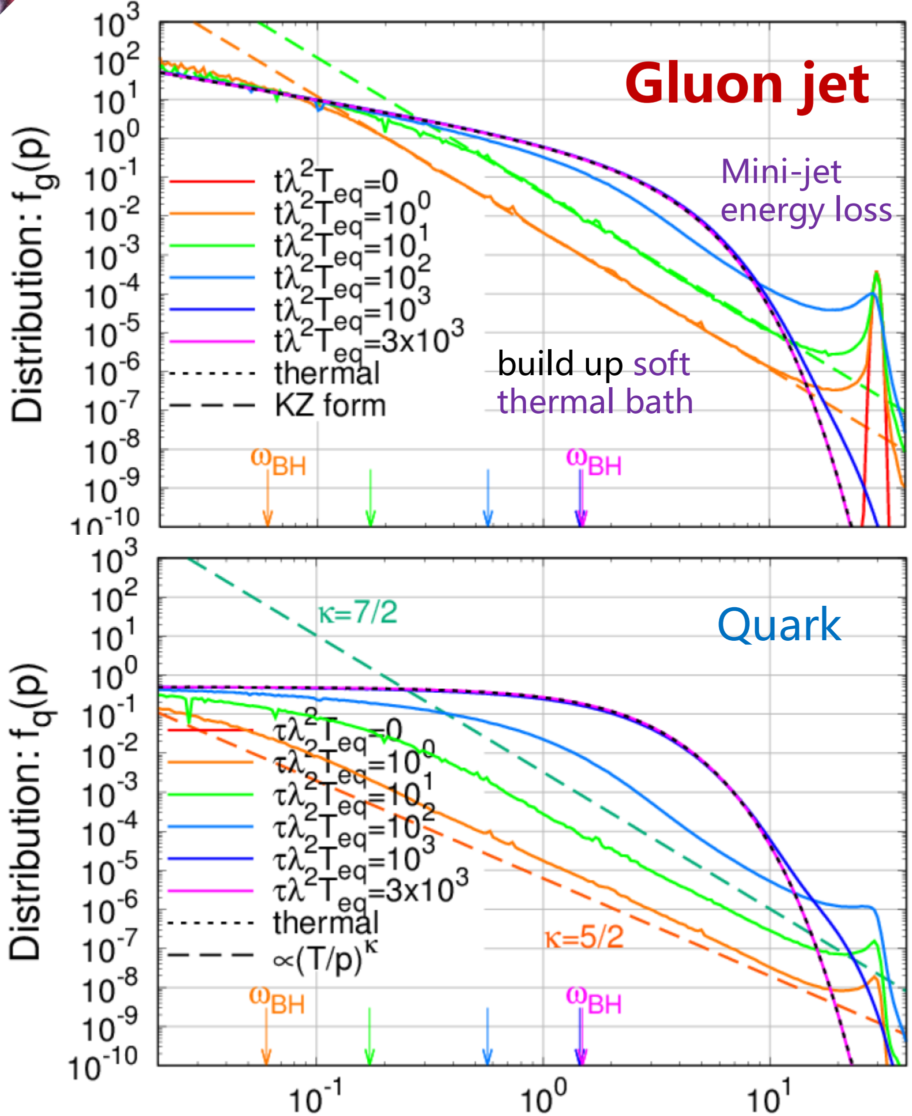
Even work for stronger coupling

t'Hooft coupling

$$\lambda = 4\pi\alpha_s N_c$$

XD, Schlichting, arXiv: 2012.09079

Under-occupied plasma



$\langle p\rangle_0 = 30, \lambda = 1$

Wave turbulence

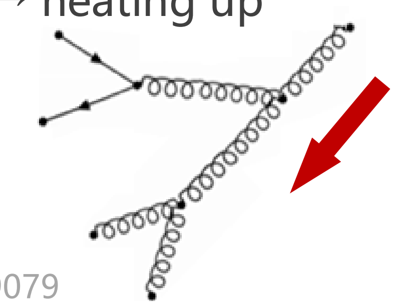
Kolmogorov-Zakharov spectrum (exponent κ=7/2 for gluon)

$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

Blaizot, Iancu, Mehtar-Tani, PRL 111, 052001 (2013)
Mehtar-Tani, Schlichting, JHEP 09, 144 (2018)

