Constraining neutrino mass using three-point mean relative velocity statistics

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Current constraints on summed neutrino mass

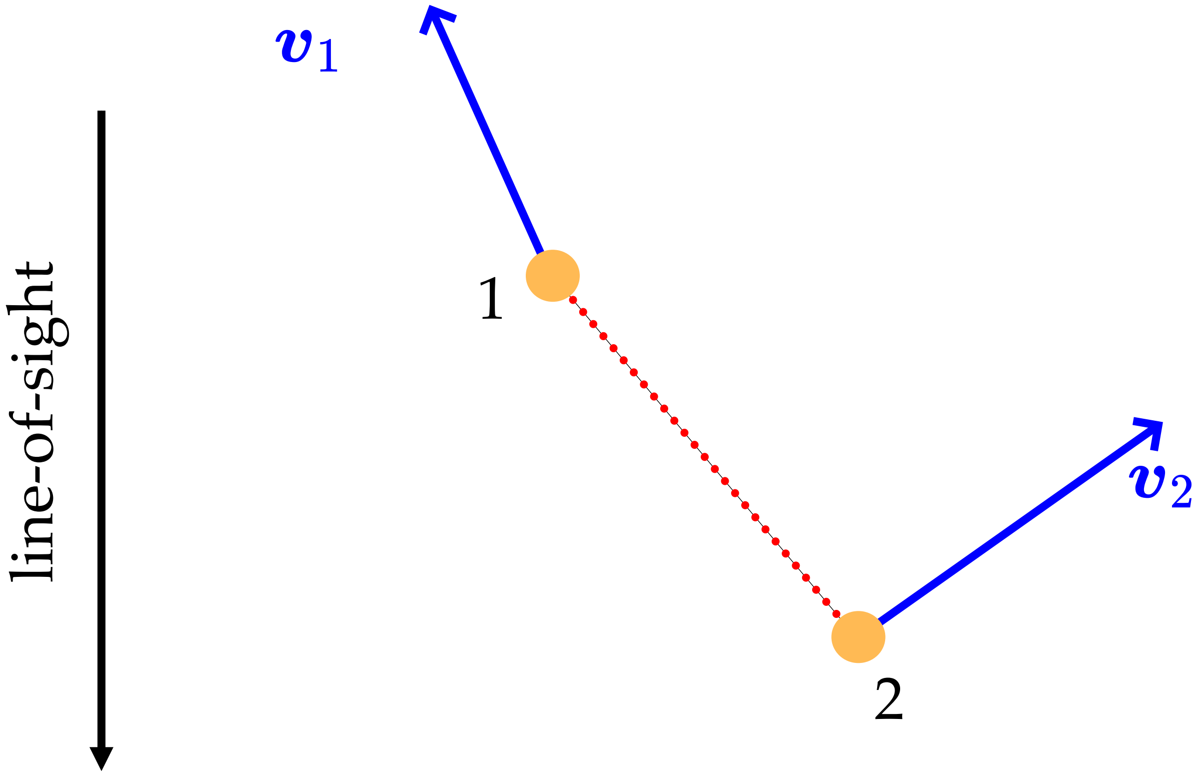
- The Karlsruhe Tritium Neutrino (KATRIN) experiment has reported the first direct detection of sub-eV neutrino mass, with an upper limit on the 'effective neutrino mass' of 0.8 eV. (Aker et al. 2021)

- By combining cosmic microwave background, baryonic acoustic oscillation, and redshift-space galaxy clustering to obtain an upper limit of 0.102 eV. (eBOSS Collaboration 2021)

- Considering extensions of the standard cosmological model, the upper limit becomes less stringent. (e.g. Vagnozzi et al. 2018; Choudhury & Hannestad 2020)

Pairwise velocity

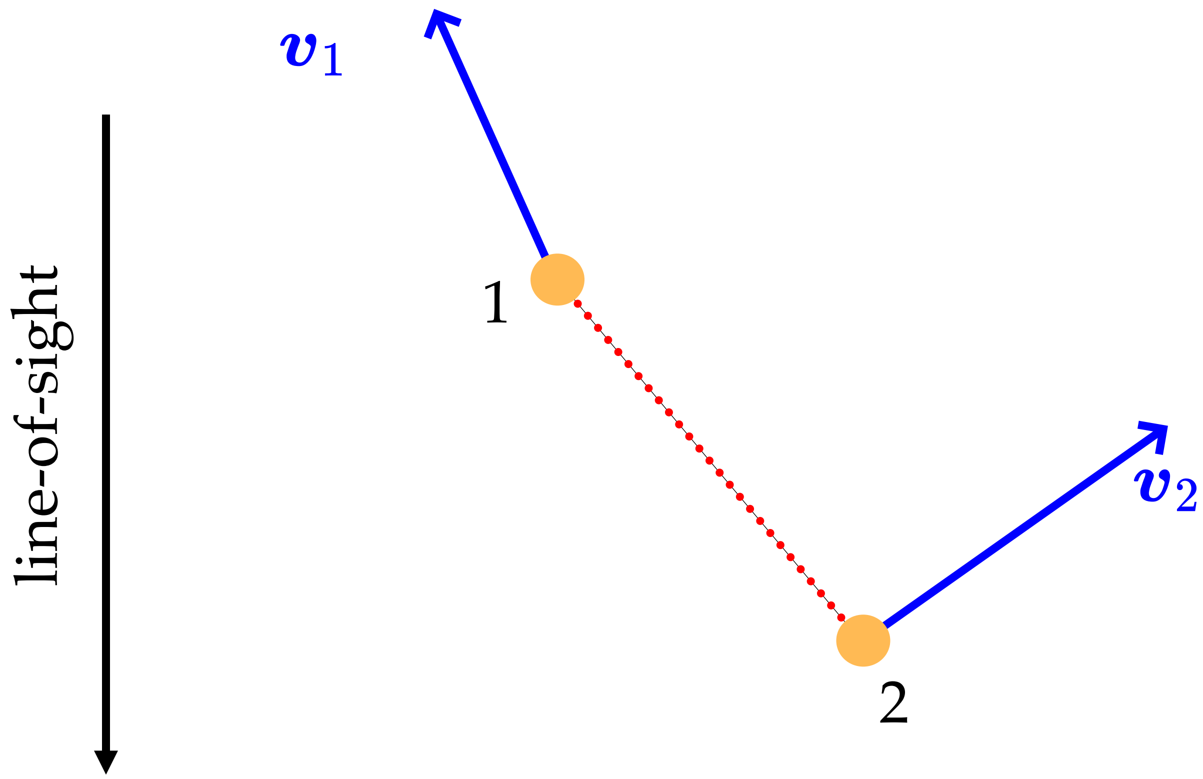
$$w_{\mathrm{r}}(r_{12}) = (\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot \hat{\boldsymbol{r}}_{12}$$



Pairwise velocity

$$w_r(r_{12}) = (\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot \hat{\boldsymbol{r}}_{12}$$

$$\langle \boldsymbol{w}_{12} | \boldsymbol{r}_{12} \rangle_{\rm p} = \frac{\langle (1+\delta_1)(1+\delta_2)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \rangle}{\langle (1+\delta_1)(1+\delta_2) \rangle}$$



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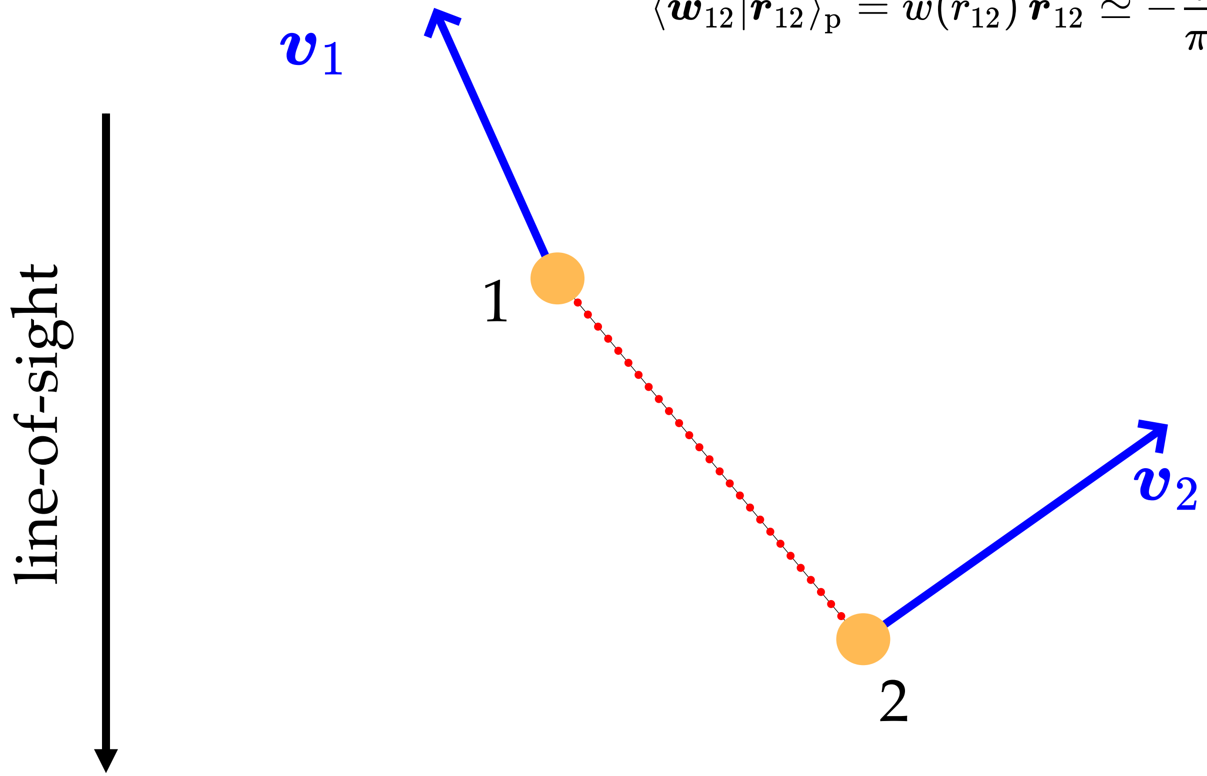
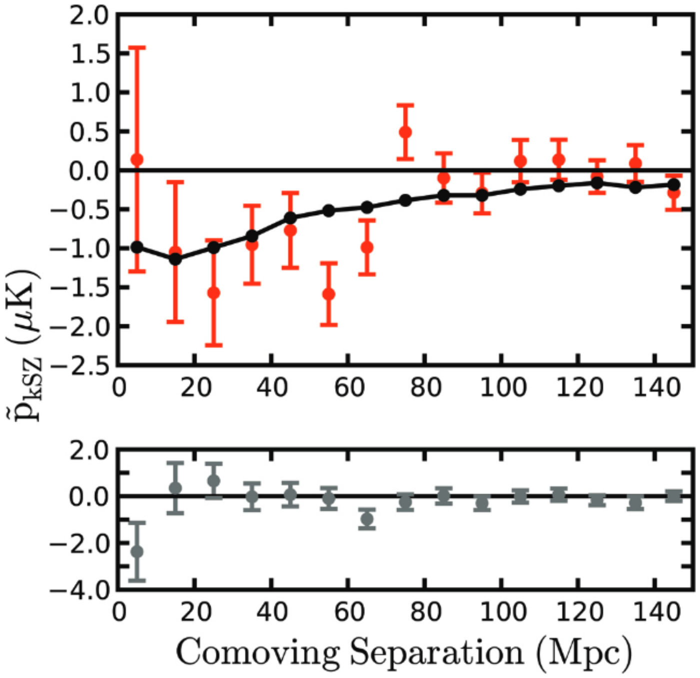
$$\langle \boldsymbol{w}_{12}|\boldsymbol{r}_{12}\rangle_{\rm p} = \bar{w}(r_{12})\,\hat{\boldsymbol{r}}_{12} \simeq -\frac{f}{\pi^2}\,\hat{\boldsymbol{r}}_{12}\int_0^\infty k\,j_1(kr_{12})\,P(k)\,{\rm d}k$$



- The first significant detection of the kinetic Sunyaev-Zeldovich (kSZ) effect was achieved through the mean pairwise mean velocity. (Hand et al. 12)



(Hand et al. 12)

