Neutrino Physics

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1. *history of neutrinos, ancient and modern*
2. *oscillations in quantum mechanics*
   - (why can one use a Schrodinger Eqn ?)
3. fermions, spinors and Lorentz-invariant m_ν
4. the scale of neutrino masses
5. leptogenesis ?

References (old)

other version of these lectures (2017 CERN school) :
https ://physicschool.web.cern.ch/ESHEP/previous_eshep.html

Giunti website "neutrino unbound" : http ://www.nu.to.infn.it/

fits : http ://www.nu-fit.org/

Raffelt talks (astropart) :http ://wwwth.mpp.mpg.de/members/raffelt/

Plots thanks to Strumia + Vissani : hep-ph/0606054

simple 3-gen probabilities for LBL :Cervera etal 0002108 (+ later versions)

current state of oscillation measurements : Gonzalez-Garcia @ CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

neutrino cosmology : Lesgourgues at CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

(hypothetical/ /known) history of neutrinos (shy in the lab, relevant in cosmo)

- ...
- inflation (gives large scale CMB fluctuations) (?driven by sneutrino ?)
- baryogenesis (excess of matter over anti-matter)via leptogenesis ?
- relic density of (cold) Dark Matter (?heavy neutrinos ?)Shaposhnikov
- Big Bang Nucleosynthesis ($H, D, ^3He, ^4He, ^7Li$ at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \to H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?thanks to ν ?) spreading heavy elements

- 1930 : Pauli hypothesises the "neutrino", to conserve E in $n \to p + e(+\nu)$
- 1953 Reines and Cowan : neutrino CC interactions in detector near a reactor
- invention of the Standard Model (SM) : massless ν
-
- **neutrinos have mass ! There is more in the Lagrangian than the SM...**

Recent history of neutrinos and men

$\sim$ 1930 :predicting the neutrino :

observe β-decay : $(A, Z) \rightarrow (A, Z+1) + e^{-} (+\bar{\nu})$

(A,Z) = nucleus of A-Z neutrons, Z protons

e^{+} has a spectrum of momenta... ?

(if 2body decay in (A,Z) restframe : (A Z+1) and e^{-} backtoback)

Pauli hypothesises wee neutral "neutrino" to conserve $\vec{p}$

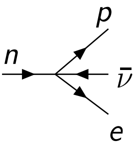
$\sim$ 1956 :confirming the neutrino

near a nuclear reactor (produces $\overline{\nu}$ flux)

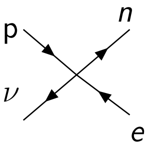
$(A, Z) \rightarrow (A, Z+1) + e + \overline{\nu}$)

Reines+Cowan detect $\overline{\nu} + p \rightarrow n + e^{+}$, $e^{+} + e^{-} \rightarrow \gamma\gamma$

$\Rightarrow \nu$ exist, and have only weak interactions



($t \longrightarrow$ in diagrams)



antiparticles

$E^2 - |\vec{p}|^2 = m^2 \Rightarrow E = \pm\sqrt{m^2 + |\vec{p}|^2}$

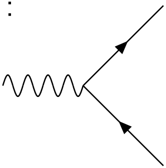
NR limit : $E \simeq m + |\vec{p}|^2/2m + \ldots$?where went -ve E solns ?

They are *antiparticles*, and *travel backwards in time*

($-\vec{p}$ in opposite spatial direction from $\vec{p}$)

but NB, retain causality : create antipart at $t = 0$:

$t \longrightarrow$



travels backwards-in-time from future

costs $+E$ in "my" frame

How to tell particle from antiparticle ?

charges reversed : electron=$e = e^- \leftrightarrow e^+ = \bar{e}$=positron

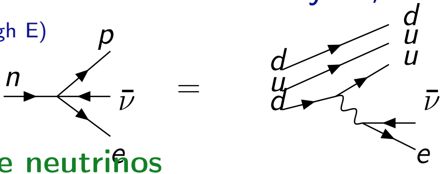
no conserved charge ? Maybe part=$\overline{\mathrm{part}}$ (like photon = γ)

Is neutrino its own antiparticle ?

• "weak" interactions are weak (at low energy) :
we stand on earth ; most ν go through
$\sim$2 second for ν to escape from sun, vs $\sim 10^3 \rightarrow 10^6$ years for γ
.

