Nonlinear aspects in galaxy clustering in modified gravity

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with Euclid TH-WG WP7
and other collaborators

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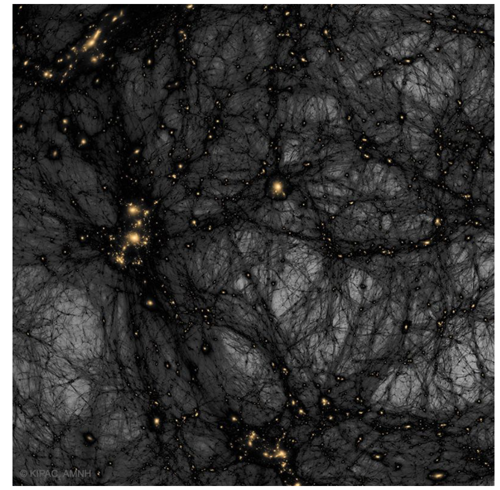
Atelier Action Dark Energy 2021

Global picture



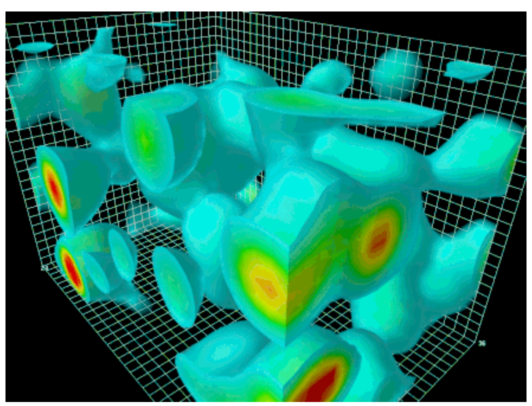
Initial Conditions

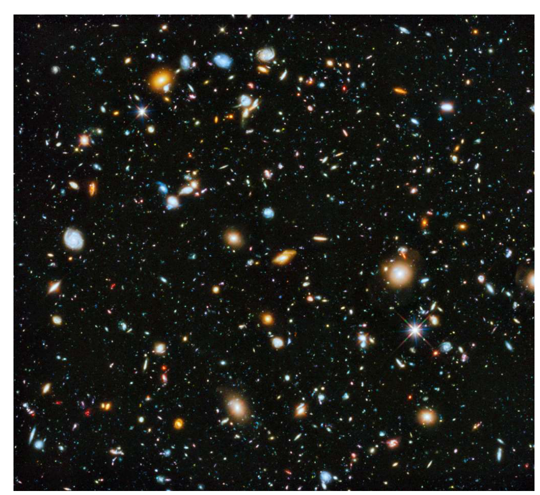
Large-Scale Structure

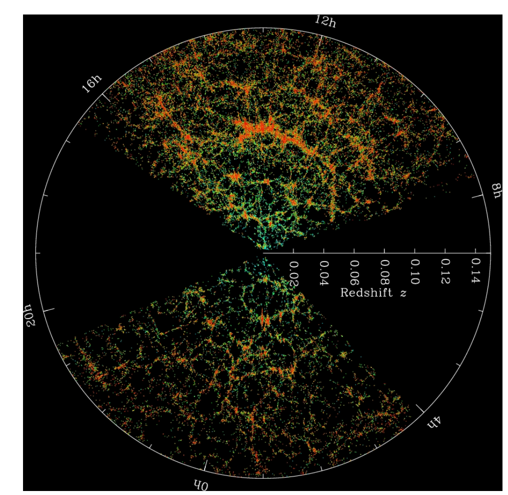
$\zeta(\boldsymbol{x}, 0)$ —SPT, EFT-of-LSS→ $\delta(\boldsymbol{x}, \tau)$ dark matter

galaxy biasing ↓

$\delta_g(\boldsymbol{x}, \tau)$ —RSD→ $\delta_g(\theta, z)$ galaxies







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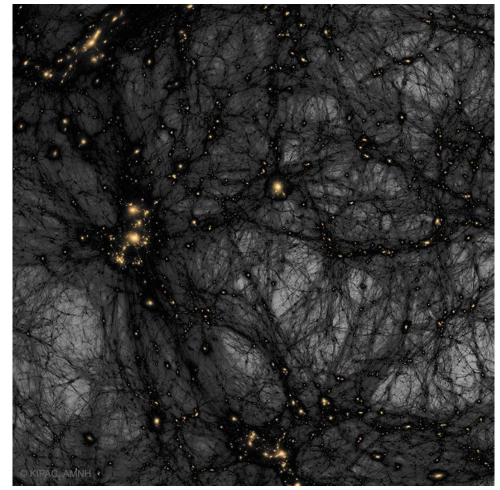

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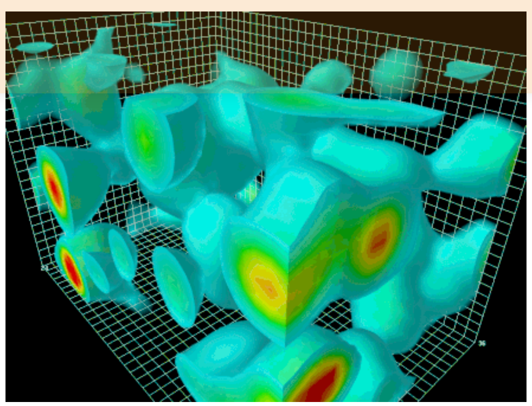
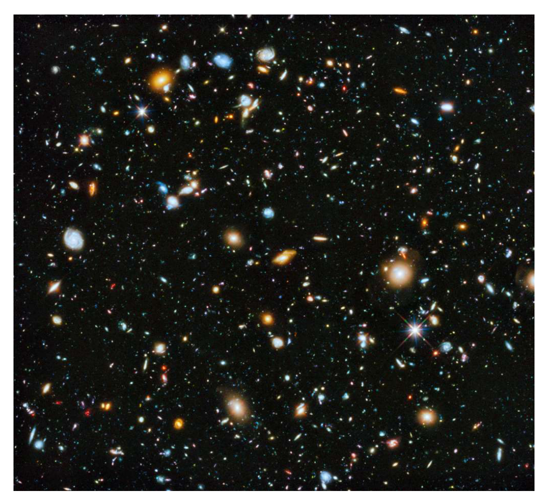
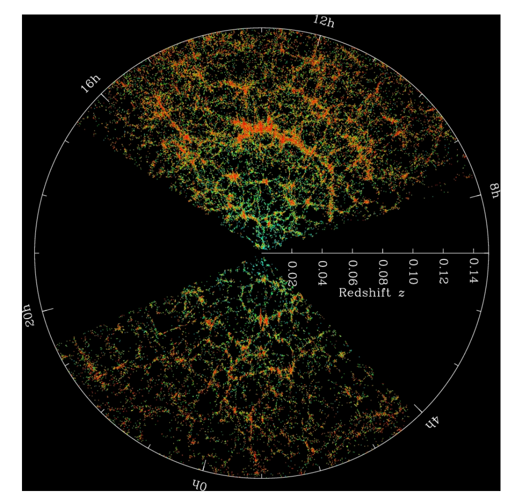