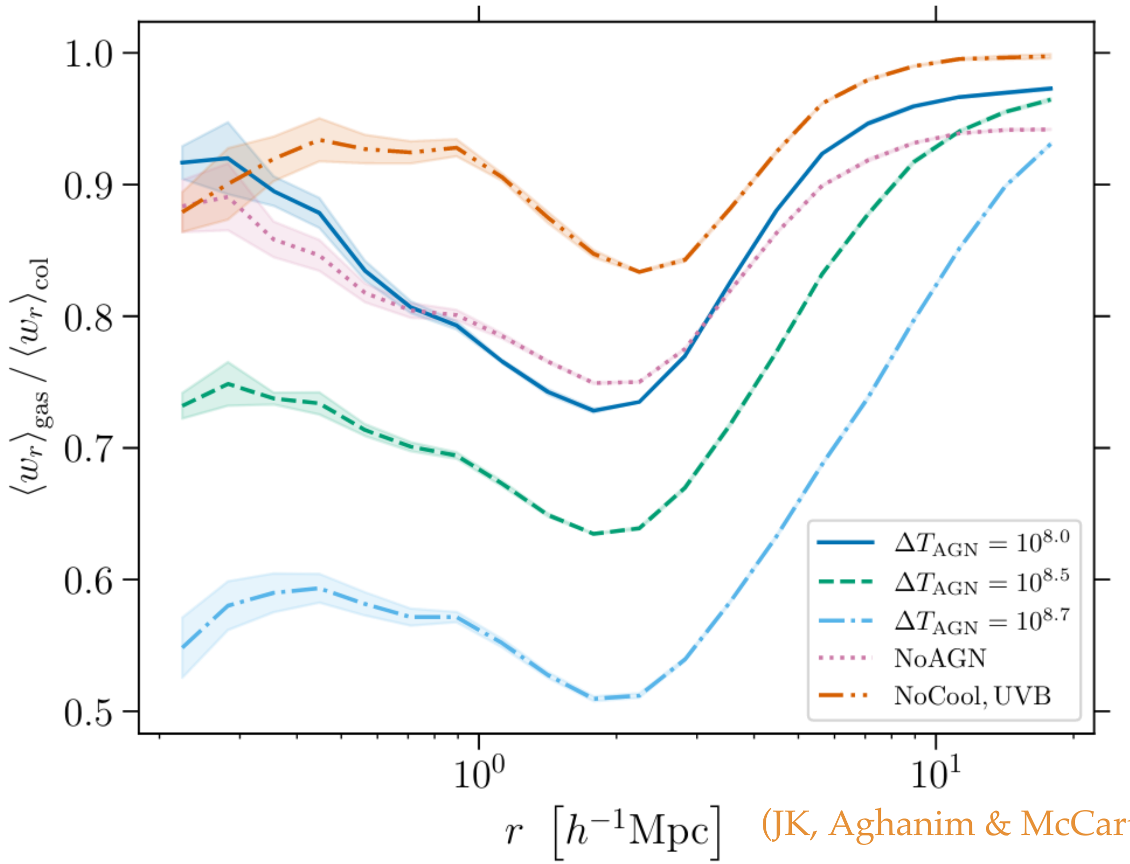
$$\frac{\Delta T^{\rm kSZ}(r_{12})}{T_{\rm cmb}} \simeq -\tau \frac{\bar{w}(r_{12})}{c}$$

- The first significant detection of the kinetic Sunyaev-Zeldovich (kSZ) effect was achieved through the mean pairwise mean velocity. (Hand et al. 12)

- The pairwise velocity measurement from kSZ experiments has been shown to be a novel probe to constrain the summed neutrino mass. (Mueller et al. 15)

- The impact of massive neutrinos and its interplay with baryonic physics at non-linear scales. (JK, Aghanim & McCarthy 20)

Impact of baryonic physics



(JK, Aghanim & McCarthy 20)

Cosmo-OWLS simulations

- Assumption of gas following the matter is not valid even at scales of roughly 20 Mpc/h.

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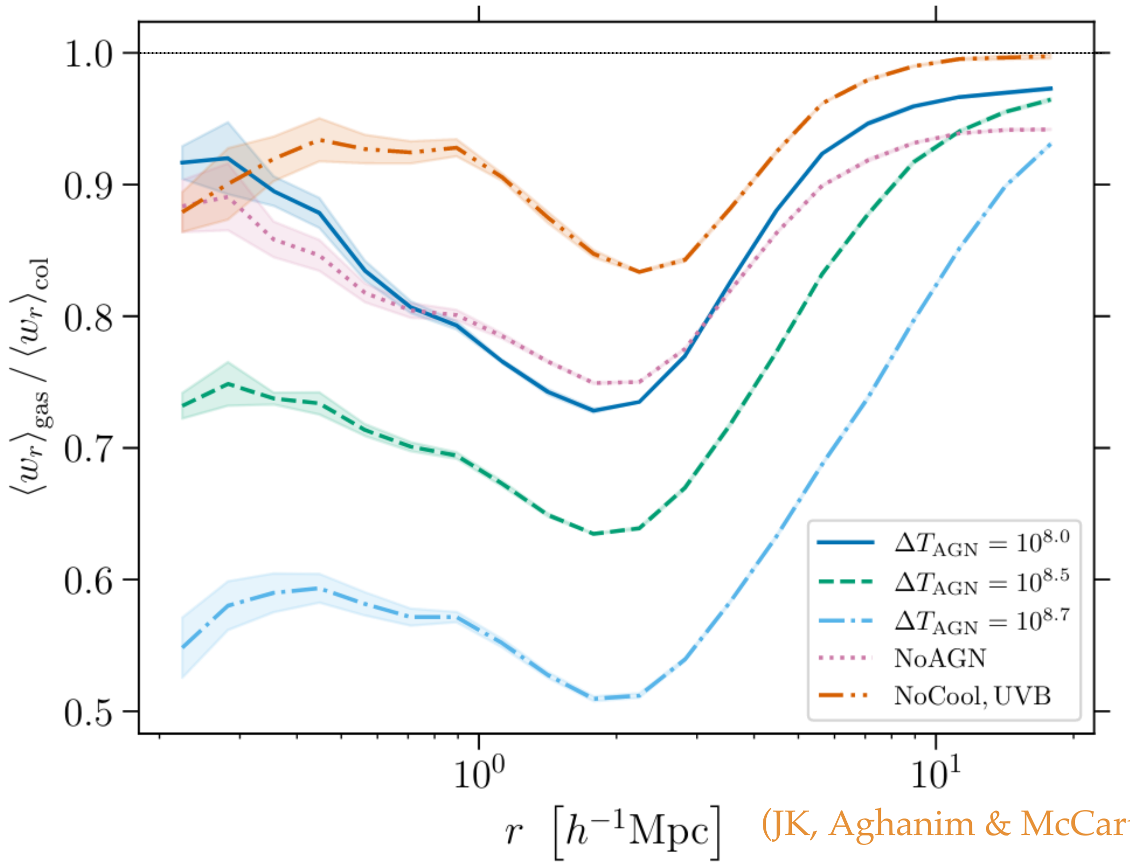
Impact of baryonic physics



(JK, Aghanim & McCarthy 20)

Cosmo-OWLS simulations

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Is there any other domain in cosmology where the pairwise velocities are important?

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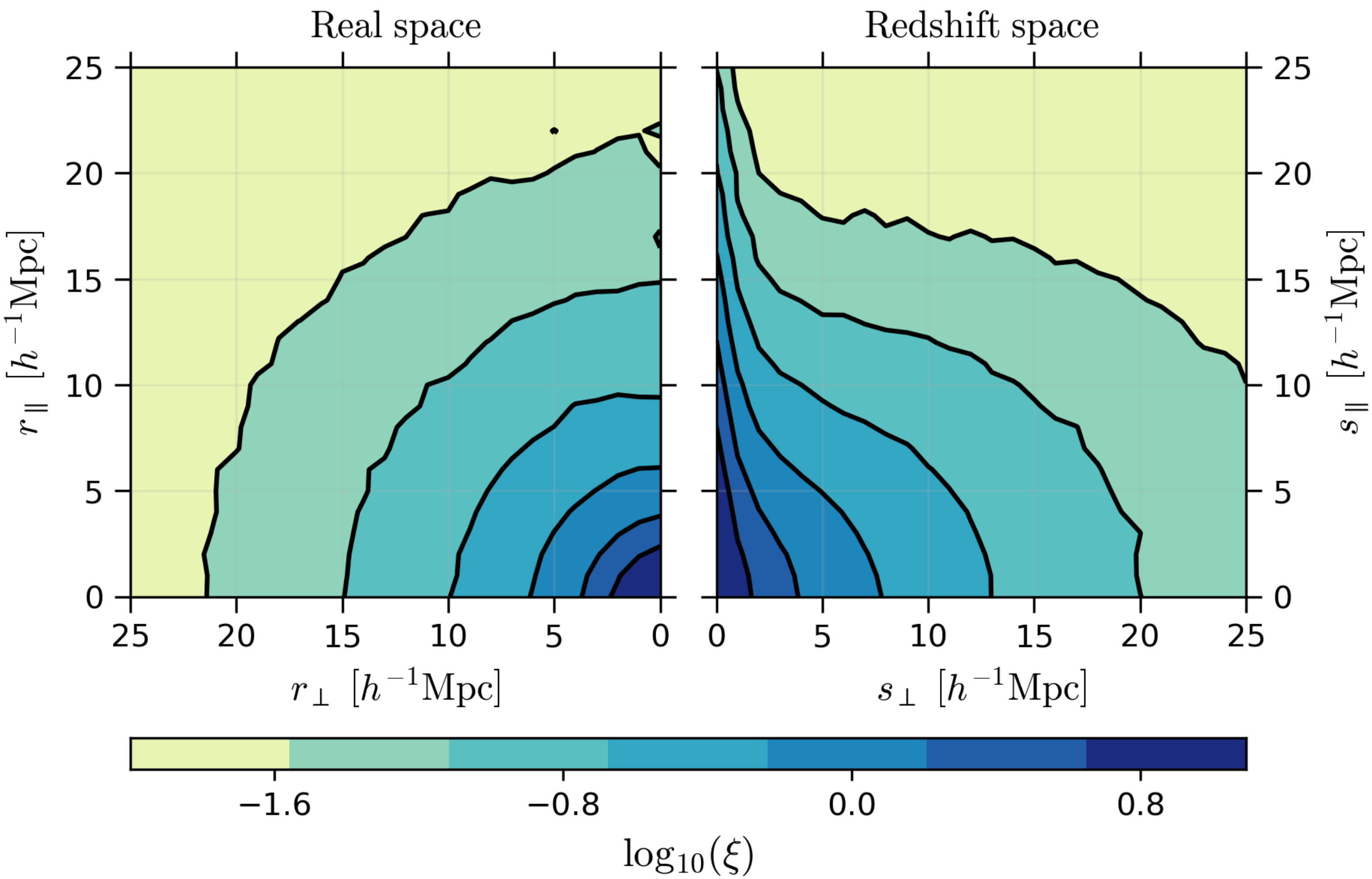
Yes, in modelling the anisotropic correlation function in redshift space!

Two-point streaming model



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Two-point streaming model

$$1 + \xi_{\rm s}(s_\perp, s_\parallel) = \int_{-\infty}^{+\infty} [1 + \xi(r)]\ \mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)\, \mathrm{d}r_\parallel$$

(Peebles 80, Fisher 95, Scoccimarro 04, JK & Porciani 18, Vlah & White 19)

$\xi_{\rm s}(s_\perp, s_\parallel)$: Anistropic redshift-space correlation function

$\xi_{\rm r}(r)$: Isotropic real-space correlation function

$\mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)$: Relative pairwise line-of-sight velocity distribution

Can we generalise streaming model to three-point and higher order statistics?

