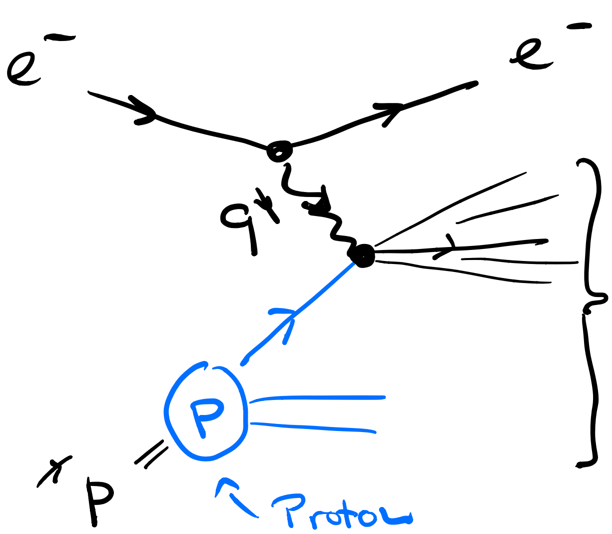
Factorization for DIS

To finish the lecture, let us briefly discuss factorization for DIS for large M_x^2, which is slightly simper than the Sudakov form factor discussed above.

Pictorially, it looks as follows:



$P_x = (p+q)^2$

$Q^2 = -q^2$

Similarly to the treatment of $\sigma(e^+e^- \to X)$ one can obtain the DIS cross section from the imaginary part of the expectation value of two vector currents

average over proton spin

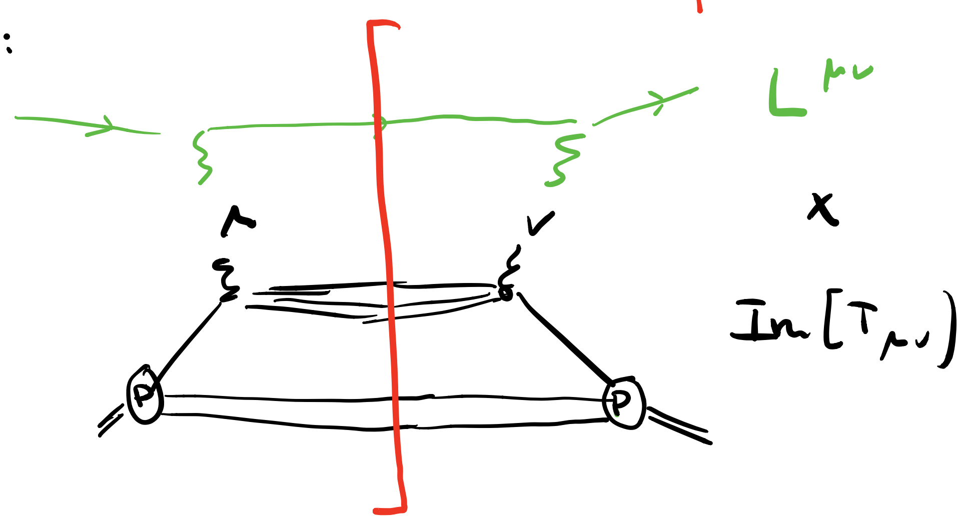
$$T_{\mu\nu}(p,q) = \frac{1}{2}\sum_{\text{spin}} i\int d^4x\, e^{iqz}$$

$$\langle P(p)|\, T\{J_\mu(x)\, J_\nu(0)\}\,|P(p)\rangle$$

Proton

$J^\mu = \sum_q e_q \bar{\psi}_q \gamma^\mu \psi_q$

Pictorially:



we then want to match

$$T\{J_\mu(x)J_\nu(0)\} = \sum_i C_i O_i$$

Operators in SCET

SCET is relevant because the process involves large energies and a small invariant mass M_p!

