Phenomenological models for the nonlinear matter power spectrum

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Atelier Dark Energy

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Halo models

Cooray & Sheth (2002)

Pairs of particles are either in the same halo or in 2 different halos:

$$P(k) = P^{1h}(k) + P^{2h}(k),$$

1-halo term:

$$P^{1h}(k) = \int \mathrm{d}m\, n(m) \left(\frac{m}{\bar{\rho}}\right)^2 |u(k|m)|^2$$

dominates at high k (small scales), associated with intra-halo regions

It depends on the halo mass function (fit to simulations) and halo profile (NFW), concentration parameter.

2-halos term:

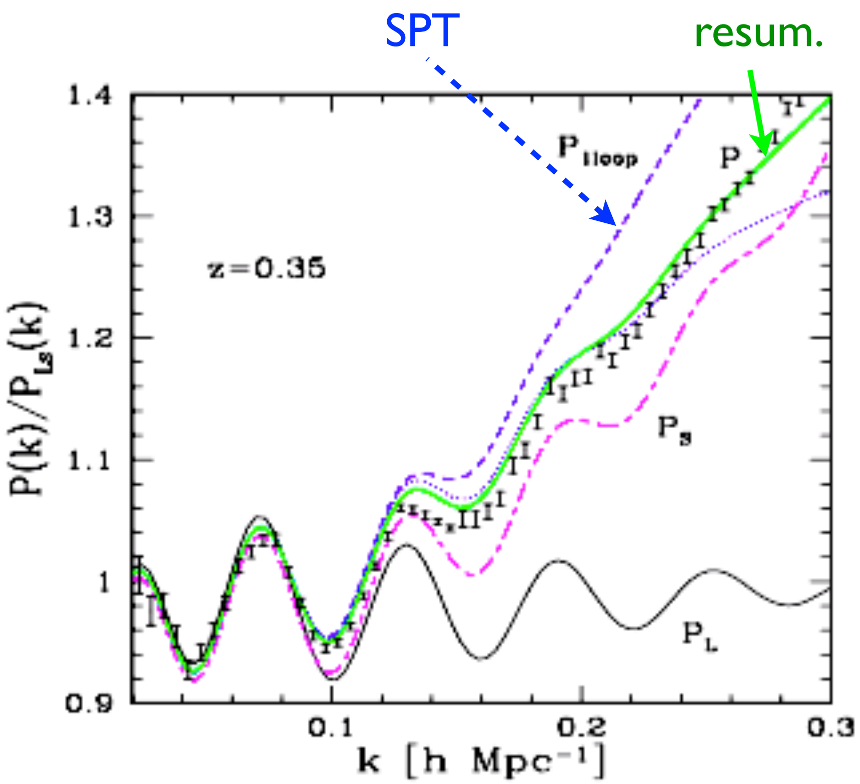
$$P^{2h}(k) = \int \mathrm{d}m_1 n(m_1) \left(\frac{m_1}{\bar{\rho}}\right) u(k|m_1) \int \mathrm{d}m_2 n(m_2) \left(\frac{m_2}{\bar{\rho}}\right) u(k|m_2) P_{hh}(k|m_1, m_2)$$

It dominates at low k (large scales), associated with the correlation function of halos.

If one considers the total matter power spectrum, it must converge to the linear matter power spectrum on large scales.

It is often replaced by the linear power spectrum, or augmented by higher-order perturbative contributions (1-loop, ...).

ratio of P(k) to a smooth reference

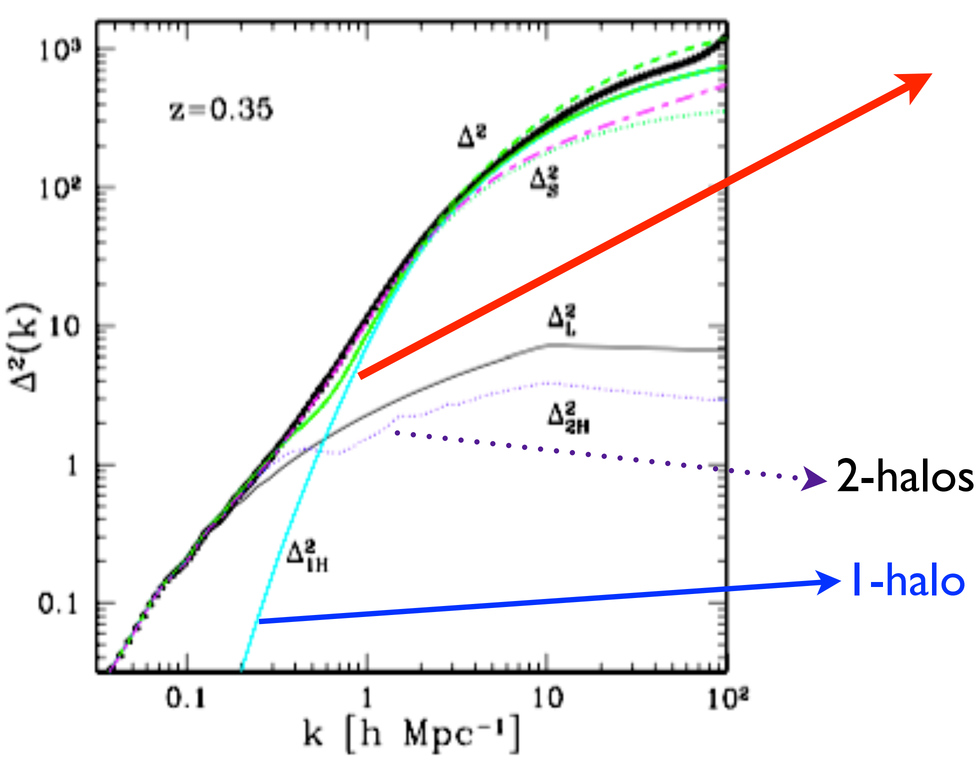


If one uses perturbation theory, one needs to go beyond SPT:
- better accuracy
- good behavior at high k for $P_{2H}(k)$: does not explode above $P_{1H}(k)$

→
- partial resummations
- Lagrangian approach
(- ad-hoc cutoff)

in combination with EFT if Eulerian EFT

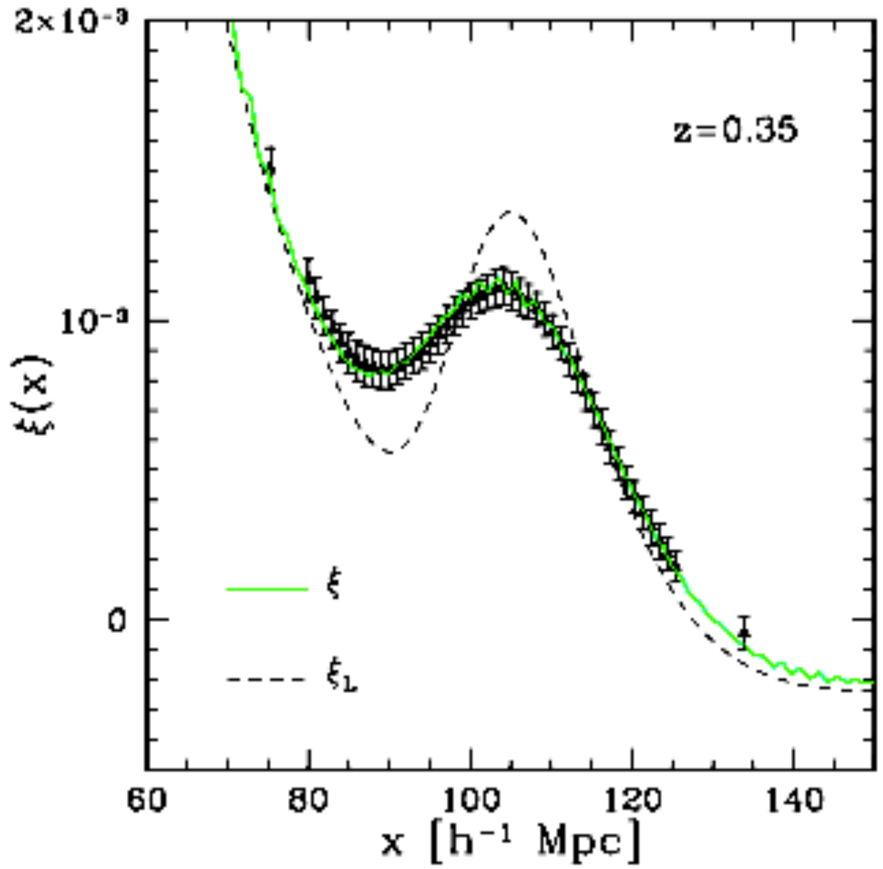
logarithmic power $\Delta^2(k) = 4\pi\, k^3 P(k)$



The transition region is difficult to get right: usually a lack of power.

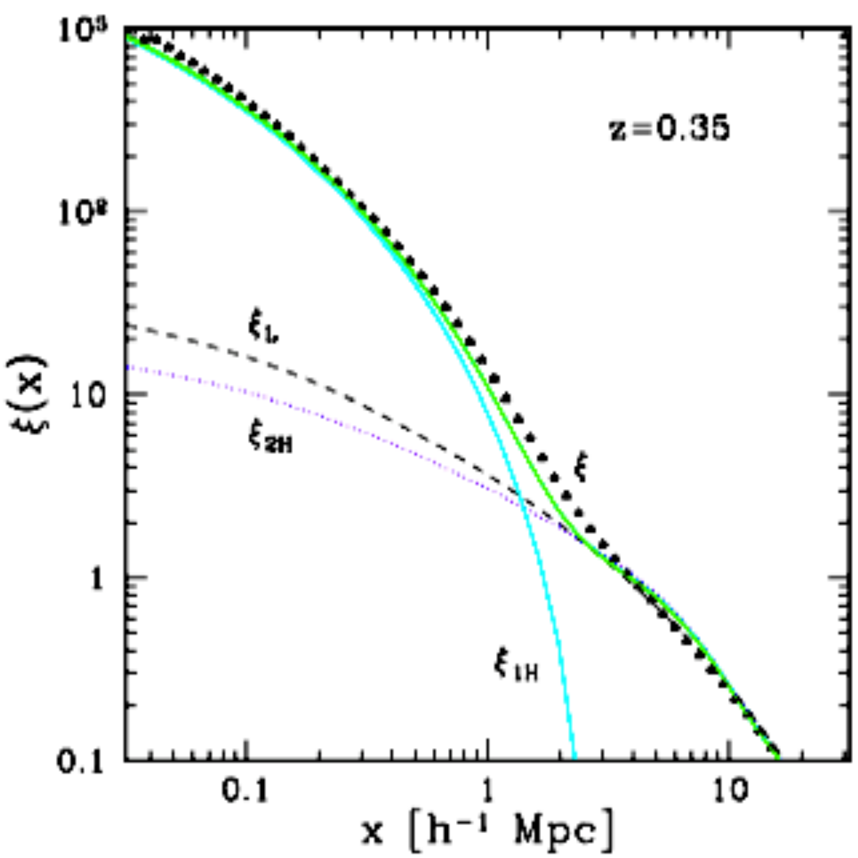
Valageas & Nishimichi (2011)

real-space two-point correlation function



- To obtain the real-space correlation one needs a well-behaved power spectrum from low to high k (so that the Fourier integral converges).

- Rather easy to recover the BAO peak, especially with Lagrangian approaches (Zeldovich approx. already works much better than linear theory).



- The lack of power at the transition also shows in the real-space correlation function.

- The separation between quasi-linear and non-linear scales/effects is cleaner in the correlation function than in the power spectrum (e.g., see the BAO peak).

