Soft-Collinear Effective Theory (SCET)

Bauer, Fleming, Pirjol, Stewart 2000, ...

Outline

* Lecture 1

    - Introduction
    - Soft Effective Theory
    - Factorization of soft γ's in QED

* Lecture 2

    - Momentum regions in the Sudakov form factor
    - Soft-Collinear Effective Theory

* Lecture 3

  - Vector current in SCET

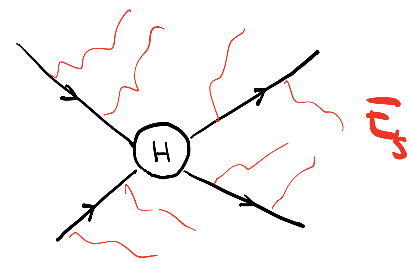
  - Factorization of Sudakov FF

  - PDF factorization in DIS

Soft-Collinear Effective Theory

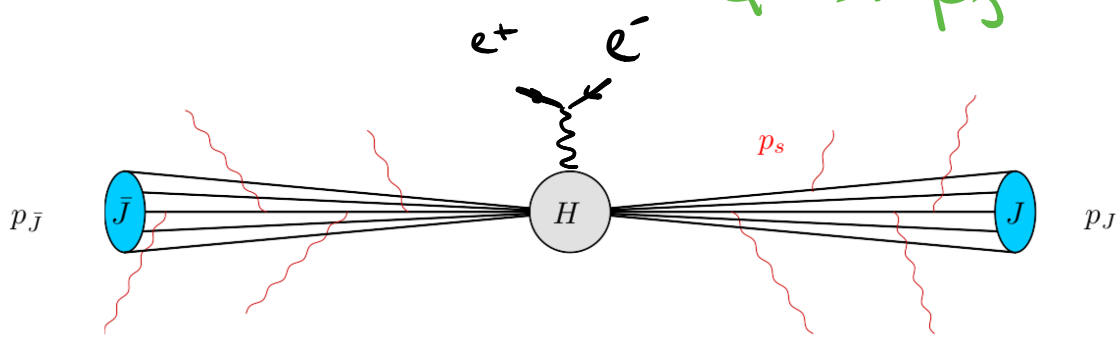
On Monday we obtained the factorization theorem

$$\sigma = H(m_e) \cdot S(E_s)$$



in massive QED. In QCD (and massless QED), often also the collinear region is relevant. Factorization then takes the schematic form

$Q^2 \gg p_J^2 \gg \Lambda_s^2$



$$\sigma = H(Q^2)\, J(p_J^2)\, J(p_{\bar{J}}^2)\, S(\Lambda_s^2)$$

with $\Lambda_s^2 \sim p_J^2 p_{\bar{J}}^2 / Q^2$ (see later)

An important complication is that we encounter an interplay of two low-energy regions. As in the QED case, we'll construct the EFT by expanding full theory diagrams. The appropriate tool is the method of regions. (Beneke, Smirnov '97) for loop & phase-space integrals:

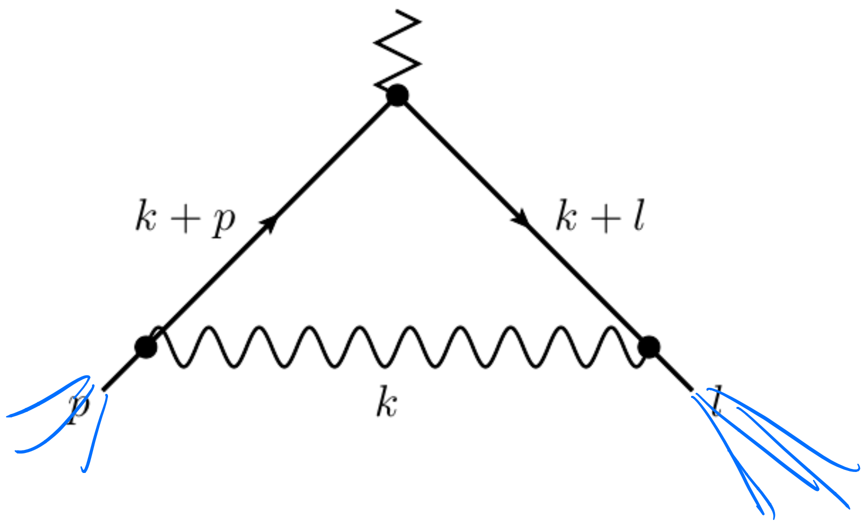
a.) Expand integrand in relevant momentum regions.