• weak interactions are mediated by W,Z

(on short distances/high E)



• at least three neutrinos

3 charged leptons= $\{e, \ \mu, \ \tau\}$. Observe each has own ν

(fermion wo strong int. ; (.5, 105, 1770 MeV)

$$\left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\}$$

Its not just zoology...

particle name $\leftrightarrow$ fn/operator of space-time pt, *eg* $\hat{\boldsymbol{\nu}}(\vec{\boldsymbol{x}}, \boldsymbol{t})$
called "field"(like Electromagnetism)

dynamics $\leftrightarrow$ 1. build Lagrangian $\mathcal{L}(\vec{x}, t)$ with fields
2. action $S = \int d^3x dt \mathcal{L}(\vec{x}, t)$, "dimensionless"
3. field is quantum :calculate amplitude

$$\mathcal{A}(\nu_1(x,t) \to \nu_2(x,t)) = \Sigma \text{ (interpolating field configs)} e^{iS}$$

(consistent with double-slit expt...) Feynman, QM via Path Integral

4. so : classical soln at min of S (constructive interference)

Lagrange Eqns $\Rightarrow$ EoM for field.

(**particle properties** $\leftrightarrow$ symmetries of Lagrangian)

To calculate in a theory, evaluate PI : $\sim$ perturb in cplg ctes.
Can read particle properties/interactions from $\mathcal{L}$.

Historical problems : neutrinos disappear...

solar ν prob. ($>$50 years, many expts)

sun ($T_{core} \sim 2$ keV, $T_{surf} \sim .5$ eV$\approx$6000 ^oK, $R \sim 6 \times 10^{10}$cm)
produces energy by a network of nuclear reactions

$$4H \rightarrow^4 He \quad (4p \rightarrow 2p + 2n + 2e^+ + 2\nu)$$

ν escape, γ diffuse to surface ($10^3 \rightarrow 10^6$yrs)
ν_e flux $\sim .3 \rightarrow .5$ expected from E output

Flux in $\sum$ flavours $\sim$ expected (SNO).

Nobel-winning plot # 2 : SNO
solar ν_e deficit, but expected $\sum \nu_\alpha$ flux(PRL 89 (2002) 011301)

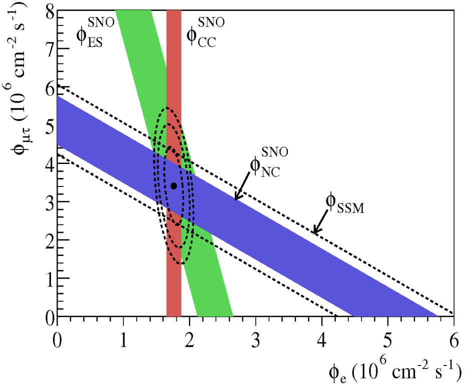
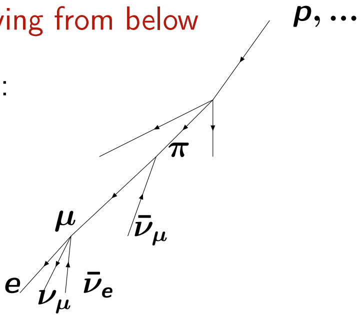


FIG. 3: Flux of ^{8}B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^{8}B flux as predicted by the SSM [11] (dashed lines) and that measured

Sudbury Neutrino Expt

Atmospheric ν problem : deficit of ν_μ arriving from below

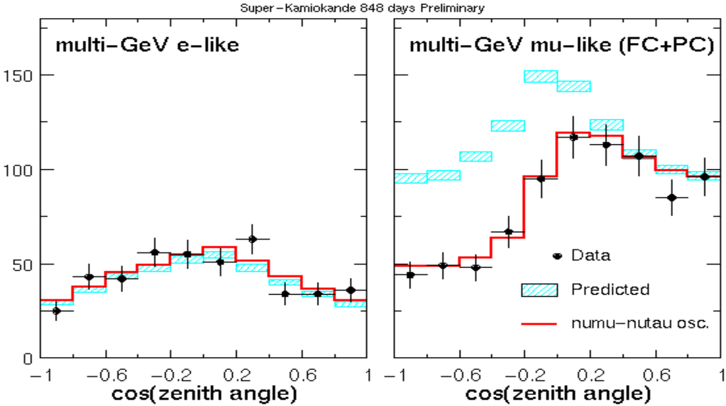


ν produced in cosmic ray interactions :
expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$

height atmosphere $\sim$ 10-100km,
$R_{earth} \sim$ 6000km

...see deficit of $\nu_\mu, \bar{\nu}_\mu$ from below

Nobel plot #1 : SK-98 :
$\nu_\mu + H_2 0 \to \mu + ..$, deficit in ν_μ from below (PRL 81 (1998) 1562-1567)



upwards $\leftrightarrow$ cos= -1 ; down $\leftrightarrow$ cos= + 1.
L : 20 km $\leftrightarrow$ 10 000 km.