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Yes we can!

n-point streaming model

- Generalised the streaming model framework to n-point statistics.

$$\mathcal{G}_n = \int \mathcal{F}_n \, \mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel} \, \mathrm{d}w_{12\parallel} \ldots \mathrm{d}w_{mn\parallel}$$

where,

(JK & Porciani 20)

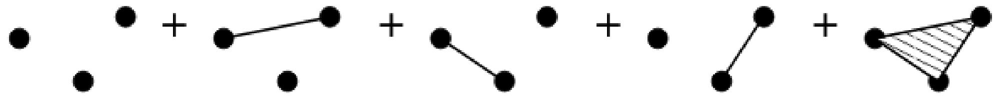
$\mathcal{G}_n$: the (anisotropic) n-point full CF in redshift space

$\mathcal{F}_n$: the (isotropic) n-point full CF in real space

$\mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel}$: the joint probability density of n-1 relative line-of-sight peculiar velocity

The equation is exact under distant observer approximation

Full correlation function in three-point

$\langle \delta_1 \, \delta_2 \, \delta_3 \rangle =$ 

Credits: Bernardeau et al. 01

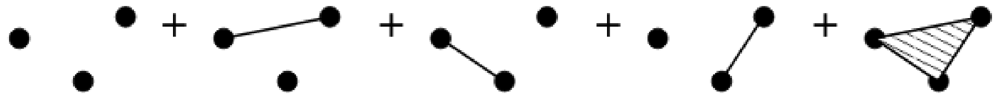
Thus the three-point streaming model is

$$
\begin{aligned}
1 + \xi_{\mathrm{s}}(s_{12\parallel}, s_{12\perp}) + \xi_{\mathrm{s}}(s_{23\parallel}, s_{23\perp}) + \xi_{\mathrm{s}}(\check{s}_{31\parallel}, s_{31\perp}) + \zeta_{\mathrm{s}}(\boldsymbol{s}_{12}, \boldsymbol{s}_{23}) \\
= \int [1 + \xi(\check{r}_{12}) + \xi(\check{r}_{23}) + \xi(\check{r}_{31}) + \zeta(\check{r}_{12}, \check{r}_{23}, \check{r}_{31})] \\
\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel} \left(s_{12\parallel} - r_{12\parallel}, s_{23\parallel} - r_{23\parallel} \,\middle|\, \check{\boldsymbol{r}}_{12}, \check{\boldsymbol{r}}_{23} \right) \, \mathrm{d}r_{12\parallel} \, \mathrm{d}r_{23\parallel}
\end{aligned}
$$

(JK & Porciani 20)

Full correlation function in three-point

$\langle \delta_1 \, \delta_2 \, \delta_3 \rangle =$



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1 + \xi_{\rm s}(s_{12\parallel}, s_{12\perp}) + \xi_{\rm s}(s_{23\parallel}, s_{23\perp}) + \xi_{\rm s}(\check{s}_{31\parallel}, s_{31\perp}) + \zeta_{\rm s}(\boldsymbol{s}_{12}, \boldsymbol{s}_{23}) \\
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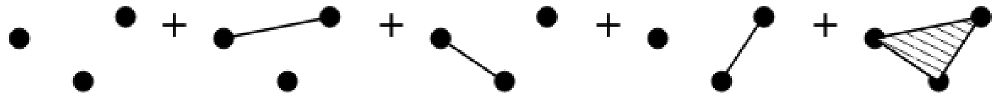
$\langle \delta_1 \, \delta_2 \, \delta_3 \rangle =$ 

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$$= \int \left[1 + \xi(\check{r}_{12}) + \xi(\check{r}_{23}) + \xi(\check{r}_{31}) + \zeta(\check{r}_{12}, \check{r}_{23}, \check{r}_{31})\right]$$
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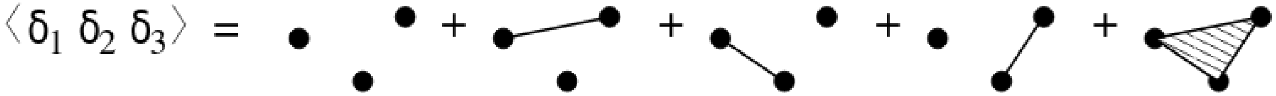
Full correlation function in three-point



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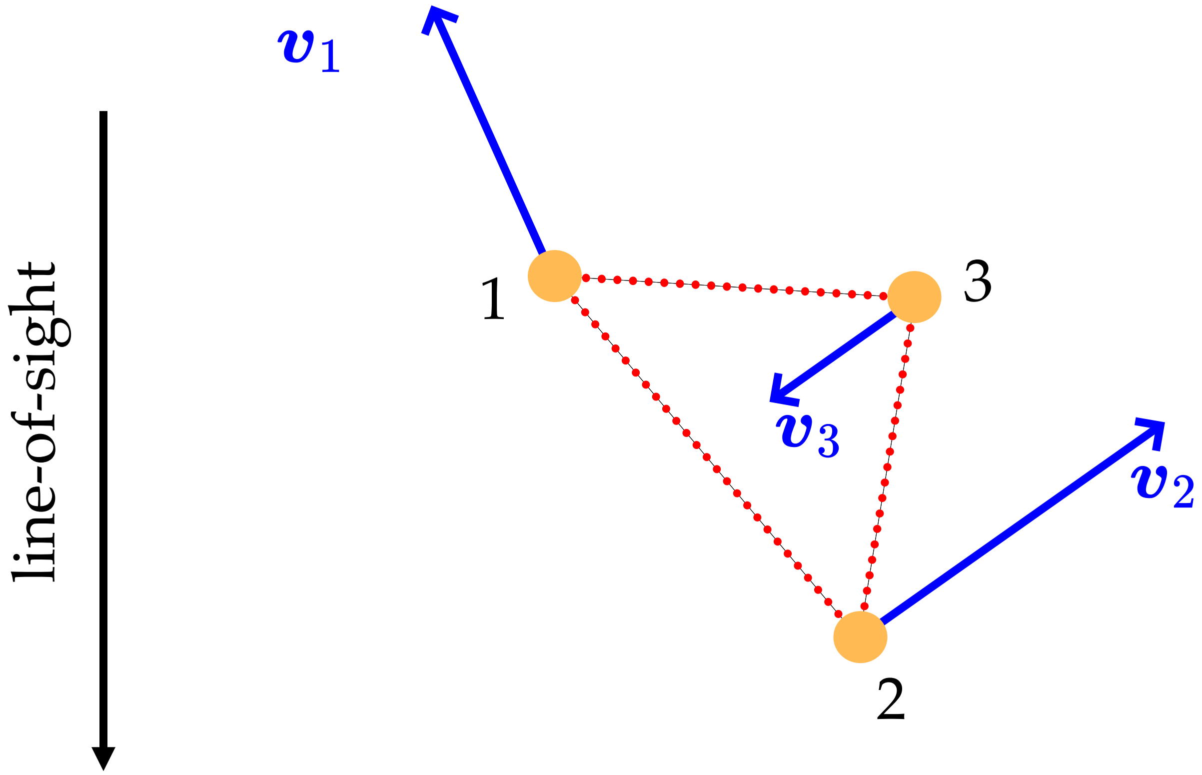
(JK & Porciani 20)

What does $\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}$ look like?

Three-point relative velocity



$$R_{12}(r_{12}, r_{23}, r_{31}) = (\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot \hat{\boldsymbol{r}}_{12}$$

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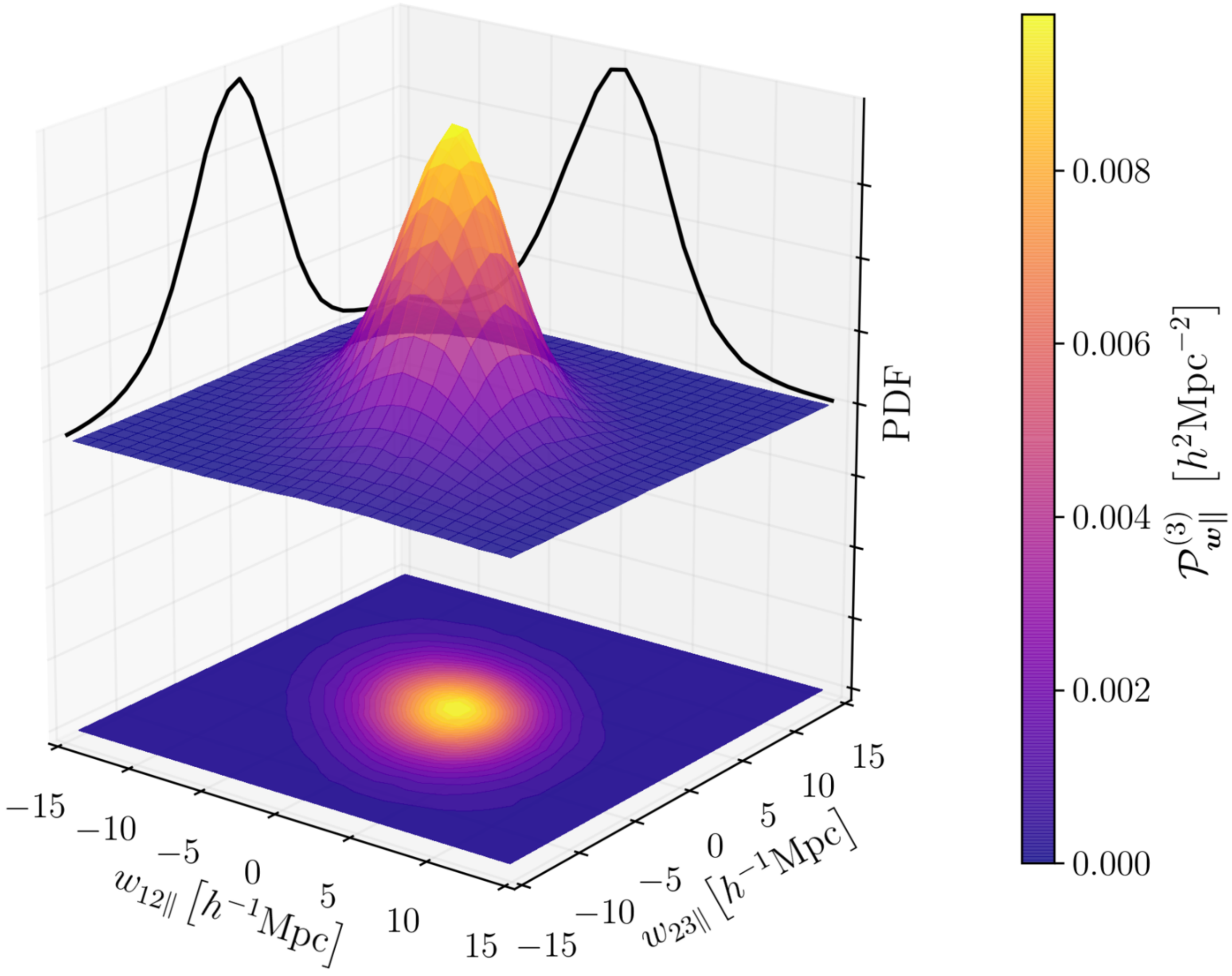
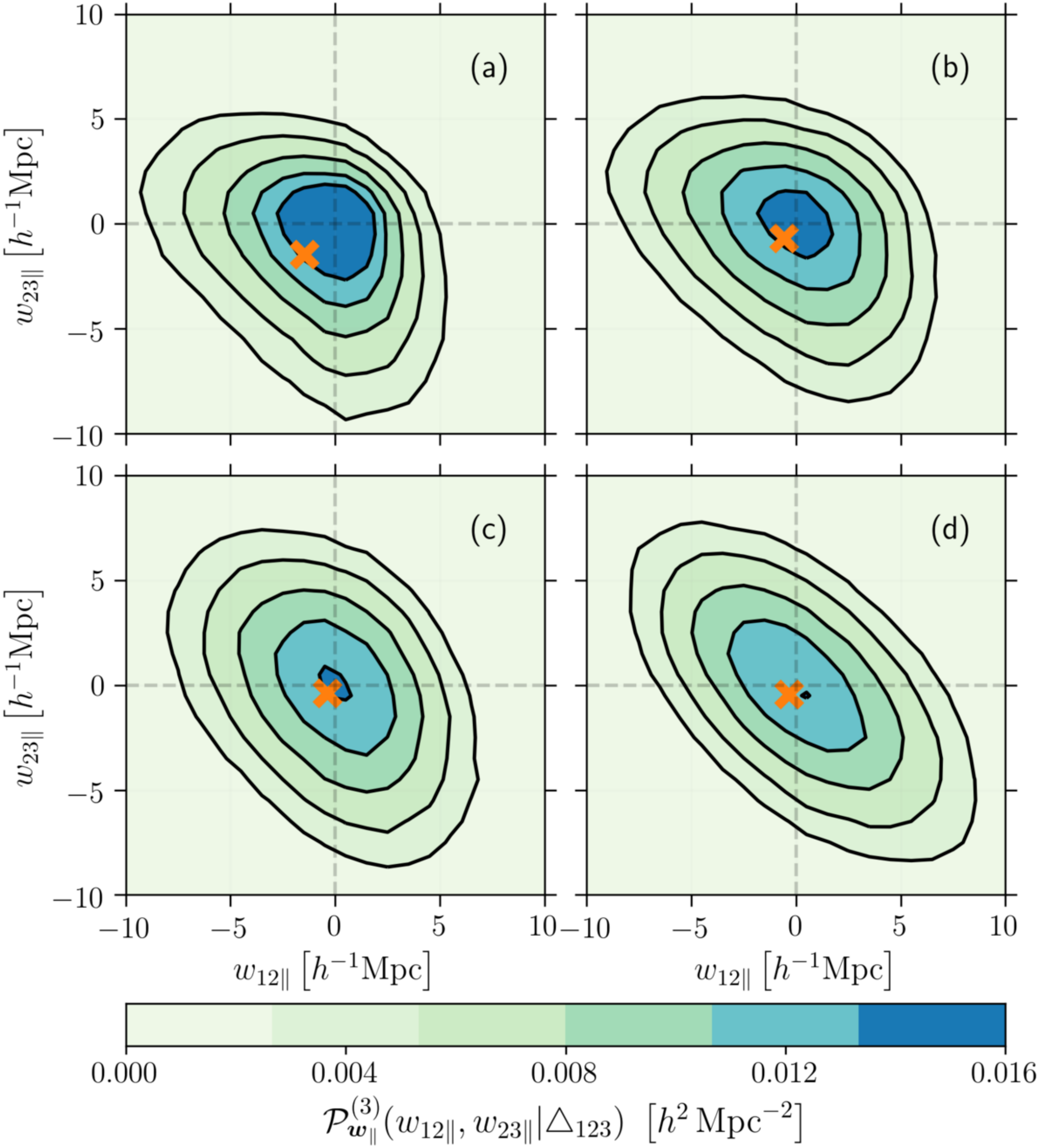