b.) Integrate over the full momentum space $\int d^dk$

c.) Add up the different contributions

Will illustrate this for Sudakov form factor and then construct SCET with field for the different regions

Method of regions for the Sudakov form factor



$$Q^2 = -(p-\ell)^2$$

$$P^2 = -p^2 \; ; \; L^2 = -\ell^2$$

$$L^2 \sim P^2 \ll Q^2$$

$$n^\mu = (1, 0, 0, 1) \approx \frac{p^\mu}{p^0}$$

$$\bar{n}^\mu = (1, 0, 0, -1) \neq \frac{e^\mu}{p^0}$$

$$n^2 = 0, \quad \bar{n}^2 = 0, \quad \bar{n}\cdot n = 2$$

$$q^\mu = n\cdot q \, \frac{\bar{n}^\mu}{2} + \bar{n}\cdot q \, \frac{n^\mu}{2} + q_\perp^\mu$$

[Check: $n\cdot q = \cancelto{0}{\bar{n}\cdot q \, \frac{n^2}{2}} + n\cdot q \, \frac{\overset{2}{\bar{n}\cdot n}}{2} + \cancelto{0}{n\cdot q_\perp}$ ✓

$$= q_+^\mu + q_-^\mu + q_\perp^\mu$$

$$q^2 = 2 q_+ \cdot q_- + q_\perp^2$$

$$= n\cdot q \, \bar{n}\cdot q \underbrace{\frac{2\, n\cdot\bar{n}}{4}}_{=1} + q_\perp^2$$

Expansion parameter

$$\lambda^2 \sim p^2/Q^2 \sim L^2/Q^2$$

$$p^2 = \lambda^2 Q^2 = \overset{O(1)}{\bar{n}\cdot p} \; n\cdot p + p_\perp^2$$

Scaling:

$$q^\mu \sim (n\cdot q, \bar{n}\cdot q, q_\perp)$$

$$p^\mu \sim (\lambda^2, 1, \lambda)Q$$

$$\ell^\mu \sim (1, \lambda^2, \lambda)Q$$

Loop momentum:

hard (h) $k^\mu \sim (1, 1, 1)Q$

coll (c) $k^\mu \sim (\lambda^2, 1, \lambda)Q$
to p^μ

coll ($\bar{c}$) $k^\mu \sim (1, \lambda^2, \lambda)Q$
to ℓ_μ

soft (s) $k^\mu \sim (\lambda^2, \lambda^2, \lambda^2)\,Q$

[Glauber $k^\mu \sim (\lambda^2, \lambda^2, \lambda)\,Q$]

$$\int d^dk \, \frac{1}{(k^2)^n} \underset{k \sim \Lambda k}{=} \Lambda^{d-2n} \int d^dk \, \frac{1}{(k^2)^n} = 0$$

$$I = \int d^dk \, \frac{1}{k^2 (k+p)^2 (k+e)^2}$$

Expand in each region

hard: $(k+p)^2 = (k+p_-)^2 + O(\lambda)$

$(k+e)^2 = (k+e_+)^2 + O(\lambda)$

coll (c) : $(k+p)^2 = (k+p)^2$ no exp!

$$(k+\ell)^2 = 2k_-\ell_+ + O(\lambda)$$

coll ($\bar{c}$) $$(k+p)^2 = 2k_+p_- + O(\lambda)$$

$(k+\ell)^2$ no. exp.

soft (s) : $$(k+p)^2 = \underset{\lambda^2}{p^2} + 2\underset{\lambda^0}{p_-}\underset{\lambda^2}{k_+} + O(\lambda^3)$$

$$(k+\ell)^2 = \ell^2 + 2\ell_+\cdot k_- + O(\lambda^3)$$

The expanded loop integrals are

$$I_h = i\pi^{-d/2}\mu^{4-d}\int d^dk \frac{1}{(k^2+i0)\,(k^2+2k_-\cdot l_+ + i0)\,(k^2+2k_+\cdot p_- + i0)}$$

$(k+l_+)^2$ $\quad$ $k \parallel l$

$$I_h = \frac{\Gamma(1+\varepsilon)}{2l_+\cdot p_-}\frac{\Gamma^2(-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{\mu^2}{\underbrace{2l_+\cdot p_-}_{\approx Q^2}}\right)^{\varepsilon}$$

$$I_c = i\pi^{-d/2}\mu^{4-d}\int d^dk \frac{1}{(k^2+i0)\,(2k_-\cdot l_+ + i0)\,[(k+p)^2 + i0]}$$

$$I_c = -\frac{\Gamma(1+\varepsilon)}{2l_+\cdot p_-}\frac{\Gamma^2(-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{\mu^2}{P^2}\right)^{\varepsilon}$$

coll scale!