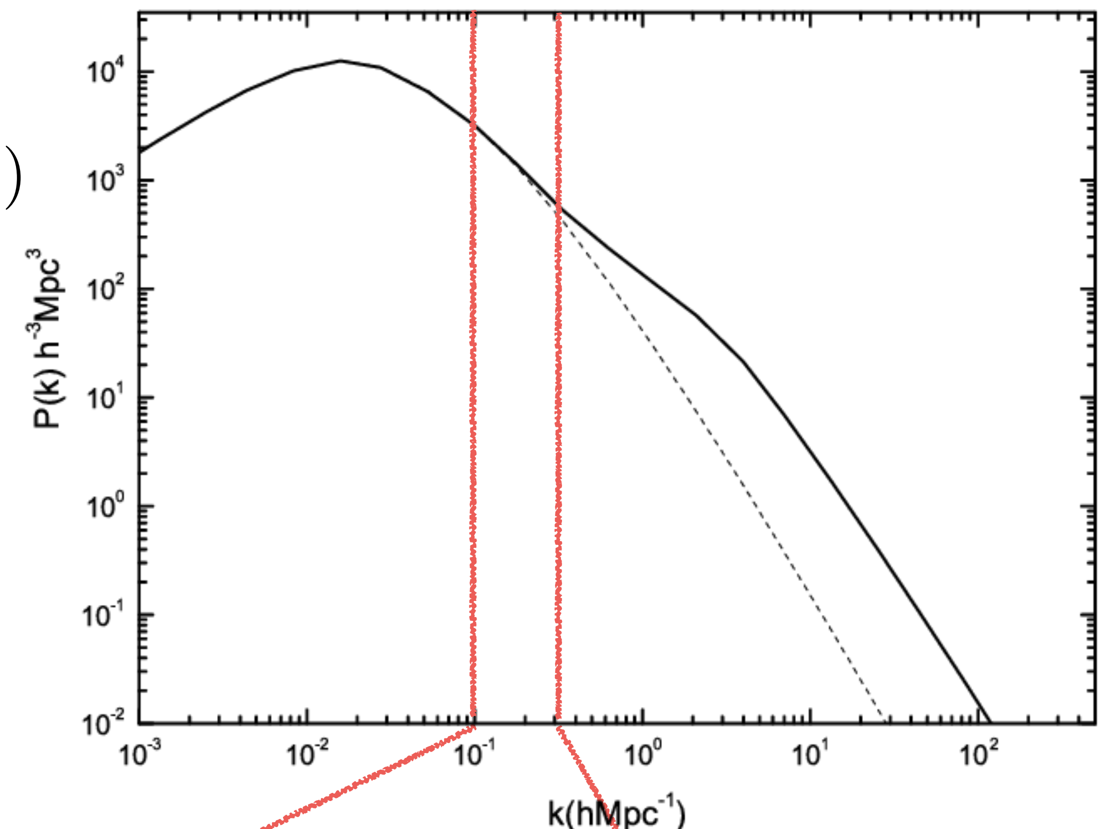




Standard perturbation theory + EFT of LSS

$$\delta(t,\vec{x}) = \rho_m(t,\vec{x})/\bar{\rho}_m(t) - 1$$

$$\langle \delta(\vec{k})\delta(\vec{k}') \rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')P(k)$$



Linear	Mildly nonlinear	Fully nonlinear + baryonic physics
$N_{\rm modes} \sim 10^6$	$N_{\rm modes} \sim 10^7$	$\delta > 1$
Linear theory	Perturbation theory	EFT (derivative expansion)
$\delta = \delta^{(1)} +$	$+\delta^{(2)} + \delta^{(3)} + \ldots$	$+c_0^2 \nabla^2 \delta^{(1)} + \ldots$

$k[h/{\rm Mpc}]$

Long-wavelength DM fluctuations computed perturbatively + finite number of unknown coefficients (counterterms) parameterising the effect of short-wavelength physics on long-wavelength one, whose k-dependence is dictated by symmetries

Standard perturbation theory + EFT of LSS

Dark matter described by continuity and Euler eqs. + Poisson eq.

$$\dot{\delta} + \frac{1}{a}\partial_i\big((1+\delta)v^i\big) = 0\ ,$$

$$\dot{v}^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\Phi = -\frac{1}{a}\frac{1}{\rho_m}\partial_j\tau^{ij}_{\rm short}\ ,$$

$$\frac{1}{a^2}\partial^2\Phi = \frac{3}{2}H^2\Omega_m\delta$$

EFT stress-energy tensor

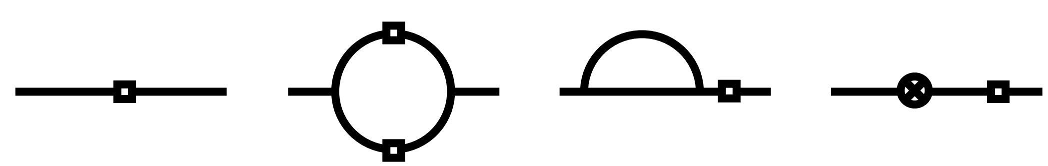
$$\partial_j\tau^{ij}_{\rm short} \to c_\delta^2\,\partial_i\delta$$

Baumann et al. 10
Carrasco, Hertzberg, Senatore 12

Power spectrum $\quad \langle\delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')P(k)$

$$\delta(\vec{k}) = \delta^{(1)}(\vec{k}) + \delta^{(2)}(\vec{k}) + \delta^{(3)}(\vec{k}) + \ldots$$

One-loop solution $\quad P^{1-{\rm loop}}(k) \ = P_{11}(k) \ + \ P_{22}(k) \ + \ P_{13}(k) \ + \ P_{13}^{ct}(k)$



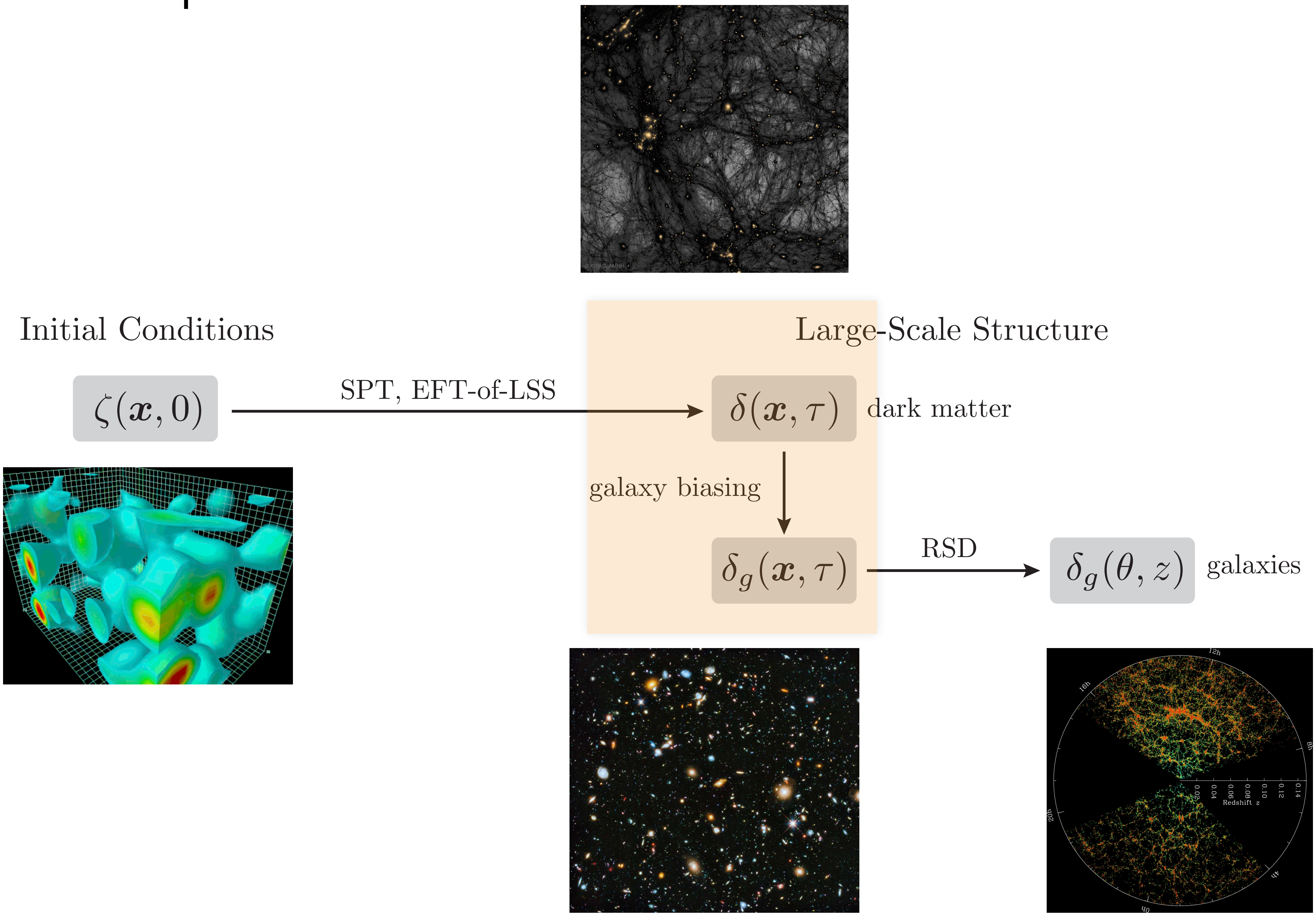
Integrals

$$P_{22}(k) = 2\int\frac{d^3q}{(2\pi)^3}P_{11}(q)P_{11}(|\vec{k}-\vec{q}|)\big[F_2(\vec{q},\vec{k}-\vec{q})\big]^2$$

$$P_{13}(k) = 6P_{11}(k)\int\frac{d^3q}{(2\pi)^3}P_{11}(q)F_3(\vec{k},\vec{q},-\vec{q})$$