PDF
0.008
0.006
0.004
0.002
0.000
$\mathcal{P}^{(3)}_{w\|}$ [h^2Mpc^{-2}]
−15
−10
−5
0
5
10
15
$w_{12\|}$ [h^{-1}Mpc]
$w_{23\|}$ [h^{-1}Mpc]



(JK & Porciani 20)

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Can we predict the velocity moments?

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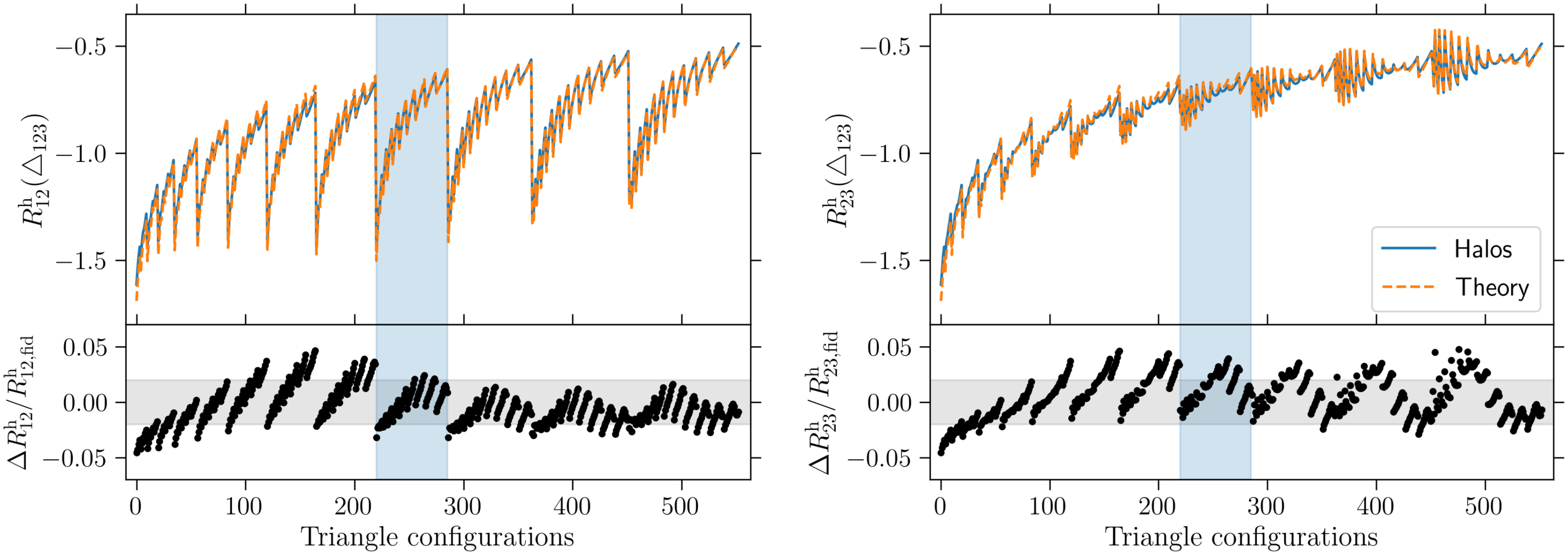
Halos - Mean relative velocity between pairs in a triplet

Triangular configurations: $50 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120\ h^{-1}\mathrm{Mpc}$

Minimum halo mass: $M_\mathrm{h} > 5 \times 10^{13}\ h^{-1} M_\odot$

15,000 simulations of the Quijote suite of simulations



$$R^\mathrm{h}_{ij}(\triangle_{123}, M_\mathrm{h}) = b(M_\mathrm{h})\, R_{ij}(\triangle_{123})$$

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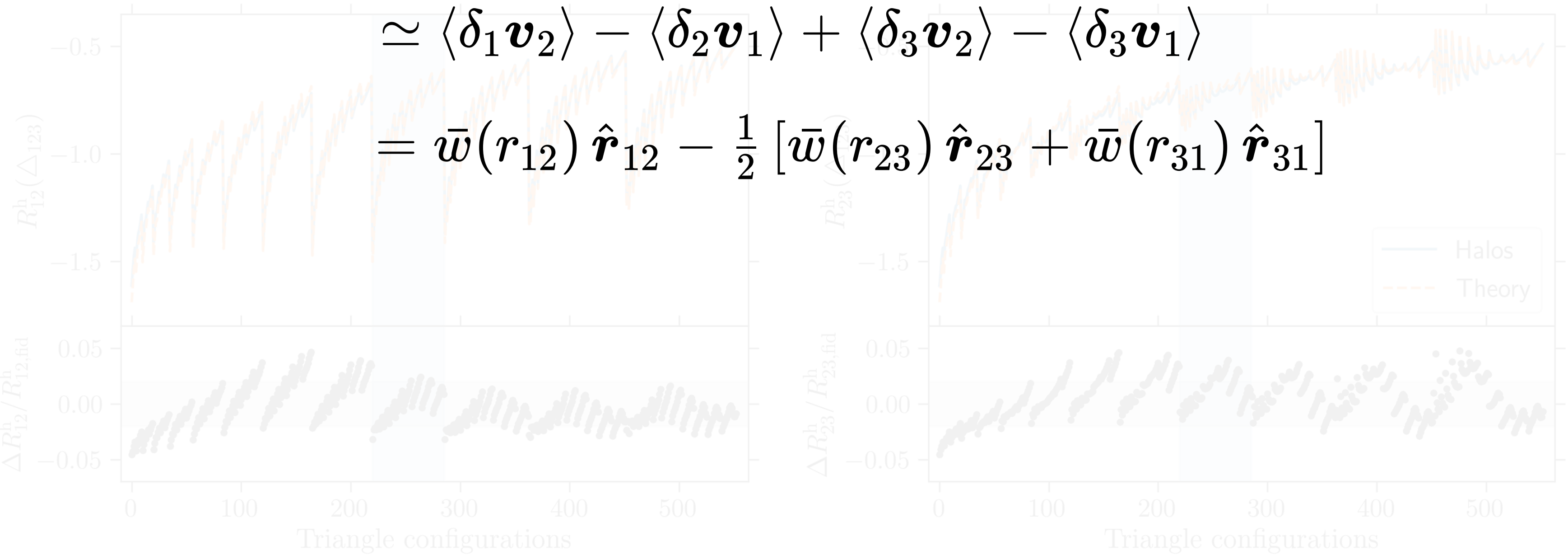
$$\langle \boldsymbol{w}_{12} | \triangle_{123} \rangle_{\mathrm{t}} = \frac{\langle (1+\delta_1)(1+\delta_2)(1+\delta_3)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \rangle}{\langle (1+\delta_1)(1+\delta_2)(1+\delta_3) \rangle}$$

$$\simeq \langle \delta_1 \boldsymbol{v}_2 \rangle - \langle \delta_2 \boldsymbol{v}_1 \rangle + \langle \delta_3 \boldsymbol{v}_2 \rangle - \langle \delta_3 \boldsymbol{v}_1 \rangle$$

$$= \bar{w}(r_{12})\, \hat{\boldsymbol{r}}_{12} - \frac{1}{2} \left[\bar{w}(r_{23})\, \hat{\boldsymbol{r}}_{23} + \bar{w}(r_{31})\, \hat{\boldsymbol{r}}_{31} \right]$$



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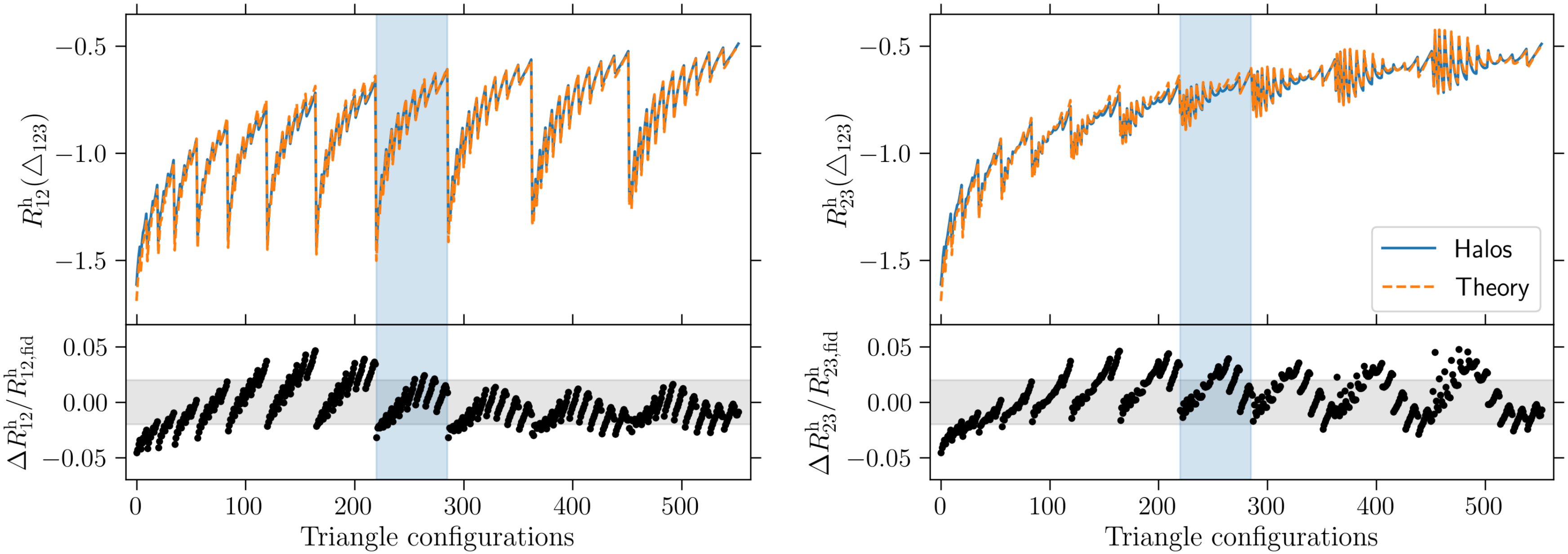


$$R_{12}(\triangle_{123}) = \bar{w}(r_{12}) - \frac{1}{2}\left[\bar{w}(r_{23})\ \cos\chi - \bar{w}(r_{31})\ \frac{r_{12} + r_{23}\cos\chi}{\sqrt{r_{12}^2 + r_{23}^2 + 2 r_{12} r_{23} \cos\chi}}\right]$$