To write down the operators, we choose the Breit frame, in which

$$q^\mu = (0,0,0,-Q) = \frac{Q}{2}(\bar{n}^\mu - n^\mu)$$

$$p^\mu = \underbrace{\bar{n}\cdot p}_{\text{large, } O(Q)} \overbrace{\frac{n^\mu}{2}}^{O(1)} + \overbrace{\frac{M_p^2}{\bar{n}\cdot p}}^{O(\lambda^2)} \frac{\bar{n}^\mu}{2}$$

$p^2 = M_p^2$!

The proton is described by a collinear field along the n-direction and we should write down all operators which have a nontrivial spin-averaged proton matrix element. The leading power operators are of the form

$$O_q = \bar{\chi}_c(s\bar{n})\,\Gamma\,\chi_c(t\bar{n})$$

$$\Gamma = \frac{\bar{n}\!\!\!/}{2}\,,\ \frac{\bar{n}\!\!\!/}{2}\gamma_5\,,\ \frac{\bar{n}\!\!\!/}{2}\gamma_\perp^\mu$$

only this one gives nonzero matrix element

Dirac basis between χ_c fields.

and (note $\bar{n}\cdot\mathcal{A} = 0$)

$$O_g = \mathcal{A}_c^{\mu}(s\bar{n})\,\mathcal{A}_c^{\nu}(t\bar{n})\,\Gamma_{\mu\nu}$$

$$\Gamma_{\mu\nu} \in \{ \underset{\substack{\uparrow \\ \text{only this} \\ \text{appears.}}}{g^{\perp}_{\mu\nu}},\ \underset{\substack{\uparrow \\ \text{power suppressed}}}{n_\mu n_\nu} \}$$

The product of currents thus has the form

$$T\{ J_\mu(z)\, J_\nu(0) \}$$

collinear quark field for flavor q

can shift one field to zero using transl. invariance

$$= \sum_q \int ds\; C^q_{\mu\nu}(z,s)\,\bar{\chi}_q(s\bar{n})\frac{\bar{n}\!\!\!/}{2}\chi_q(0)$$

$$- \int ds\; C^g_{\mu\nu}(z,s)\,\mathcal{A}_\perp^{\alpha,a}(s\bar{n})\,\mathcal{A}^a_{\perp\alpha}(0)$$

$$+ \mathcal{O}(\lambda^3)$$

This is reminiscent of the OPE, except that the operators are smeared along the n lightcone, which reflects the fact that the $\bar{n}\cdot\partial$ derivatives are unsuppressed. Note that the soft fields decouple also from the operators O_q and O_g. To see this note that

$\bar{n}\cdot x \, \frac{n^\mu}{2}$

$$\chi_c(x) \to S_n(x_-)\, \chi_c^{(0)}(x)$$

$$\to \chi_c(s\bar{n}) \to S_n(0)\, \chi_c(s\bar{n})$$

Therefore only collinear fields contribute.

The matrix elements of O_q & O_g define the Parton Distribution Functions

$$\frac{1}{2}\sum_{\text{spin}} \langle P(p)|\, \bar{\chi}_q^{(0)}(s\bar{n})\, \frac{\bar{n}\!\!\!/}{2}\, \chi_q^{(0)}(0)\,|P(p)\rangle$$

$$= \bar{n}\cdot p \int_{-1}^{1} d\xi\, f_{q/p}(\xi)\, e^{i\xi s\bar{n}\cdot p}$$

Note: $f_{\bar{q}/p}(\xi) = -f_{q/p}(-\xi)$

is the anti-quark PDF

Intuitively, the PDF can be understood by noting that χ annihilates a quark inside the proton. This quark carries a fraction ξ of the total proton momentum.

Similarly:

$$\frac{1}{2}\sum_{\text{spin}} \langle P(p)| - A_\perp^{\mu,a}(s\bar{n})\, A^a_{\perp,\mu}(0)|P(p)\rangle$$

gluon PDF

$$= \int_{-1}^{1} \frac{d\xi}{\xi}\, f_{g/p}(\xi)\, e^{i\xi s\bar{n}\cdot p}$$

charge-conjugation

$$f_{g/p}(-\xi) = f_{g/p}(\xi)$$

Since the soft fields decouple and $\mathcal{L}_c \hat{=} \mathcal{L}_{QCD}$, these SCET matrix elements are equivalent to the PDFs defined in QCD.