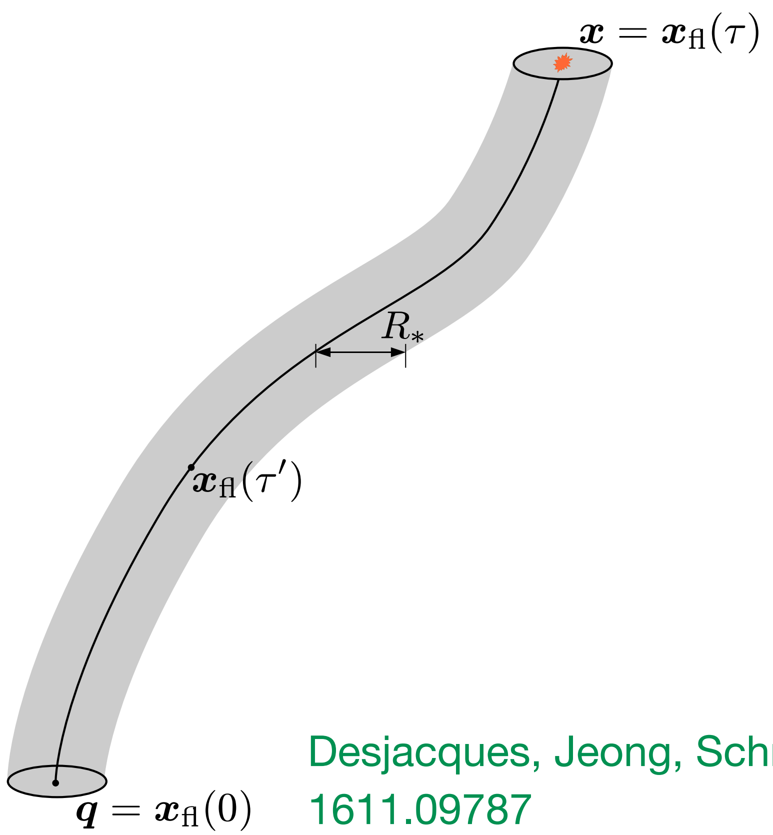
$$P_{13}^{ct}(k) = c_\delta^2\,\frac{k^2}{k_{\rm NL}^2}P_{11}(k)$$

Global picture



Galaxy biasing

Long-wavelength fluctuations of galaxies are described as biased tracers of the long-wavelength fluctuations of DM + DM counterterms.



Desjacques, Jeong, Schmidt
1611.09787

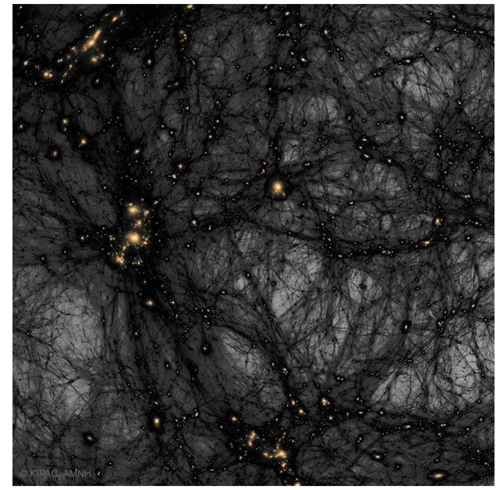
Controlled expansion (in perturbation theory and in derivatives)

$$\delta_g(x,t) = \sum_n \int dt' K_n(t,t')\, \tilde{\mathcal{O}}_n(x_{\rm fl}, t') = \sum_{n,m} b_{n,m}(t)\, \mathcal{O}_{n,m}(x,t)$$

For the one-loop power spectrum we need

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + b_{\nabla^2\delta}\nabla^2\delta\ , \qquad \mathcal{G}_2 \equiv \left(\frac{\partial_i\partial_j}{\nabla^2}\delta\right)^2 - \delta^2$$

Global picture



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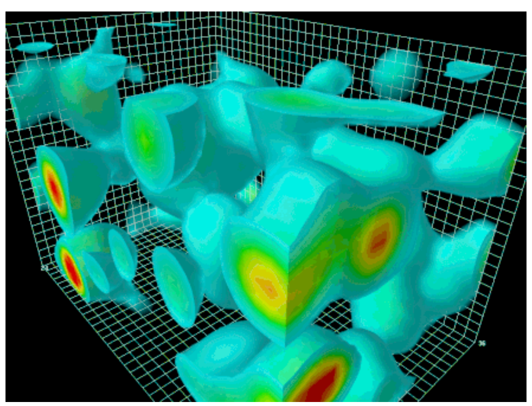
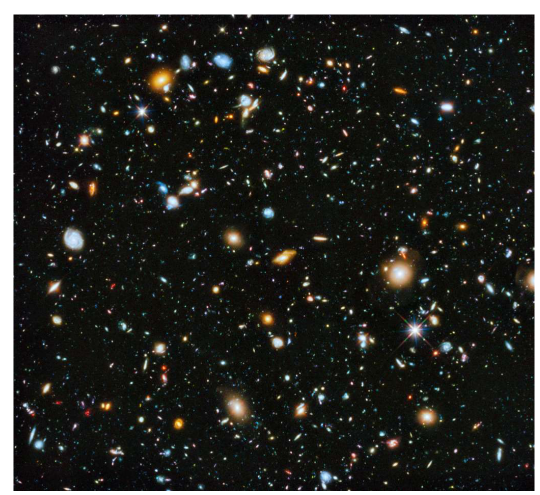
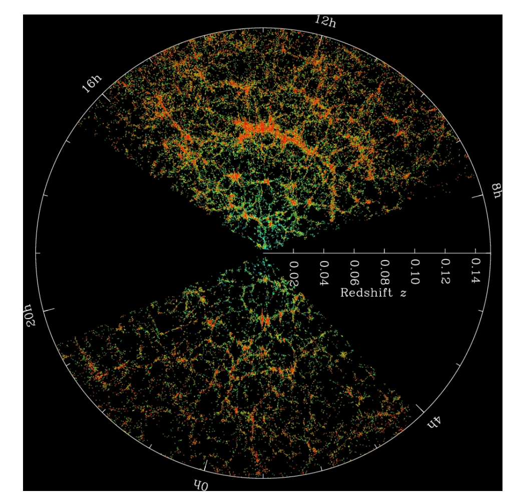


