Soft - Collinear Effective Theory (SCET)

Bauer, Fleming, Pirjol, Stewart 2000, ...

Outline

* Lecture 1
  - Introduction
  - Soft Effective Theory
  - Factorization of soft γ's in QED

* Lecture 2
  - Momentum regions in the Sudakov form factor
  - Soft-Collinear Effective Theory

* Lecture 3

  - Soft collinear interactions
  - Vector current in SCET
  - Factorization & Sudakov FF
  - PDF factorization in DIS

Time permitting....

" It is better to uncover a little than to cover a lot. "

V. Weisskopf

Summary of Wednesday:

$$k^\mu = n\cdot k \,\frac{\bar n^\mu}{2} + \bar n \cdot k\, \frac{n^\mu}{2} + k_\perp^\mu$$

Scalings:

		$(n\cdot k\,,\ \bar n\cdot k\,,\ k_\perp)$
hard	h	$(1\,,\ 1\,,\ 1)\,Q$
collinear to p^μ	c	$(\lambda^2\,,\ 1\,,\ \lambda)\,Q$
collinear to ℓ^μ	$\bar c$	$(1\,,\ \lambda^2\,,\ \lambda)\,Q$
soft	s	$(\lambda^2\,,\ \lambda^2\,,\ \lambda^2)\,Q$

Split QED fields into modes

$$\psi \longrightarrow \overset{\lambda^1}{\underset{\substack{\uparrow \\ \frac{\not n \not{\bar n}}{4}\psi_c}}{\xi_c}} + \overset{\lambda^2}{\underset{\substack{\uparrow \\ \frac{\not{\bar n} \not n}{4}\psi_c \\ \text{integrated out!}}}{\eta_c}} + \overset{\lambda^3}{\psi_s} + \text{"}\bar c\text{"}$$

$$\mathcal{L}_{SCET} = \mathcal{L}_c + \mathcal{L}_s + \mathcal{L}_{c+s} + \cdots$$

$$\mathcal{L}_c = \bar{\psi}_s \, i \not{D}_s \, \psi_s - \frac{1}{4} G^{s}_{\mu\nu,a} G_s^{\mu\nu,a}$$

$$\mathcal{L}_c = \bar{\xi}_c \frac{\not{\bar{n}}}{2} \left[in\cdot D_c + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp \right] \xi_c$$

Soft-Collinear Interactions

* $\psi_s \sim \lambda^3$. No soft quarks at leading power.

* $\bar{n}\cdot A_s \ll \bar{n}\cdot A_c$, $A_{\perp,s} \ll A_{\perp c}$

$\longrightarrow$ Only $n\cdot A_s$ at leading power!

Substitute $n\cdot A_c \to n\cdot A_c + n\cdot A_s$ in $\mathcal{L}_c$ to obtain $\mathcal{L}_{c+s}$.

$$S_{c+s} = \int d^4x \, \bar{\xi}_c(x) \frac{\bar{n}\!\!\!/}{2} n\cdot A_s(x) \, \xi_c(x)$$

Counting: $\hat{p}_{c_1} + \hat{p}_s + \hat{p}_{c_2}$

$$p^\mu \sim (\lambda^2, 1, \lambda)$$

$$x^\mu \sim (1, \lambda^{-2}, \lambda^{-1})$$

Derivative expansion

$$x_\perp \cdot \partial_\perp \phi_s(x) \sim \lambda$$

$$\frac{1}{\lambda} \quad \lambda^2$$

$$x_+ \cdot \partial_- \phi_s(x) \sim \lambda^2$$

$$1 \cdot \lambda^2$$

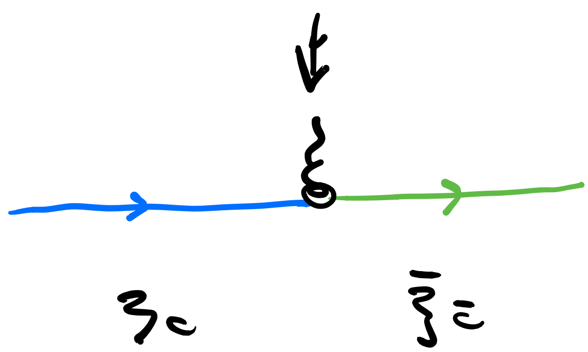
$$x_- \cdot \partial_+ \phi_s(x) \sim \lambda^0$$