Super-Kamioka-Nucleon-Decay-Expt

Oscillations of massive ν

a relativistic muon decays at the top of the atmosphere, produces a ν.
Suppose massive ν_2, ν_3, but not reconstruct $(E_\nu, \vec{k}_\nu)$ well enough to identify if ν is ν_3 or ν_2...
The ν travels to the SK detector, where it produces another μ

$\Rightarrow$ must sum in *amplitude* possibility to travel as ν_2 or ν_3
$\Leftrightarrow$ neutrino propagation is a quantum process

neutrinos "oscillate"(QM version : easy to rederive)

A relativistic neutrino, with momentum $\vec{k}$, is produced in muon decay at $t = 0$ (at Tokai/edge atmosphere). Describe as a quantum mechanical state :

$$|\nu(t=0)\rangle = |\nu_\mu\rangle$$

It travels a distance L in time t to the detector (SuperK)

$$|\nu(t)\rangle$$

where it produces an μ in CC scattering. With what probability ?

$$\mathcal{P}_{\mu\to\mu}(t) = |\langle\nu_\mu|\nu(t)\rangle|^2 \quad = \quad ?$$

1. Suppose massive neutrinos (two generations for simplicity). Flavour and mass eigenstates related by : $\nu_\alpha = U_{\alpha i}\nu_i$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

2. Suppose time evolution in the mass basis described by

$$i\frac{d}{dt}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{bmatrix} E_2 & 0 \\ 0 & E_3 \end{bmatrix}\begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \qquad , \quad E_i^2 = k^2 + m_i^2$$

3. If produce relativistic ν_μ at $t = 0$, then at t later :

$$|\nu(t)\rangle = \sum_j U_{\mu j}|\nu_j(t)\rangle = \sum_j U_{\mu j} e^{-iE_j t}|\nu_j\rangle$$

Amplitude for neutrino to produce charged lepton α in CC scattering in detector after t :

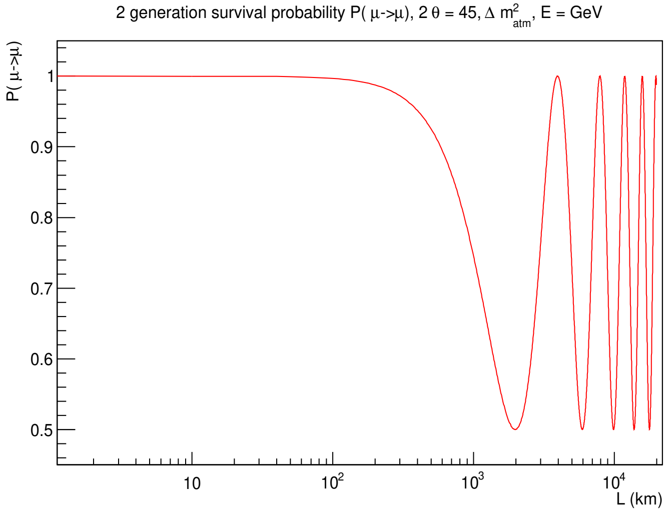
$$|\langle \nu_\alpha | \nu(t) \rangle| = \left| \sum_j U_{\mu j} e^{-iE_j t} U^*_{\alpha j} \right|$$

