Collective flow in single hit QCD kinetic theory

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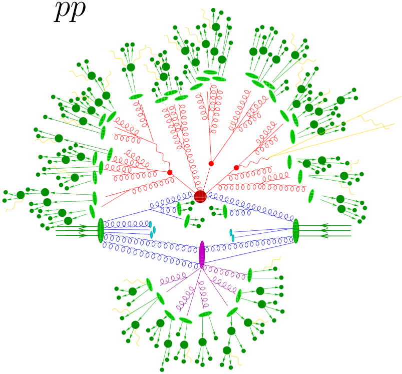
16-04-2021

European Research Council
Established by the European Commission
Grant:collectiveQCD

arXiv:2104.08179
Aleksi Kurkela, Aleksas Mazeliauskas, RT

TWO PICTURES OF HADRON COLLISIONS

pp



SHERPA, JHEP 02 (2009)

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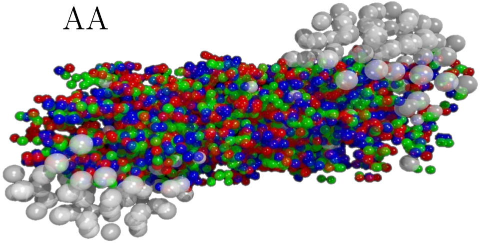
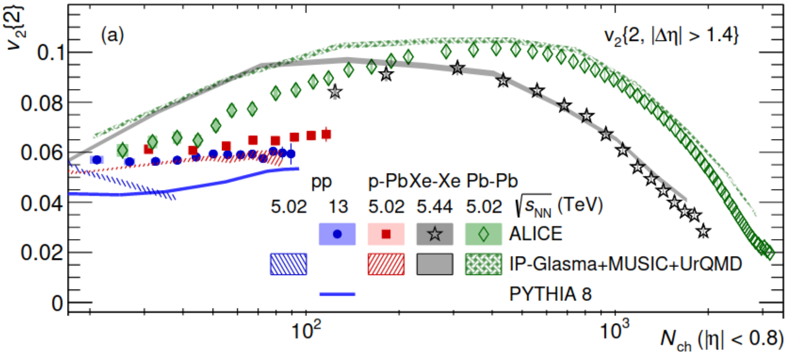


UrQMD, Marcus Bleicher

COLLECTIVITY IN SMALL SYSTEMS

ALICE, Phys.Rev.Lett. 123 (2019)



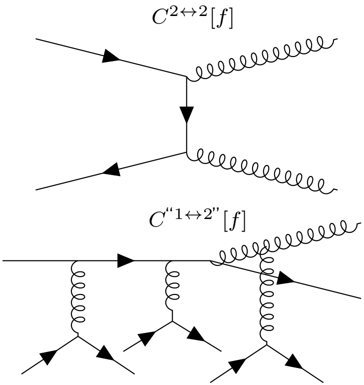
QCD EFFECTIVE KINETIC THEORY

- Boltzmann equation

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})\, f(\mathbf{x}, \mathbf{p}, t) = -C[f(\mathbf{x}, \mathbf{p}, t)].$$

  Relevant LO scattering processes[1]:
  - $2 \leftrightarrow 2$ elastic scattering.
  - "$1 \leftrightarrow 2$" collinear radiation.
- Out-of-equilibrium QGP description.
- Bottom-up thermalization in large systems[2].

We use EKT to study collective flow in small systems.



[1] P. Arnold, G.D. Moore, and L.G. Yaffe, Journal of High Energy Physics (2003).
[2] R. Baier, A.H. Mueller, D. Schiff, D.T. Son, Phys. Lett. B 502 (2001)

SINGLE HIT APPROXIMATION

Expand in the number of scatterings: $f = f^{(0)} + f^{(1)} + \dots$.