galaxy biasing ↓

$\delta_g(\boldsymbol{x}, \tau)$ —RSD→ $\delta_g(\theta, z)$ galaxies





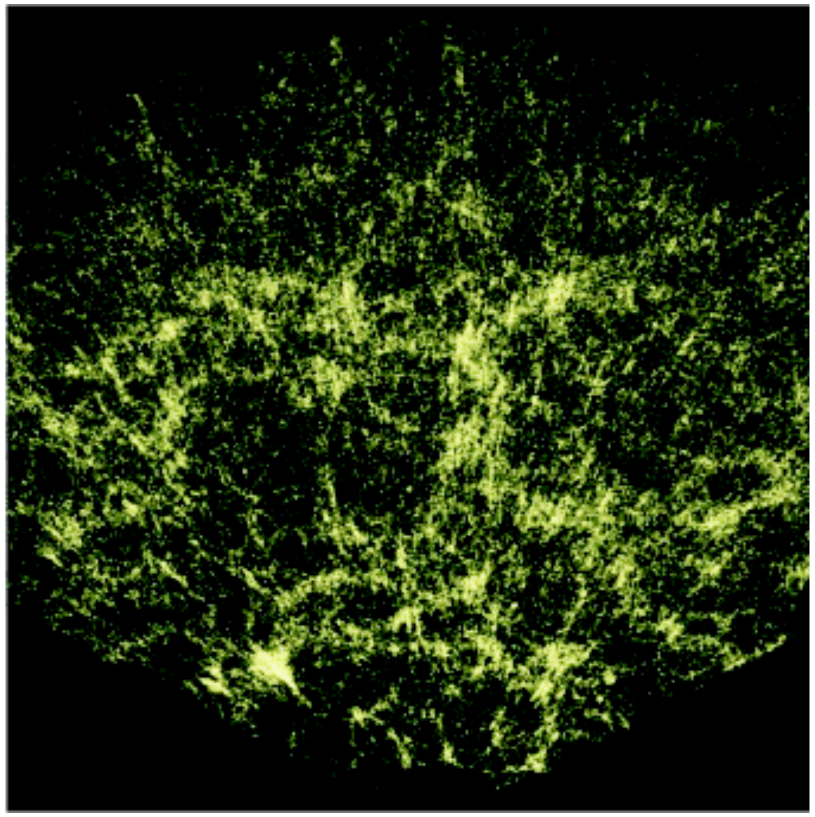
Redshift-Space Distortions

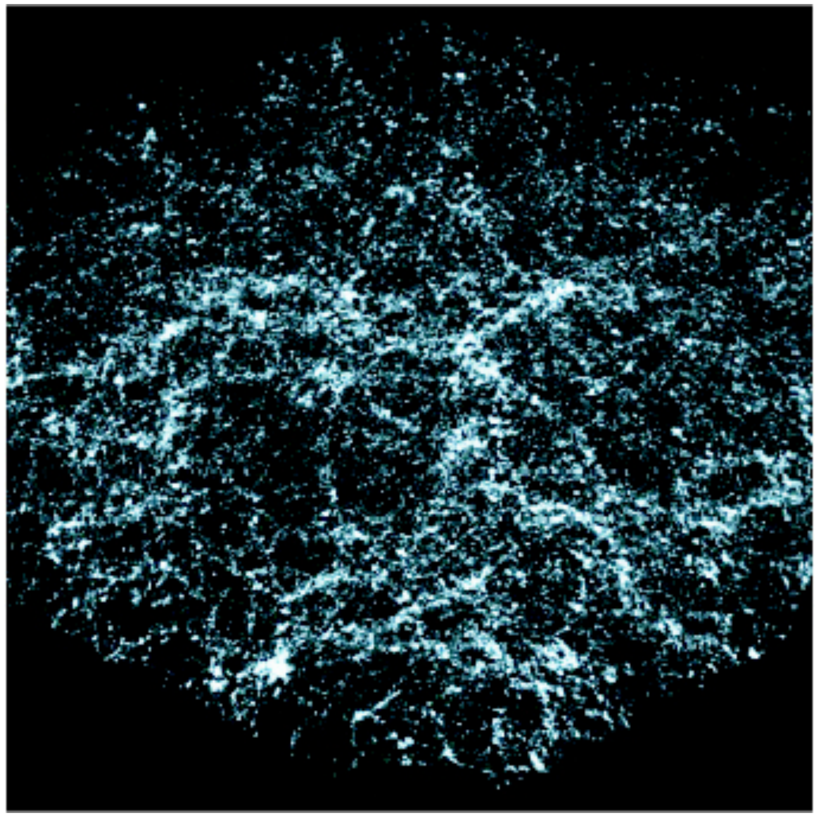
Galaxies are measured in redshift space but we can relate the density in redshift space and real space by mass conservation

$$1+\delta_s(\vec{x}_s) = [1+\delta(\vec{x}(\vec{x}_s))]\left|\frac{\partial \vec{x}_s}{\partial \vec{x}}\right|^{-1}_{\vec{x}(\vec{x}_s)}$$

Kaiser ‘87

$$\vec{x}_s = \vec{x} + \frac{\vec{v}\cdot\hat{z}}{H_0}\hat{z}$$





One-loop power spectrum

$$\begin{aligned} P_g(k,\mu) &= Z_1(\mu)^2 P_{11}(k) \\ &+ 2\int \frac{d^3q}{(2\pi)^3}\, Z_2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q},\mu)^2 P_{11}(|\boldsymbol{k}-\boldsymbol{q}|)P_{11}(q) \\ &+ 6Z_1(\mu)P_{11}(k)\int \frac{d^3q}{(2\pi)^3}\, Z_3(\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k},\mu)P_{11}(q) \\ &+ 2Z_1(\mu)P_{11}(k)\left(c_{\rm ct}\frac{k^2}{k_{\rm M}^2} + c_{r,1}\mu^2\frac{k^2}{k_{\rm M}^2} + c_{r,2}\mu^4\frac{k^2}{k_{\rm M}^2}\right) \\ &+ \frac{1}{\bar{n}_g}\left(c_{\epsilon,1} + c_{\epsilon,2}\frac{k^2}{k_{\rm M}^2} + c_{\epsilon,3} f\mu^2\frac{k^2}{k_{\rm M}^2}\right). \end{aligned}$$

D’Amico et al. 1909.05271

(see also Ivanov et al. 1909.05277)

Fast loop evaluation

Scan parameter space with MCMC: need to compute loop-integrals quickly

$$P_{22}(k) = 2\int \frac{d^3q}{(2\pi)^3} P_{11}(q) P_{11}(|\vec{k}-\vec{q}|) \left[F_2(\vec{q},\vec{k}-\vec{q})\right]^2$$

$$F_2(\vec{q},\vec{k}-\vec{q}) = \frac{5}{14} + \frac{3k^2}{28q^2} + \frac{3k^2}{28|\vec{k}-\vec{q}|^2} - \frac{5q^2}{28|\vec{k}-\vec{q}|^2} - \frac{5|\vec{k}-\vec{q}|^2}{28q^2} + \frac{k^4}{14|\vec{k}-\vec{q}|^2 q^2}$$

Bottleneck is the power spectrum shape. Solution **FFTLog**: decompose it as Fourier series of *log(k)*

Cosmology dependent

$$\bar{P}_{\rm lin}(k_n) = \sum_{m=-N/2}^{m=N/2} c_m\, k_n^{\nu+i\eta_m}$$

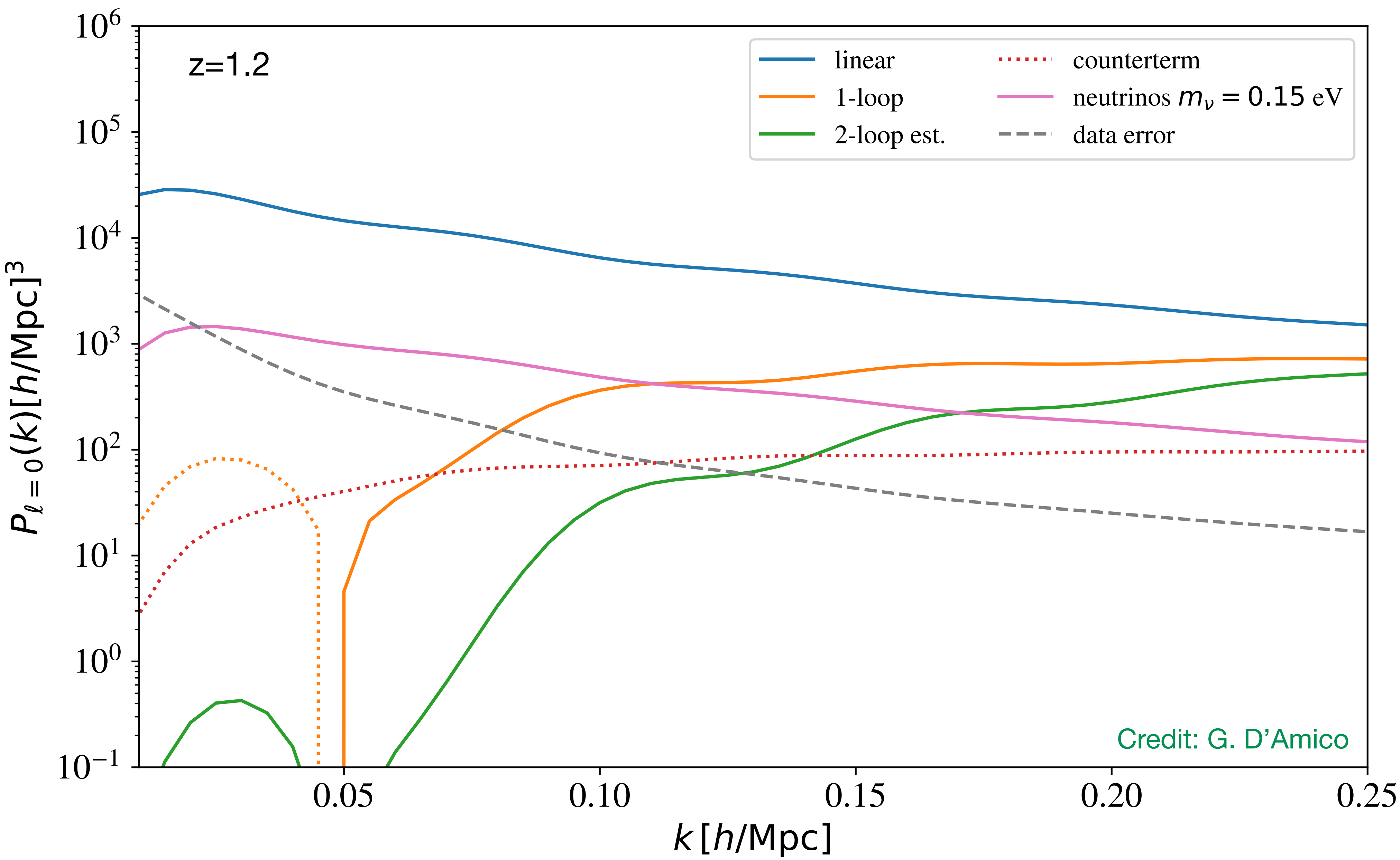
Simonovic et al. 1708.08130

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1}|\boldsymbol{k}-\boldsymbol{q}|^{2\nu_2}} \equiv k^{3-2\nu_{12}} \frac{1}{8\pi^{3/2}} \frac{\Gamma(\frac{3}{2}-\nu_1)\Gamma(\frac{3}{2}-\nu_2)\Gamma(\nu_{12}-\frac{3}{2})}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(3-\nu_{12})}$$

Precomputed Feynman loop-integrals of a massless QFT (cosmology independent)

Loop-integrals become matrix multiplications

Controlled accuracy



Smooth and clustering dark energy

Dark matter described by continuity and Euler equations + Poisson eq.

$$\dot{\delta} + \frac{1}{a}\partial_i\big((1+\delta)v^i\big) = 0\,,$$

$$\dot{v}^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\Phi = -\frac{1}{a}\frac{1}{\rho_m}\partial_j\tau^{ij}_{\rm short}\,,$$

$$\frac{1}{a^2}\partial^2\Phi = \frac{3}{2}H^2\Omega_m\delta$$

EFT stress-energy tensor

$$\partial_j\tau^{ij}_{\rm short} \to c^2_\delta\,\partial_i\delta$$

Smooth dark energy component: $c_s^2 \simeq 1\,,\qquad w \geq -1$ no DE perturbation, same PT.

Clustering dark energy component: $c_s^2 \ll 1\,,\qquad w \neq -1$ comoving fluids, same PT.

Sefusatti, FV et al. 1101.1026

$$\dot{\delta}_m + \frac{1}{a}\partial_i\left((1+\delta_m)v^i\right) = 0\,,$$

$$\dot{\delta}_Q - 3wH\delta_Q + \frac{1}{a}\partial_i\left((1+w+\delta_Q)v^i\right) = 0\,,$$

$$\frac{1}{a^2}\partial^2\Phi = \frac{3}{2}H^2\Omega_m\left(\delta_m + \frac{\Omega_Q}{\Omega_m}\delta_Q\right)$$

Perturbation theory, biasing and RSD essentially the same (except exact time dependence vs Einstein-de-Sitter approximation)

MG: Scale independent models

Scale independent models: nDGP, Galileons, Horndeski, beyond Horndeski, etc.

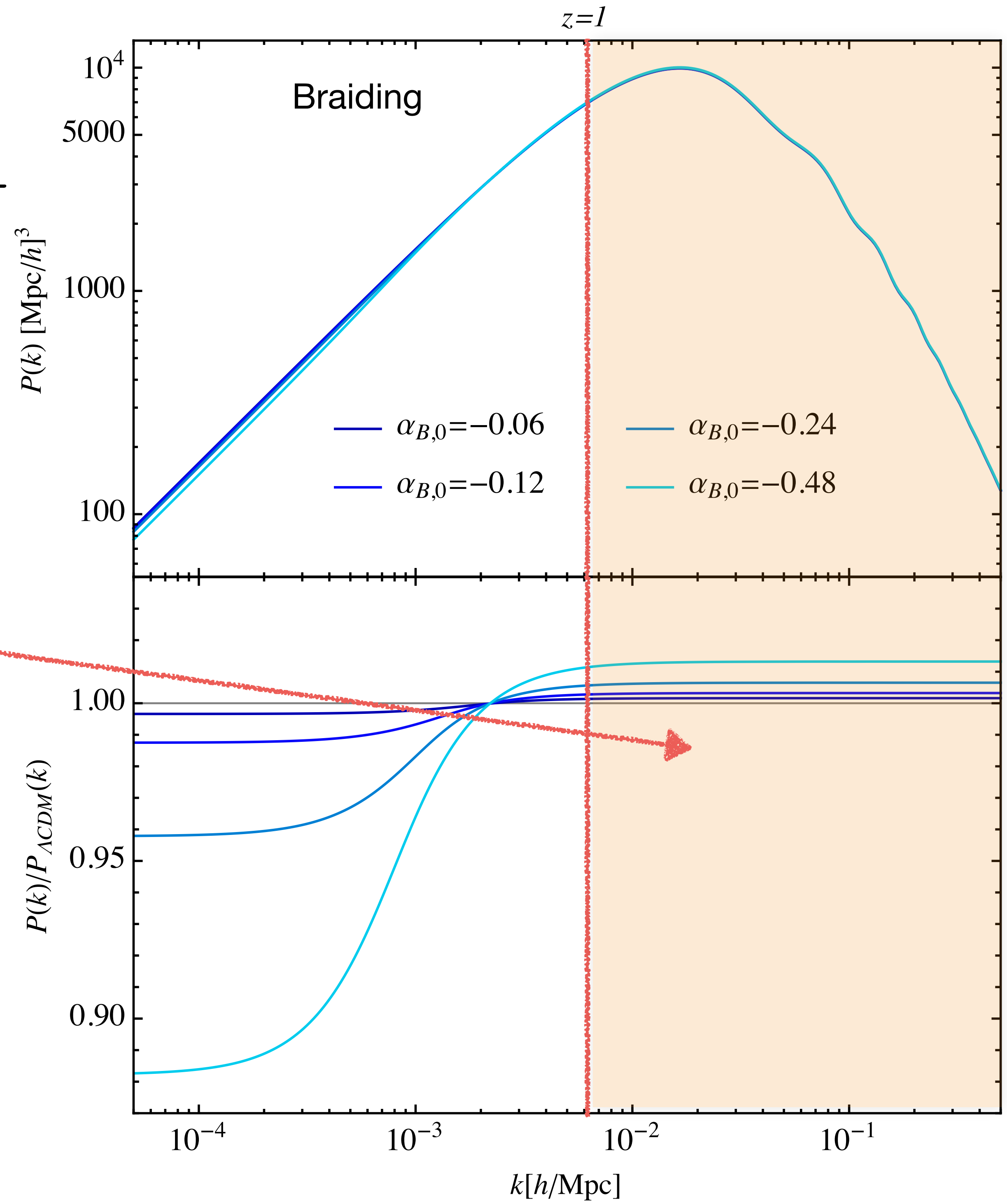
$$m_\phi \ll k_{\rm f.m.}/a$$

Ex: braiding (kinetic mixing) $\mathcal{L} \supset \dot{\Psi}\dot{\pi}\ ,\ \partial\Psi\partial\pi$

Scalar field clustering on short scales:

$$\nabla^2\pi = \alpha \cdot 4\pi G\delta$$

Quasi-static approximation on short scales



D'Amico, Huang, Mancarella, FV 1609.01272

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Same equations for the dark matter fluid (continuity + Euler). **Poisson equation modified:**

Time-dependent gravitational constant

$$k^2\Phi = 4\pi G_{\rm eff}\delta + \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\delta(\vec{k}-\vec{k}_1-\vec{k}_2)M_2(\vec{k}_1,\vec{k}_2)\Phi(\vec{k}_1)\Phi(\vec{k}_2)$$

$$+\int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}\delta(\vec{k}-\vec{k}_1-\vec{k}_2-\vec{k}_3)M_3(\vec{k}_1,\vec{k}_2,\vec{k}_3)\Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)$$

NEW PT nonlinear couplings

Theory dependent

$$G_{\rm eff} = G_{\rm eff}(a)$$

$$M_2(\vec{k}_1,\vec{k}_2) = \mu_2(a)\left(k_1^2k_2^2 - (\vec{k}_1\cdot\vec{k}_2)^2\right)$$

$$M_3(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \ldots$$

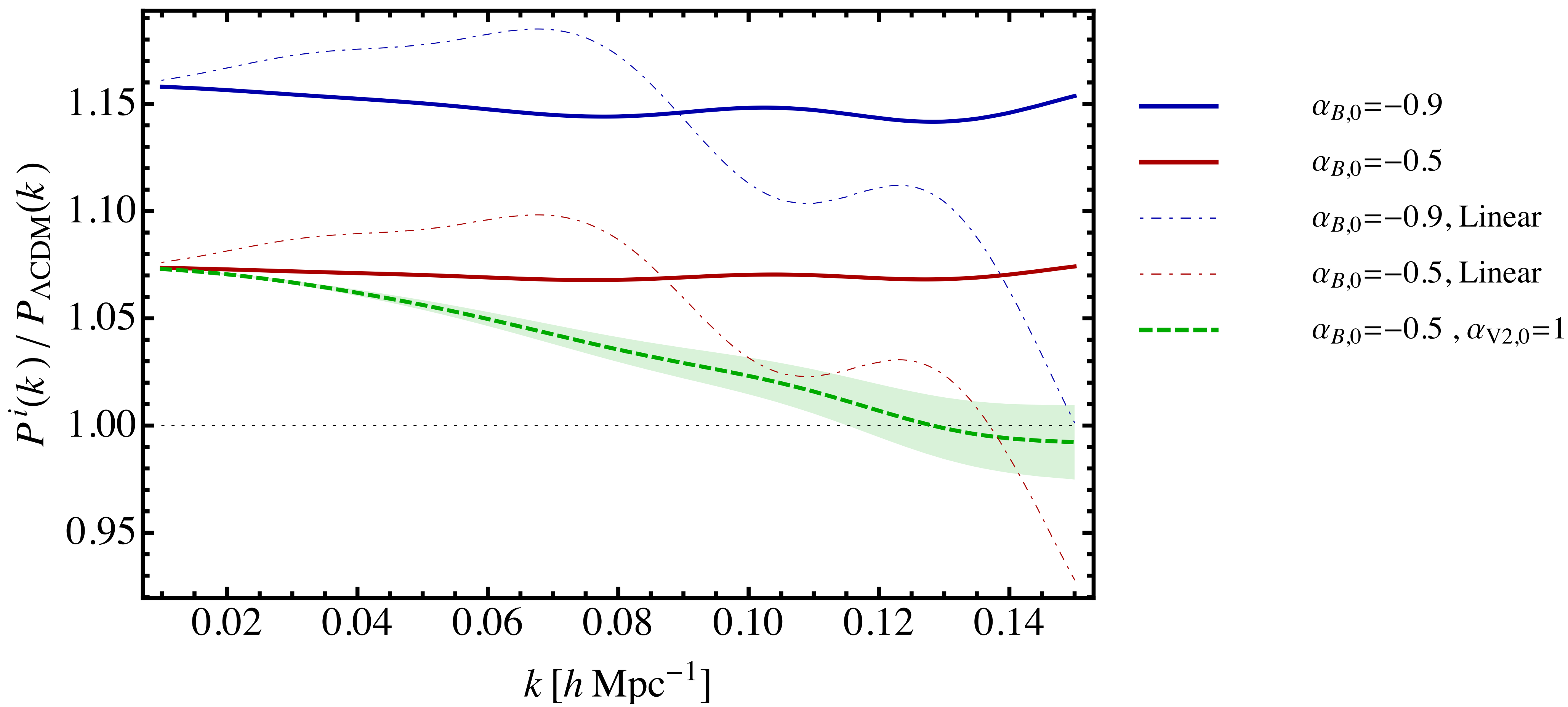
MG: Scale independent models

Sca

Ex: I

Sam

k^2



Cusin, Lewandowski, FV 17

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$$M_3(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \ldots$$

EFT of Dark Energy

Bridge models and observations
in a minimal and systematic way

Space of theories

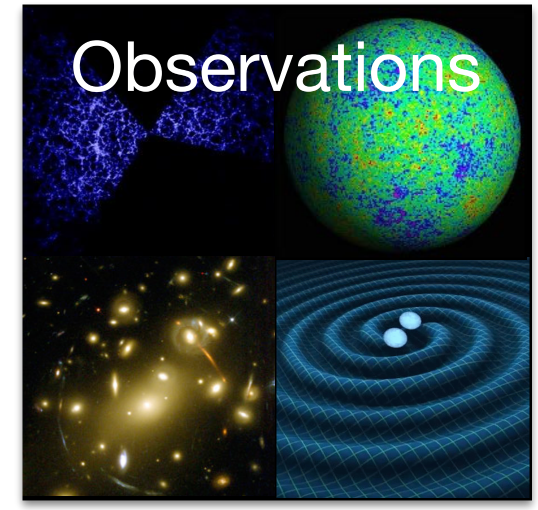
$$G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\Box\phi - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right] + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X) \times \left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] + \dots$$

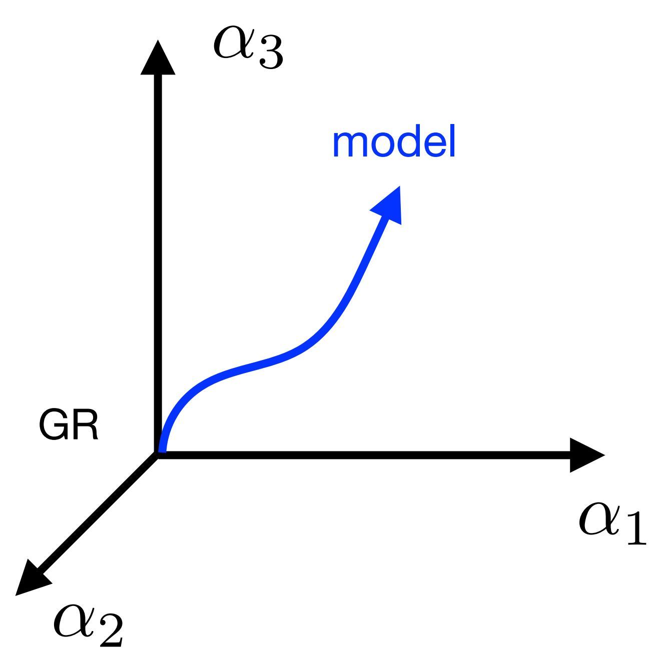
←

EFT of DE

$\alpha_1(t),\ \alpha_2(t),\ \dots$

→





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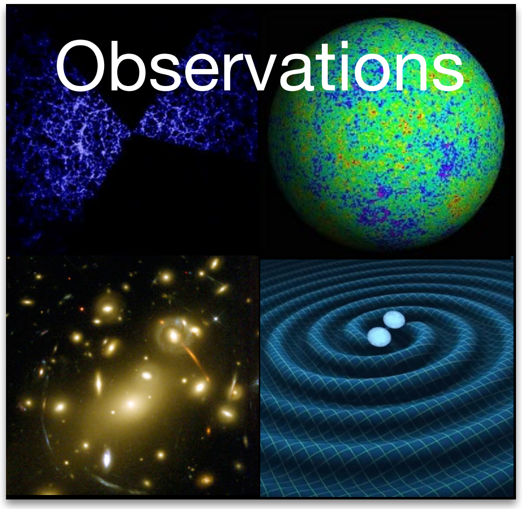
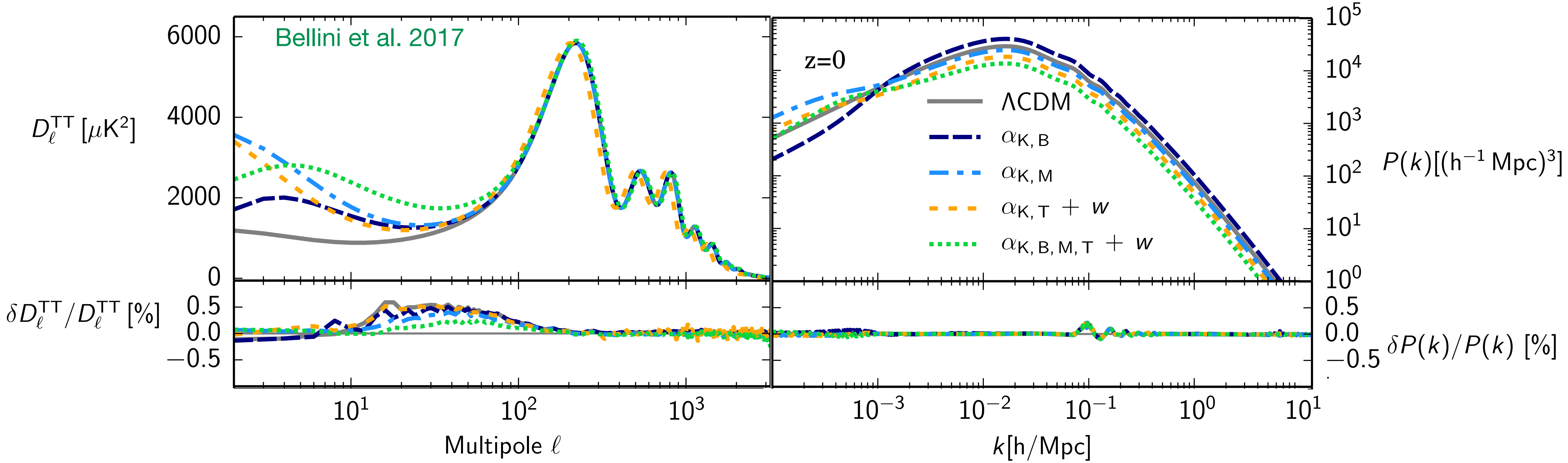
← **EFT of DE** →

$$\alpha_1(t),\ \alpha_2(t),\ \dots$$



$$\mu = \mu\big(k; \alpha_1(t), \alpha_2(t), \dots\big)$$
$$\Sigma = \Sigma\big(k; \alpha_1(t), \alpha_2(t), \dots\big)$$

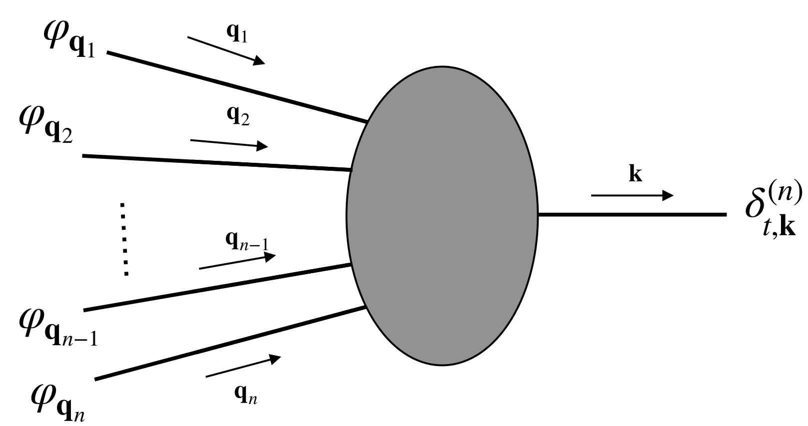
$$\nabla^2\Phi = 4\pi G\,\mu\,\delta\rho_{\rm m}$$
$$\nabla^2(\Phi+\Psi) = 8\pi G\,\Sigma\,\delta\rho_{\rm m}$$



Symmetries

Structure of PT kernels dictated by **symmetries** (e.g. translation, rotations, Bose, mass and momentum conservation, etc.)

D'Amico et al., in preparation



$$\delta^{(n)}_{\mathbf{k}}(\eta) \equiv \frac{1}{n!}\int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta_D\left(\mathbf{k}-\sum_{i=1}^{n}\mathbf{q}_i\right)F^{(n)}(\mathbf{q}_1,\cdots,\mathbf{q}_n;\eta)\varphi_{\mathbf{q}_1}(\eta)\cdots\varphi_{\mathbf{q}_n}(\eta),$$

Time-dependent translation symmetry (**Equivalence Principle**)

$$\tilde{x}^i = x^i + n^i(t)\ ,\quad \tilde{t} = t\ ,$$

$$\tilde{\varphi}_a(\tilde{x}^j,t) = \varphi_a(x^j,t) + h^i_{\varphi_a}(t)\tilde{x}^i\ ,$$
$$\tilde{\delta}(\tilde{x}^j,t) = \delta(x^j,t)\ ,$$
$$\tilde{v}^i(\tilde{x}^j,t) = v^i(x^j,t) + a\dot{n}^i(t)\ ,$$

Same symmetries for tracers (excluding mass and momentum conservation): **same PT structure for bias and RSD**

Horndeski vs beyond Horndeski

Horndeski theories: time-dependent translation symmetry (EP)

PT kernels, bias, RSD enjoy the same structure as Standard Perturbation Theory. Example:

$$F_2(\vec{k},\vec{q}) = (1+c_0)\frac{17}{21} + \frac{\mu}{2}\left(\frac{k}{q}+\frac{q}{k}\right) + (1+c_2)\left(\mu^2-\frac{1}{3}\right)\,, \qquad \mu \equiv \hat{k}\cdot\hat{q}$$

Fixed by symmetries

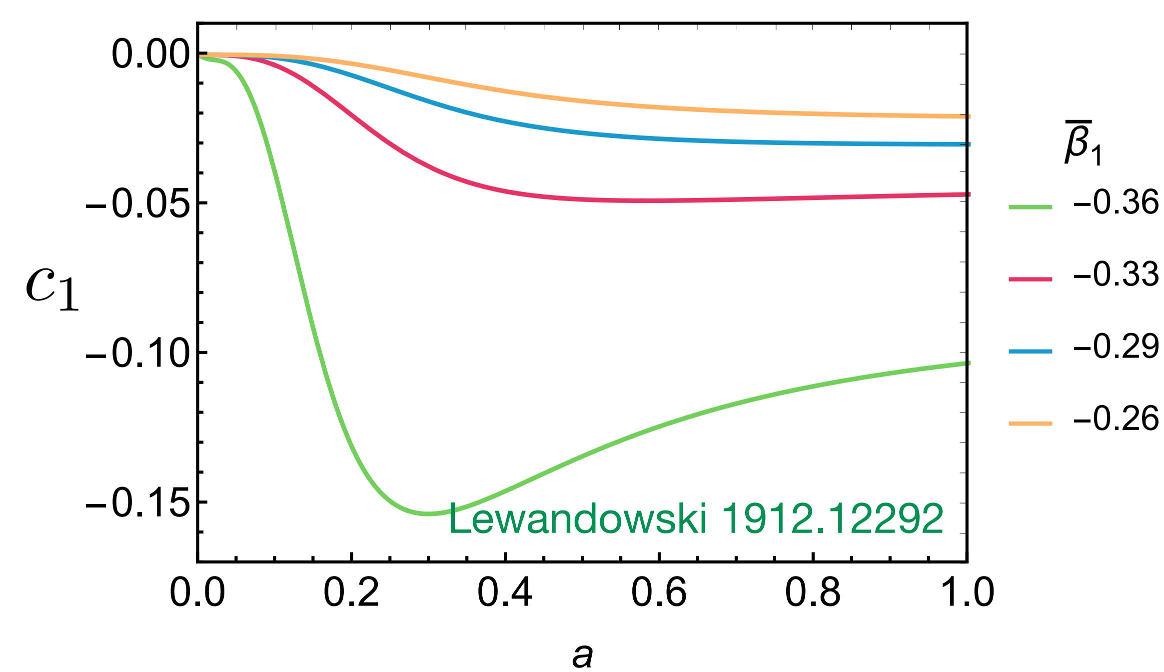
Beyond Horndeski theories: ~~time-dependent translation symmetry (EP)~~

New features in PT kernels, bias, RSD, loops…

Hirano et al. 1801.07885;
Crisostomi, Lewandowski, FV 1909.07366

$$F_2(\vec{k},\vec{q}) = (1+c_0)\frac{17}{21} + (1+c_1)\frac{\mu}{2}\left(\frac{k}{q}+\frac{q}{k}\right) + (1+c_2)\left(\mu^2-\frac{1}{3}\right)\,, \qquad \mu \equiv \hat{k}\cdot\hat{q}$$

Anomalous dipole



MG: Scale dependent models

Scale dependent models: f(R), chameleon, etc. $m_\phi \gtrsim k_{\rm f.m.}/a$

Growth depends on scale:

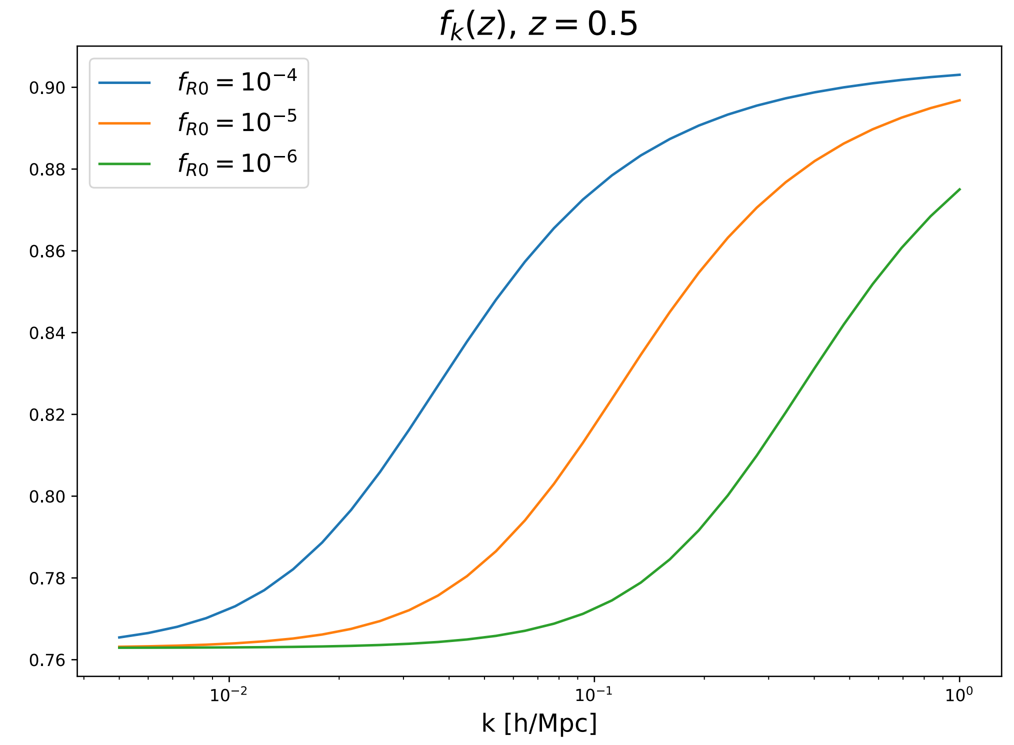
$$\frac{d^2}{d\tau^2}D(k,\tau) + \mathcal{H}\frac{d}{d\tau}D(k,\tau) - \frac{3}{2}\Omega_m(a)\mathcal{H}^2\left[1+\alpha(\tau)\frac{k^2}{k^2+a^2m^2(\tau)}\right]D(k,\tau) = 0$$

Two regimes:

$k/a \gg m_\phi$ $\quad 1+\alpha \quad$ enhancement

$k/a \ll m_\phi$ $\quad$ GR

$$f \equiv \frac{d\ln D}{\ln a}$$



Bias expansion should be scale dependent

$$\delta_g(\mathbf{k},t) = \sum_n b_n(k,t)\mathcal{O}_n(\mathbf{k},t)$$

Captured by higher-order operators?

$$\nabla^2\delta\,, \quad \nabla^4\delta\,,\ldots$$

f(R): One-loop galaxy PS

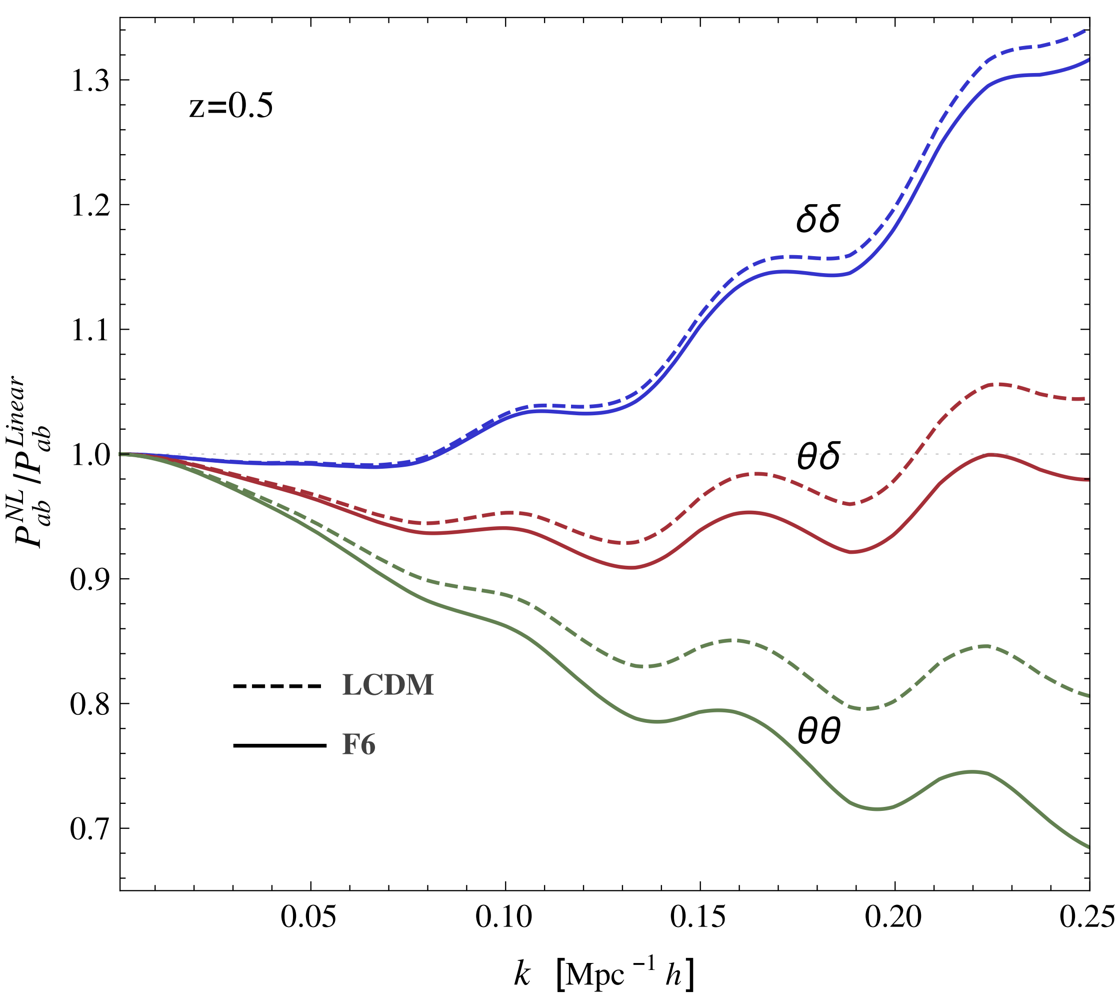
PT kernels are non-standard but can be computed straightforwardly:

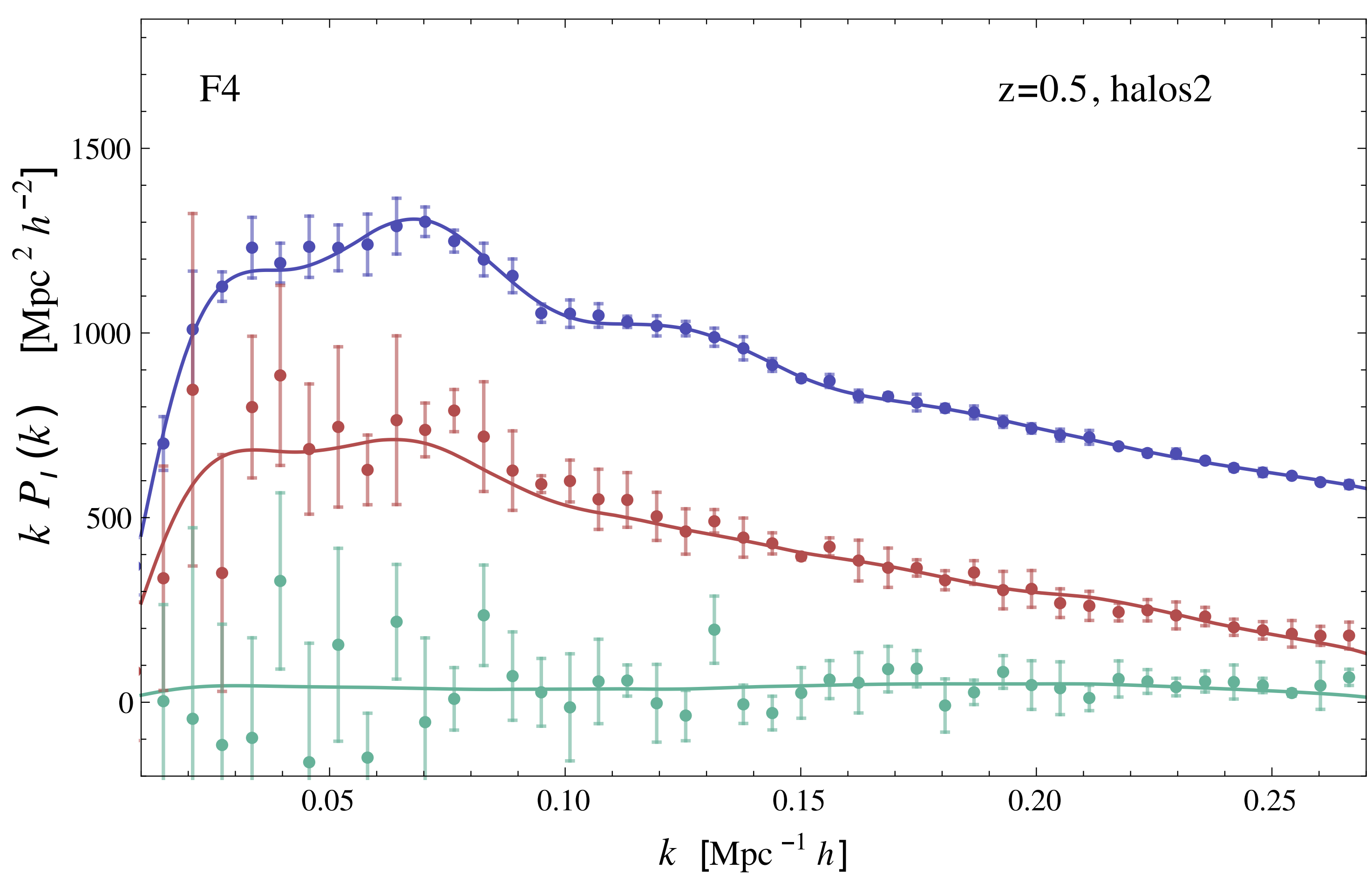
$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{1}{2} + \frac{3}{14}\mathcal{A} + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1 k_2}\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right) + \left(\frac{1}{2} - \frac{3}{14}\mathcal{B}\right)\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2}$$

$$\mathcal{A}(\mathbf{k}_1, \mathbf{k}_2, t) = \frac{7D^{(2)}_{\mathcal{A}}(\mathbf{k}_1, \mathbf{k}_2, t)}{3D_+(k_1, t)D_+(k_2, t)}, \qquad \mathcal{B}(\mathbf{k}_1, \mathbf{k}_2, t) = \frac{7D^{(2)}_{\mathcal{B}}(\mathbf{k}_1, \mathbf{k}_2, t)}{3D_+(k_1, t)D_+(k_2, t)},$$

Full calculation on one-loop PS in redshift space with EFT counterterms:

Aviles et al. 2012.05077





Conclusions and challenges

LCDM, smooth and clustering DE

- Galaxy clustering in the mildly nonlinear regime modelled by perturbation theory + finite number of free parameters, **with controlled accuracy**.
- Challenge: Going **beyond one loop** (two loops). