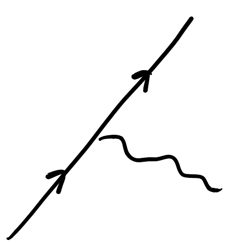
Organisation

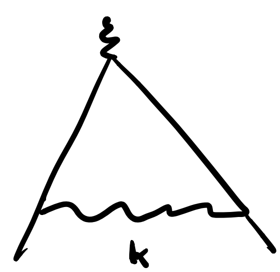
* Blackboard-style lecture, will post notes on indico page

* just unmute & ask questions, (or use chat)

* 2×45mins, with 15 minute break.

* I'll assume some QFT knowledge





$\int d^dk$ dim.reg. $d=4-2\varepsilon$

* Vaccination on Thursday ...

Soft-Collinear Effective Theory (SCET)

Bauer, Fleming, Pirjol, Stewart 2000, ...

Outline

* Lecture 1
  - Introduction
  - Soft Effective Theory
  - Factorization of soft γ's in QED
* Lecture 2
  - Momentum regions in the Sudakov form factor
  - Soft-Collinear Effective Theory
  - Factorization of Sudakov FF

* lecture 3

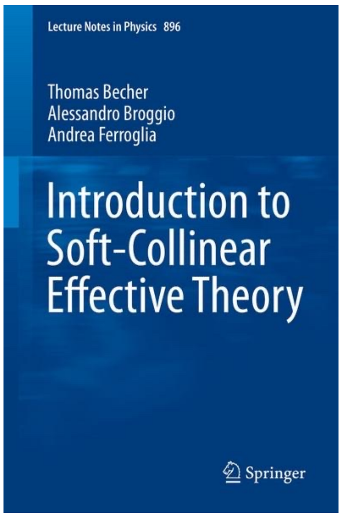
 - Applications: Factorization and resummation

Endless possibilities

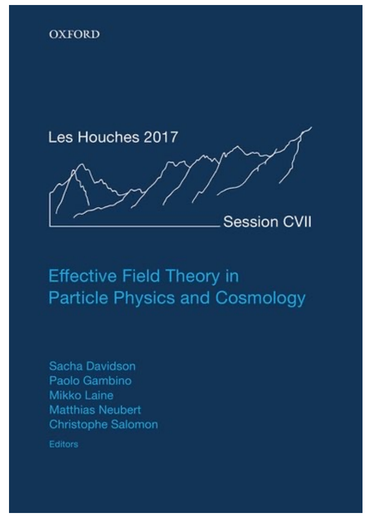
1. * Event-shapes in e^+e^-
   * Jet cross sections
2. * PDF factorization
3. * Transverse momentum resummation
   * Threshold resummation
4. * Shape function in $\bar{B} \to X_s \gamma$, $B \to X_u \ell \nu$ at small q^2.

* Exclusive heavy-to-light decays $B \to K^{(*)} e^+ e^-$, ...

Literature / lecture notes



1410.1892, 172p



1803.04310, 57p

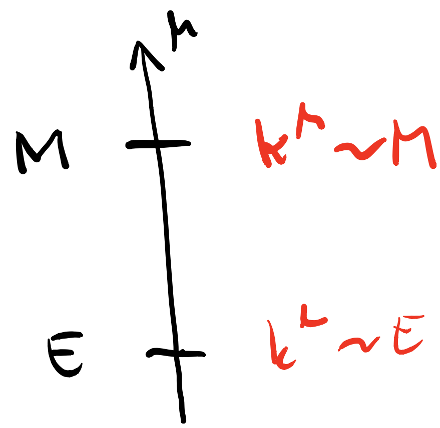
will follow this!

Other resources:

- TASI lecture notes (unpublished) by Bauer and Stewart
- As scales become separated. EFT lectures by Cohen 1903.03622

Modern versus traditional EFTs

Traditional
Euclidean EFT,
e.g. integrating out
heavy particle



$$\mathcal{L}_{\text{eff}} = \sum_i C_i(M)\, O_i$$

order by dimension
derivative expansion

only light dofs

$$\sigma = \sum_i C_i(M,\mu) \langle O_i(\mu) \rangle$$

Factorization $\quad$ $k^\mu \sim M$ $\quad$ $k^\mu \sim E$

Renormalization group evolution

$$\frac{d}{d\ln\mu} C_i = C_j \, \Gamma_{ji}$$

anomalous
dimension

$\leadsto$ Resummation of $\alpha_s^n \ln^m\left(\frac{M}{E}\right)$

Complications in modern Minkowskian EFTs:

* Not all components of momentum scale the same!

e.g. nonrelativistic particles

$$E \sim M \gg |\vec{p}|$$

$\rightarrow$ Reference vectors to split momenta into different components

e.g. $p_b^\mu = m_b \cdot v^\mu + r^\mu$

reference vectors $v^\mu \sim (1,0,0,0)$

* Cannot simply integrate out particles. E.g. b-quark in HQET

⇝ split field into "modes" corresponding to different mom. regions.

e.g. $\phi = \phi_h + \phi_c + \phi_s$

⇝ Sometimes several fields for single particle!