Free streaming $\quad (\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^{(0)} = 0.$

1st scattering $\quad (\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f^{(1)} = -C[f^{(0)}].$

$$f^{(1)}(\tau) = -\int_{\tau_0}^{\tau} d\tau' C[f^{(0)}](\tau').$$

INITIAL CONDITIONS

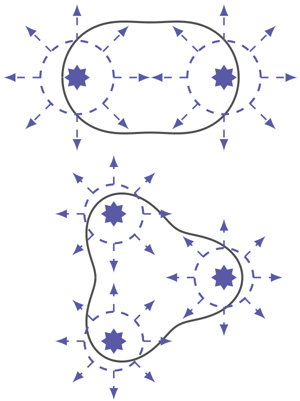
Linearized perturbation of symmetric background,

$$f^{(0)} = \bar{f} + \delta f = \bar{f}(\mathbf{p}, \mathbf{x}) \left(1 + \epsilon \frac{r^n}{R_0^n} \cos(n\phi_{\mathbf{x}})\right).$$

CGC motivated momentum distribution,

$$\bar{f} = \frac{A}{p_\xi} e^{-\frac{2p_\xi^2}{3}}, \quad p_\xi \equiv \frac{\sqrt{p_\perp^2 + \xi^2 p_z^2}}{Q(\mathbf{x}_\perp)}, \quad Q(\mathbf{x}_\perp) = Q_0 e^{-\frac{|\mathbf{x}_\perp|^2}{4R_0^2}}.$$

Initially $\bar{f}$ isotropic in $\mathbf{p}_\perp$.



ANISOTROPIC ENERGY FLOW

We find Fourier harmonics of transverse energy flow

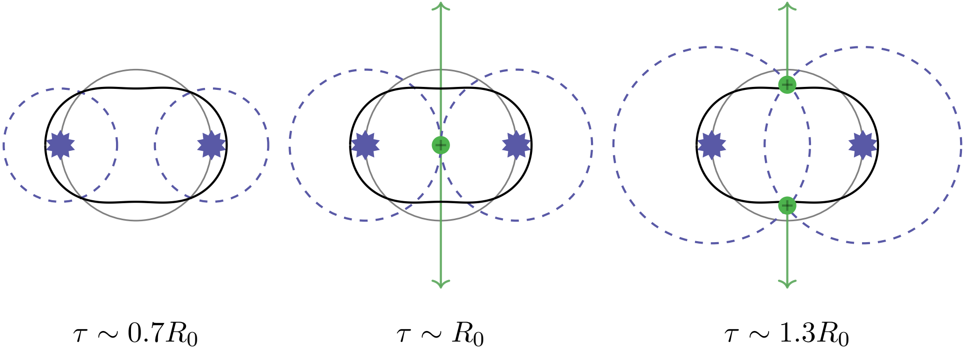
$$\frac{dE_\perp}{d\eta d\phi_p} = \tau \int d^2\mathbf{x}_\perp \int \frac{p_\perp dp_\perp dp_z}{(2\pi)^3} p_\perp f(\tau, \mathbf{x}, \mathbf{p}) = \frac{dE_\perp}{2\pi d\eta}\left(1 + 2\sum_{n=1} v_n \cos[n(\phi_\mathbf{p})]\right).$$

In single hit approximation

$$v_n = -\int_{\tau_0}^{\tau} d\tau' \tau' \int \frac{d^2\mathbf{x}_\perp}{2\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p_\perp \cos[n(\phi_\mathbf{p})]\delta C_{2\leftrightarrow 2}\left[\bar{f}, \delta f\right]}{dE_\perp/(2\pi d\eta)|_{\bar{f}}}.$$