So in 2 generation case, using $t = L$, $E_3 - E_2 \simeq \frac{m_3^2 - m_2^2}{2E} \equiv \frac{\Delta^2_{32}}{2E}$:

$$\begin{aligned}
\mathcal{P}_{\mu\to\tau}(t) &= \left| \sin\theta \cos\theta \left(e^{i\Delta^2_{32}L/4E} - e^{-i\Delta^2_{32}L/4E} \right) \right|^2 \\
&= \sin^2(2\theta) \sin^2\left(L \frac{\Delta^2_{32}}{4E} \right)
\end{aligned}$$

$$\mathcal{P}_{\mu\to\mu}(t) = 1 - \sin^2(2\theta) \sin^2\left(L \frac{\Delta^2}{4E} \right) = 1 - \sin^2(2\theta) \sin^2\left(1.27 \frac{L}{km} \frac{\Delta^2}{\mathrm{eV}^2} \frac{\mathrm{GeV}}{4E} \right)$$

$E = \nu$ energy, L source-detector distance, $\Delta^2_{32} \sim 10^{-3}\mathrm{eV}^2$
$E \sim 10$ GeV for atmospheric νs ; $L : 20km \to 10000km$



$$\mathcal{P}_{\mu\to\mu}(L) = 1 - \sin^2(2\theta)\sin^2\left(1.27\frac{L}{km}\frac{\Delta^2}{\mathrm{eV}^2}\frac{\mathrm{GeV}}{4E}\right)$$

$$\begin{aligned} \Delta^2_{32} &= 2.5\times 10^{-3}\mathrm{eV}^2 \\ E &\sim 0.6\mathrm{GeV(T2K)} \\ &\sim \mathrm{MeV(reactors)} \\ &\sim 10\mathrm{GeV(atmosphe}\end{aligned}$$

Schrodinger Eqn for relativistic particles?
is ok : have Eqn for the number operator $\hat{n}_p \equiv \hat{a}_p^\dagger \hat{a}_p$:

$$i\frac{\partial}{\partial t}\hat{n} = [\hat{H}, \hat{n}]$$

...take expectation values and get QM version.
quantum coherence over km?
• $m_\nu \ll$, so $\Delta_{expt}\sqrt{E_\nu^2 - |\vec{p}_\nu|^2} \gg m_\nu$ (decoherence slide)
• recall ν only interact *weakly*, can cross earth without interaction (no "observations" to collapse wavefns)
But...there is forward scattering $\Rightarrow$ effective contribution to m_ν from matter in sun, earth and supernovae (more later, maybe)

decoherence of neutrinos for large $L/E \gg 1/\Delta^2$

- at production, 2 superposed wavepackets of masses m_2,m_3.
- group velocity of packets

$$v_i = \frac{\partial E}{\partial p} = \frac{p}{E} \simeq 1 - \frac{m_i^2}{2E^2}$$

- after distance L, packets have separated by

$$(v_2 - v_3)L \simeq \frac{\Delta_{23}^2}{E^2} L \simeq \frac{L}{\ell_{osc}} \frac{1}{E}$$

- no interference if larger than size of packets $\sim 1/(\delta E)$ where packet energy uncertain by δE. so no oscillations once

$$\frac{L}{\ell_{osc}} \gtrsim \frac{E}{\delta E}$$

can make similar estimate doing sum over paths, phases should sum coherently

Massive ν in the Standard Model

From antique 2-flavour QM calculation and astro problems to $\geq$ three light ν in a lively exptal programme using reactors, accelerators and astro sources

What masses ?

oscillations say there are mass differences : (global fits of www.nu-fit.org)

$$
\begin{aligned}
|\Delta^2_{atm}| = |\Delta^2_{3j}| &= |m_3^2 - m_j^2| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2 \\
&\gg \Delta m^2_{21} \simeq 7.50 \pm 0.2 \times 10^{-5}\ \mathrm{eV}^2 \\
\sqrt{\Delta m^2_{31}} &\simeq 0.05\ \mathrm{eV} \qquad\qquad \sqrt{\Delta m^2_{21}} \simeq 0.008\ \mathrm{eV}
\end{aligned}
$$

mass scale $\lesssim$ eV from

- cosmology : massive ν are DM today, and affect CMB.
- spectrum of e in β decay : Katrin expt
- $0\nu2\beta$... if ν own antiparticle

And there are mixing angles

In 2 flavour, wrote :

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

but there are three lepton flavours in SM, should write

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

U

Can write as :

$$U_{\alpha i} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}}_{\text{atm.} + \text{LBL disa.}} \underbrace{\begin{bmatrix} c_{13} & & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}}_{\text{reac.disa.} + \text{LBL app.}} \underbrace{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{sol} + \text{reac.disa.}} P$$

$$= \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix} P$$

$$\theta_{23} \simeq \pi/4 \pm \pi/40 \quad \theta_{12} \simeq \pi/6 \quad \theta_{13} \simeq 8^o$$

(global fits of www.nu-fit.org)

Where to put U in SM ?