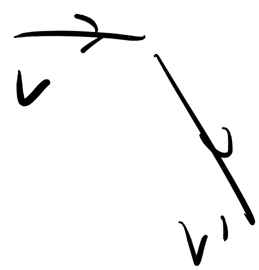
$$1/\lambda^2 \quad \lambda^2$$

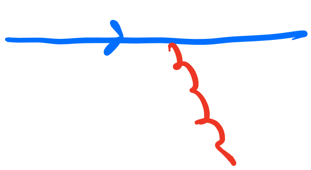
$$\rightsquigarrow \quad S_{c+s} = \int d^4x \; \bar{\xi}_c(x) \frac{\bar{n}\!\!\!/}{2} \, n\cdot A_s(x_-) \, \xi_c(x)$$

$$+ \; O(\lambda)$$

summary: $x^\mu \longrightarrow x_-^\mu$ in soft fields in s-c interactions







$\mathcal{L}_{SCET}$: Summary

After performing the multipole expansion, we arrive at the final form of the leading-power SCET Lagrangian

$$\mathcal{L}_{\text{SCET}} = \bar{\psi}_s i \not{D}_s \psi_s + \bar{\xi}_c \frac{\not{\bar{n}}}{2} \left[in \cdot D + i \not{D}_{c\perp} \frac{1}{i\bar{n} \cdot D_c} i \not{D}_{c\perp} \right] \xi_c - \frac{1}{4} \left(F^{s,a}_{\mu\nu} \right)^2 - \frac{1}{4} \left(F^{c,a}_{\mu\nu} \right)^2 . \tag{3.38}$$

This expression involves the collinear and soft covariant derivatives

$$iD^s_\mu = i\partial_\mu + gA^s_\mu(x) , \qquad iD^c_\mu = i\partial_\mu + gA^c_\mu(x) , \tag{3.39}$$

as well as the mixed derivative

$$in \cdot D = in \cdot \partial + g\, n \cdot A_c(x) + g\, n \cdot A_s(x_-) . \tag{3.40}$$

The associated field-strength tensors are

$$igF^{s,a}_{\mu\nu} t^a = \left[iD^s_\mu, iD^s_\nu \right] , \qquad igF^{c,a}_{\mu\nu} t^a = \left[iD_\mu, iD_\nu \right] , \tag{3.41}$$

where the derivative appearing in the second commutator in (3.41) is defined as

$$D^\mu = n \cdot D \frac{\bar{n}^\mu}{2} + \bar{n} \cdot D_c \frac{n^\mu}{2} + D^\mu_{c\perp} . \tag{3.42}$$

As stated above, we omitted the terms involving the $\bar{c}$ fields for brevity in the Lagrangian (3.38). They have the same form as the ones involving c fields, but with $n \leftrightarrow \bar{n}$ and $x_- \leftrightarrow x_+$.

$n \leftrightarrow \bar{n}$

$\mathcal{L}_c \leftrightarrow \mathcal{L}_{\bar{c}}$

Vector current in SCET

$$J^\mu = \bar{\psi}\gamma^\mu\psi \xrightarrow{\text{tree level!}} J^\mu_{SCET} = \bar{\xi}_c \gamma^\mu \xi_c$$

$$\bar{\xi}_c \gamma^\mu \xi_c = \overbrace{\bar{\xi}_c \left[n^\mu \cdot \frac{\bar{\not{n}}}{2}}^{=0} + \bar{n}^\mu \overbrace{\frac{\not{n}}{2} + \gamma_\perp^\mu \right] \xi_c}^{=0}$$

$$= \bar{\xi}_c \gamma_\perp^\mu \xi_c$$

Problem: Operators with derivatives not always suppressed.

$$\bar{n}\cdot\partial\,\phi_c \sim \lambda^0 Q\,\phi_c$$

$\rightsquigarrow$ Need arbitrarily high derivatives

$$\phi_c(x + t\bar{n})$$

$$= \sum_{n=0}^{\infty} \frac{t^n}{n!}(\bar{n}\cdot\partial)^n \phi_c(x)$$

$$\int dt\, C(t)\, \phi_c(x + t\bar{n})$$

n.D for gauge th!