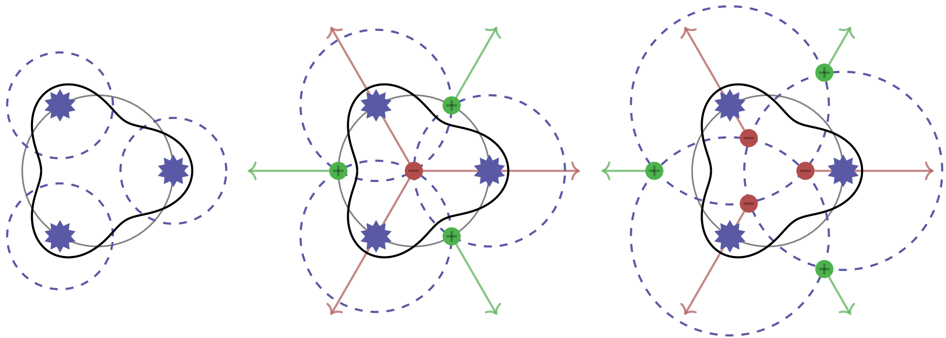
ESCAPE MECHANISM

ELLIPTIC FLOW



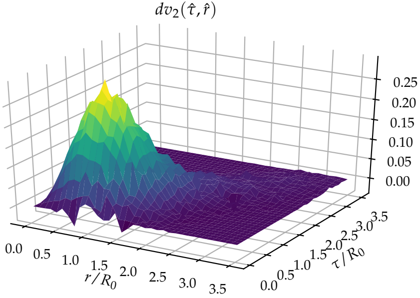
ESCAPE MECHANISM

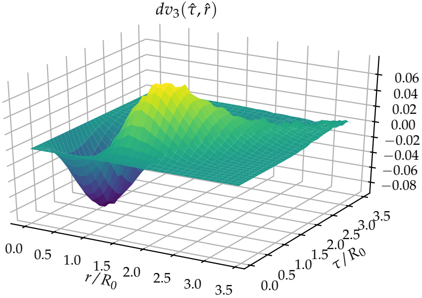
TRIANGULAR FLOW



$\tau \sim 0.7R_0$ $\tau \sim R_0$ $\tau \sim 1.3R_0$

COLLECTIVE FLOW RESPONSE





$$v_n = \int_{\tau_0/R_0}^{\infty} d\hat{\tau}' \int d\hat{r}\; dv_n(\hat{\tau}', \hat{r})$$

CONFORMAL SCALING

Due to conformal scaling parameters appear only in certain combinations

$$\hat{R} = \lambda^2 R_0 Q_0 \hat{A}, \quad \hat{A} \equiv \frac{A\tau_0}{\xi R_0}, \quad \hat{m}_g^2 = \frac{m_g^2}{Q_0^2}\frac{\tau_0}{R_0}.$$

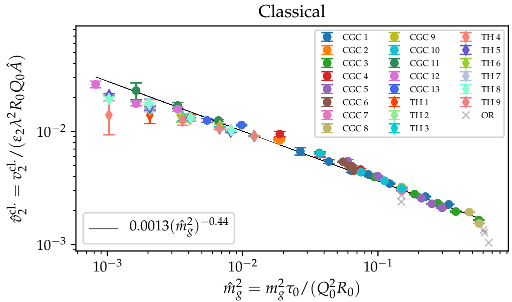
EKT single hit flow response given by the scaling formula

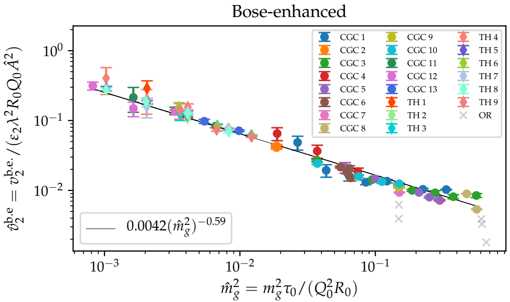
$$\frac{v_n}{\varepsilon_n} = \hat{R}\left[\hat{v}_n^{\mathrm{cl.}}(\hat{m}_g^2) + \hat{A}\,\hat{v}_n^{\mathrm{b.e.}}(\hat{m}_g^2)\right].$$

- $\hat{v}_n^{\mathrm{cl.}}$ - contribution from classical loss/gain term.
- $\hat{v}_n^{\mathrm{b.e.}}$ - contribution from Bose-enhanced loss/gain term.