Transition region

1) Smoothing the transition with a fitting parameter:

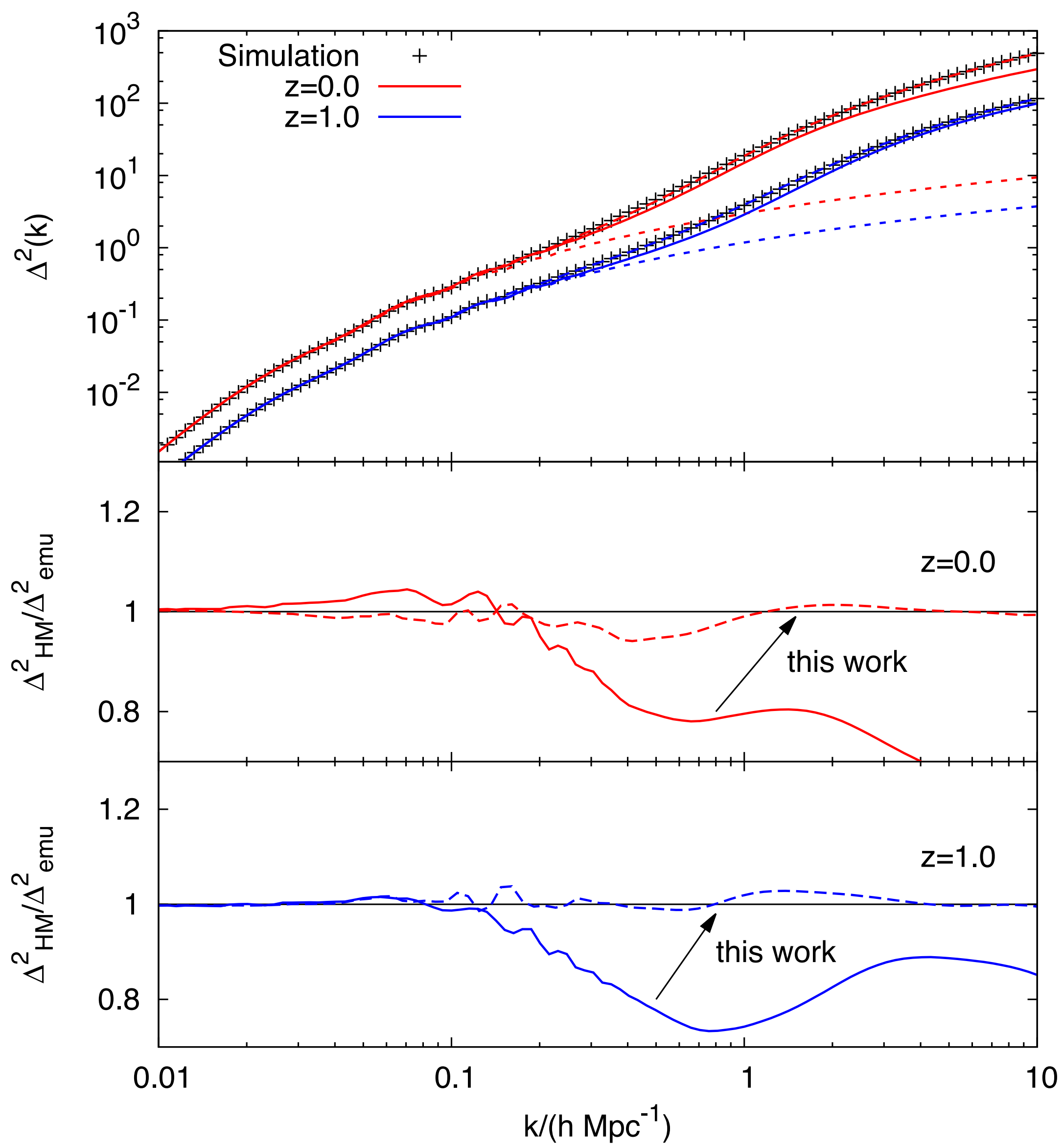
Mead et al. (2015)

$$\Delta^2(k) = [(\Delta'^2_{2H})^\alpha + (\Delta'^2_{1H})^\alpha]^{1/\alpha}$$

α Quasi-linear one- to two-halo term softening

$2.93 \times 1.77^{n_{\rm eff}}$

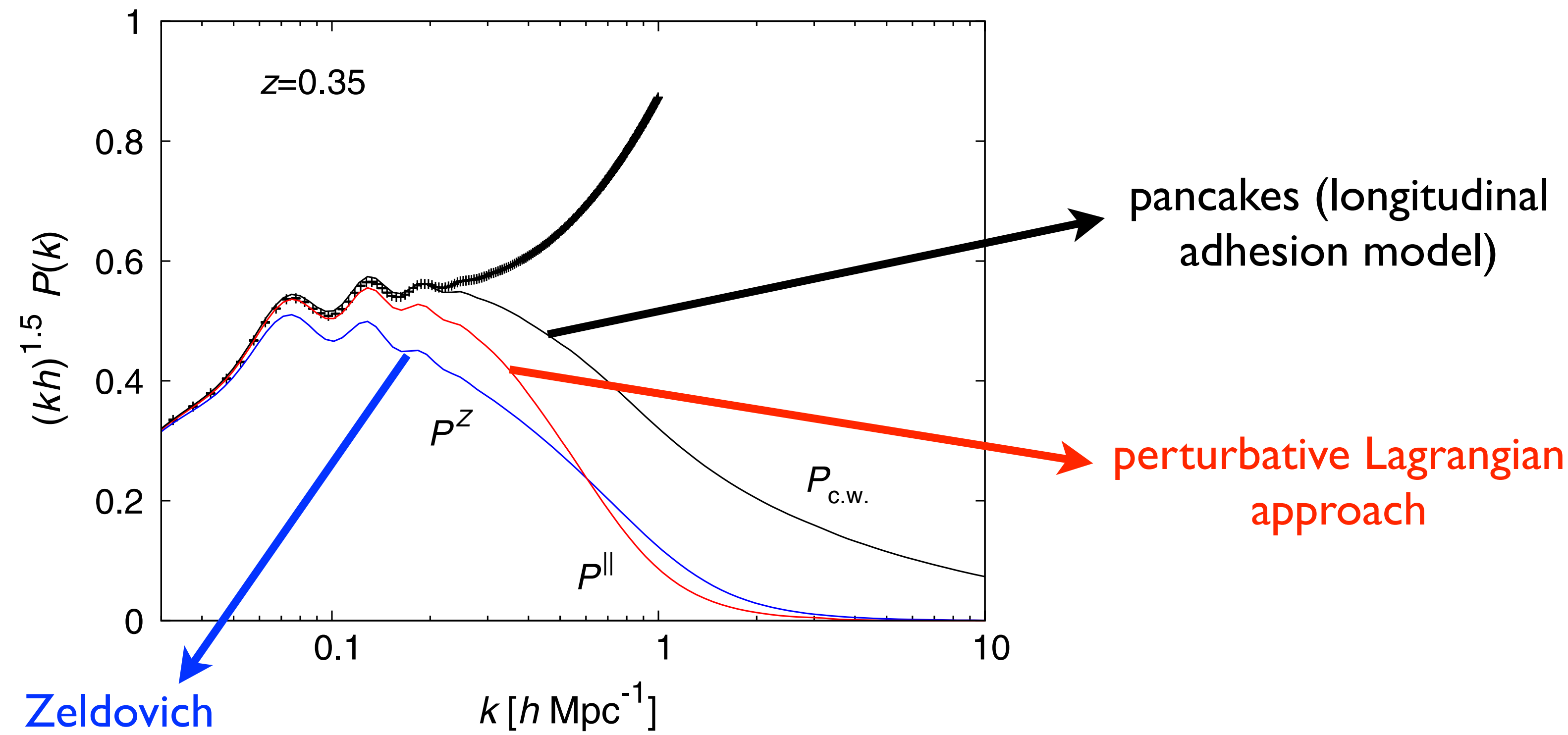
$$3 + n_{\rm eff} \equiv -\left.\frac{{\rm d}\ln\sigma^2(R)}{{\rm d}\ln R}\right|_{\sigma=1}$$

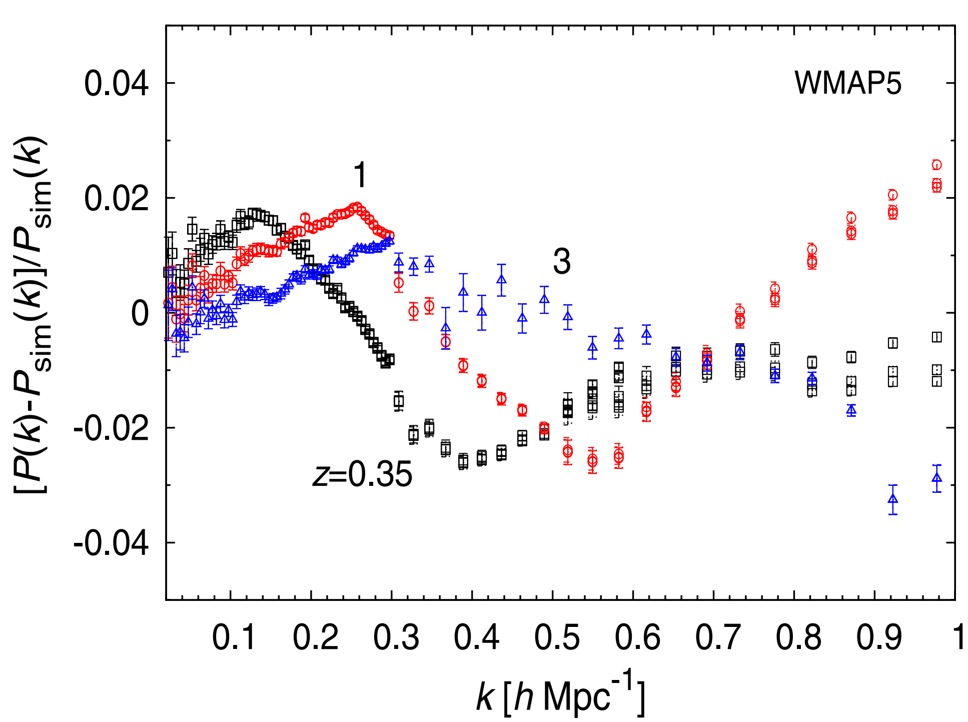


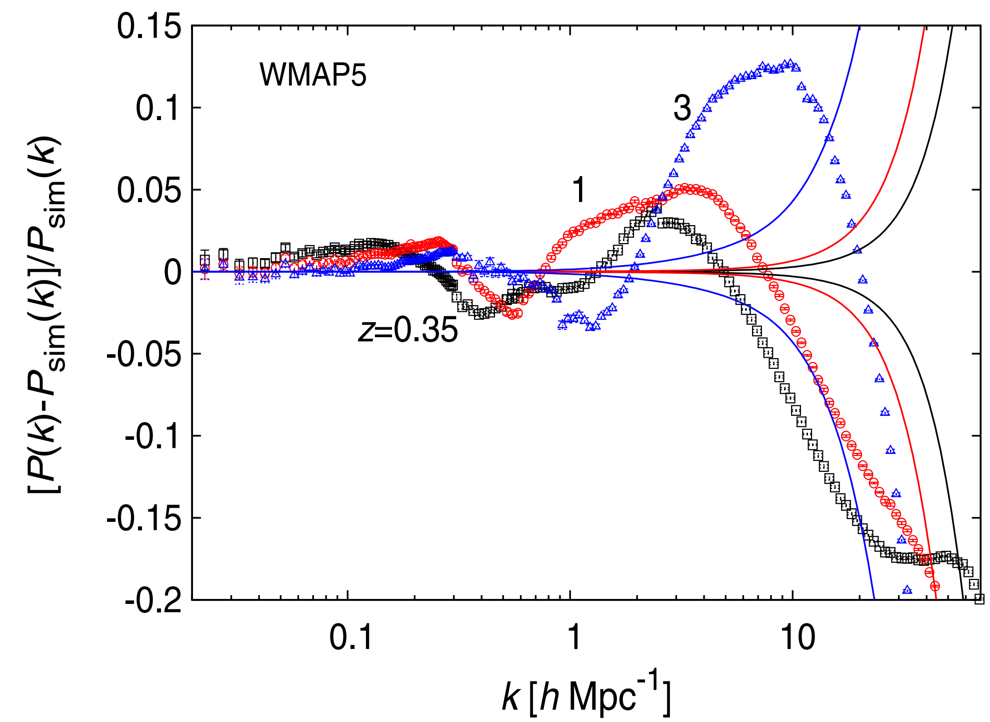
2) Formation of pancakes (adhesion model along the pair longitudinal direction):

$$\mathcal{P}_{\parallel}^{\rm ad}(\kappa_{\parallel}) = a_1 \Theta(\kappa_{\parallel} > 0) \mathcal{P}_{\parallel}(\kappa_{\parallel}) + a_0 \delta_D(\kappa_{\parallel}),$$

Valageas et al. (2013)







3) Halo-void-dust model

Voivodic et al. (2020)

$$\rho(\mathbf{x}) = \sum_i^{\text{halos}} \rho_h(\mathbf{x} - \mathbf{x}_i | M_i) + \sum_j^{\text{voids}} \rho_v(\mathbf{x} - \mathbf{x}_j | M_j) + \rho_d(\mathbf{x})$$

$$P_{mm}(k) = P_{mm}^{1H}(k) + P_{mm}^{2H}(k) + P_{mm}^{1V}(k) + P_{mm}^{2V}(k) + 2P_{mm}^{HV}(k) + 2P_{mm}^{HD}(k) + 2P_{mm}^{VD}(k) + P_{mm}^{2D}(k)\,,$$

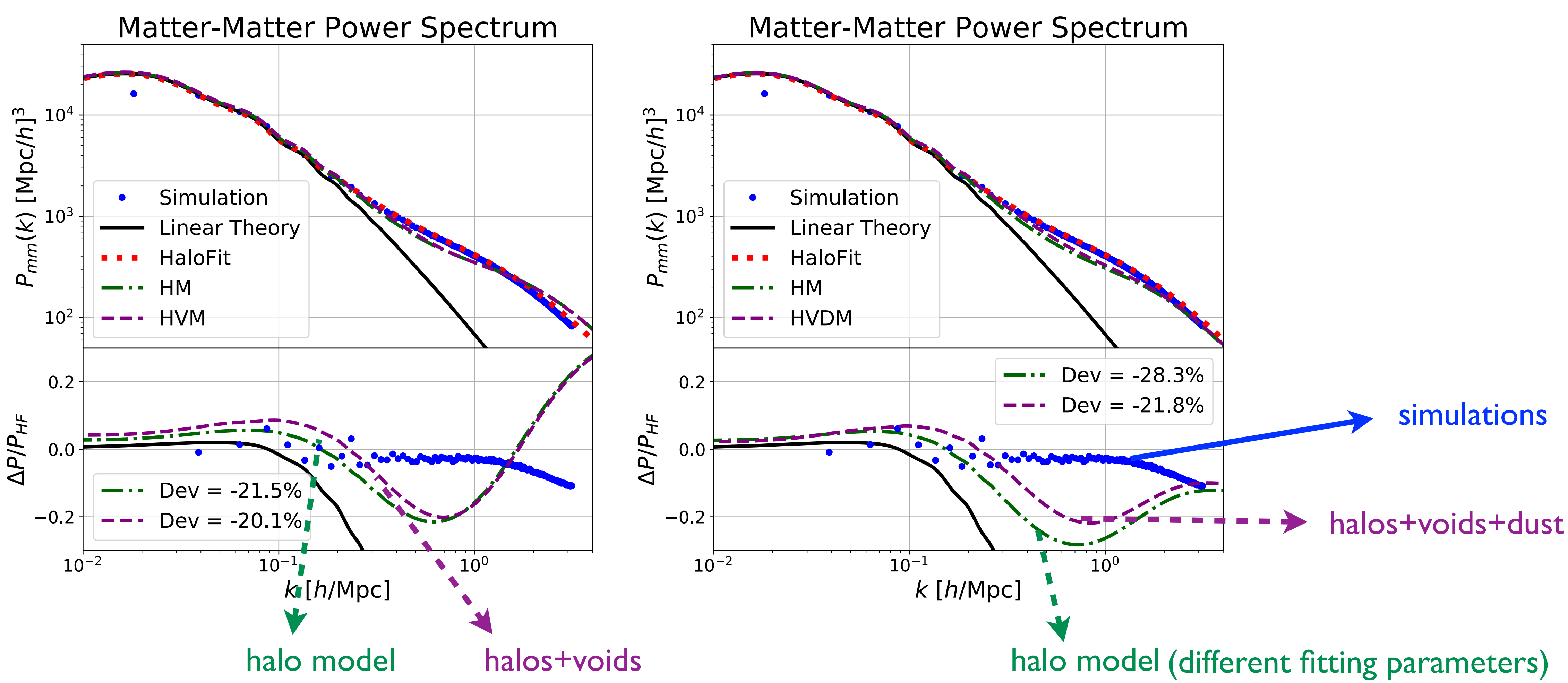
void profile:

$$\frac{\rho_v(r|r_v)}{\bar{\rho}_m} = \frac{1}{2}\left[1 + \tanh\left(\frac{y - y_0}{s(r_v)}\right)\right]$$

void multiplicity function:

$$\frac{dn_v}{d\ln R} = \frac{f_v(\sigma)}{V(R)} \frac{d\ln\sigma^{-1}}{d\ln R}$$

$$f_v^{1\mathrm{SB}}(\sigma) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\delta_v}{\sigma} \exp\left(-2\delta_v^2/2\sigma^2\right)$$



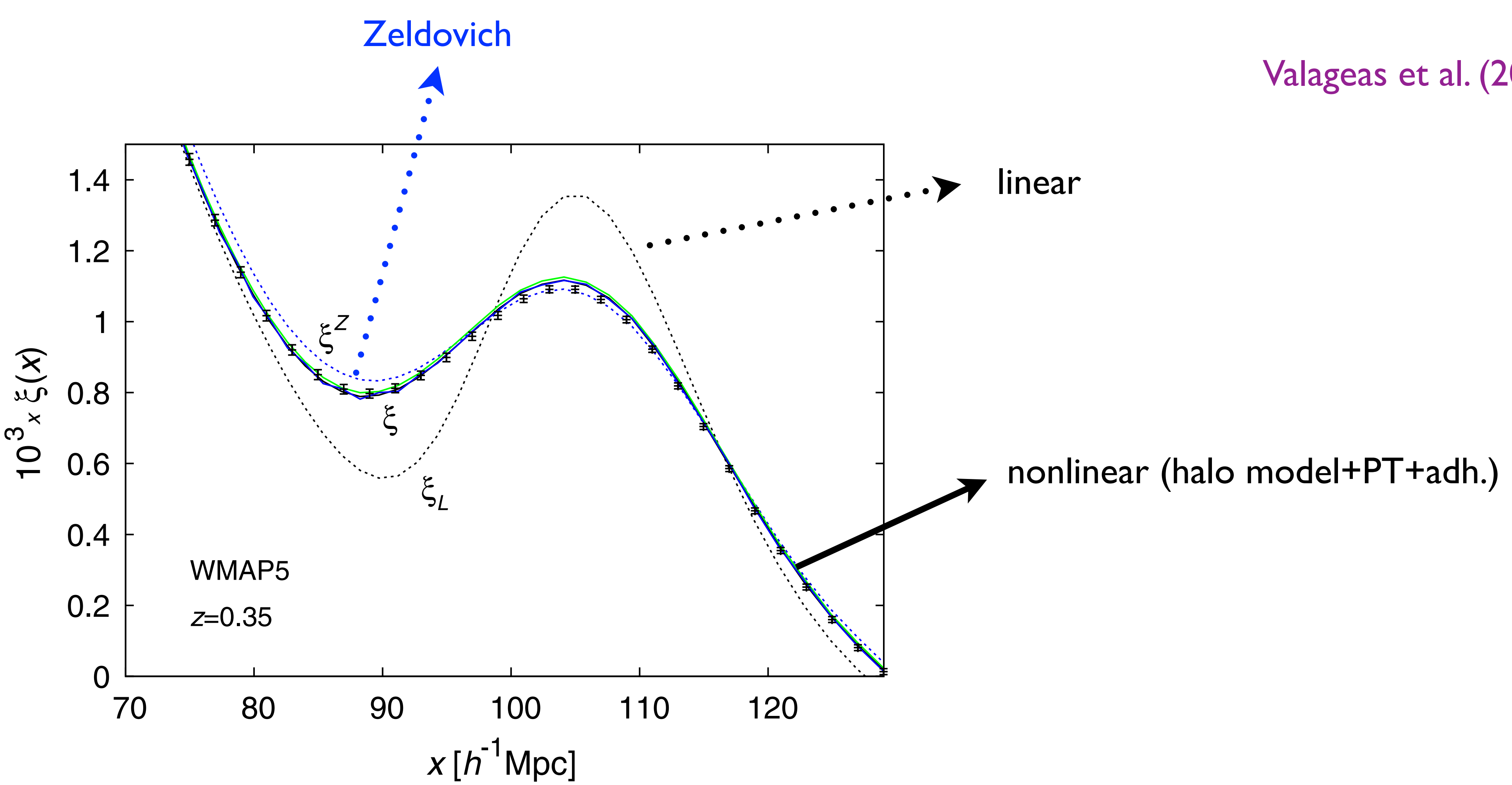
BAO damping

1) Resummations or Lagrangian approaches:

The BAO peak is rather easy to obtain from Lagrangian approaches, or resummations that capture the large-scale (IR) displacements (Lagrangian approach in disguise).

This is clear from the comparison between the Eulerian linear theory and the (Lagrangian linear) Zeldovich approximation.

Valageas et al. (2013)



2) Fit the damping

Mead et al. (2015)

$$\Delta^2_{\rm lin}(k) \rightarrow {\rm e}^{-k^2\sigma_{\rm v}^2}\Delta^2_{\rm lin}(k)$$

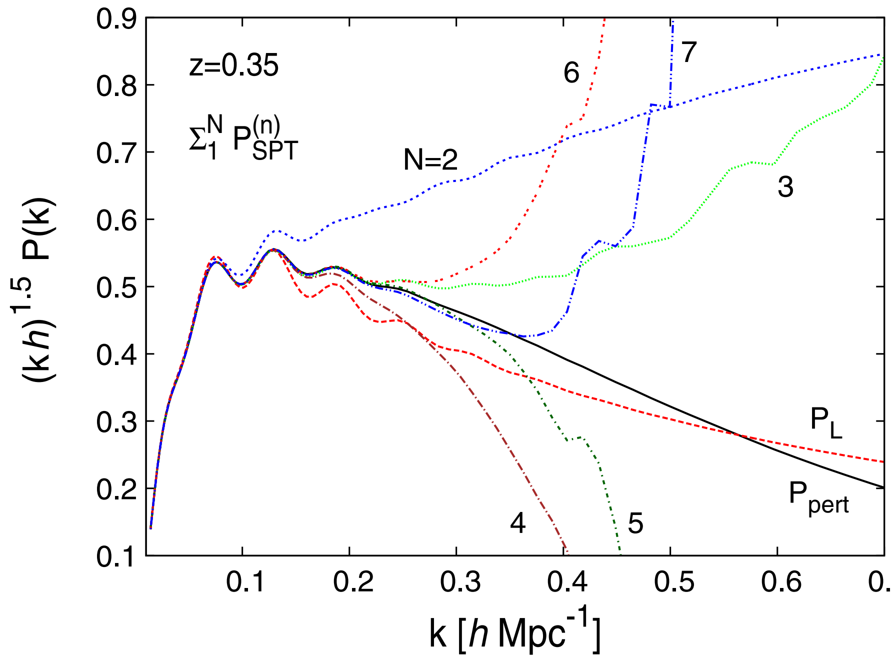
Originates from a Lagrangian picture: damping from large-scale displacements

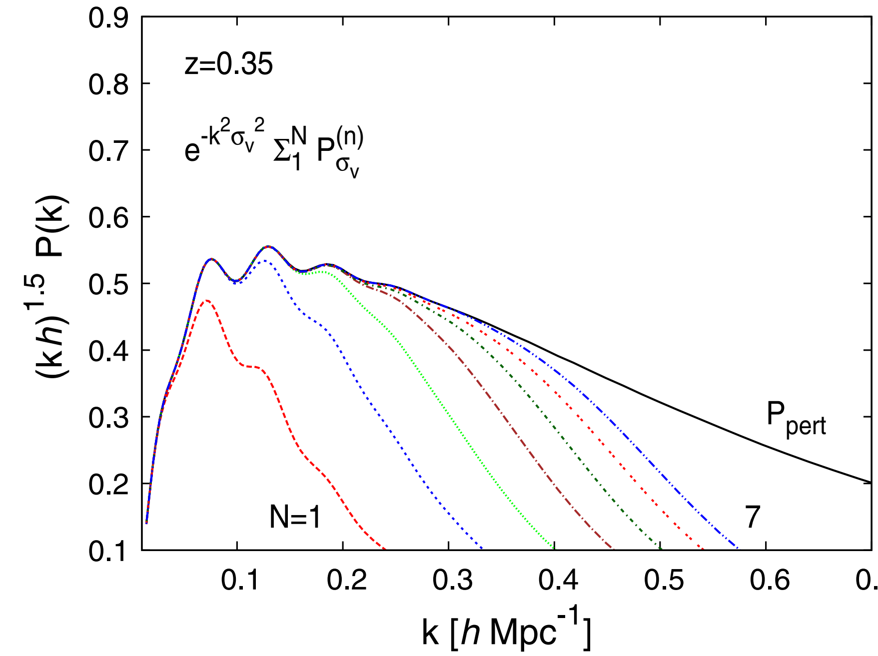
Crocce et al. (2006)

Re-ordering of PT: $P_{\rm pert}(k) = e^{-k^2\sigma_v^2}\sum_{n=1}^{\infty} P^{(n)}_{\sigma_v}(k) \quad \text{with} \quad P^{(n)}_{\sigma_v} \propto (P_L)^n,$

all PT orders are now positive:

the “convergence” (to the pert. contrib.) is better controlled





Valageas et al. (2013)

However, this is not fully correct: does not apply to equal-time statistics → over-damping

Mead et al. (2015)

$$\Delta'^2_{\rm 2H}(k) = \left[1 - f\tanh^2\left(k\sigma_{\rm v}/\sqrt{f}\right)\right]\Delta^2_{\rm lin}(k) \qquad k\sigma_{\rm v} \gg 1 \qquad \bar{\Delta}^2_{\rm 2H} = (1-f)\Delta^2_{\rm lin}$$

Low-k damping of the 1-halo term

$$P^{1h}(k) = \int \mathrm{d}m\, n(m) \left(\frac{m}{\bar{\rho}}\right)^2 |u(k|m)|^2$$ goes to constant at $k \to 0$

$P_L(k) \propto k^n \to 0 \quad \text{at} \quad k \to 0$

The 1-halo term would dominate on very large scales, which is not physical.

In fact, any nonlinear redistribution of matter should give rise to a contribution that decays as

Peebles (1974)

- matter conservation: $\propto k^2$
- momentum conservation: $\propto k^4$

Such global constraints are "forgotten" in the standard halo model.

Using a compensated filter would spoil the 2-halo term. Cooray & Sheth (2002)

$$P^{2h}(k) = \int \mathrm{d}m_1 n(m_1) \left(\frac{m_1}{\bar{\rho}}\right) u(k|m_1) \int \mathrm{d}m_2 n(m_2) \left(\frac{m_2}{\bar{\rho}}\right) u(k|m_2) P_{hh}(k|m_1, m_2)$$

1) Mass conservation easily taken into account through a Lagrangian derivation of the halo model

Valageas & Nishimichi (2011)

$$P_{1\mathrm{H}}(k) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} F_{1\mathrm{H}}(q) \left\langle \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\Delta\boldsymbol{x}} - \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{q}} \right\rangle_{1\mathrm{H}} \quad \Rightarrow \quad P_{1\mathrm{H}}(k) = \int_0^\infty \frac{\mathrm{d}\nu}{\nu} f(\nu) \frac{M}{\overline{\rho}(2\pi)^3} \left(\tilde{u}_M(k)^2 - \tilde{W}(kq_M)^2 \right)$$

counter-term

2) Exclusion effects in the 2-halos term

Smith et al. (2011)

$$\xi^{\mathrm{hh}}_{\mathrm{cent}}(r|M_1, M_2) = -1 \qquad (r < r_{\mathrm{vir},1} + r_{\mathrm{vir},2}),$$

$$P^{\mathrm{hh}}_{\mathrm{cent}}(k|R) = \int_{r_{\mathrm{vir},1}+r_{\mathrm{vir},2}}^{\infty} d^3\mathbf{r}\, b(M_1) b(M_2) \xi(r|R) j_0(kr) + \int_0^{r_{\mathrm{vir},1}+r_{\mathrm{vir},2}} d^3\mathbf{r}(-1) j_0(kr)$$

$$= P^{\mathrm{NoExc,hh}}_{\mathrm{cent}}(k) - P^{\mathrm{Exc,hh}}_{\mathrm{cent}}(k).$$

counter-term

3) Redefine the halo number density field and its noise

Ginzburg et al. (2017)

$$\delta_i(\boldsymbol{x}) = \left(b_i + \tilde{\epsilon}_{\delta i}(\boldsymbol{x}) - b_i \tilde{\epsilon}_{\delta m}(\boldsymbol{x})\right) \delta(\boldsymbol{x}) + \tilde{\epsilon}_{0i}(\boldsymbol{x}) - b_i \tilde{\epsilon}_{0m}(\boldsymbol{x}) \qquad \delta_m(\boldsymbol{x}) = \delta(\boldsymbol{x}).$$

It is OK to have shot noise for the halo power spectrum (discrete distribution).
Separate the cases of continuous dark matter and discrete halos.

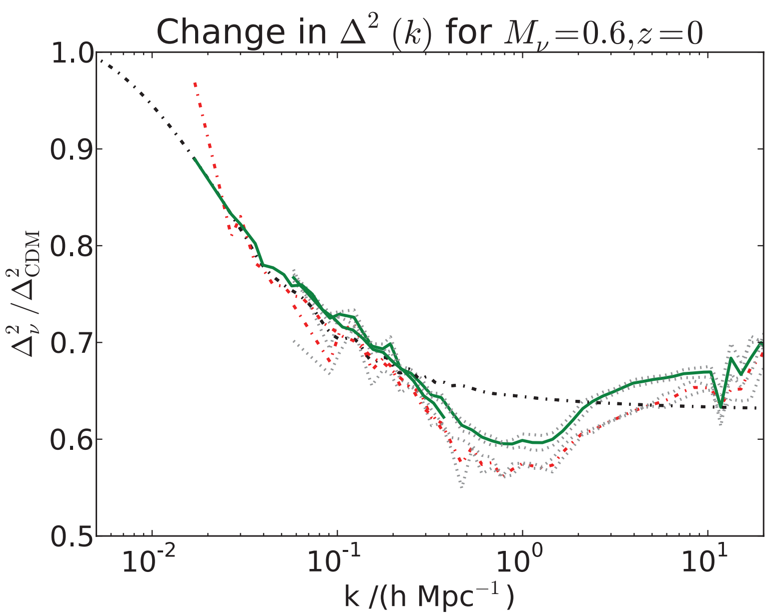
4) Introduce a free parameter to cut the 1-halo term at low k

Mead et al. (2015)

$$\Delta_{1\mathrm{H}}^{\prime 2} = [1 - \mathrm{e}^{-(k/k_*)^2}]\Delta_{1\mathrm{H}}^2$$

Baryonic feedback, neutrinos, ...

1) Multiplicative factors



Bird et al. (2012)

suppression of power due to neutrinos measured in numerical simulations, with or without baryons:

change on the ratio with/without baryons less than 1% for *k*<8 *h*/Mpc.

Harnois-Deraps et al. (2015)

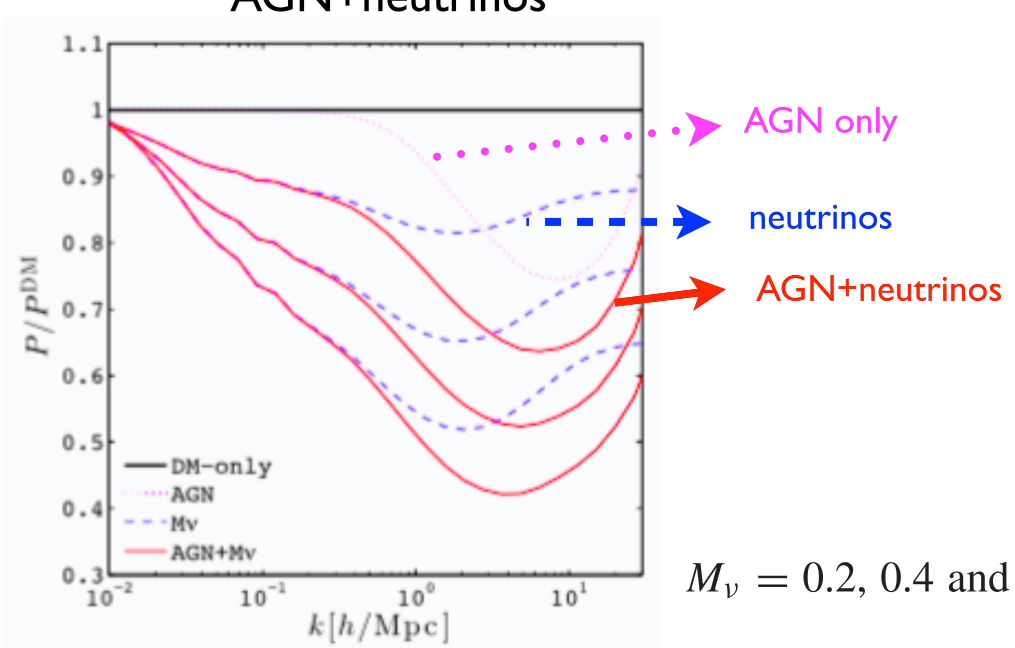
combine the ratios:

$$P^{\mathrm{DM}+\nu+b(m)}(k,z) = P^{\mathrm{DM}}(k,z) \times b_m^2(k,z) \times b_{M_\nu}^2(k,z).$$

$$P^{\mathrm{DM}+\nu+b(\mathrm{m})+\mathrm{MG}} = P^{\mathrm{DM}} \times b_{M_\nu}^2 \times b_{\mathrm{m}}^2 \times b_{\mathrm{MG}(\alpha)}^2$$

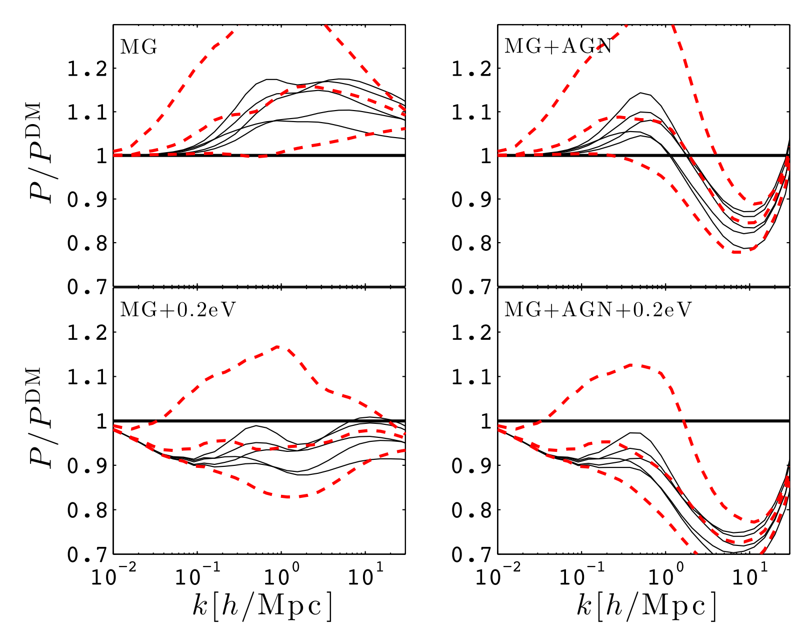
$$b_m^2(k,z) \equiv \frac{P_{\mathrm{OWL}}^{\mathrm{DM}+b(m)}(k,z)}{P_{\mathrm{OWL}}^{\mathrm{DM}}(k,z)} \qquad b_{M_\nu}^2(k,z) \equiv \frac{P^{\mathrm{DM}+M_\nu}(k,z)}{P^{\mathrm{DM}}(k,z)}$$

AGN+neutrinos



$M_\nu = 0.2$, 0.4 and 0.6 eV,

AGN+neutrinos+modified gravity



2) Include in the halo model parameters

Mead et al. (2015, 2016)

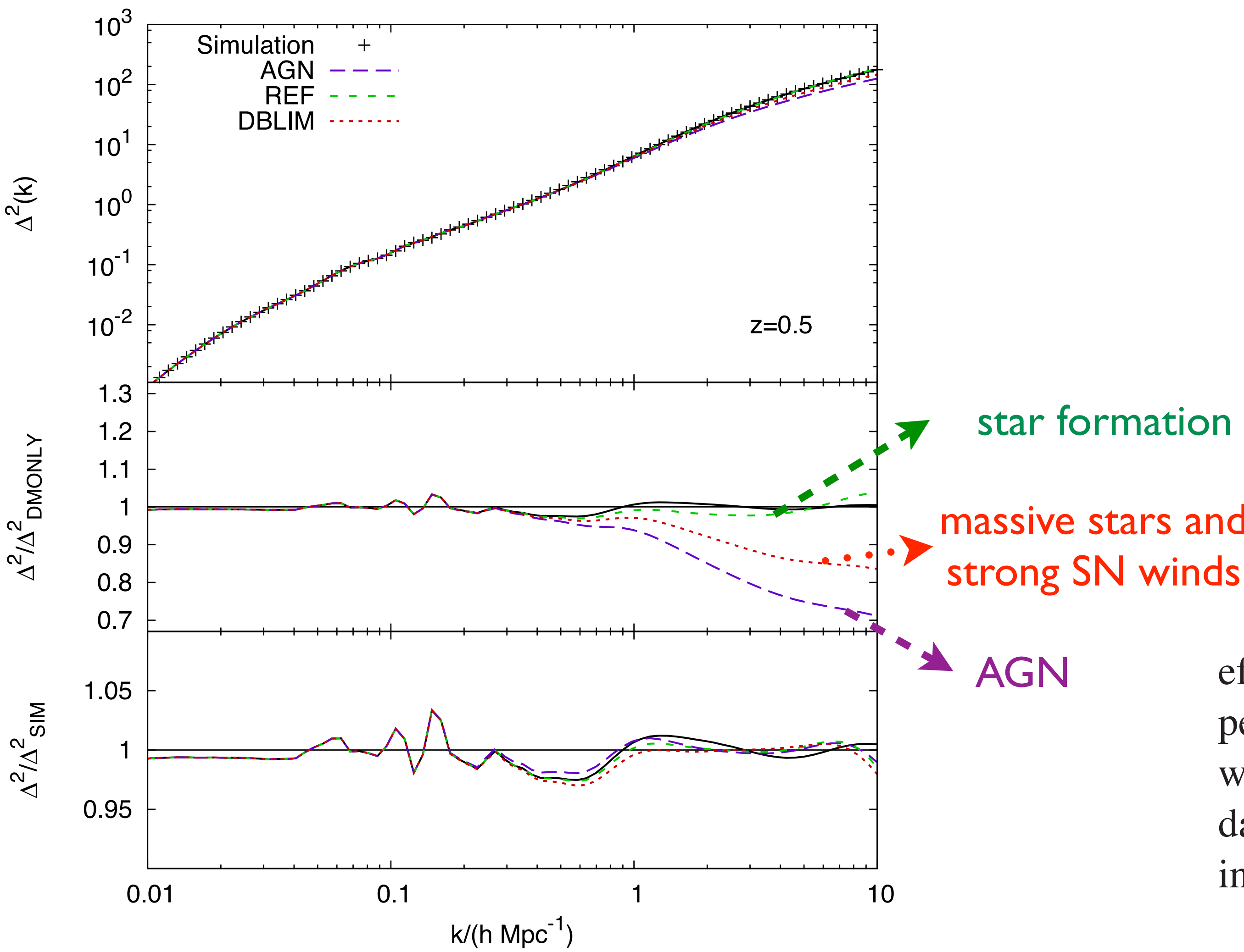
Include baryonic effects through the concentration parameter A and the halo profile extent

$$c(M,z) = A\frac{1+z_\mathrm{f}}{1+z} \qquad\qquad W(k,M) \rightarrow W(\nu^{\eta}k, M)$$

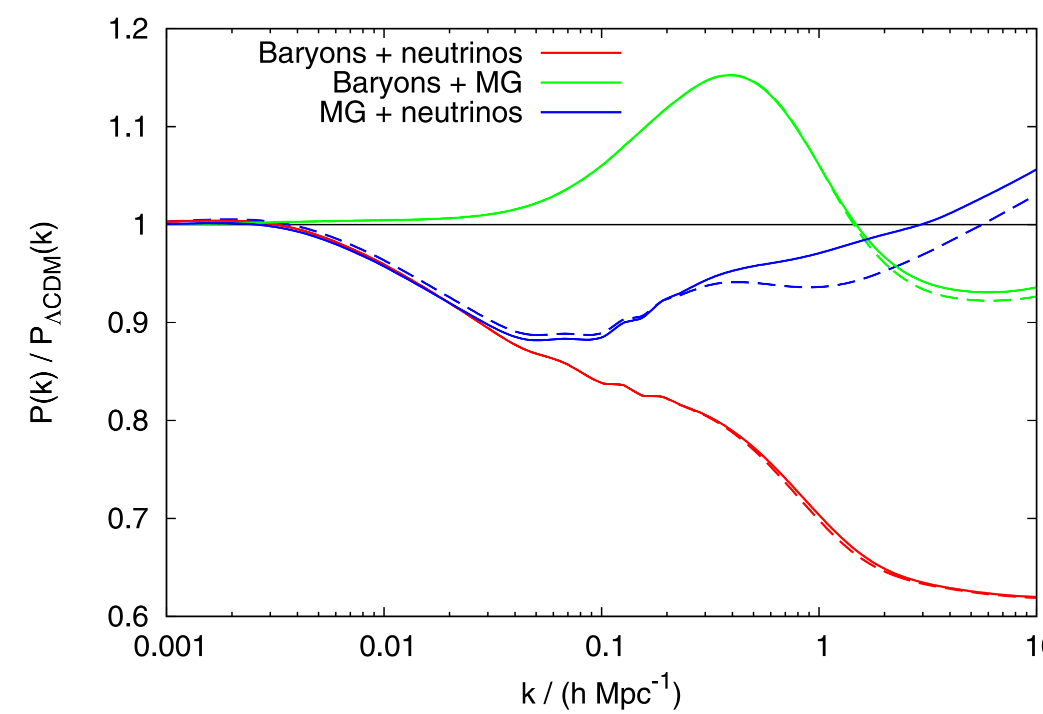
$\eta > 0$ higher mass ($\nu > 1$) haloes are puffed out, while lower mass haloes are contracted, both at constant virial radius: $\eta > 0$ decreases the power whereas $\eta < 0$ increases it.

Model	η_0	A
All COSMIC EMU simulations	0.60	3.13
DMONLY (*WMAP*3 from OWLS)	0.64	3.43
AGN	0.76	2.32
REF	0.68	3.91
DBLIM	0.70	3.01

decrease of power on small scales
due to baryonic feedback



approximate factorization



We show combinations of two of the three effects where the ratio is computed both by assuming the effects act independently (dashed line) and via the full halo-model calculation (solid line) where the effects are treated in tandem. Differences between each pair of dashed and solid lines indicate the extent to which the effects can be treated independently.

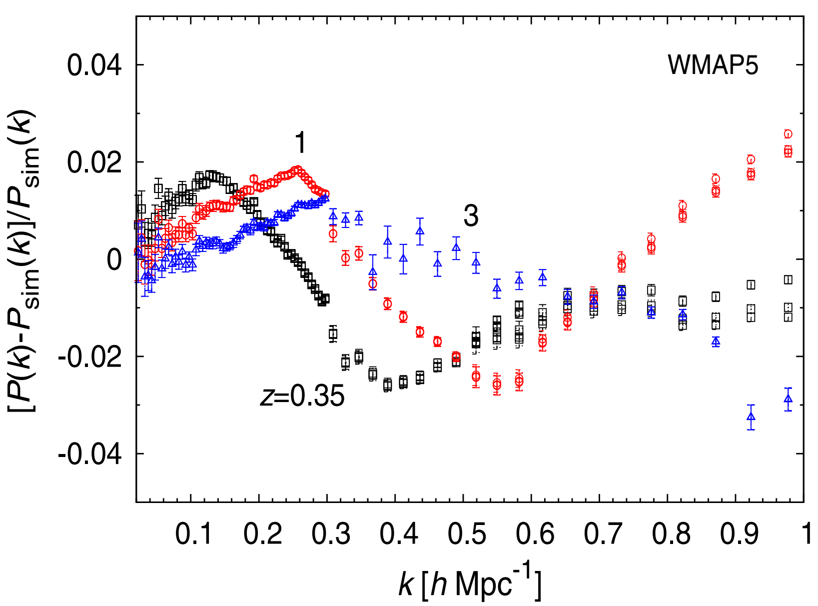
Some results

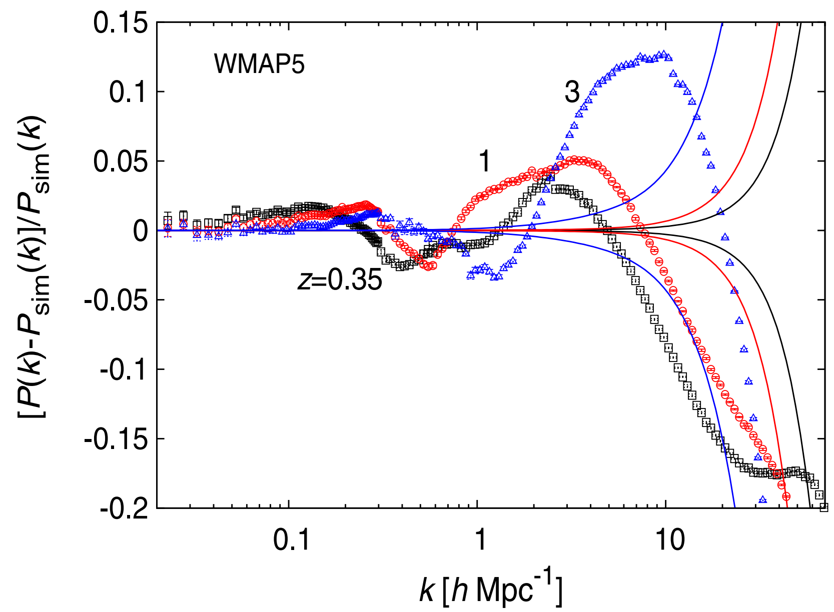
1) Lagrangian approach: SPT+adhesion+1-halo term

LCDM relative deviation from simulations

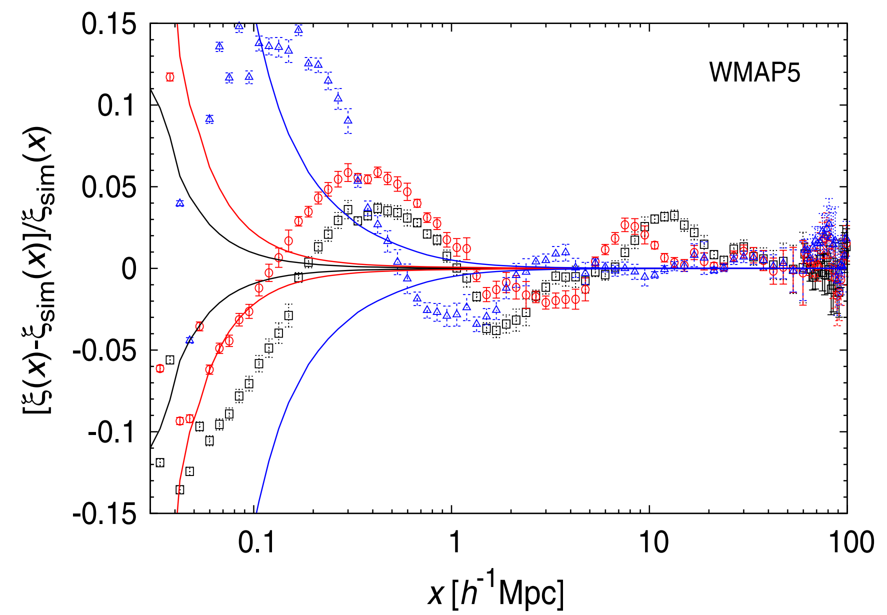
Valageas et al. (2013)

$P(k)$





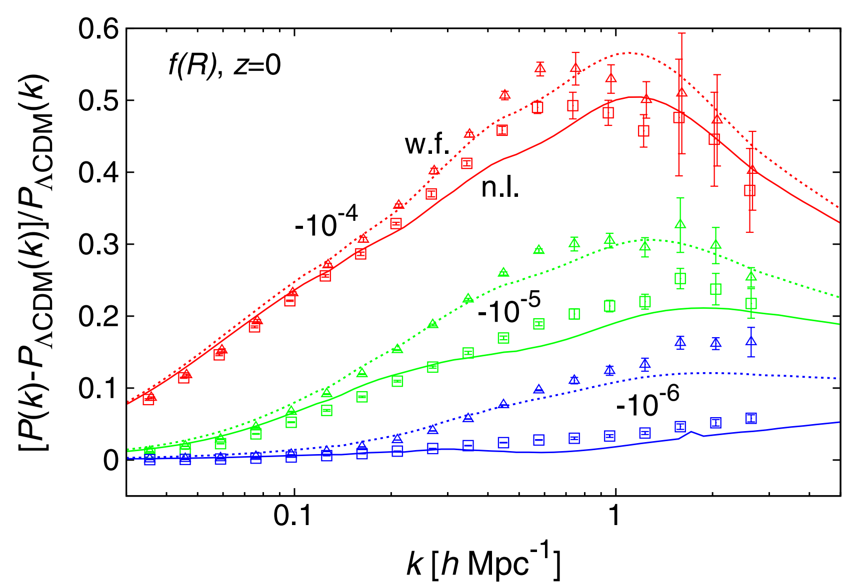
$\xi(x)$

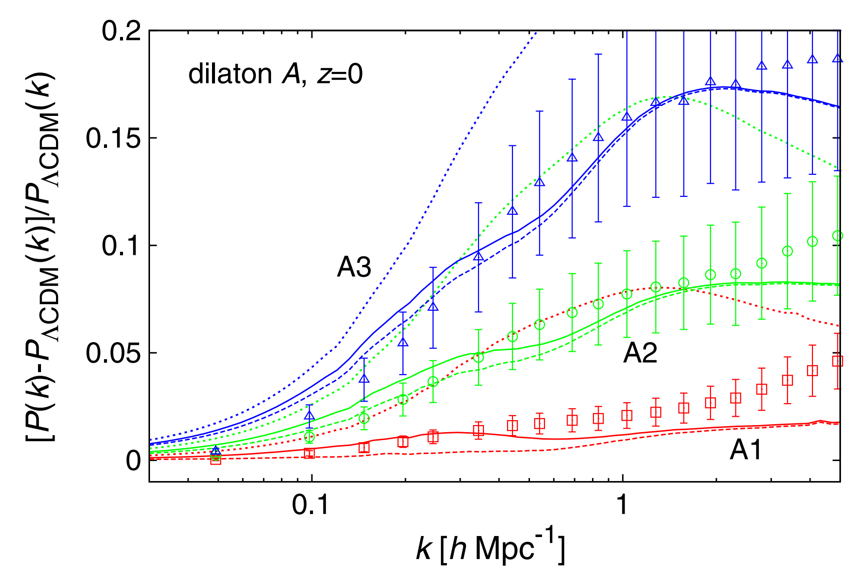


Modified gravity

relative deviation from LCDM: predictions and simulations

impact through:
- linear growth factor+SPT
- halo density threshold
(spherical collapse)

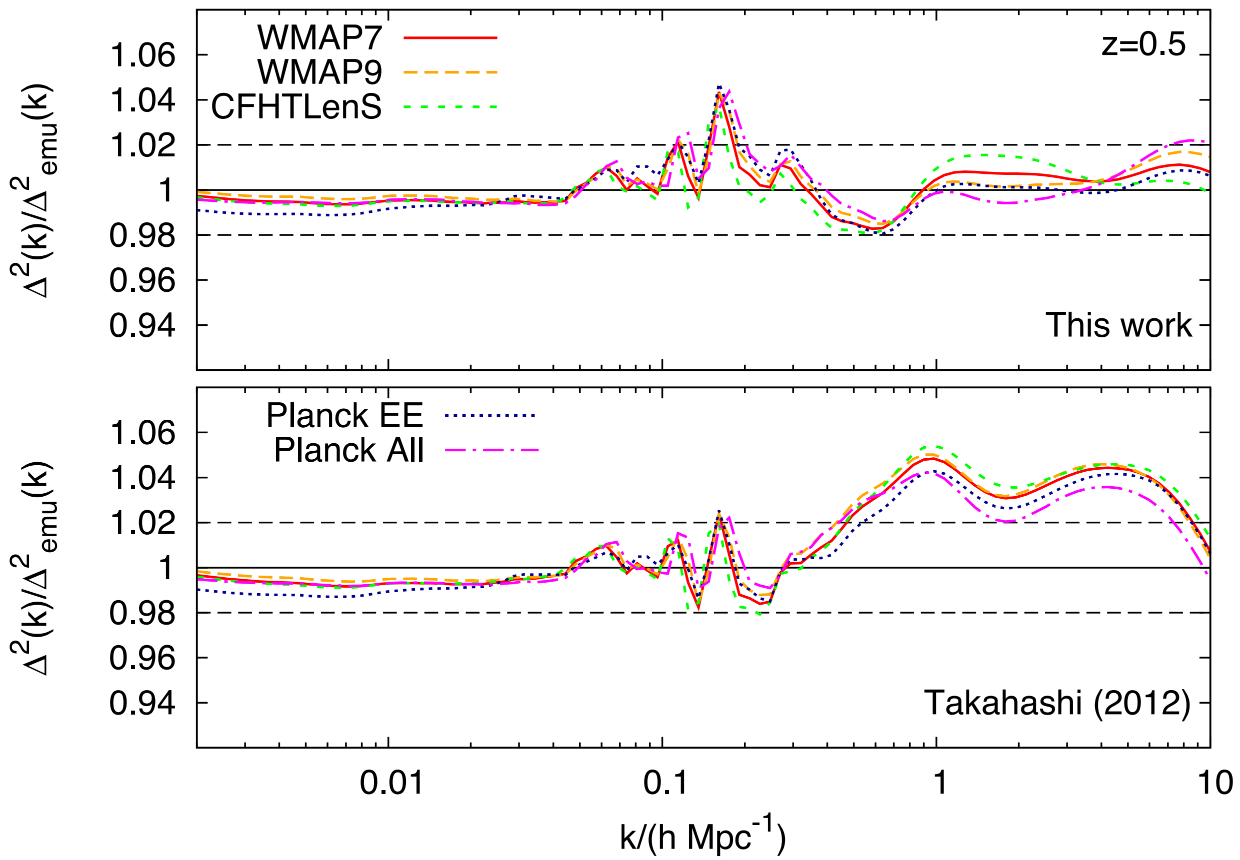




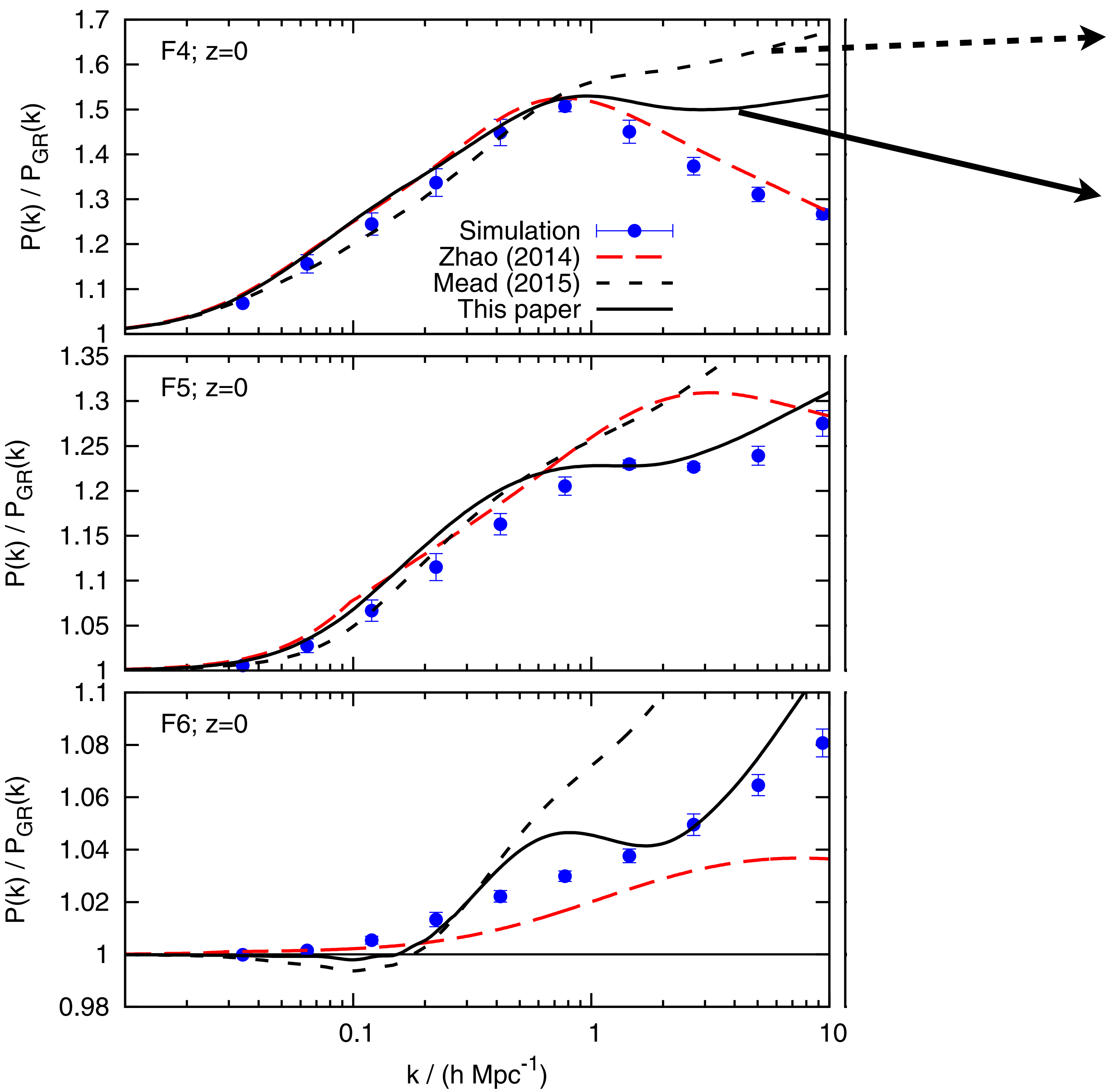
2) Halo model with some more fitting parameters

Mead et al. (2015)

Parameter	Description	Original value	Fitted value
Δ_v	Virialized halo overdensity	200	$418 \times \Omega_m^{-0.352}(z)$
δ_c	Linear collapse threshold	1.686	$1.59 + 0.0314 \ln \sigma_8(z)$
η	Halo bloating parameter	0	$0.603 - 0.3\,\sigma_8(z)$
f	Linear spectrum transition damping factor	0	$0.188 \times \sigma_8^{4.29}(z)$
k_*	One-halo damping wavenumber	0	$0.584 \times \sigma_v^{-1}(z)$
A	Minimum halo concentration	4	3.13
α	Quasi-linear one- to two-halo term softening	1	$2.93 \times 1.77^{n_{\rm eff}}$



Mead et al. (2016) modified gravity: *f(R)* → impact through the linear growth factor



LCDM-type halo model

also fit the halo density contrast

$$\delta_c \propto \text{constant}$$

$$\Delta_v \propto (G_{eff}/G)^{-0.5}$$

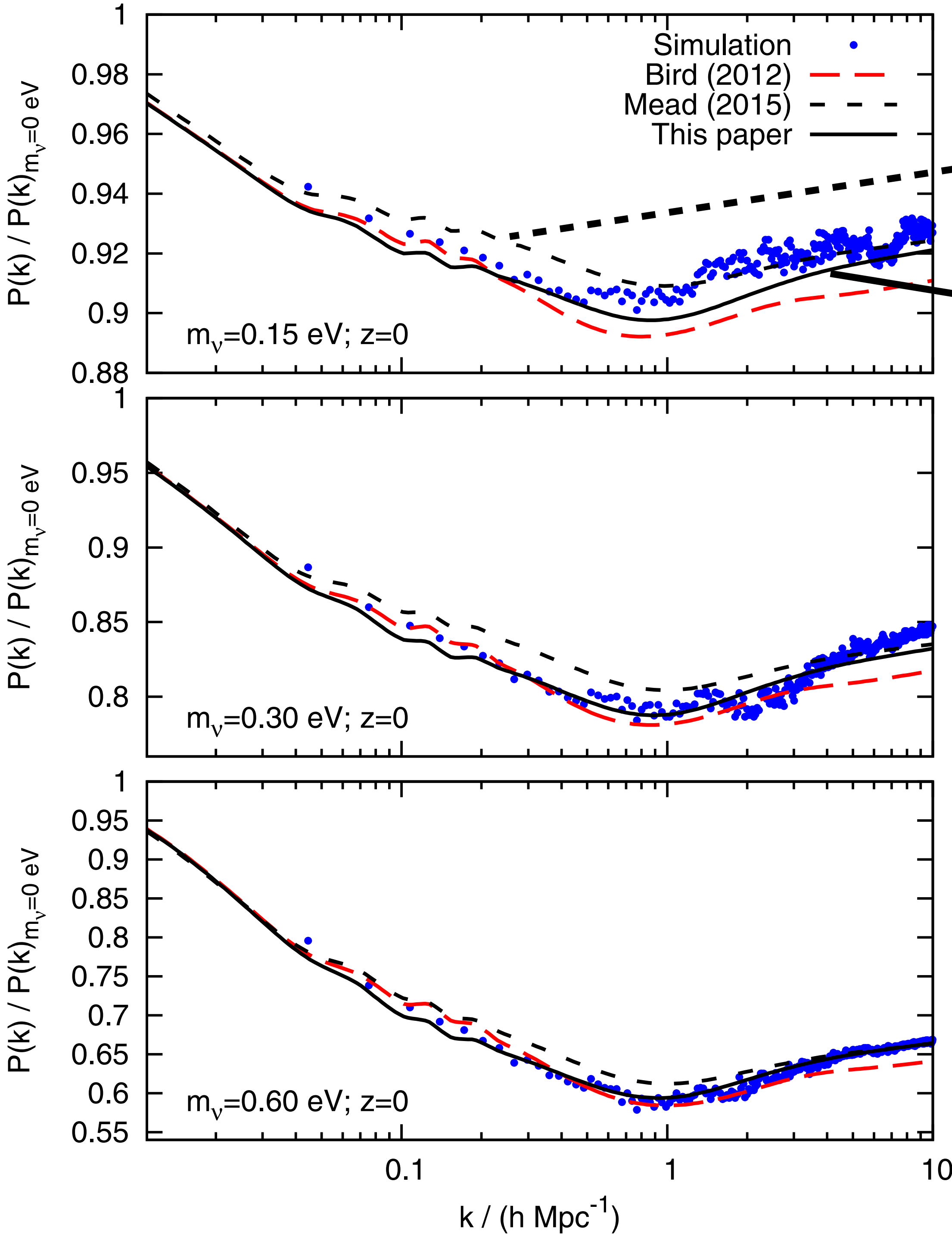
$G_{eff}/G = 4/3$ at low halo mass

$G_{eff}/G = 1$ at high halo mass (screening)

mass at the screening transition:

$$\log_{10}\left(\frac{M_s}{h^{-1}\,M_\odot}\right) = 18.9 + \frac{3}{2}(5 + \log_{10}|f_R(a)|) - \frac{1}{2}\log_{10}\Omega_m$$

Mead et al. (2016) massive neutrinos → impact through the linear growth factor



LCDM-type halo model

also fit the halo density contrast

$$\delta_c \propto 1 + 0.262 f_\nu,$$

$$\Delta_v \propto 1 + 0.916 f_\nu.$$

“React” approach

Cataneo et al. (2019)

Instead of trying to predict the power spectrum for any cosmology, it may be easier to predict the deviation from a reference LCDM power spectrum.

Familiar idea, for instance to predict the halo mass function for non-Gaussian initial conditions, use the ratio to LCDM predicted by a Press-Schechter-like modeling.

$$\mathcal{R}(k, z) \equiv \frac{P^{\mathrm{real}}(k, z)}{P^{\mathrm{pseudo}}(k, z)}$$

P^{real} → cosmology we want to predict

P^{pseudo} → reference LCDM cosmology

$$P^{\mathrm{real}} = \mathcal{R} \times P^{\mathrm{pseudo}}$$

The reference cosmology is LCDM with a linear power spectrum normalized at the redshift we want to predict (i.e. including linear modified-gravity effects, etc...):

$$P_{\mathrm{L}}^{\mathrm{pseudo}}(k, z_{\mathrm{f}}) = P_{\mathrm{L}}^{\mathrm{real}}(k, z_{\mathrm{f}}).$$

→ automatically exact for the LCDM reference and at linear order for any cosmology

We can hope to bypass the difficulties associated with a full computations, such as the 2-halos __ 1-halo transition, the low-k asymptote, ...

$$\mathcal{R}(k,z) = \frac{[(1-\mathcal{E})e^{-k/k_\star} + \mathcal{E}]P_{\rm L}^{\rm real}(k,z) + P_{\rm 1h}^{\rm real}(k,z)}{P_{\rm L}^{\rm real}(k,z) + P_{\rm 1h}^{\rm pseudo}(k,z)}$$

$$P_{\rm L}^{\rm pseudo}(k,z_{\rm f}) = P_{\rm L}^{\rm real}(k,z_{\rm f}).$$

(i) On large linear scales $\mathcal{R} \to 1$ by definition

(ii) On small non-linear scales $\mathcal{R} \approx P_{\rm 1h}^{\rm real} / P_{\rm 1h}^{\rm pseudo}$

smoothness factor $\mathcal{E}(z) = \frac{P_{\rm 1h}^{\rm real}(k \to 0, z)}{P_{\rm 1h}^{\rm pseudo}(k \to 0, z)}$ to preserve the step at the 2-halos __ 1-halo transition

transition rate $k_\star$ $\mathcal{R}(k_0, z|k_\star) = \frac{P_{\rm SPT}^{\rm real}(k_0,z) + P_{\rm 1h}^{\rm real}(k_0,z)}{P_{\rm SPT}^{\rm pseudo}(k_0,z) + P_{\rm 1h}^{\rm pseudo}(k_0,z)}$

1-loop SPT $P_{\rm SPT}(k,z) = P_{\rm L}(k,z) + P_{22}(k,z) + P_{13}(k,z) + P_{13}^{\Psi}(k,z),$ $k_0 = 0.06\, h\,{\rm Mpc}^{-1}$

captures the deviation from the LCDM reference on weakly non-linear scales, through the matching to 1-loop SPT at k_0

Modified gravity (for instance) enters through the linear growth rate, 1-loop SPT, the halo mass function (spherical collapse for the critical and virial density contrasts) and the halo profiles (concentration parameter).