$$= \sum_{n=0}^{\infty} \frac{C_n}{n!} (\bar{n}\cdot\partial)^n \phi_c(x)$$

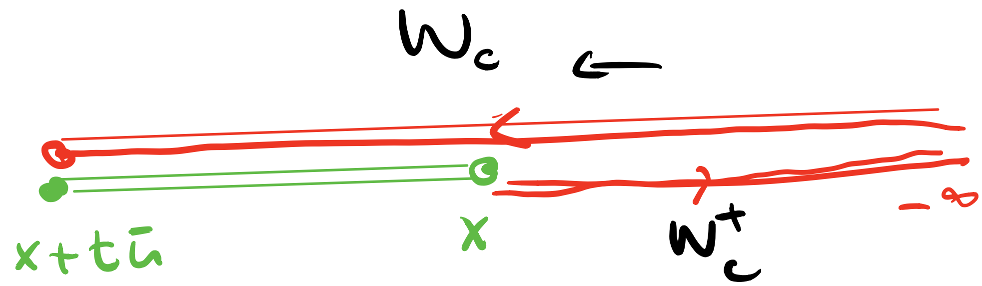
where $C_n = \int dt\, C(t)\, t^n$

$$\bar{\xi}_c(x + t\bar{n})\, \mathcal{U}_c(x+t\bar{n}, x)\, \xi_c(x)$$

$$\mathcal{U}_c(x+t\bar{n}, x)$$

$\bar{n}\cdot A_c^a t^a$

$$= \mathbb{P} \exp\left[i g \int_0^t dt'\, \bar{n}\cdot A_c(x+t'\bar{n}) \right]$$



$$\mathcal{U}_c(x+t\bar{n},x) = W_c(x+t\bar{n})W_c^\dagger(x)$$

Define building blocks:

$$\mathcal{X}_c(x) = W_c^\dagger(x)\,\xi_c(x)$$

$$\mathcal{A}_c^\mu = W_c^\dagger\left(D_c^\mu W_c\right)$$

$$J^\mu_{SCET}(0) = \int ds\,\int dt\, C_V(s,t)\,\bar{\mathcal{X}}_{\bar{c}}(sn)\,\gamma_\perp^\mu\,\mathcal{X}_c(t\bar{n})$$

Tree-level: $C_V(s,t) = \delta(s)\,\delta(t)$

$$\chi_c(t\bar{n}) = \sum_{n=0}^{\infty} \frac{t^n}{n!} (\bar{n}\cdot D)^n \xi_c^{(0)}$$

$$C_V(s,t) \longrightarrow \overset{\text{moments}}{C_V^{nm}}$$

Take matrix element:

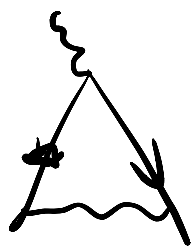
$$\langle q(\ell) | J^\mu_{SCET}(0) | q(p) \rangle$$

$$= \int ds \int dt \, C_V(s,t) \, \bar{u}(\ell) \gamma_\perp^\mu u(p) e^{it\bar{n}\cdot p} \cdot e^{-is n\cdot \ell}$$

$$= \tilde{C}_V(\bar{n}\cdot p \, n\cdot\ell) \, \bar{u}(\ell) \gamma_\perp^\mu u(p)$$

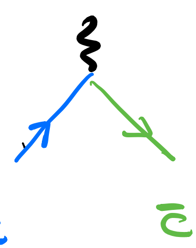
$$\tilde{C}_V\left(\overbrace{Q^2}\right)$$

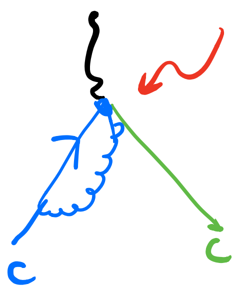
QCD:



$=$

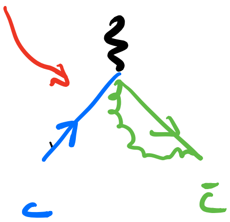
on-shell FF!

$\tilde{C}_V(Q^2)$ 

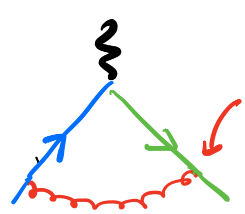
$+$ 

$(p^2)^{\varepsilon}$

Gluon from Wilson line!