Previously wrote

$$\left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\}$$

write ν in mass eigenstates too(propagate eigenstates of Hamiltonian...)

$$\ell^e_L \equiv \begin{pmatrix} U_{ei}\nu^i_L \\ e_L \end{pmatrix} , \ \ell^\mu_L \equiv \begin{pmatrix} U_{\mu j}\nu^j_L \\ \mu_L \end{pmatrix} , \ell^\tau_L \equiv \begin{pmatrix} U_{\tau k}\nu^k_L \\ \tau_L \end{pmatrix}$$

3×3 mixing matrix $U_{\alpha,i}$ appears at $W^\pm$ vertices (like CKM)

$$\to \quad -i\,\frac{gU^*_{ej}}{\sqrt{2}}\overline{\nu^j_L}\gamma^\mu W^+_\mu e_L + ...$$

but flavour-diagonal Z vertex :

$$\propto \sum_\alpha -i\,\frac{g}{2}U^*_{\alpha j}\overline{\nu^j_L}\gamma^\mu Z^+_\mu U_{\alpha k}\nu^k_L = \delta_{jk}\frac{g}{2}\overline{\nu^j_L}\gamma^\mu Z^+_\mu \nu^k_L$$

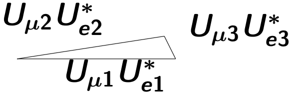
The drunken Unitarity triangle

Not hear much about "leptonic unitarity triangle"
1.not measure elements at tree in CC
2. Also, it drinks.
Amplitude to oscillate from flavour α to β over distance L :

$$\mathcal{A}_{\alpha\beta}(L) = U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{\alpha 3} U^*_{\beta 3} e^{-i(m_3^2 - m_1^2)L/(2E)}$$

at $L = 0$ unitarity : $\Rightarrow \mathcal{A}_{\alpha\beta} = 1$ for $\alpha = \beta$
$\mathcal{A}_{\alpha\beta} = 0$ for $\alpha \neq \beta$

$\Leftrightarrow$ unitarity triangle(in complex plane)



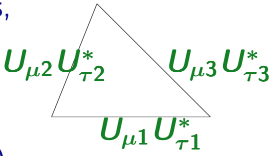
At $L = t \neq 0$, two of the vectors rotate in the complex plane,
with frequencies $(m_j^2 - m_1^2)/2E$
oscillations $\leftrightarrow$ time-dependent non-unitarity

About two- flavour analyses : atm/LBL ν_μ disappearance

Amplitude to oscillate from flavour μ to τ over distance L :

$$\mathcal{A}_{\mu\tau}(L) = U_{\mu 1} U^*_{\tau 1} + U_{\mu 2} U^*_{\tau 2} e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{\mu 3} U^*_{\tau 3} e^{-i(m_3^2 - m_1^2)L/(2E)}$$

At $L \sim E/(m_3^2 - m_1^2)$, vector "3" rotates, frequency $(m_j^2 - m_1^2)/2E$



$\Rightarrow$ "Atmospheric" neutrinos, also LBL (ν_μ disappearance via Δm^2_{31} oscillations) :

$$\mathcal{A}_{\mu\tau}(L) \simeq U_{\mu 1} U^*_{\tau 1} + U_{\mu 2} U^*_{\tau 2} + U_{\mu 3} U^*_{\tau 3} e^{-i(m_3^2 - m_1^2)L/(2E)}$$

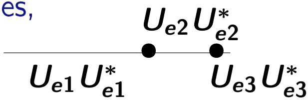
$U_{\mu 3} U^*_{\tau 3}$ oscillates on timescale $t = L \sim (m_3^2 - m_1^2)/E$
$U_{\mu 2} U^*_{\tau 2} \sim$ stationary, measure θ_{23}

About two- flavour analyses : solar and Kamland

Amplitude to oscillate from flavour e to e over distance L :

$$\mathcal{A}_{ee}(L) = U_{e1}U^*_{e1} + U_{e2}U^*_{e2}e^{-i(m_2^2-m_1^2)L/(2E)} + U_{e3}U^*_{e3}e^{-i(m_3^2-m_1^2)L/(2E)}$$