Plugging into $T_{\mu\nu}$, we get

$$T_{\mu\nu} = i\int d^4z\, e^{iqz} \,.$$

$$\langle P(p) | T\{ J_\mu(z)\, J_\nu(0) \} | P(p) \rangle$$

$$= \int_{-1}^{1} d\xi \sum_q \tilde{C}_q^{\mu\nu}(Q^2, \bar{n}\cdot p\, \xi)\, \bar{n}\cdot p\, f_{q/p}(\xi)$$

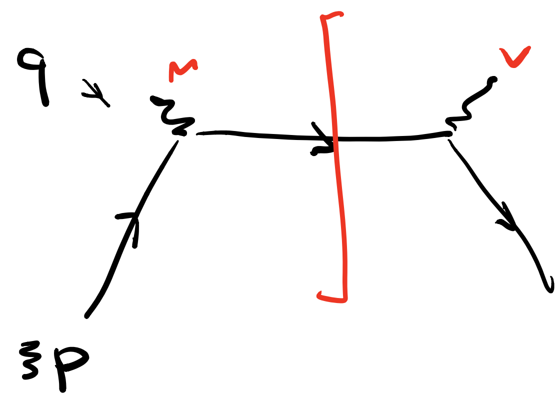
$$+ \text{"glue"}$$

To obtain $\tilde{C}_q^{\mu\nu}$, one performs a matching computation. The easiest way to obtain the coefficient is to work with partonic states, i.e. external quarks or gluons.

For an external quark of flavor q and momentum $\xi \cdot p$, we have

$$f_{q'/q}(\xi) = \delta(1-\xi)\, \delta_{qq'}$$

and the Wilson coefficient is equivalent to the diagram



In summary, one finds the following:

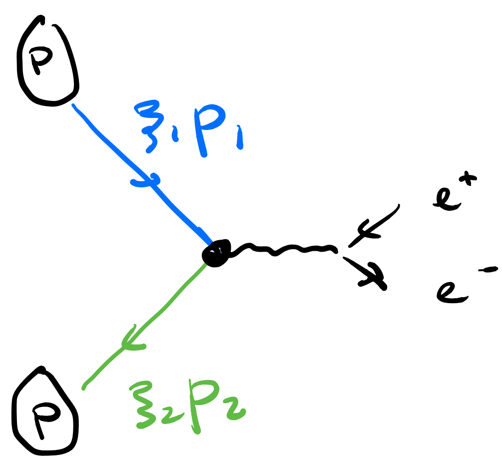
$$\sigma_{e^-p\to e^-X} = \sum_{i=q,\bar{q},g} \int_0^1 d\xi \; f_{i/p}(\xi) \cdot \underbrace{\hat{\sigma}_{e^-i\to e^-X}}$$

partonic amplitude
the incoming parton
has momentum

$\xi \cdot p$

proton momentum

A similar result holds for pp collisions, but with two PDFs for two incoming partons, e.g



Finally, let us remark that the the PDF matrix elements suffer from UV divergencies and need renormalization. The renormalized PDFs depend on the scale μ and fulfill a RG equation

$$\frac{d}{d\ln\mu} f_{i/p}(\xi,\mu) = \sum_j P_{i\leftarrow j} \otimes f_j$$

$$= \sum_j \int_\xi^1 dx \, P_{i\leftarrow j}\left(\xi/x, \alpha_s(\mu)\right) f_{j/p}(x,\mu)$$

The different PDFs mix under renormalization.

The PDFs contain non-perturbative bound-state dynamics. There are methods to extract information on them using lattice QCD, but as of now most information on them is obtained by assuming a functional form (at some reference scale μ_0) and then fitting to experimentally measured cross sections.

The paper hep-ph/0202088 (Bauer, Fleming, Pirjol, Rothstein, Stewart) provides many further examples of factorization theorems derived in SCET.