* Nonlocalities associated with directions of large momentum flow.

PDFs involve operators separated along light-cone.

All of these difficulties are present in SCET.

To simplify things, we'll first discuss SET, i.e. Soft Effective Theory and derive a factorization theorem for

soft photon radiation

$$\sigma(e^- e^- \to e^- e^- + X_{soft})$$

in QED. We'll show that

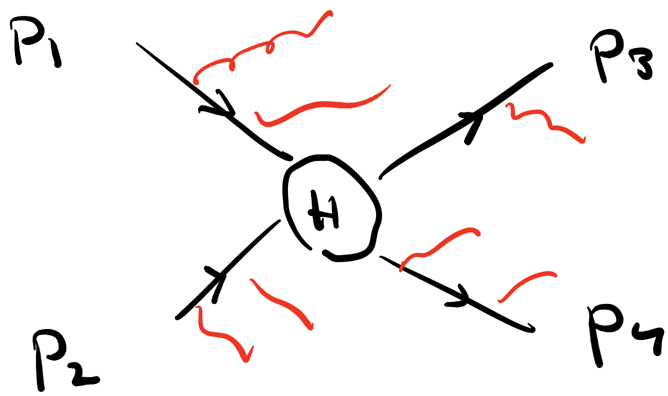
$= \{v_1, \dots, v_n\}$ directions of e^-

$$\sigma = \mathcal{H}(m_e, \{\underline{v}\})\, \mathcal{S}(E_{soft}, \{\underline{v}\})$$

This fits nicely with T. Mannel's lecture since SET $\hat{=}$ H"Q"ET.

more precisely heavy e^- theory!

Soft-Effective Theory



$E_s \ll m_e$

$\lambda = E_s / m_e$

EFT for γ's

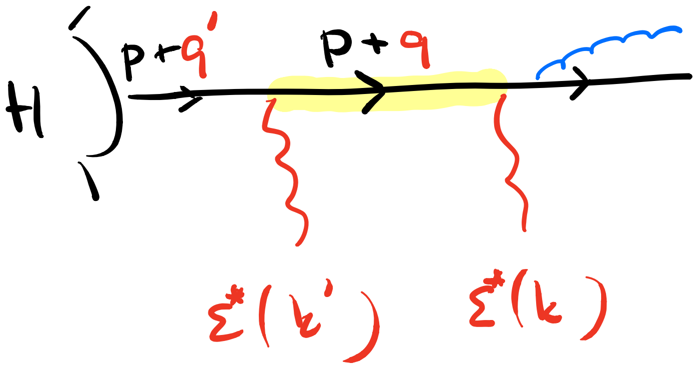
$\partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathcal{L}_\gamma^{eff} = \overset{= -1/4}{C_4}\, F_{\mu\nu} F^{\mu\nu} + \frac{1}{m_e^2} \mathcal{L}^{(6)}$$

$d = 4$

Euler-Heisenberg.

$F_{\mu\nu}\tilde{F}^{\mu\nu}$



$p^\mu = m v^\mu$

$q' = k + k'$

$q = k$

$p^\mu = m v^\mu$ with $v^2 = 1$.

$$\Delta(p+q) = \frac{i(\not{p} + \cancel{\not{q}} + m)}{(p+q)^2 - m^2 + i0}$$

$$p^2 + 2p\cdot q + \cancel{q^2} \to 0$$

$$= i\,\underbrace{\frac{\not{v}+1}{2}}_{P_v}\cdot\frac{1}{v\cdot q + i0}$$

Properties:

$\not{v} P_v = P_v$; $P_v^2 = P_v$;

$$P_v \not{\varepsilon}^* P_v = P_v \, \varepsilon^* \cdot v$$

$$= \bar{u}(p) P_v (-ie\, \varepsilon^*(k) \cdot v) P_v \frac{i}{v \cdot q}$$

$$\cdot (-ie\, \varepsilon^*(k') \cdot v) P_v \frac{i}{v \cdot q'} (\cdots)$$

Construct $\mathcal{L}_{eff}$ for this amplitude:

Propagator : $\dfrac{i}{v \cdot q}$

Vertex : $-ie v^\mu$

like HQET !

$$\mathcal{L} = \bar{h}_v \, i v \cdot D \, h_v$$

h_v is auxiliary field with $P_v h_v = h_v$.

Can use $h_v = P_v \psi$.