At $L \sim 2E/(m_2^2 - m_1^2)$, vector 2 rotates,
frequency $(m_2^2 - m_1^2)/2E$
vec. 3 spins rapidly



$\Rightarrow$ "Solar" + "KamLAND" (reactor $\overline{\nu_e}$ for $L \sim 100$ km) neutrinos
$\Leftrightarrow$ ν_e disappearance over long baselines $L \sim (m_2^2 - m_1^2)/2E$
two-ν approx works because θ_{13} is small ($U_{e3} = sin\theta_{13}$) :

$$\mathcal{A}_{ee} \simeq |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2-m_1^2)\tau/(2E)}$$

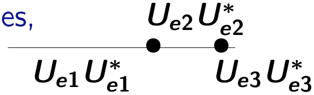
measure θ_{12}

About two- flavour analyses : θ_{13} at reactors

Amplitude to oscillate from flavour e to e over distance L :

$$\mathcal{A}_{ee}(L) = U_{e1}U_{e1}^* + U_{e2}U_{e2}^* e^{-i(m_2^2-m_1^2)L/(2E)} + U_{e3}U_{e3}^* e^{-i(m_3^2-m_1^2)L/(2E)}$$

At short enough L, only third vector rotates, frequency $(m_3^2 - m_1^2)/2E$



$\Rightarrow$ reactor θ_{13} by $\overline{\nu_e}$ disappearance ; select short baseline such that only $|U_{e3}(t)|^2$ moves

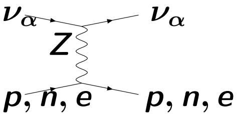
$$\begin{aligned} \mathcal{A}_{ee} &\simeq (|U_{e1}|^2 + |U_{e2}|^2) + |U_{e3}|^2 e^{-i(m_3^2-m_1^2)L/(2E)} \\ &= c_{13}^2(c_{12}^2 + s_{12}^2) + s_{13}^2 e^{-i(m_3^2-m_1^2)L/(2E)} \end{aligned}$$

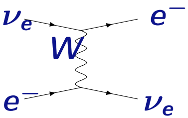
Flavour transition in matter

oscillations and adiabatic

Flavour transitions in matter

Coherent forward scattering of ν in matter give extra contribution to the Hamiltonian :





To see : use $\mathcal{H}_{\mathrm{mat}} = \mathcal{H}_0 + \mathcal{H}_{int}$ in QFT oscillation derivation,

$$\mathcal{H}_{int} \simeq 2\sqrt{2}G_F \int d^4x(\overline{\hat{\nu}_e}(x)\gamma^\alpha P_L \hat{\nu}_e)(\overline{\hat{e}}\gamma_\alpha P_L \hat{e}(x))$$

evaluated in a medium with electrons (NC irrelevant ; same for all ν generations = add unit matrix to H. And no μ or τ in the matter.)

$$\langle \mathrm{medium}|\overline{e}\gamma_\alpha P_L e(x)|\mathrm{medium}\rangle \to \delta_{\alpha 0}\frac{n_e}{2}$$

H_{mat} in flavour basis $(\nu_e, (\nu_\tau - \nu_\mu)/\sqrt{2})$, $V_e = \sqrt{2}G_F n_e$:

$$H_{\mathrm{mat}} = \ldots + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \Delta^2/(2E) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix}$$

Oscillations in matter — ctd

$H_{\rm mat}$ in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$:

$$H_{\rm mat} = \ldots + \begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V_e & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix}$$

With $U_{mat}^T H_{mat} U_{mat}^* =$ diagonal :

$$\tan(2\theta_{\rm mat}) = \frac{\Delta^2 \sin(2\theta_{21})}{2EV_e - \Delta^2\cos(2\theta_{21})} \xrightarrow{2EV_e \to \Delta^2 c2\theta} \text{large}$$

$$\Delta^2_{\rm mat} = \sqrt{(\Delta^2 c2\theta - 2EV)^2 + (\Delta^2 s2\theta)^2}$$

- for $V_e \ll \frac{\Delta^2}{2E}\cos(2\theta_{21})$, matter effects negligeable
- $\theta_{mat} \to \pi/4$ ("resonance") at $V_e = \frac{\Delta^2}{2E}\cos(2\theta_{21})$
- $V \gg \frac{\Delta^2}{2E}\cos(2\theta_{21})$: $\nu_e \sim$ mass eigenstate