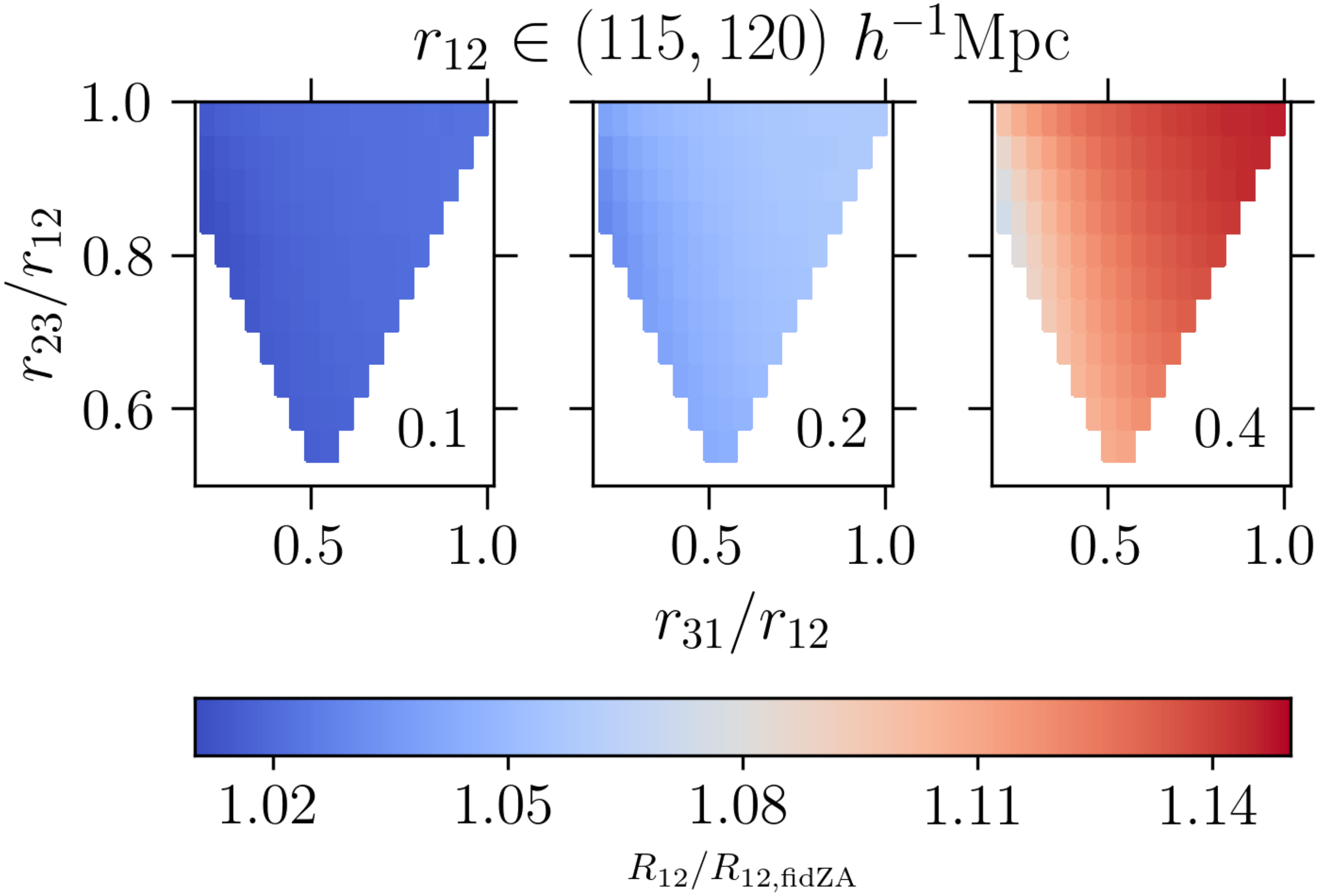
Can we use the relative velocity statistics to constrain cosmology?

Halos - Mean relative velocity between pairs in a triplet

- Fisher matrix formalism
- Compute the covariance matrix for the relative velocity statistics using 15,000 simulations from the Quijote suite.
- Derivatives also computed directly using the Quijote suite.

- The Quijote simulations is a suite of 44,100 full N-body simulations. (Villaescusa-Navarro et al. 20)
- This work runs parallel to the quantification of the information content from the halo redshift-space bispectrum using the Quijote suite. (Hahn et al. 20)

Effect of massive neutrinos



(JK & Aghanim 21)

$$F_{\alpha\beta} = \frac{\partial \boldsymbol{S}}{\partial \theta_\alpha} \cdot \hat{\mathbf{C}}^{-1} \cdot \frac{\partial \boldsymbol{S}^{\mathsf{T}}}{\partial \theta_\beta}$$

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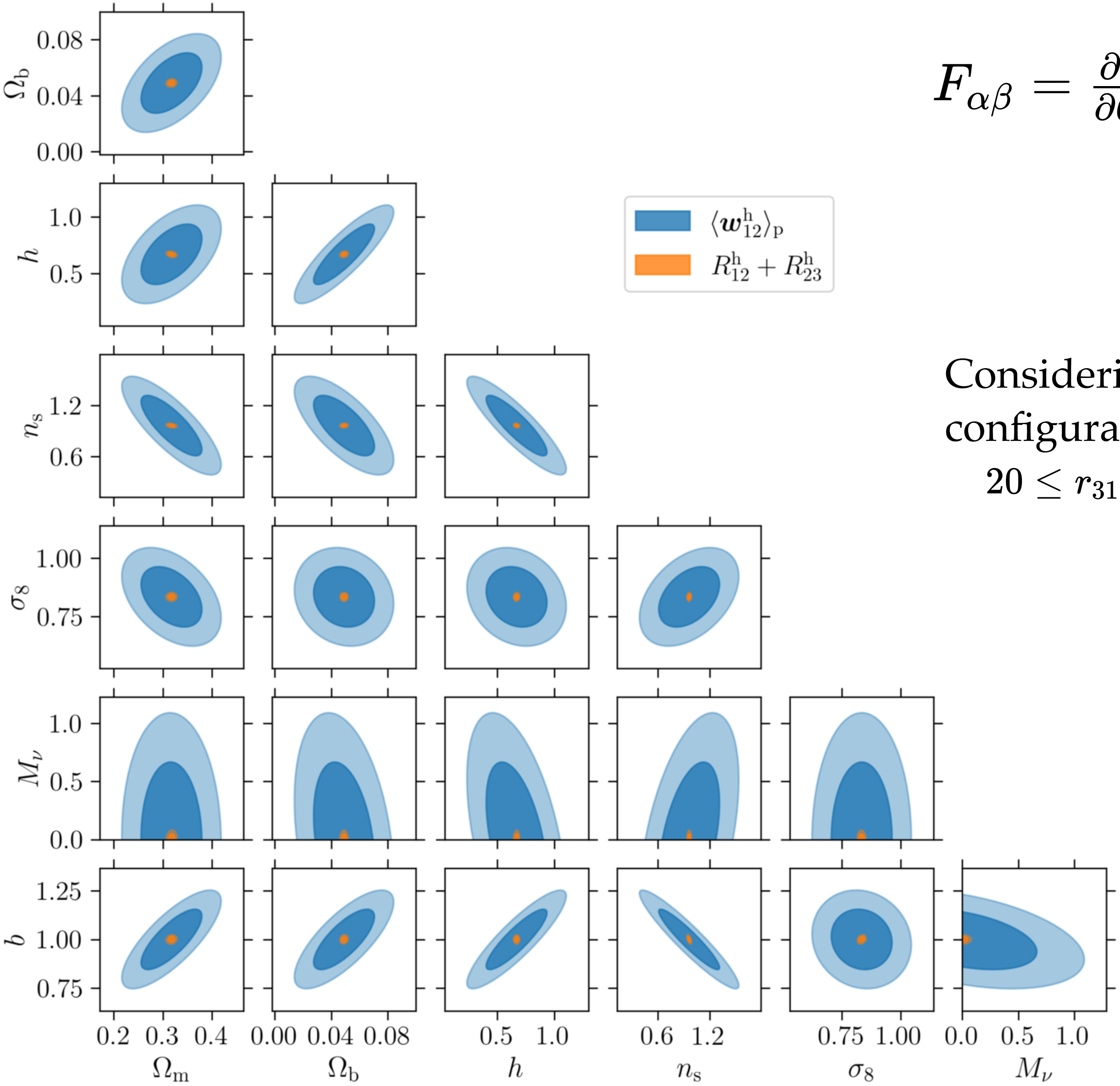
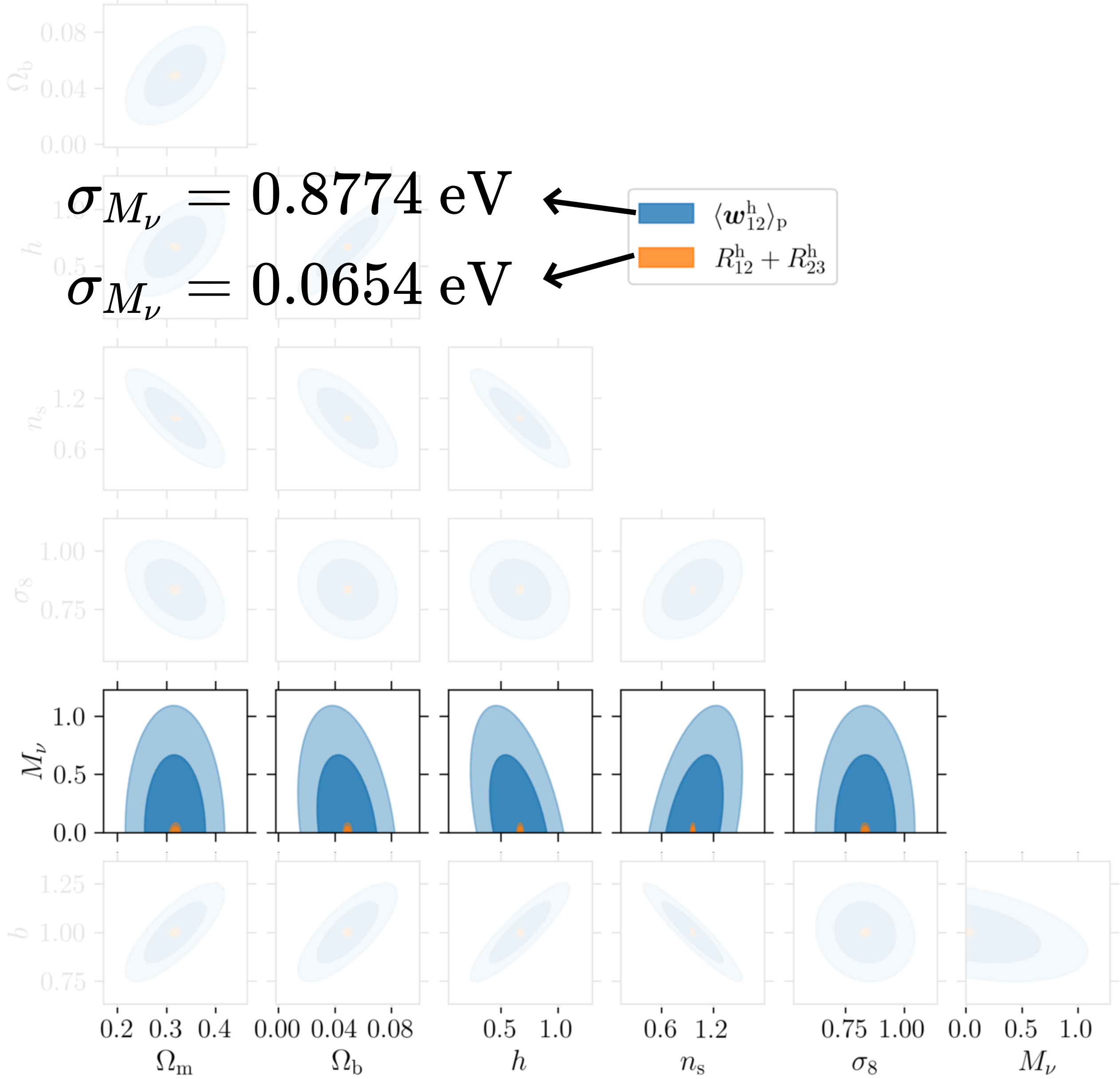


Considering all triangular configurations:

$$20 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120$$

$$\sigma_{M_\nu} = 0.8774 \text{ eV}$$

$$\sigma_{M_\nu} = 0.0654 \text{ eV}$$



But is this really competitive with clustering statistics?

Summary statistics	Matter density	Baryon density	Hubble parameter (h)	Spectral index	Sigma 8	Summed neutrino mass (eV)
mean relative velocity	0.0091	0.0024	0.0226	0.0221	0.0156	0.0655
power spectrum multipoles	**2.6**	**4.8**	**4.9**	**5.7**	**2.3**	**4.5**
bispectrum monopole	**1.2**	**1.7**	**1.7**	**1.5**	**0.9**	**0.8**

(Hahn et al. 20)

* Numbers within the orange box denotes the factor of improvement of 1 sigma constraint from three-point relative velocity with respect to the corresponding clustering statistics.