CONFORMAL SCALING

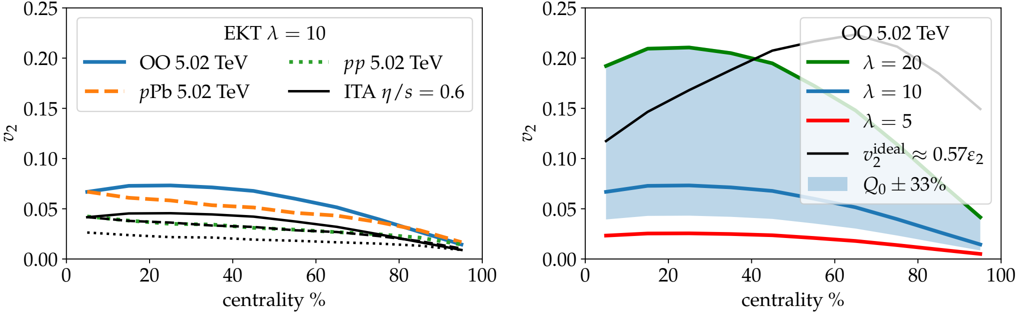
Test scaling for many different configurations with numerical simulations.





$$\frac{v_n}{\varepsilon_n} = \hat{R}\left[\hat{v}_n^{\mathrm{cl.}}(\hat{m}_g^2) + \hat{A}\,\hat{v}_n^{\mathrm{b.e.}}(\hat{m}_g^2)\right]$$

ELLIPTIC FLOW IN SMALL SYSTEMS



$$v_2 \approx \varepsilon_2 \hat{R} \left[1.3(\hat{m}_g^2)^{-0.44} + \hat{A} 4.2(\hat{m}_g^2)^{-0.59} \right] \cdot 10^{-3}$$

Pocket formula used with realistic input parameters for small systems.

CONCLUSIONS

- First study of QCD EKT in small systems.
- Non-trivial scaling properties of the collective flow.
- Identified both similarities and differences between toy models.
- Reasonable order of magnitude of v_2, smaller than ideal hydrodynamics.

Outlook:

- $p_\perp$-dependence.
- v_3, v_4, ...
- Higher order in scatterings.

CONCLUSIONS

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Outlook:

- $p_\perp$-dependence.
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Thank you for listening.

BACKUP: KINETIC THEORY

Defines collision term,

$$\left(\partial_t + \overline{v} \cdot \nabla_{\overline{x}}\right) f = -C^{2\leftrightarrow 2}[f] - C^{''1\leftrightarrow 2''}[f]$$

where e.g.

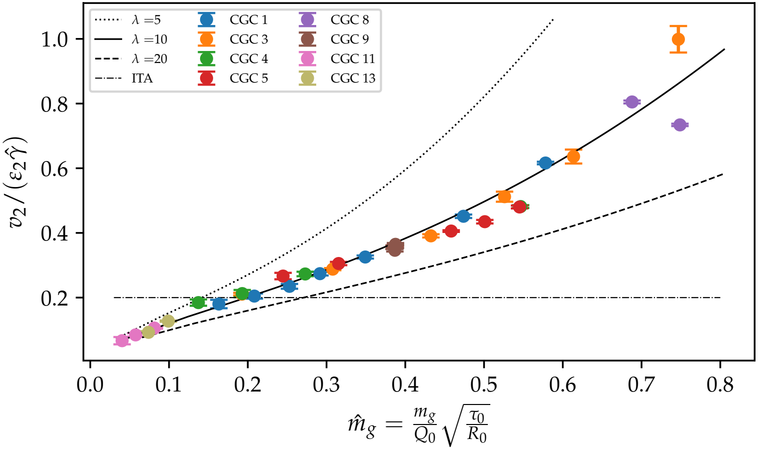
$$C^{s}_{2\leftrightarrow 2}[f](\bar{\mathbf{p}}) = \frac{1}{2}\frac{1}{\nu_s}\frac{1}{4}\sum_{abcd}(2\pi)^3\int_{\mathbf{pkp'k'}}|\mathcal{M}^{ab}_{cd}|^2(2\pi)^4 \times \delta^{(4)}(P+K-P'-K')$$
$$\times \left\{(f^a_{\mathbf{p}} f^b_{\mathbf{k}}(1 \pm f^c_{\mathbf{p'}})(1 \pm f^d_{\mathbf{k'}})) - (f^c_{\mathbf{p'}} f^d_{\mathbf{k'}}(1 \pm f^a_{\mathbf{p}})(1 \pm f^b_{\mathbf{k}}))\right\}$$
$$\times \left[\delta(\bar{\mathbf{p}}-\mathbf{p})\delta^{as} + \delta(\bar{\mathbf{p}}-\mathbf{k})\delta^{bs} - \delta(\bar{\mathbf{p}}-\mathbf{p'})\delta^{cs} - \delta(\bar{\mathbf{p}}-\mathbf{k'})\delta^{ds}\right]$$