What is V_e ?

$$
\begin{aligned}
H_{\rm mat} &= \ldots + \begin{bmatrix} -\frac{\Delta^2}{4E}\cos 2\theta + V_e & \frac{\Delta^2}{4E}\sin 2\theta \\ \frac{\Delta^2}{4E}\sin 2\theta & \frac{\Delta^2}{4E}\cos 2\theta \end{bmatrix} \\
\tan(2\theta_{\rm mat}) &= \frac{\Delta^2 \sin(2\theta_{21})}{2EV_e - \Delta^2\cos(2\theta_{21})} \\
\Delta m^2_{21} &\simeq 7.5 \pm \times 10^{-5}\ {\rm eV}^2 \\
V_e &= \sqrt{2} G_F n_e \simeq 8\ {\rm eV} \frac{\rho Y_e}{10^{14} g/cm^3} \\
Y_e &= \frac{n_e}{n_n + n_p}\ ,\ \rho = \begin{cases} 10 g/cm^3 & \text{earth} \\ 100 g/cm^3 & \text{sun} \\ 10^{14} g/cm^3 & \text{SN} \end{cases}
\end{aligned}
$$

For $\bar{\nu}$ V_e of opposite sign ! (because $\langle out|\bar{\hat{\nu}}\hat{\nu}|in\rangle \sim \langle out|\hat{a}^\dagger \hat{a} + \hat{b}\hat{b}^\dagger|in\rangle$)

$\Rightarrow$ solar matter effect for ν_e,not $\bar{\nu}_e$, fixes sign of $m_2^2 - m_1^2 > 0$.

Mass scale

First of 3 probes of the mass scale :cosmology

• a late contribution to DM in cosmology :

relic ν free-stream til they become non-rel. (after recomb. for $\Sigma \lesssim$ eV), then contribute to DM $\propto \sum_i |m_i| \equiv \Sigma$.

• Σ has effects on CMB :

Relativistic $\rightarrow$ non-rel transition affects CMB propagation...parameter in cosmological fits :

Lesgourgues book

$$\begin{array}{lll} \sum & \lesssim & 0.1 \rightarrow .6\ eV \quad now:\ PLANCK, +LSS/Ly\alpha\ (in\ \Lambda CDM) \\ & \lesssim & 0.6\ eV \qquad now:\ PLANCK + BAO\ (in\ 12\ param\ \Lambda CDM \\ & \rightarrow & \lesssim 2m_{atm} \qquad cosmo.indep.\ (Planck + EUCLID...) \\ & & \sim m_{atm} \qquad \Lambda CDM \end{array}$$

DiValentino etal
1507.06646

beta decay

m_ν^2 distorts e spectrum in $n \to p + e + \bar{\nu}$ $\Leftrightarrow$bound

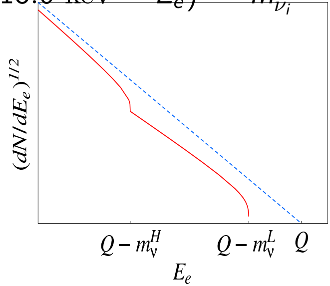
Consider Tritium β decay :

$^3H \to ^3 He + e + \bar{\nu}_e$, $Q = E_e + E_\nu = 18.6\mathrm{eV}$

where $E_e = Q - E_\nu \le Q -$ "m_{e_ν}"

Endpoint of e spectrum :

$$\frac{dN_e}{dE_e} \propto \sum_i |U_{ei}|^2 \sqrt{(18.6\ \mathrm{keV} - E_e)^2 - m_{\nu_i}^2}$$



Current bound : $m_{\nu_e} \lesssim 2$ eV, Katrin sensitivity ~ 0.3 eV.

Single β decay kinematically forbidden for some nuclei
(eg ${}^{76}_{32}Ge$ lighter than ${}^{76}_{33}As$, so ${}^{76}_{32}Ge \to^{76}_{34} Se + ee\bar{\nu}_e\bar{\nu}_e$. $\tau \sim 10^{21}$ yrs)

for majorana neutrinos, or other LNV, but not Dirac neutrinos.

Neutrino*less* double beta decay : $(Z, A) \rightarrow (Z+2, A) + 2e$