What about the optical depth degeneracy?

We saw earlier that $\frac{\Delta T^{\mathrm{kSZ}}(r_{12})}{T_{\mathrm{cmb}}} \simeq -\tau \frac{\bar{w}(r_{12})}{c}$ thus $\Delta T^{\mathrm{kSZ}} \propto \tau f \sigma_8^2$

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- One of the attempt to break this degeneracy has been proposed by using fast radio bursts (FRB).

  (Madhavacheril et al. 19)

- However I introduce a new estimator which is independent of optical depth, and also sigma 8.

  (Kuruvilla, 2021, to be submitted)

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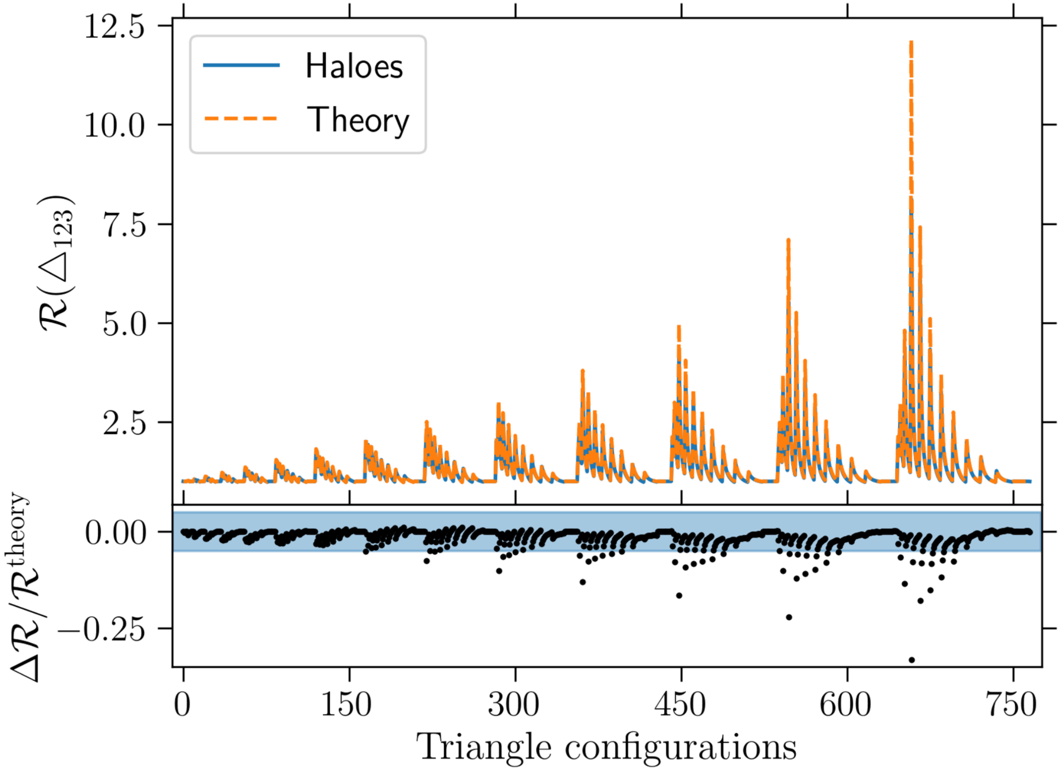
(Kuruvilla, 2021, to be submitted)

$$\mathcal{R}(\triangle_{123}) = \frac{\Delta T_{12}^{\mathrm{kSZ}}(\triangle_{123})}{\Delta T_{23}^{\mathrm{kSZ}}(\triangle_{123})} \equiv \frac{R_{12}^{\mathrm{h}}(\triangle_{123})}{R_{23}^{\mathrm{h}}(\triangle_{123})} = \frac{R_{12}(\triangle_{123})}{R_{23}(\triangle_{123})}$$

Triangular configurations: $40 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120\ h^{-1}\mathrm{Mpc}$

Minimum halo mass: $M_\mathrm{h} > 5 \times 10^{13}\ h^{-1} M_\odot$



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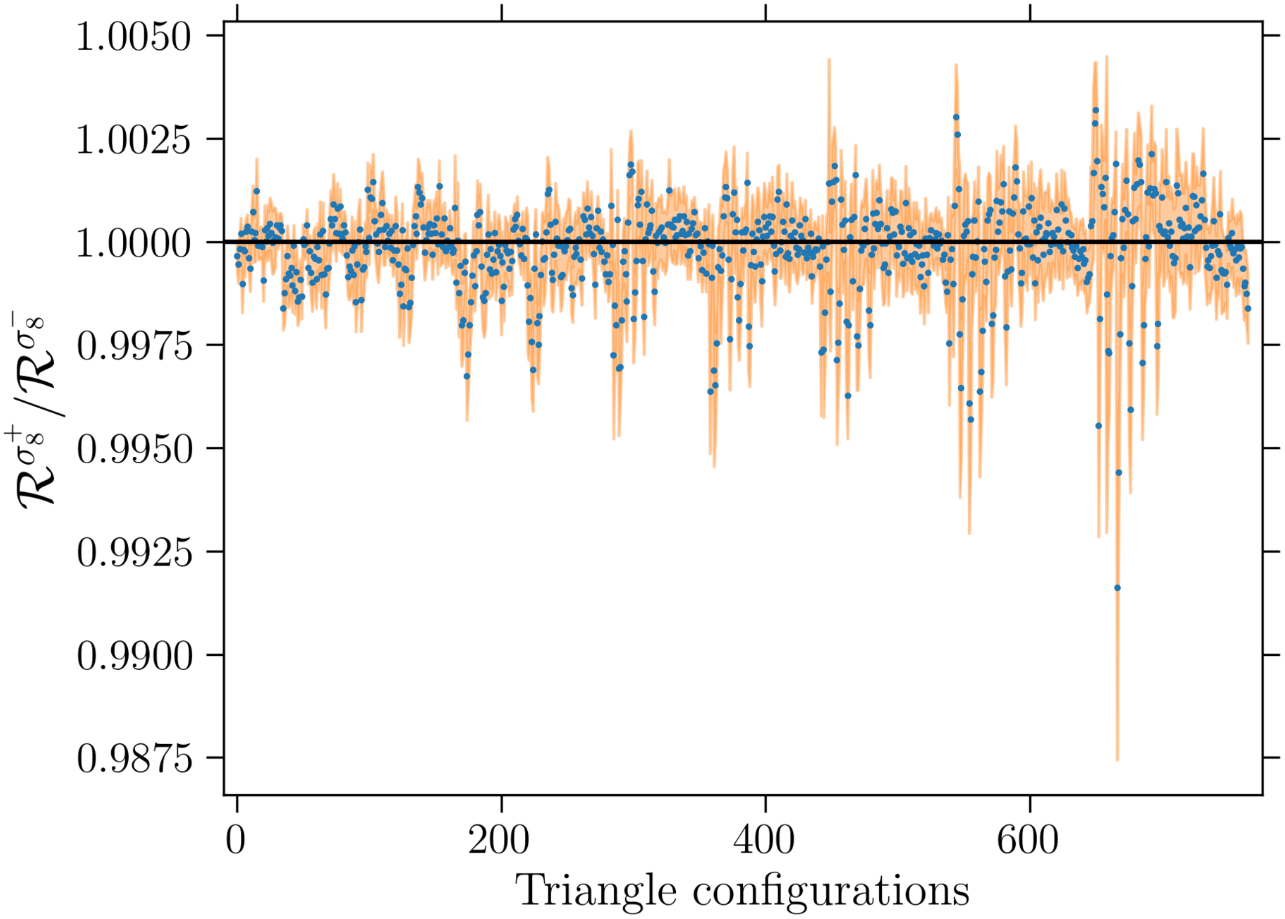
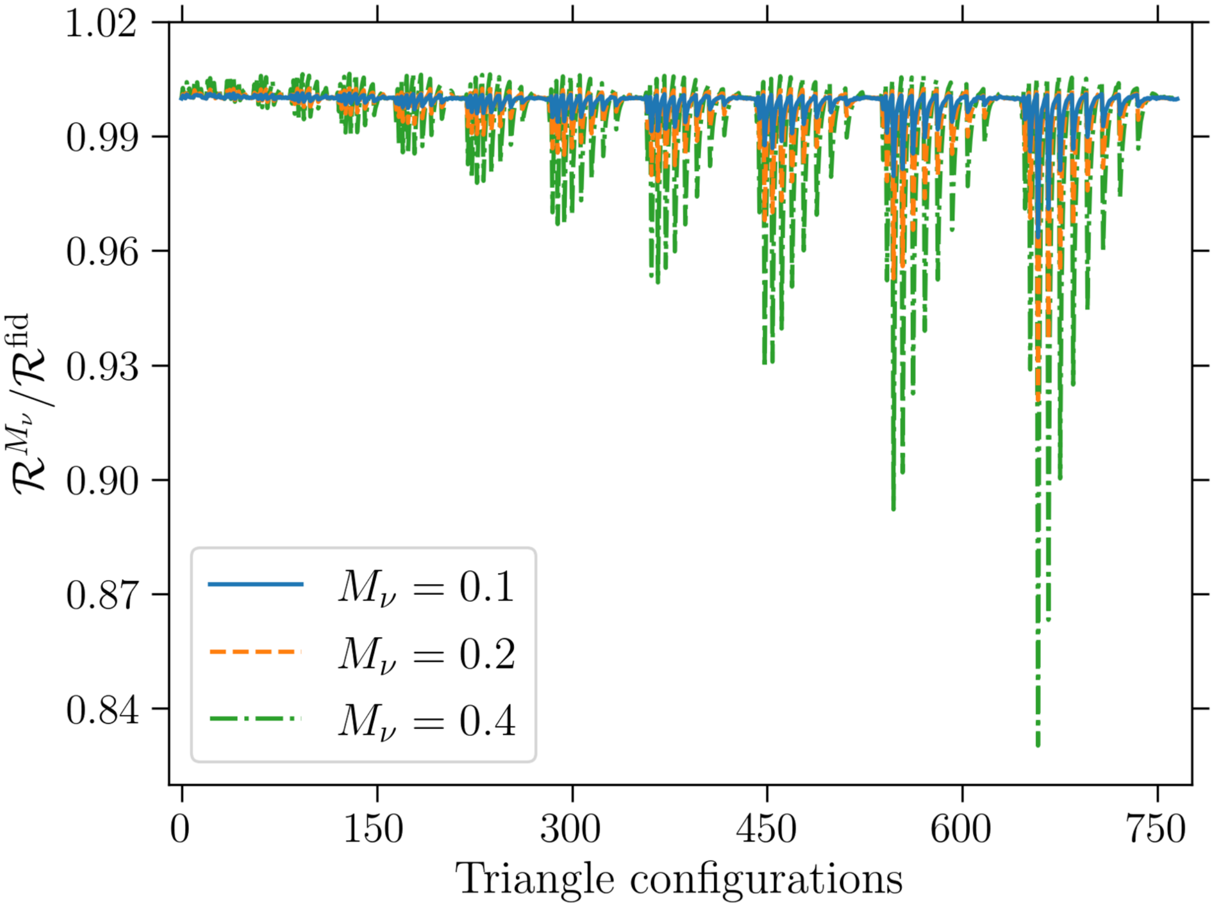


1.0050
1.0025
1.0000
0.9975
0.9950
0.9925
0.9900
0.9875
$\mathcal{R}^{\sigma_8^+}/\mathcal{R}^{\sigma_8^-}$
0
200
400
600
Triangle configurations