BACKUP: LINEAR

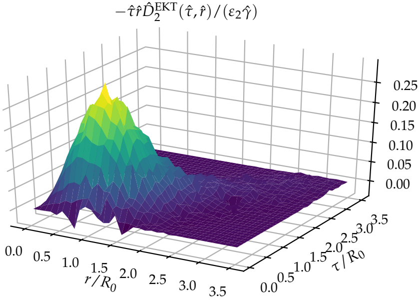
$$v_n = -\int_{\tau_0}^{\tau} d\tau'\tau' \int r dr D_n(\tau', r)$$

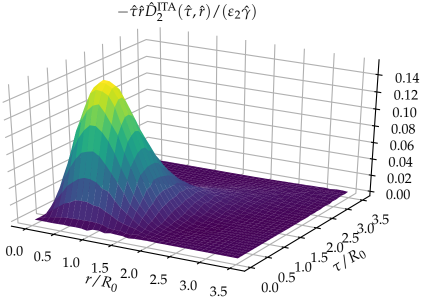
$$D_n(\tau, r) = \epsilon \frac{1}{16\nu_g} \left(\frac{dE_\perp}{2\pi d\eta}\bigg|_{\bar{f}} \right)^{-1} \int_{\mathbf{pkp'k'}} |\mathcal{M}(\bar{m}_g^2)|^2 (2\pi)^4 \delta^{(4)}(P + K - P' - K')$$
$$\times \left[\frac{\delta|\mathcal{M}(\bar{m}_g^2, \delta m_g'^2)|^2}{|\mathcal{M}(\bar{m}_g^2)|^2} \left\{ \bar{f}_{\mathbf{p}} \bar{f}_{\mathbf{k}} [1 + \bar{f}_{\mathbf{p'}}][1 + \bar{f}_{\mathbf{k'}}] - \bar{f}_{\mathbf{p'}} \bar{f}_{\mathbf{k'}} [1 + \bar{f}_{\mathbf{p}}][1 + \bar{f}_{\mathbf{k}}] \right\} \right.$$
$$\times \left(p_\perp \cos(n\phi_{\mathbf{p}}) + k_\perp \cos(n\phi_{\mathbf{k}}) - k'_\perp \cos(n\phi_{\mathbf{k'}}) - p'_\perp \cos(n\phi_{\mathbf{p'}}) \right)$$
$$+ \left\{ \bar{f}_{\mathbf{p}} \bar{f}_{\mathbf{k}} [1 + \bar{f}_{\mathbf{p'}}][1 + \bar{f}_{\mathbf{k'}}] \left(\frac{\bar{f}_{\mathbf{p'}}}{1 + \bar{f}_{\mathbf{p'}}} C_{\mathbf{p'}} + \frac{\bar{f}_{\mathbf{k'}}}{1 + \bar{f}_{\mathbf{k'}}} C_{\mathbf{k'}} + C_{\mathbf{p}} + C_{\mathbf{k}} \right) \right.$$
$$\left. \left. - \bar{f}_{\mathbf{p'}} \bar{f}_{\mathbf{k'}} [1 + \bar{f}_{\mathbf{p}}][1 + \bar{f}_{\mathbf{k}}] \left(C_{\mathbf{p'}} + C_{\mathbf{k'}} \frac{\bar{f}_{\mathbf{p}}}{1 + \bar{f}_{\mathbf{p}}} C_{\mathbf{p}} + \frac{\bar{f}_{\mathbf{k}}}{1 + \bar{f}_{\mathbf{k}}} C_{\mathbf{k}} \right) \right\} \right].$$

BACKUP: EKT VS ITA



BACKUP: EKT VS ITA





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