Bottom-up thermalization

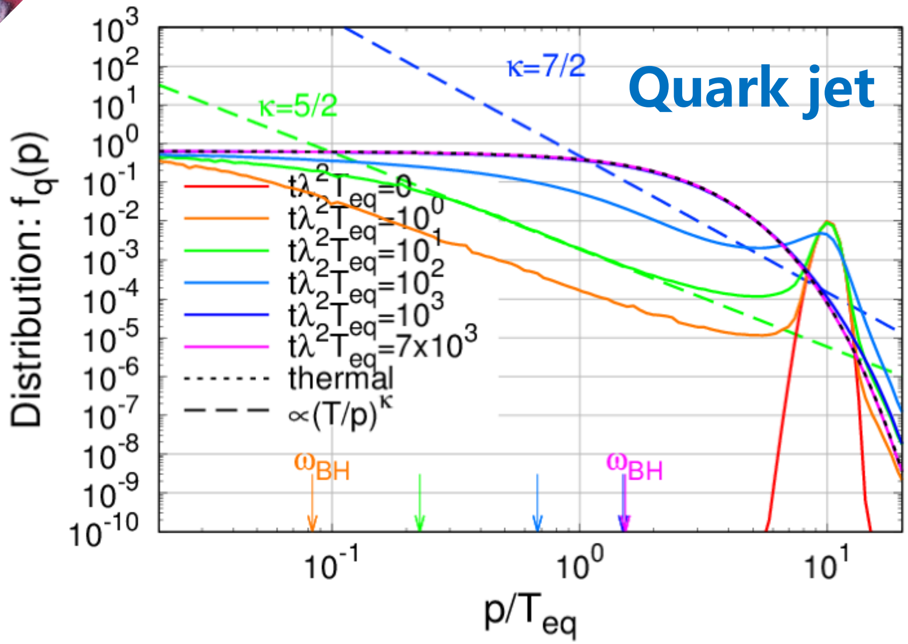
1. Emission of (soft) quarks and gluon
2. Radiative breakup by multiple branchings → build up soft thermal bath
3. Mini-Jet energy loss → heating up thermal bath

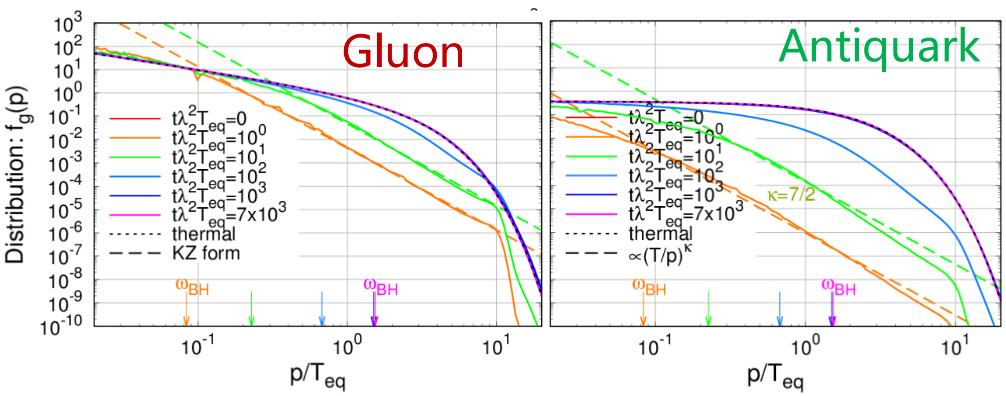


Baier, et al. PLB 502 (2001) 51

XD, Schlichting, arXiv: 2012.09079

Under-occupied plasma





$\langle p\rangle_0 = 10, \lambda = 1$

XD, Schlichting, arXiv: 2012.09079

Wave turbulence

1. Quark follows κ=5/2 to κ=7/2
2. Gluon follows κ=7/2
3. Antiquark follows gluon (secondary production)

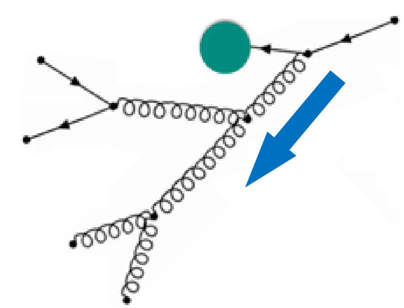
$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