Effect of summed neutrino mass



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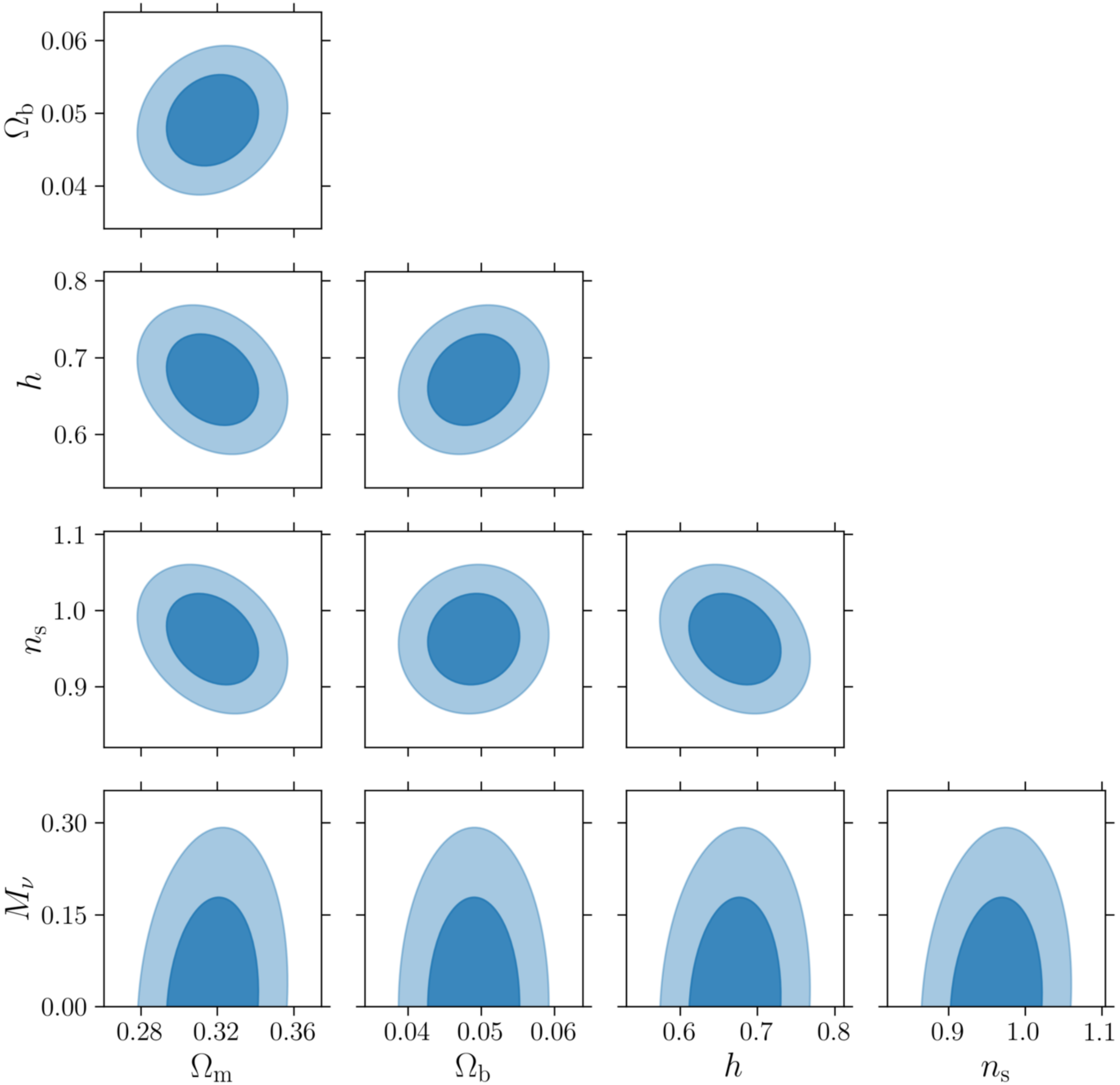
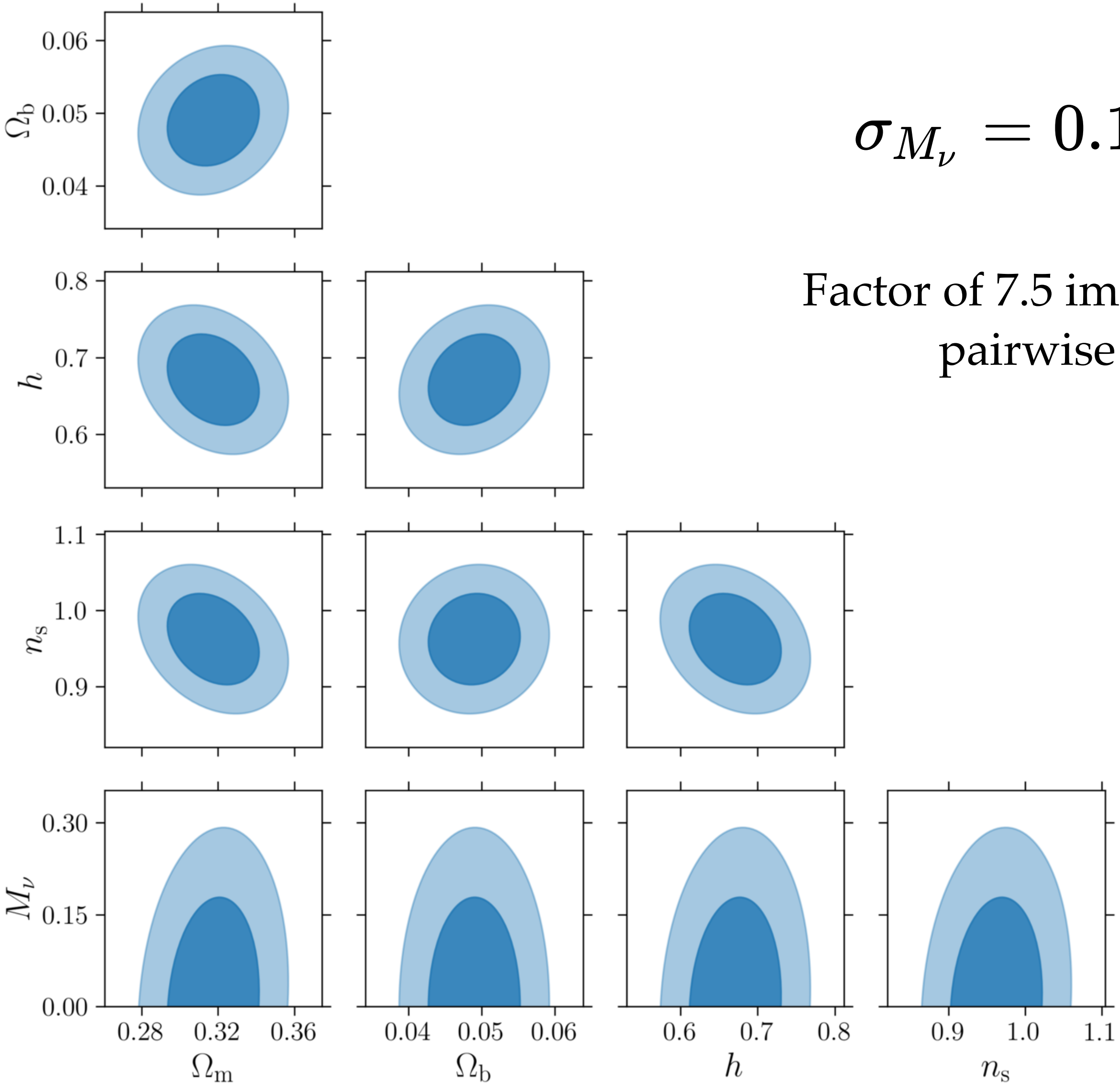

0.06
0.05
0.04
Ω_b
0.8
0.7
0.6
h
1.1
1.0
0.9
n_s
0.30
0.15
0.00
M_ν
0.28
0.32
0.36
Ω_m
0.04
0.05
0.06
Ω_b
0.6
0.7
0.8
h
0.9
1.0
1.1
n_s



$$\sigma_{M_\nu} = 0.1175 \text{ eV}$$

Factor of 7.5 improvement from pairwise velocities

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Conclusions

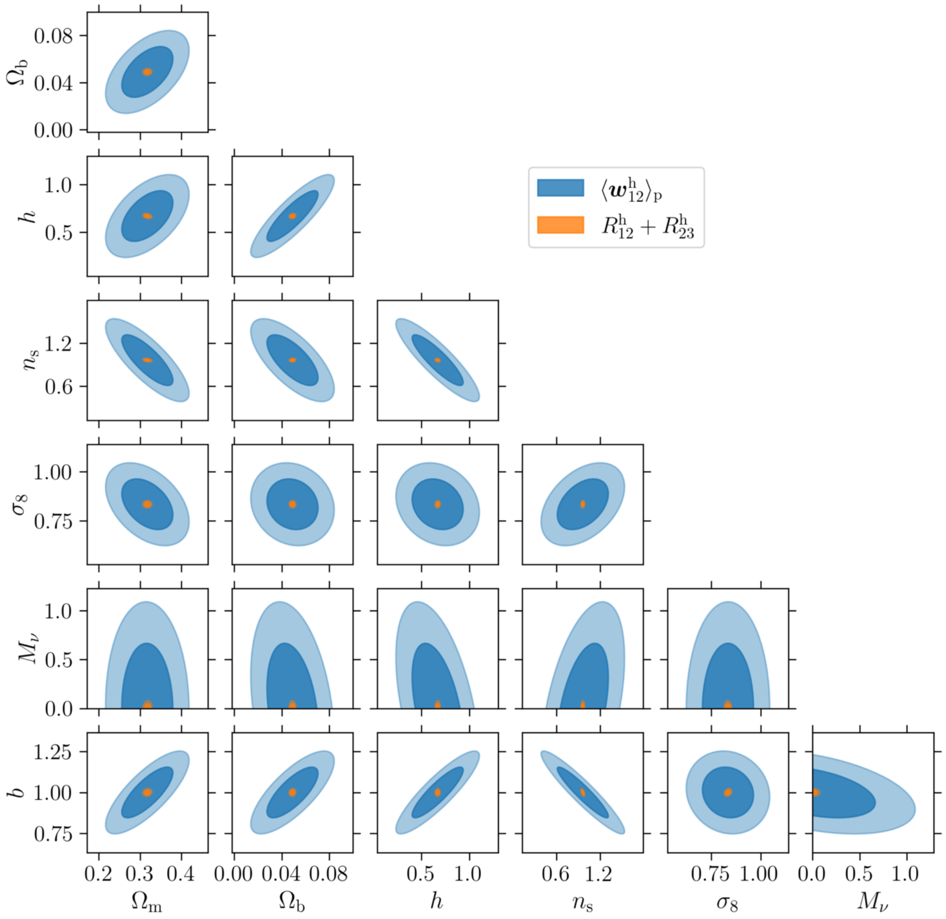
- Cosmological constraints from the mean three-point relative velocity statistics are competitive with those obtained from the bispectrum, while having sizeable improvements with respect to the power spectrum.

(Kuruvilla & Aghanim 2021, submitted to A&A)

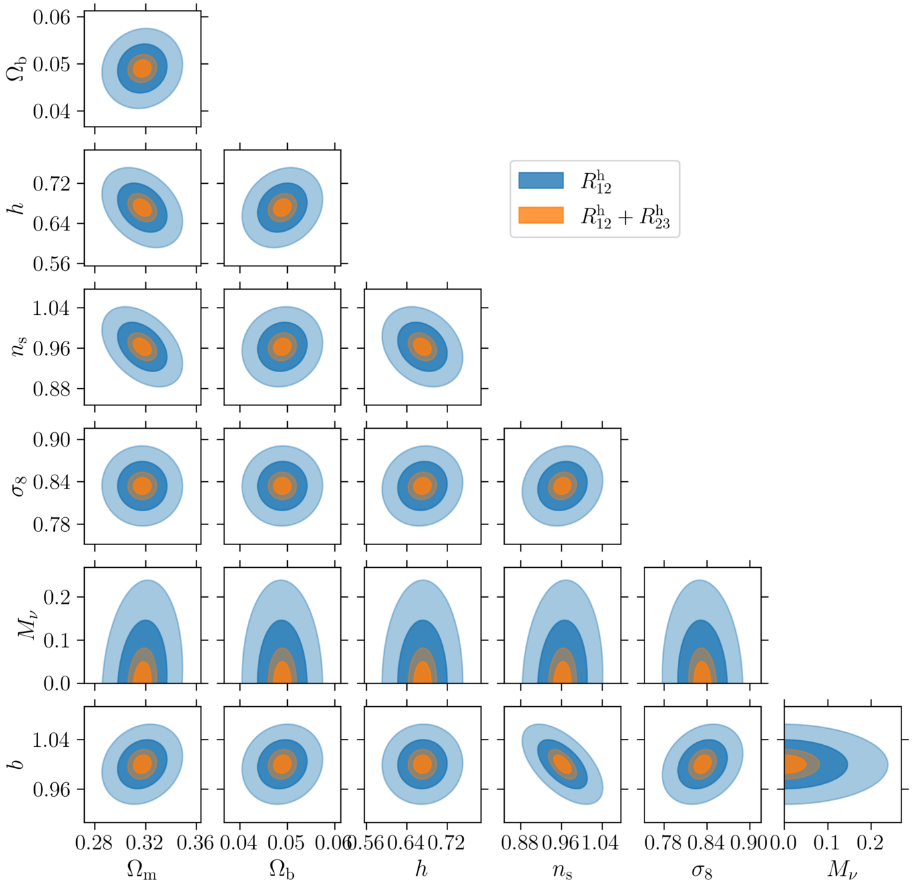
- Introduced a new estimator which enables to constrain the summed neutrino mass independent of optical depth and sigma 8.