$(L^2)^{-\varepsilon}$

$+$ 

$s+\bar{c}$ interaction

soft gluon

Decoupling trafó & factorization

$$\xi_c(x) = S_n(x_-)\,\xi_c^{(0)}(x)$$

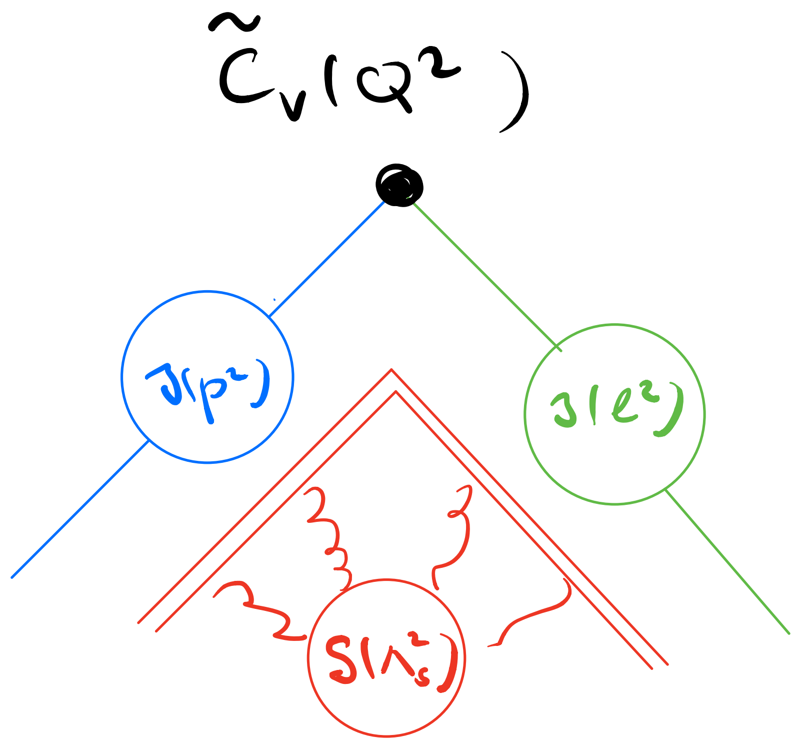
$$A_c^\mu(x) = S_n(x_-)\,A_c^{(0)\mu}(x)\,S_n^\dagger(x_-)$$

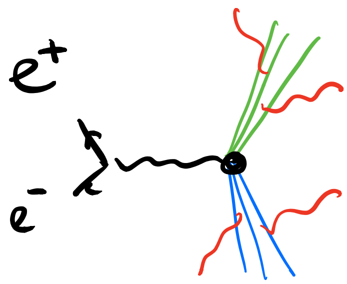
Soft Wilson line along n

$$\bar{\xi}_c \frac{\bar{n}\!\!\!/}{2} n\cdot D\, \xi_c$$

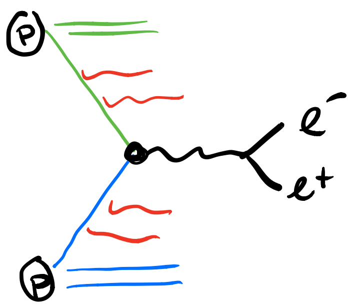
$$= \bar{\xi}_c^{(0)} \frac{\bar{n}\!\!\!/}{2} n\cdot D_c^{(0)}\, \xi_c^{(0)}$$

$$J^{\mu}_{SCET} = \int ds \int dt \, C_V(s,t) \, \bar{\chi}^{(0)}_{\bar{c}}(sn) \, S^{+}_{\bar{n}}(0) \gamma^{\mu}_{\perp} S_{n}(0) \, \chi^{(0)}_{c}(t\bar{n})$$

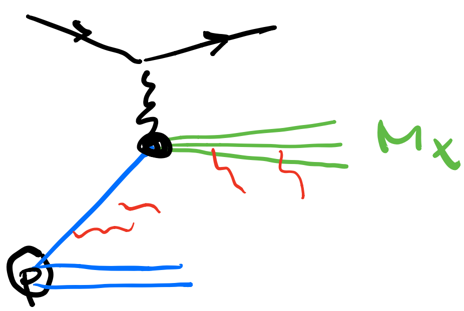

$\tilde{C}_V(Q^2)$
$J(p^2)$
$J(\ell^2)$
$S(\Lambda_s^2)$



Event-shapes in 2-jet production (e.g. thrust)



Drell-Yan process ($pp \to e^+e^- + X$) near threshold.



Deep Inelastic Scattering ($e^-p \to e^- + X$) for small M_X