Bottom-up thermalization

Same pattern as for

in-medium jet energy loss and equilibration

with unified description of soft and hard sectors



Soudi, Schlichting, 2008.04928

Hydrodynamization in HICs

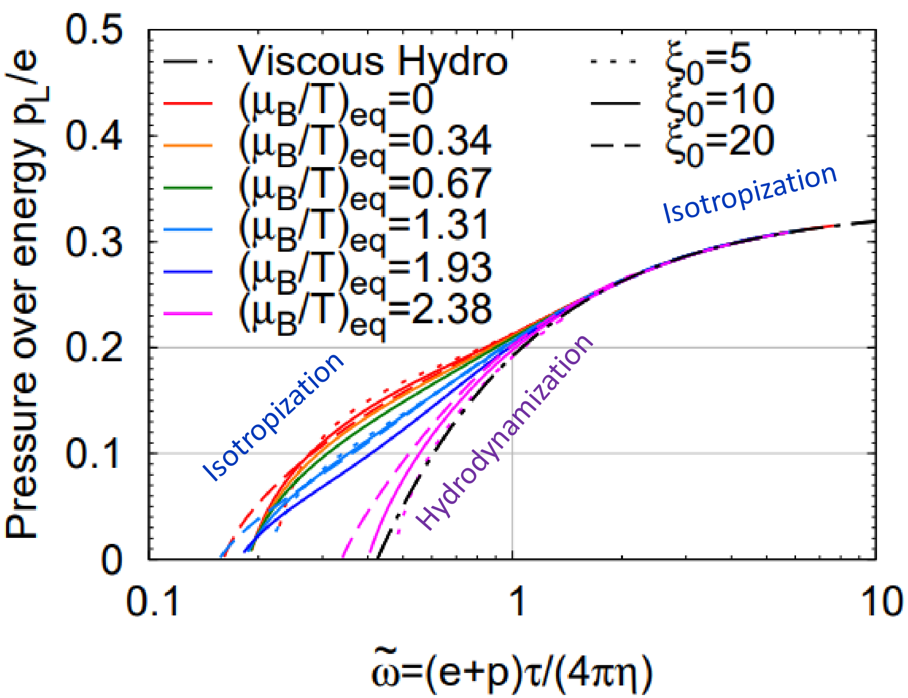
Isotropization

longitudinal pressure/energy density

0 (initial) → 1/3 (final equilibrium)

Hydrodynamic constitutive relation:

$$\frac{p_L}{e} = \frac{1}{3} - \frac{16\eta}{9(e+p)\tau}$$



Universal scaling

$$\tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

Recast:

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}}$$

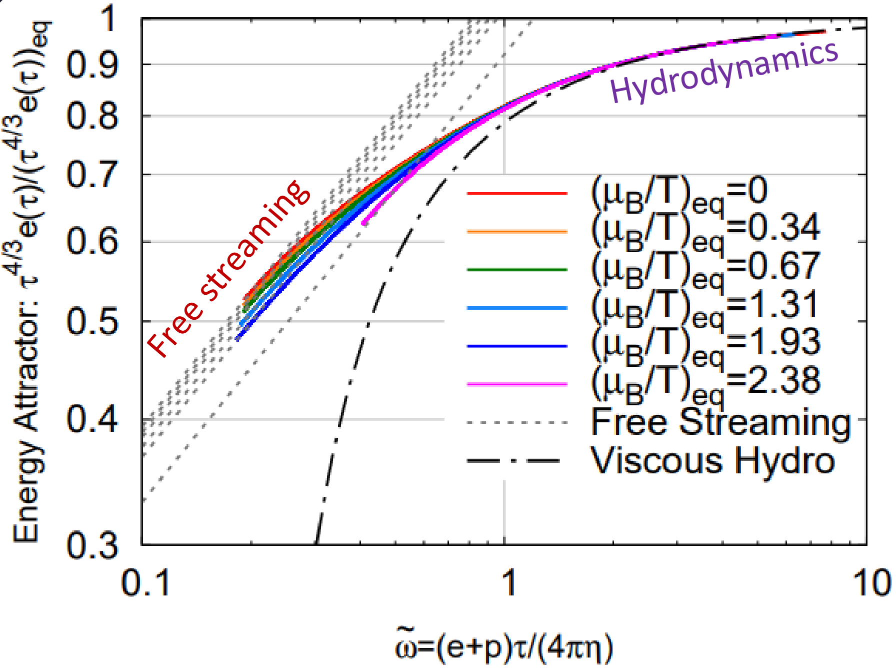
Pressure attractor

Effective constitutive relation from EKT

$$\frac{p_L}{e} = f(\tilde{\omega})$$

Kurkela, Mazeliauskas, PRD 99 (2019) 054018
KOMPOST, PRC 99 (2019) 034910
XD, Schlichting, arXiv: 2012.09068