(Kuruvilla 2021, to be submitted)

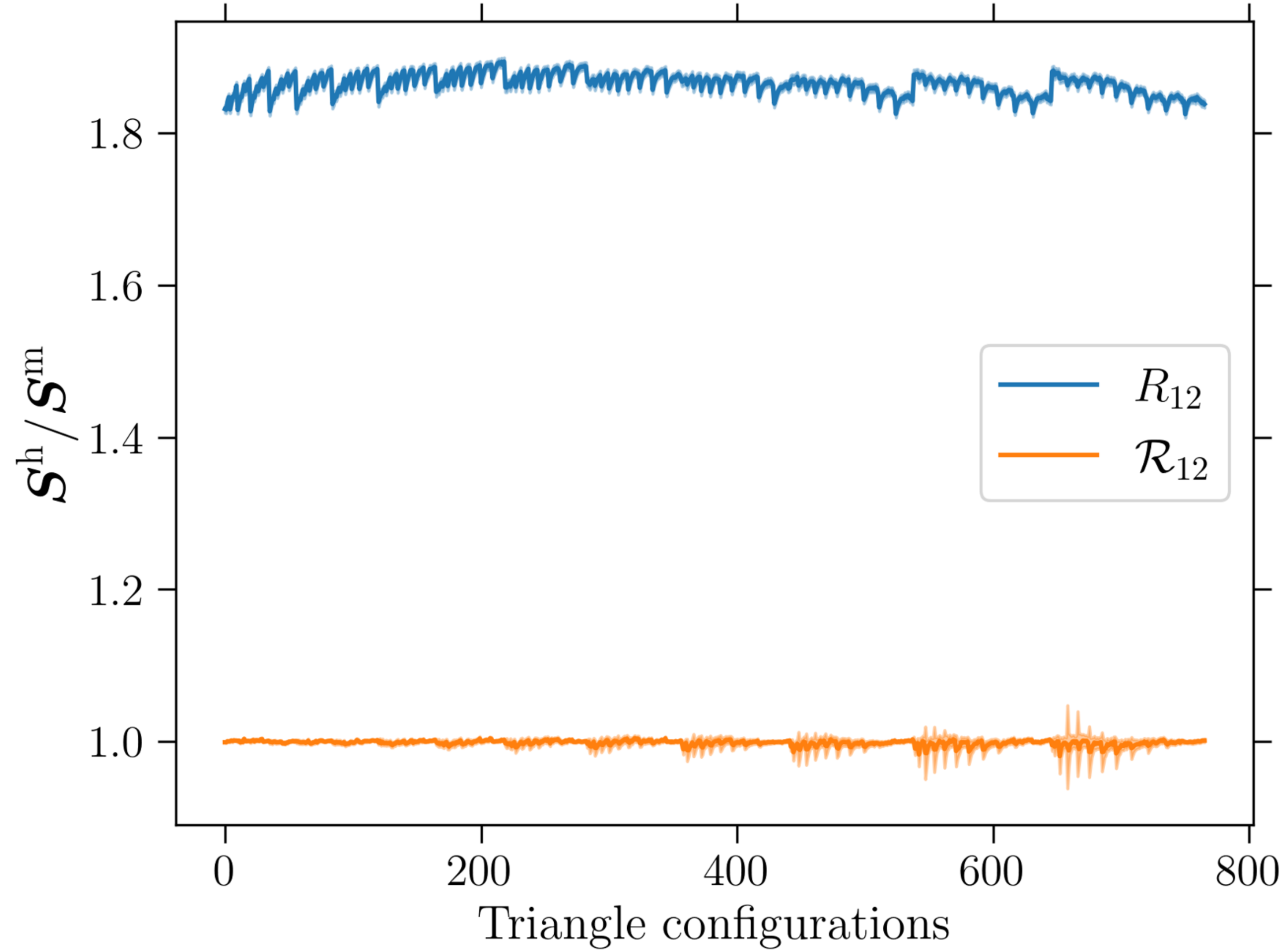
Thank you for listening!

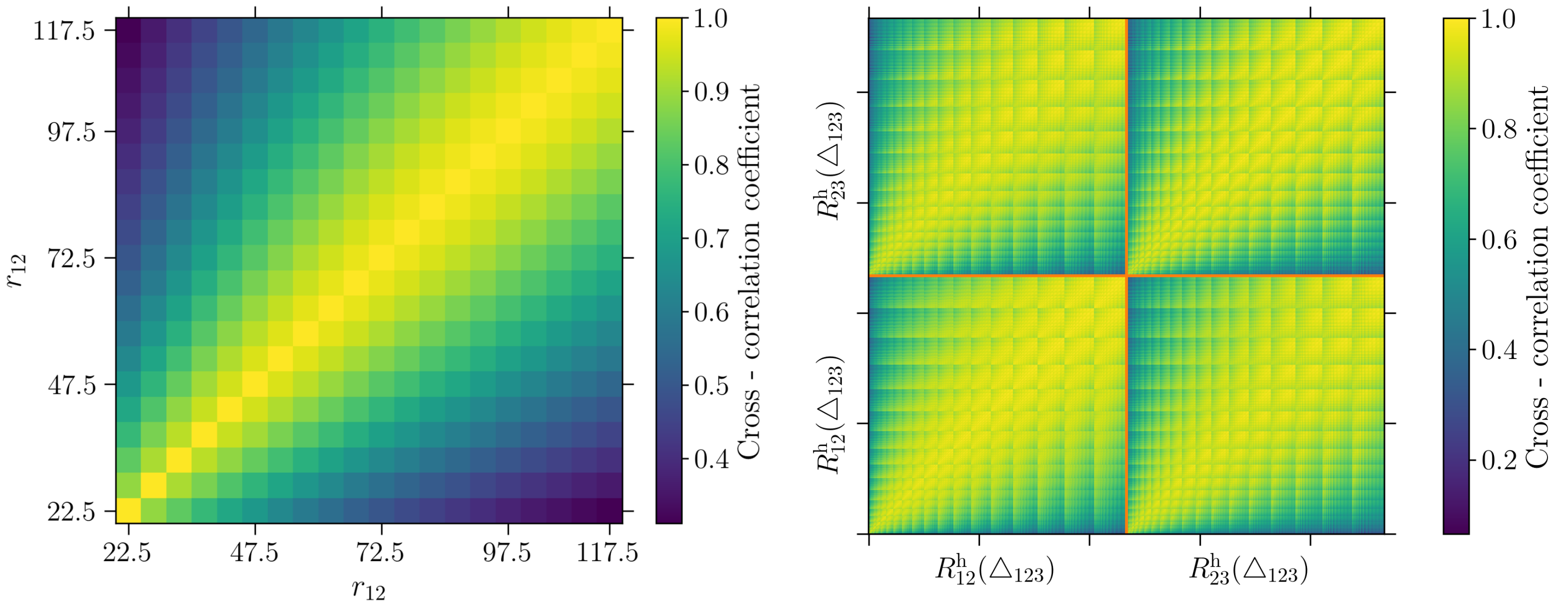


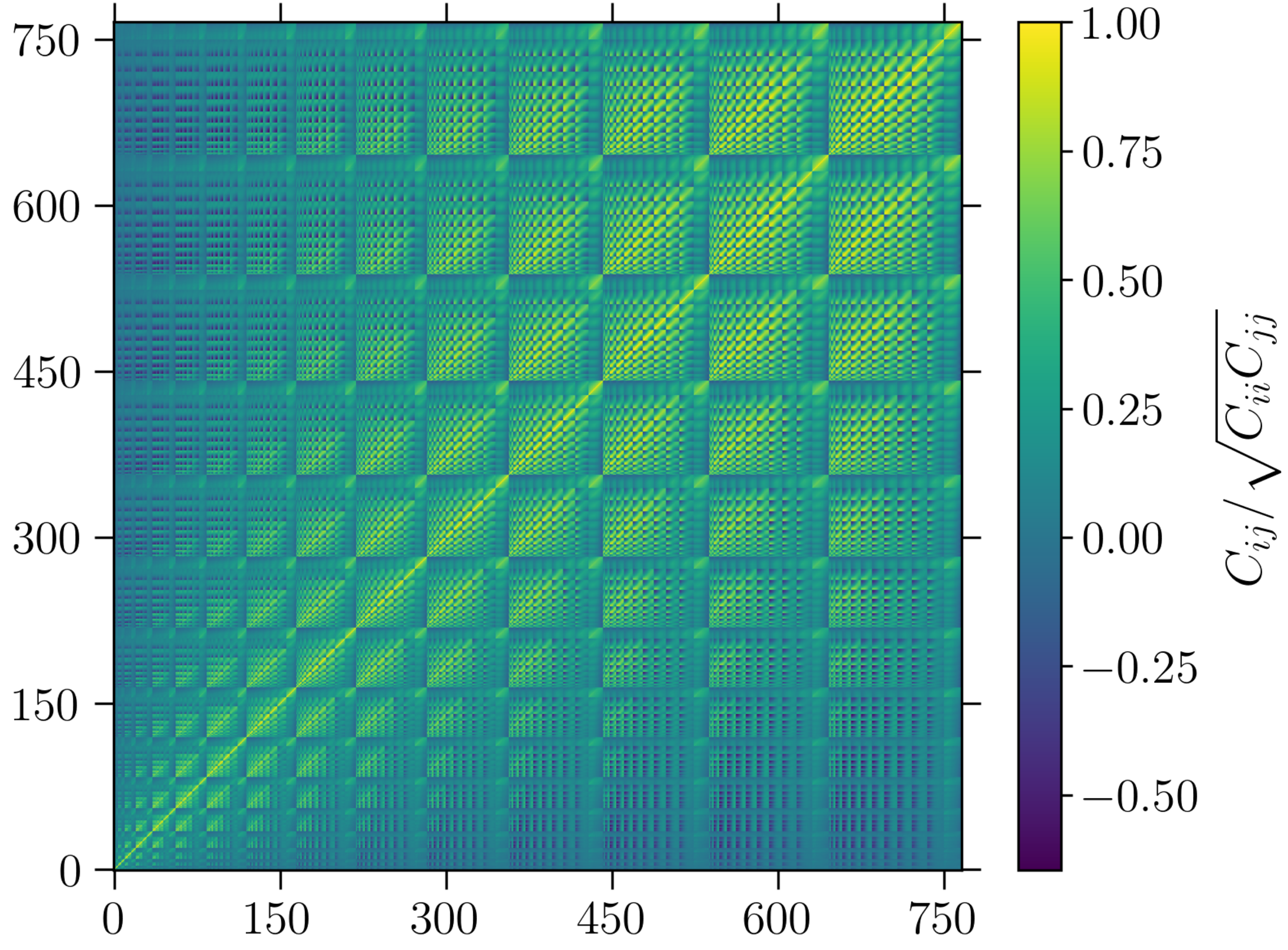
$\langle \boldsymbol{w}^{\mathrm{h}}_{12} \rangle_{\mathrm{p}}$
$R^{\mathrm{h}}_{12} + R^{\mathrm{h}}_{23}$
Ω_{b}
h
n_{s}
σ_8
M_ν
b
Ω_{m}



R^{h}_{12}
$R^{\mathrm{h}}_{12} + R^{\mathrm{h}}_{23}$
Ω_{b}
h
n_{s}
σ_8
M_ν
b
Ω_{m}


$S^{\mathrm{h}}/S^{\mathrm{m}}$
1.8
1.6
1.4
1.2
1.0
0
200
400
600
800
Triangle configurations
R_{12}
$\mathcal{R}_{12}$


117.5
97.5
72.5
47.5
22.5
r_{12}
22.5
47.5
72.5
97.5
117.5
r_{12}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Cross - correlation coefficient
$R^{h}_{23}(\triangle_{123})$
$R^{h}_{12}(\triangle_{123})$
$R^{h}_{12}(\triangle_{123})$
$R^{h}_{23}(\triangle_{123})$
1.0
0.8
0.6
0.4
0.2
Cross - correlation coefficient



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