Need 4 auxilary fields:

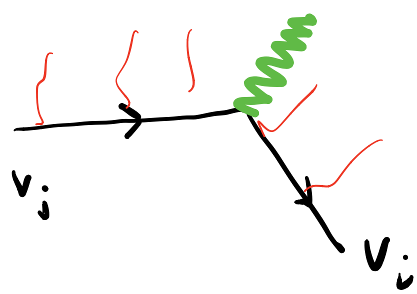
$iD_\mu = i\partial_\mu - eA_\mu$

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{4} \bar{h}_{v_i} v_i \cdot D\, h_{v_i} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$+ \Delta \mathcal{L}_{\text{int}}$$

Interaction terms:

$$\underset{\overset{\shortparallel}{0}}{C_\Gamma(v_i \cdot v_j)}\, \bar{h}_{v_i} \Gamma h_{v_j}$$



Does not happen!

$\{v_1, v_2, v_3, v_4\}$

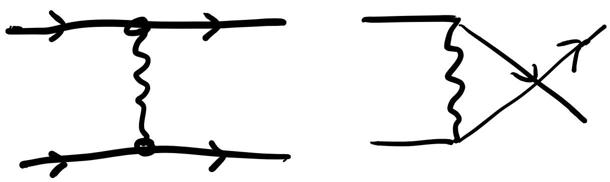
Dirac Ind.

$$\Delta\mathcal{L}_{int} = C_{\alpha\beta\gamma\delta}(m_e, \{\underline{v}\})\, \bar{h}_{v_4}^{\delta}\, \bar{h}_{v_3}^{\gamma}\, h_{v_1}^{\alpha}\, h_{v_2}^{\beta}$$

$$\sim \frac{\alpha}{m_e^2}$$

Matching:

QED



$$\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}$$

$$C_{\alpha\beta\gamma\delta}(m_e, \{\underline{v}\})$$

Wilson coeff. is scattering amp w/o external photons!

Define Wilson

$$S_i(x) = \exp\left[-ie\int_{-\infty}^{0} ds\, v_i \cdot A(x+sv_i)\right]$$

$$v_i \cdot D\, S_i(x) = 0$$

Redefine

$$h_{v_i}(x) = S_i(x)\, h^{(0)}_{v_i}(x)$$

Lagrangian becomes

$$\bar{h}_{v_i}\, iv \cdot D\, h_{v_i} = \bar{h}^{(0)}_{v_i}\, S^{+}\, iv D\, S\, h^{(0)}_{v_i}$$

$$= \bar{h}^{(0)}_{v_i} \, i v \cdot \partial \, h^{(0)}_{v_i} .$$

ϕ_{v_i}

χe - interactions move to $\mathcal{L}_{int}$!

$$\mathcal{L}_{int} = C_{\alpha\beta\gamma\delta} \, h^{(0)\alpha}_{v_1} \, h^{(0)\beta}_{v_2} \, \bar{h}^{(0)\gamma}_{v_3} \, \bar{h}^{(0)\delta}_{v_4}$$

$$S^{+}_{v_4} S^{+}_{v_3} S_{v_2} S_{v_1}$$

Now compute $\sigma (e^- e^- \to e^- e^- + X_{soft})$

$$\mathcal{M} = \bar{u}_\delta(p_3) \, \bar{u}_\gamma(p_4) \, u_\alpha(p_1) \, u_\beta(p_2)$$

$$C_{\alpha\beta\gamma\delta} (m_e , \{ \underline{v} \})$$

$$\langle X_s \mid S^{+}_{v_4} S^{+}_{v_3} S_{v_1} S_{v_2} \mid 0 \rangle$$

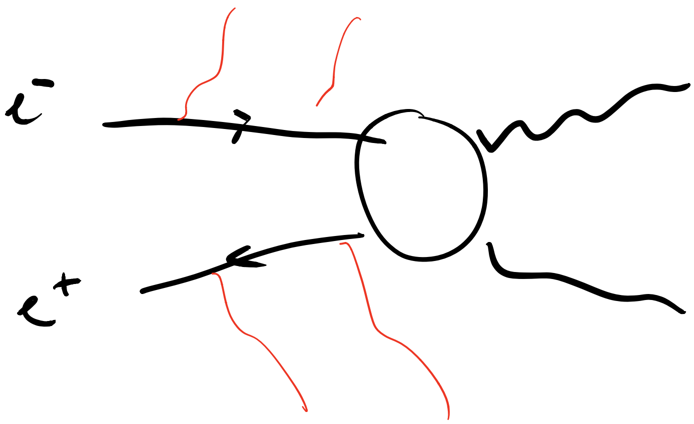
For cross section, we have

$$\boxed{\sigma = H(\{v\}, m_e, \mu) \times S(E_s, \{v\}, \mu)}$$

$$S = \sum_{X_S} \left| \langle X_s | S^{\dagger}_{v_4} S^{\dagger}_{v_3} S_{v_1} S_{v_2} | 0 \rangle \right|^2 \Theta(E_s - E_X)$$

$\mathcal{S}$ exponentiates!

$$\mathcal{S} = \exp\left[\frac{\alpha}{4\pi} \mathcal{S}^{(1)} \right]$$


e^-
e^+