Attractor in Hydrodynamization



Energy attractor

$$\mathcal{E}\left(\tilde{\omega}=\frac{(e+p)\tau}{4\pi\eta}\right)=\frac{\tau^{4/3}e}{(\tau^{4/3}e)_{eq}}$$

Asymptotically

$$\mathcal{E}(\tilde{\omega}\gg 1)\approx 1-\frac{2}{3\pi\tilde{\omega}}$$ Hydrodynamics

$$\mathcal{E}(\tilde{\omega}\gg 1)\approx C_\infty^{-1}\tilde{\omega}^{4/9}$$ Free streaming

Universal non-equilibrium attractor

Pre-equilibrium description connects initial to hydro in HICs

$$(\tau^{4/3}e)_{\tilde{\omega}}=\left(4\pi\frac{\eta T_{eff}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{\frac{1}{9}}(\tau e)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}}=(\tau\Delta n_f)_0$$

Two-way

Provide input for hydrodynamics

Learn the past !(pre-eq, initial)

Giacalone, Mazeliauskas, Schlichting PRL 123(2019) 26
XD, Schlichting, arXiv: 2012.09068

Pre-equilibrium QGP trajectory

Fix the final equilibrium quantities

From EKT: entropy

$$(\tau s)_{eq} = \frac{\tau\left(e + p - \sum_f \mu_f \Delta n_f\right)}{T}$$

From data: charged particle multiplicity

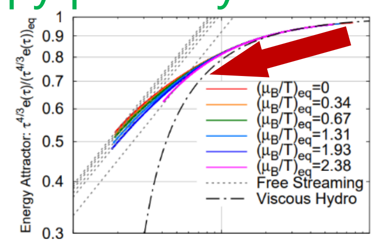
$$\frac{dN_{ch}}{d\eta} = \frac{N_{ch}}{JS}(\tau s)_{eq} S_T \approx 0.12(\tau s)_{eq} S_T$$

From EKT: net baryon number

$$\Delta n_B = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

From data: entropy per baryon

$$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B}\right)_{eq}$$



Learn the pre-equilibrium QGP

1. Apply non-equilibrium attractor

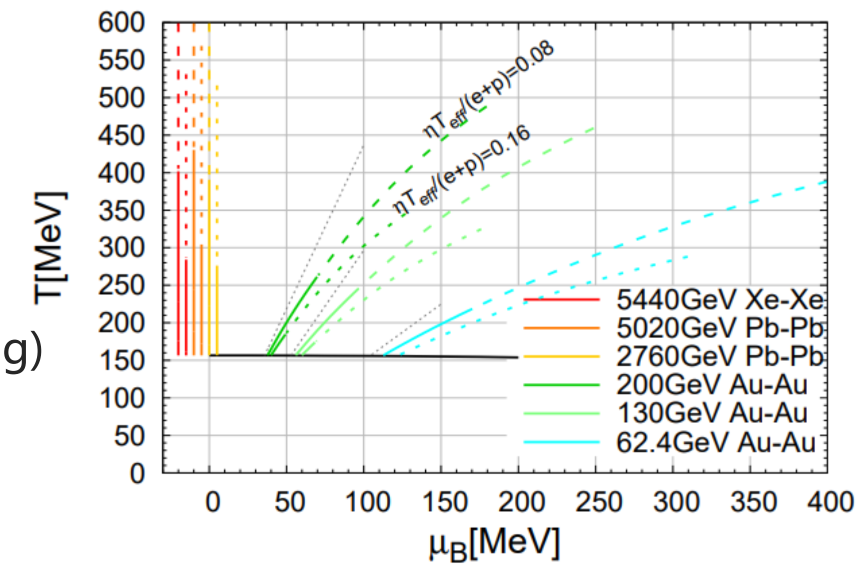
$$\left(\tau^{4/3} e\right)_{\tilde{\omega}} = \mathcal{E}(\tilde{\omega})\left(\tau^{4/3} e\right)_{eq}$$

$$\left(\tau \Delta n_f\right)_{\tilde{\omega}} = \left(\tau \Delta n_f\right)_{eq}$$

2. Define effective T and μ_B (Landau matching)

Non-equilibrium QGP trajectory

(at large baryon density)



XD, Schlichting, arXiv: 2012.09068

Pre-equilibrium di-lepton production

Electromagnetic probes

Coquet, XD, Ollitrault, Schlichting, Winn, arXiv: 2104.07622

Produced through-out HICs, not interacted with QGP, photon, di-lepton

Di-lepton production proportional to exp(-M/T), important at early stage of HICs

$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x,p_1) f_{\bar{q}}(x,p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-} \delta^{(4)}(K - P_1 - P_2)$$

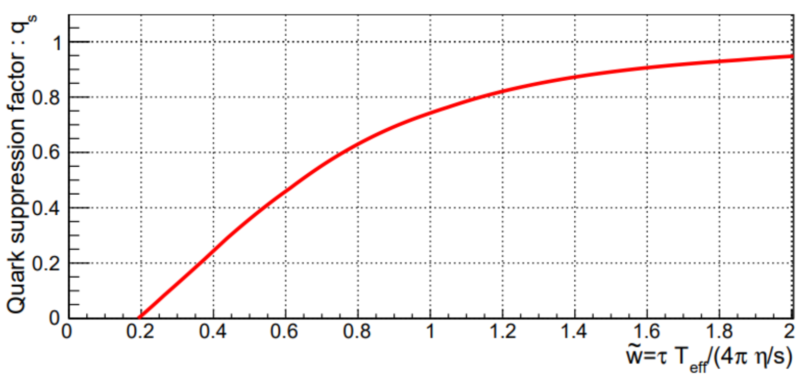
Pre-equilibrium quark suppression

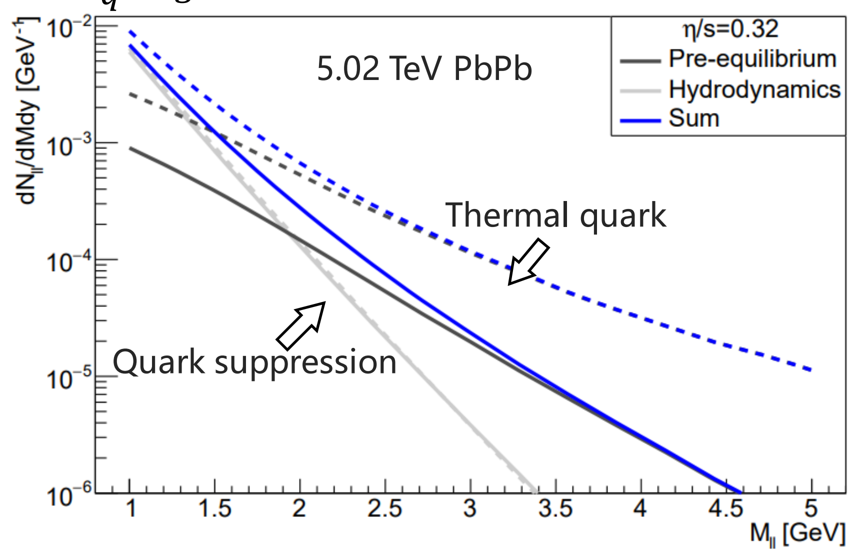
- Thermal di-lepton production: $f_{q/\bar{q}}(x,p)$ Fermi-Dirac distribution
- Pre-equilibrium di-lepton production: $q(\tau) = \frac{e_g^{eq}}{e_q^{eq}} \frac{e_q(\tilde{\omega})}{e_g(\tilde{\omega})}$

Chemical equilibration of QGP

Quark abundance increases

(YM plasma → QCD plasma)





Conclusions

- **Pre-equilibrium QCD plasmas**
  - QCD effective kinetic theory numerical solver at finite density

- **Turbulence in pre-equilibrium QCD plasmas**
  - Self-similar scaling equilibration
  - Kolmogorov-Zakharov spectrum
  - Bottom-up thermalization

- **Pre-equilibrium QCD plasmas and early stage of HICs**
  - Non-equilibrium effective constitutive relation from EKT
  - Universal attractor solution and its applications:
  - Pre-equilibrium QGP Trajectory in HICs
  - Pre-equilibrium di-lepton production in HICs

Based on
[1] XD, Schlichting, arXiv: 2012.09068
[2] XD, Schlichting, arXiv: 2012.09079
[3] Coquet, XD, Ollitrault, Schlichting, Winn, arXiv: 2104.07622