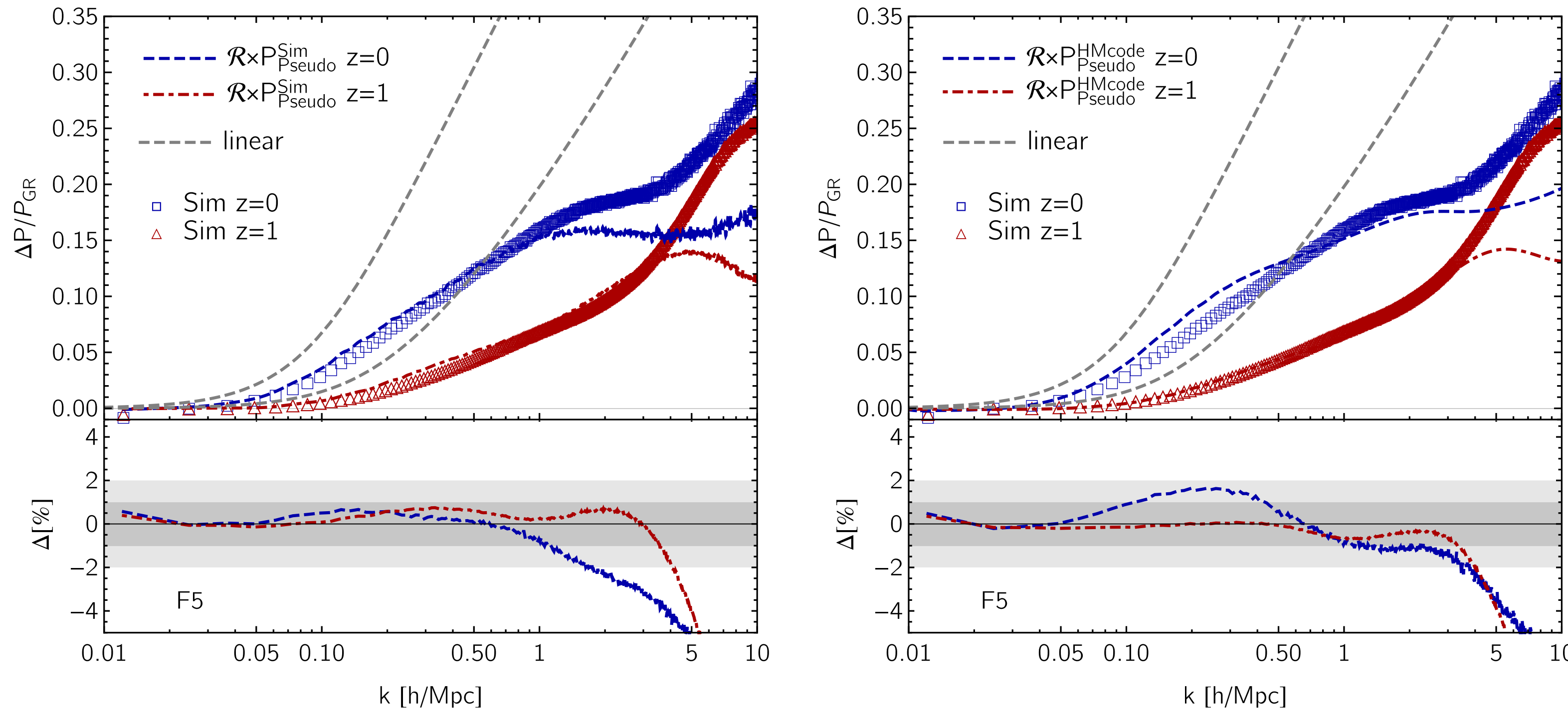
$$c_{\rm vir}(M_{\rm vir},z) = \frac{c_0}{1+z}\left(\frac{M_{\rm vir}}{M_*}\right)^{-\alpha} \qquad \nu(M_*) = 1 \qquad c_{\rm vir} \to \frac{c_0}{1+z}\left(\frac{M_{\rm vir}}{M_*}\right)^{-\alpha} \frac{g_{\rm DE}(z\to\infty)}{g_\Lambda(z\to\infty)}$$

Modified gravity: *f(R)* $|f_{R_0}| = 10^{-5}$

Cataneo et al. (2019)

relative deviation from LCDM

relative deviation from simulations



using simulations for the reference P(k)

using Halo Model for the reference P(k)

→ 1% accuracy up to $k \sim 1\ h$/Mpc

fast, no need for many PT or complicated integrals (only 1-loop SPT at 1 wavenumber)

massive neutrinos:

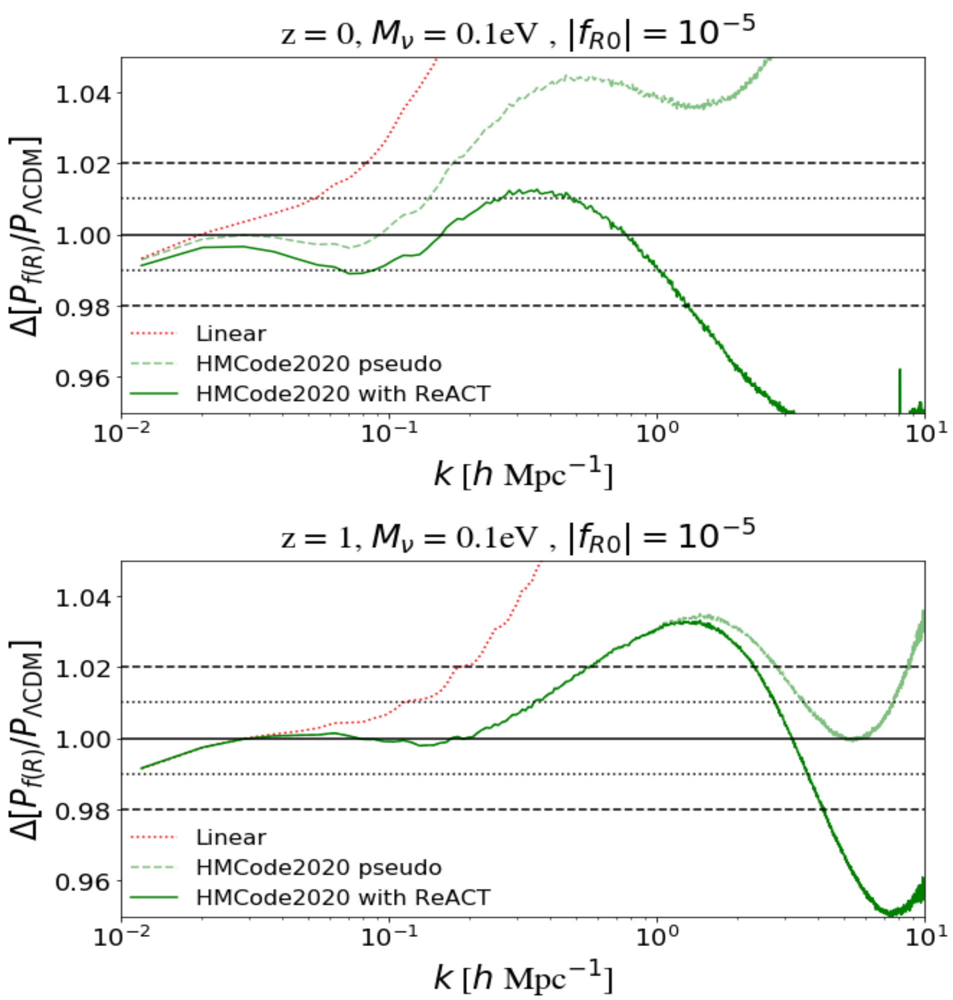
Bose et al. (2021)

$$\mathcal{R}(k) = \frac{(1-f_\nu)^2\, P^{(\mathrm{cb})}_{\mathrm{HM}}(k) + 2 f_\nu\,(1-f_\nu)\, P^{(\mathrm{cb}\nu)}_{\mathrm{HM}}(k) + f_\nu^2 P^{(\nu)}_{\mathrm{L}}(k)}{P^{(\mathrm{m})}_{\mathrm{L}}(k) + P^{\mathrm{pseudo}}_{1\mathrm{h}}(k)}$$

neutrinos at the linear level, halo model for CDM+baryons

modified gravity *f(R)* + neutrinos:

relative deviation from LCDM, normalized to the simulations



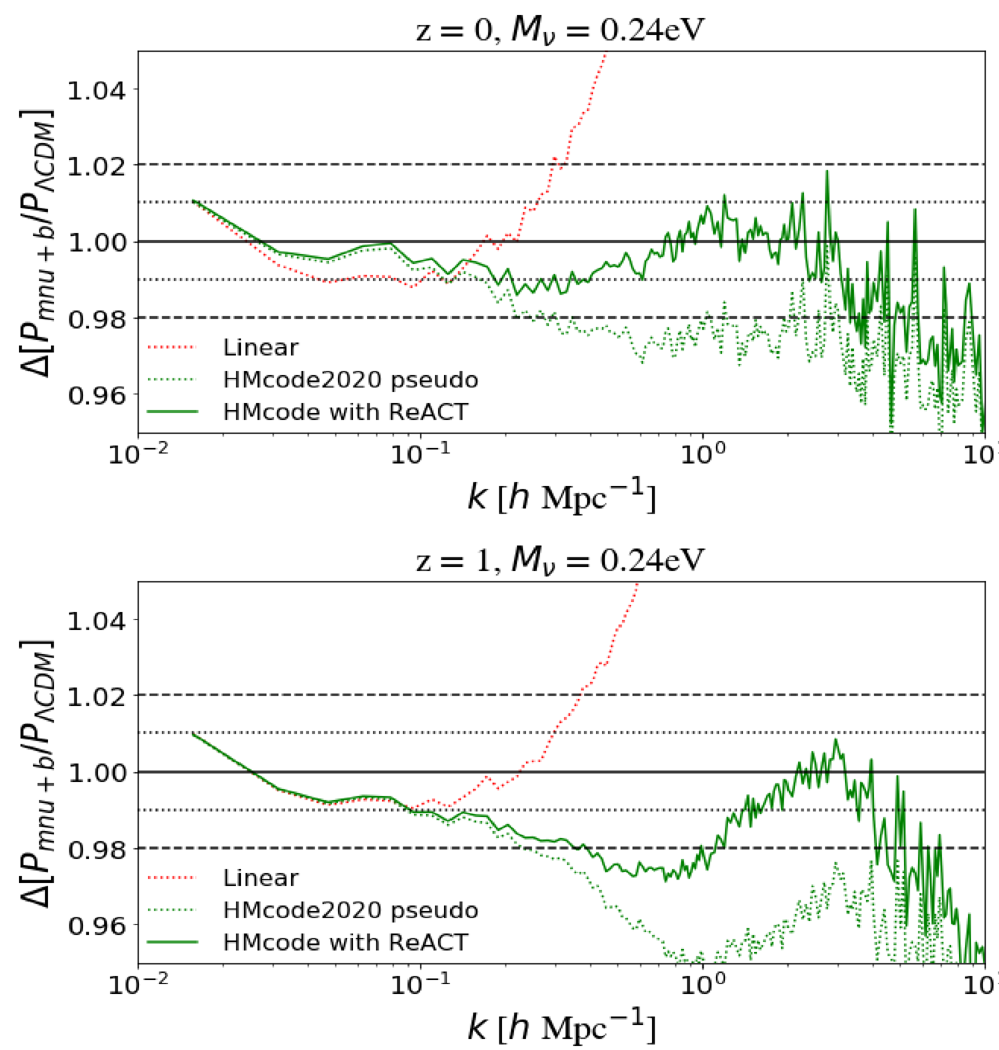
5% accuracy up to $k \sim 5\ h/\mathrm{Mpc}$

neutrinos + bayonic feedback:

Bose et al. (2021)

baryonic feedback is included in the reference pseudo spectrum, obtained from simulations or from a halo model (fitted to LCDM hydro simulations)

3% accuracy up to $k \sim 5\ h/\mathrm{Mpc}$



baryonic feedback can be considered independently of massive neutrinos or modified gravity (its relative impact does not depend much on cosmology): the feedback parameters are kept as in LCDM.

This uses the Halo Model from Mead et al. (2021)

Parameter	Equation	BAHAMAS formula
B_0	20	$3.44–0.496\theta$
B_z	20	$-0.0671–0.0371\theta$
$f_{*,0}/10^{-2}$	25	$2.01–0.30\theta$
$f_{*,z}$	25	$0.409+0.0224\theta$
$\log_{10}(M_{b,0}/h^{-1}\,\mathrm{M_\odot})$	24	$13.87+1.81\theta$
$M_{b,z}$	24	$-0.108+0.195\theta$