$$\begin{aligned} I_s &= i\pi^{-d/2}\mu^{4-d}\int d^dk \frac{1}{(k^2+i0)\,(2k_-\cdot l_+ + l^2 + i0)\,(2k_+\cdot p_- + p^2 + i0)} \\ &= -\frac{\Gamma(1+\varepsilon)}{2l_+\cdot p_-}\Gamma(\varepsilon)\Gamma(-\varepsilon)\left(\frac{2l_+\cdot p_-\mu^2}{L^2P^2}\right)^{\varepsilon}. \end{aligned}$$

$$\left(\frac{\mu^2}{\Lambda_s^2}\right)^{\varepsilon}$$

$$\Lambda_s^2 = \frac{L^2P^2}{Q^2}$$

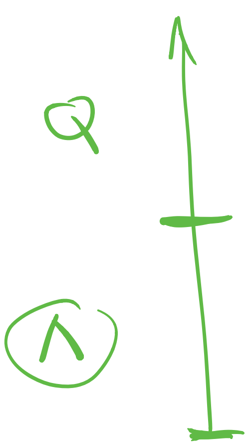
$$I_h = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{6}\right)$$

$$I_c = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln\frac{\mu^2}{P^2} - \frac{1}{2}\ln^2\frac{\mu^2}{P^2} + \frac{\pi^2}{6}\right)$$

$$I_{\bar{c}} = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln\frac{\mu^2}{L^2} - \frac{1}{2}\ln^2\frac{\mu^2}{L^2} + \frac{\pi^2}{6}\right)$$

$$I_s = \frac{\Gamma(1+\varepsilon)}{Q^2}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln\frac{\mu^2 Q^2}{L^2 P^2} + \frac{1}{2}\ln^2\frac{\mu^2 Q^2}{L^2 P^2} + \frac{\pi^2}{6}\right)$$

$$I_{\text{tot}} = \frac{1}{Q^2}\left(\ln\frac{Q^2}{L^2}\ln\frac{Q^2}{P^2} + \frac{\pi^2}{3}\right).$$



Effective Lagrangian

$$\psi \to \psi_c + \psi_{\bar{c}} + \psi_s$$

$$A^\mu \to A^\mu_c + A^\mu_{\bar{c}} + A^\mu_s$$

Scaling of fields?

$$\langle 0 | T\{ A^a_\mu(x) A^b_\nu(0) \} | 0 \rangle$$

$$= \int \frac{d^4k}{(2\pi)^4} e^{ikx} \frac{i\delta^{ab}}{k^2} \left(-g^{\mu\nu} + \xi \frac{k^\mu k^\nu}{k^2} \right)$$

$\Rightarrow$ $A_\mu \sim k_\mu$

$$(n A_s, \bar{n} A_s, A_\perp) \sim (\lambda^2, \lambda^2, \lambda^2)$$

$$(n A_c, \bar{n} \cdot A_c, A_c^\perp) \sim (\lambda^2, 1, \lambda)$$

$$\langle 0 | T[\psi_s(x)\,\bar{\psi}_s(x)] | 0 \rangle =$$

$$\int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{i\not{k}}{k^2}$$

$$(\lambda^2)^4 \cdot \frac{1}{\lambda^2} = \lambda^6$$

$$\rightarrow \quad \underline{\psi_s \sim \lambda^3}$$

Collinear quarks

$$\not{k} = \underset{\lambda^2}{k\cdot n}\,\frac{\bar{\not{n}}}{2} + \underset{\lambda^0}{k\cdot\bar{n}}\,\frac{\not{n}}{2} + \underset{\lambda}{\not{k}_\perp}$$

$$\psi_c = \xi_c + \eta_c = P_+\psi_c + P_-\psi_c$$

$$P_+ = \frac{\not{n}\bar{\not{n}}}{4}\;;\quad P_- = \frac{\bar{\not{n}}\not{n}}{4}$$

$$P_+^2 = P_-^2 = 1$$

$$P_+ + P_- = 1$$

$$\left[\; \frac{\not{n}\not{\bar n}}{4} + \frac{\not{\bar n}\not{n}}{4} = \frac{2\{\not{\bar n}, \not{n}\}}{4} = \frac{2\,\overbrace{n\cdot\bar n}^{2}}{4} = 1 \right.$$

$$\langle 0 | T[\xi_c(x)\,\bar\xi_c(0)]\,|0\rangle$$

$$= \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{i}{k^2}\, \frac{\not{n}\not{\bar n}}{4}\, \overset{k\cdot\bar n \frac{\not n}{2}}{\not{k}}\, \frac{\not{\bar n}\not{n}}{4}$$

$$\not{k} = \underset{\lambda^2}{k\cdot n \frac{\not{\bar n}}{2}} + \underset{\lambda^0}{k\cdot \bar n \frac{\not{n}}{2}} + \underset{\lambda}{k_\perp}$$

$$\sim \lambda^4 \cdot \frac{1}{\lambda^2} \cdot \lambda^0 \sim \lambda^2$$

$$\rightsquigarrow \quad \xi_c \sim \lambda$$

$$\eta_c \sim \lambda^2$$

Insert into QCD action

$$S_{QCD} = S_s + S_c + S_{\bar{c}} + S_{c+s} + \dots$$

$$S_s = \int \underset{\left(\frac{1}{\lambda^2}\right)^4}{\underset{\uparrow}{d^4x}} \; \underset{\lambda^3}{\underset{\uparrow}{\bar{\psi}_s}} \; i \underset{\lambda^2}{\underset{\uparrow}{\not{D}_s}} \; \underset{\lambda^3}{\underset{\uparrow}{\psi_s}} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

$$S_c = \int d^4x \, (\bar{\xi}_c + \bar{\eta}_c)$$
$$\left[in\cdot D_c \frac{\bar{n}\!\!\!/}{2} + i\bar{n}\cdot D_c \frac{n\!\!\!/}{2} + iD\!\!\!/_\perp \right]$$
$$(\xi_c + \eta_c)$$

$$= \int d^4x \left[\bar{\xi}_c \, in\cdot D_c \frac{\bar{n}\!\!\!/}{2} \xi_c + \bar{\eta}_c \, i\bar{n}\cdot D_c \frac{n\!\!\!/}{2} \eta_c \right.$$
$$\left. + \bar{\xi}_c \, iD\!\!\!/_\perp \eta_c + \bar{\eta}_c \, iD\!\!\!/_\perp \xi_c \right]$$

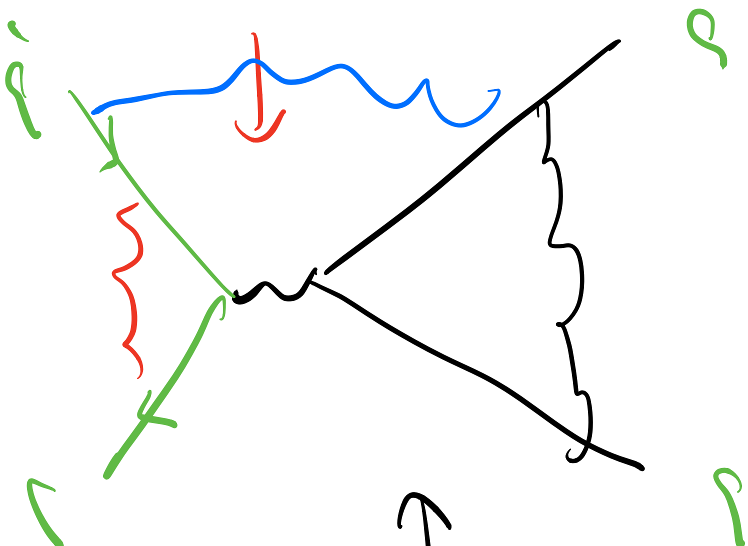
shift

$$\eta_c \longrightarrow \eta_c - \frac{n\!\!\!/}{2} \frac{1}{i\bar{n}\cdot D_c} iD\!\!\!/_{c\perp} \xi_c$$

Q

$$\mathcal{L}_c = \bar{\xi}_c \frac{\bar{n}\!\!\!/}{2} \left[in\cdot D_c + iD\!\!\!/_\perp^c \frac{1}{i\bar{n}\cdot D_c} iD\!\!\!/_\perp^c \right] \xi_c$$
$$+ \bar{\eta}_c \frac{n\!\!\!/}{2} i\bar{n}\cdot D_c \eta_c$$

det



$i\pi$