$$X(z) = X_0 \times 10^{zX_z}$$

subgrid heating parameter of the simulations

$$\theta = \log_{10}(T_{\rm AGN}/10^{7.8}\,{\rm K})$$

concentration parameter

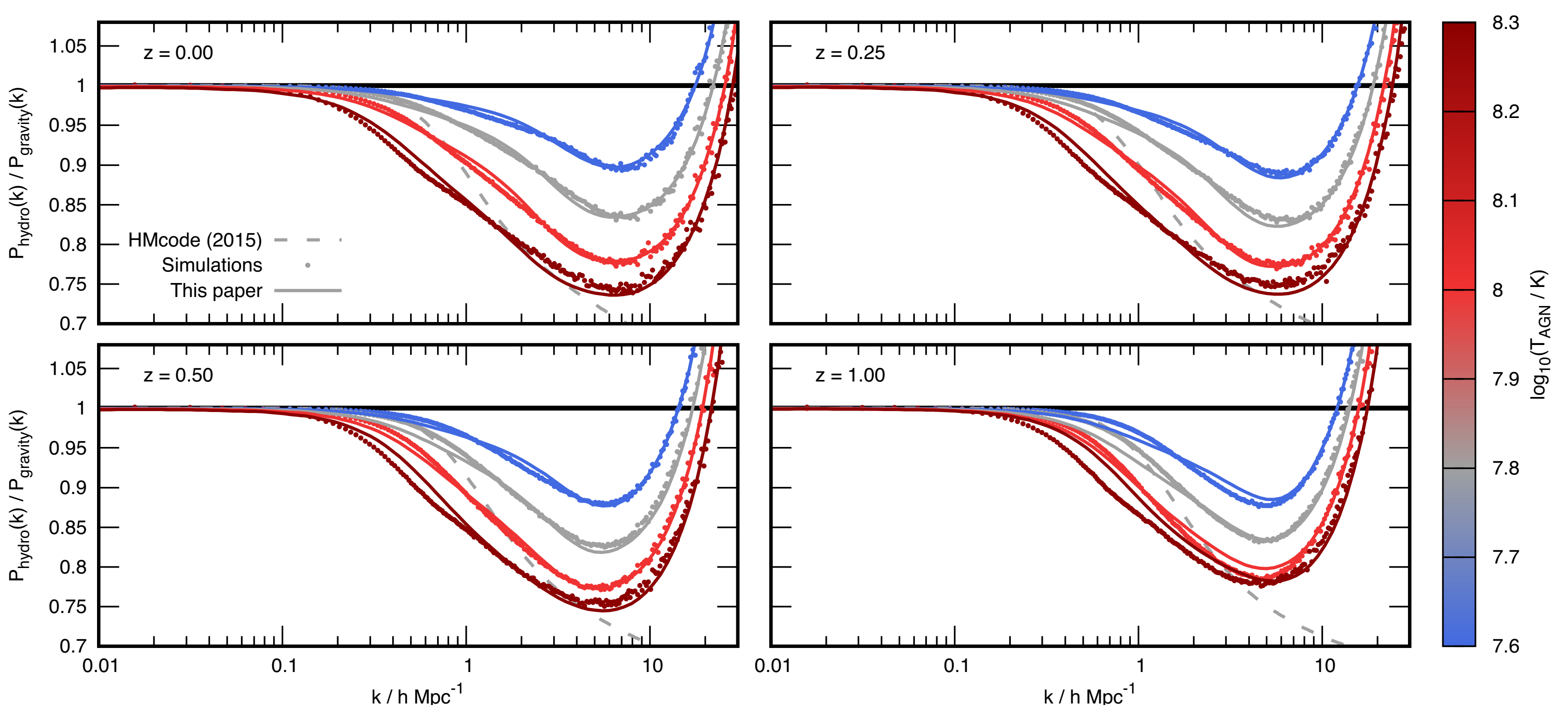
$$c(M, z) = B\left[\frac{1+z_f(M,z)}{1+z}\right]$$

halo profile (window function)

$$\tilde{W}(M,k) = \left[\frac{\Omega_c}{\Omega_m} + f_g(M)\right] W(M,k) + f_*\frac{M}{\bar{\rho}}.$$

$$f_g(M) = \left(\frac{\Omega_b}{\Omega_m} - f_*\right)\frac{(M/M_b)^\beta}{1+(M/M_b)^\beta}$$

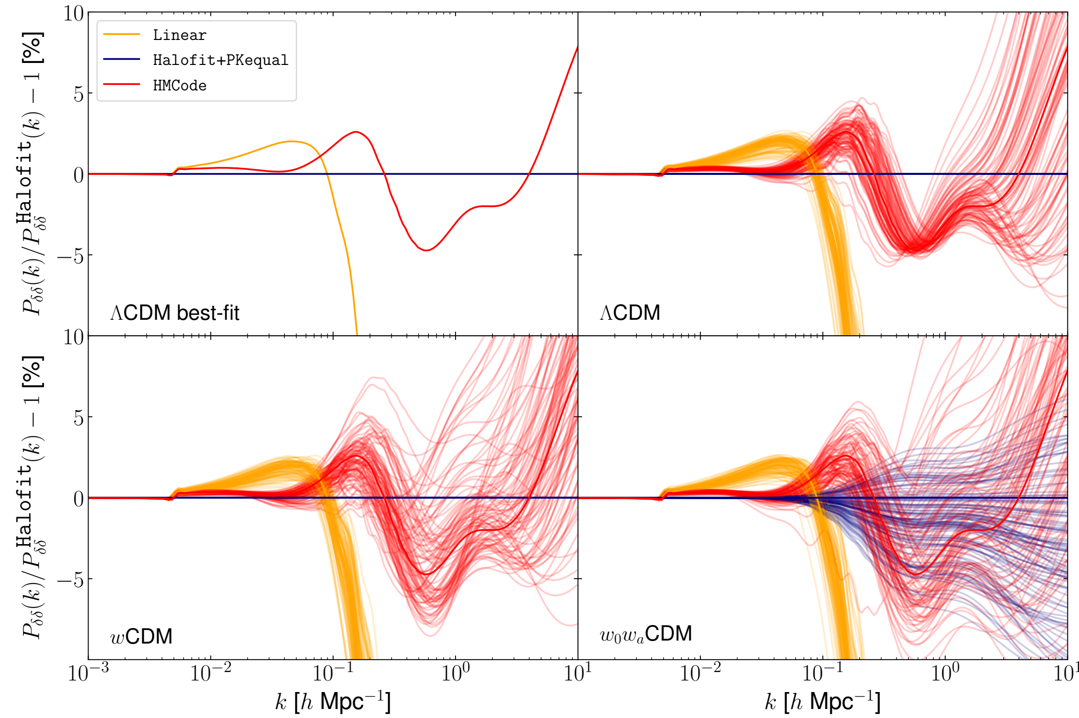
decrease of *P(k)* at *k* ~ 7 *h*/Mpc by baryonic feedback



Impact of the scatter between halo models

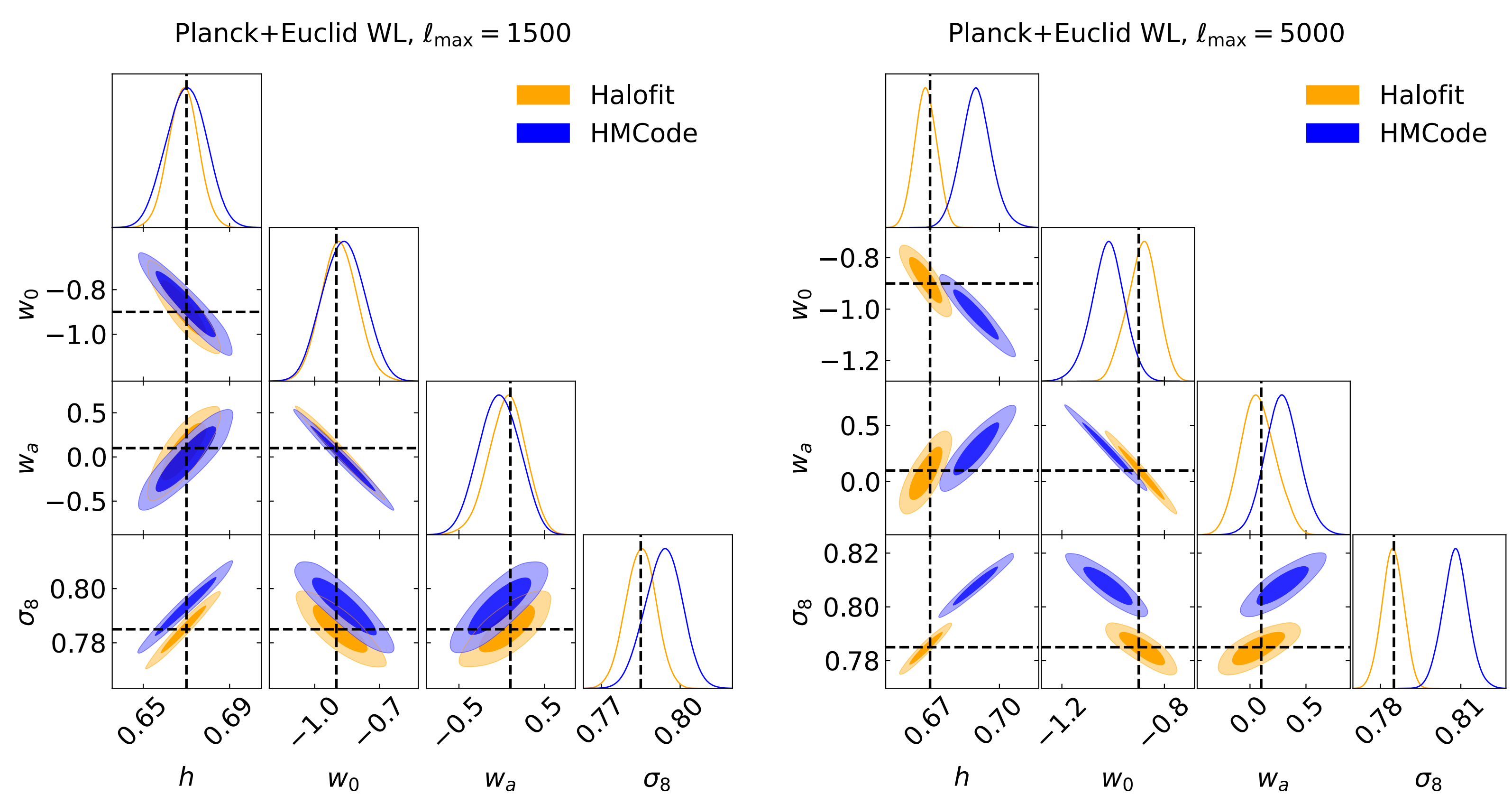
Martinelli et al. (2021)

relative difference between
Halofit,
Halofit+PKequal (quintessence),
HMCode.



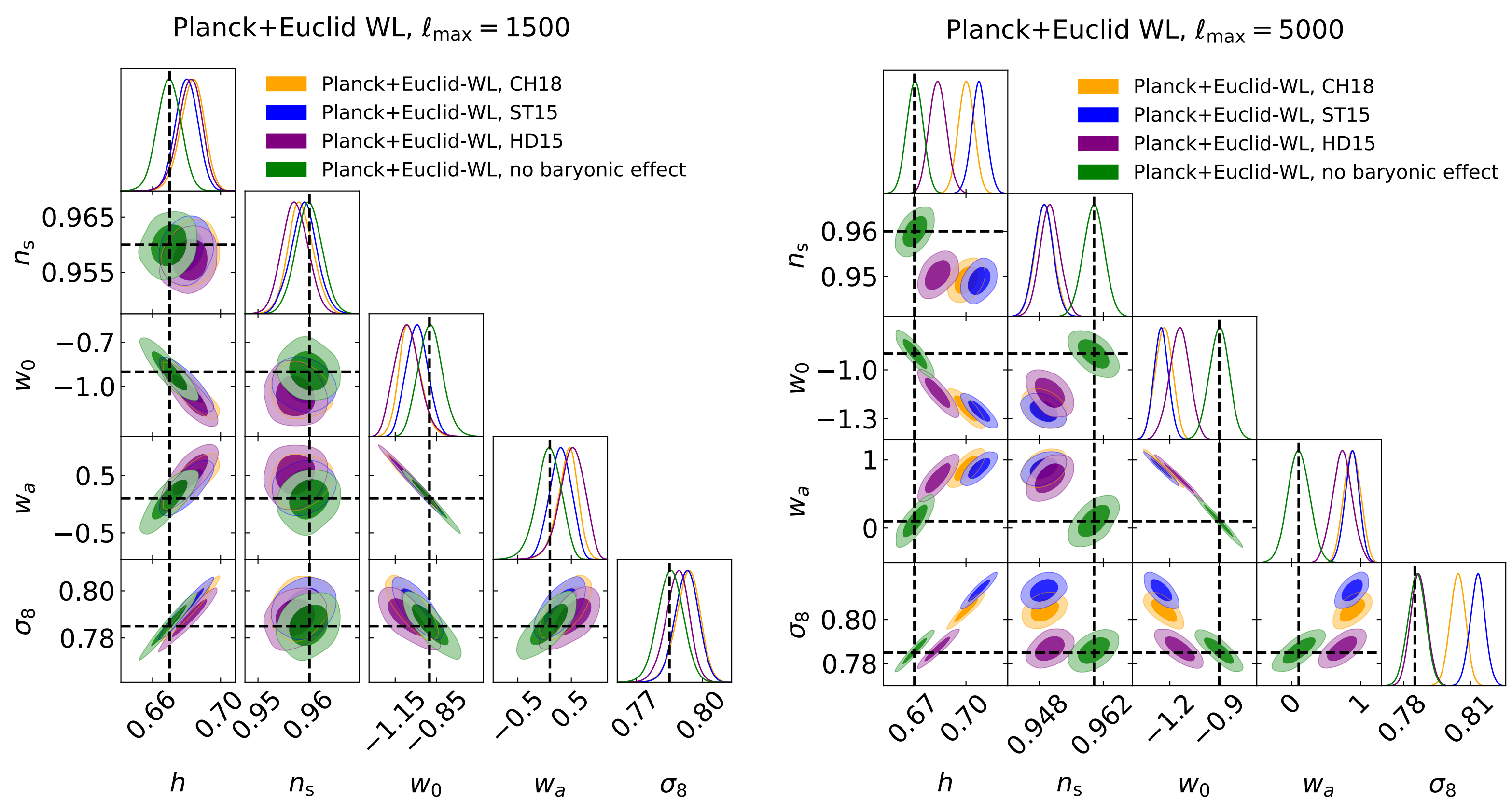
varying the
cosmological
parameters

Impact on the
measurement of
cosmological
parameters



systematic bias of the
order of the statistical
uncertainty

Impact of baryonic feedback



On small scales, ignoring baryonic feedback leads to large errors.

Sensitivity of semi-analytical models

Valageas (2013)

relative importance of *perturbative*, 2-halos *non-perturbative*, and *1-halo* contributions

impact of a 10% variation of the concentration parameter, or of using another published fit.

impact of a 10% variation of the halo mass function, or of using 2 other published fits.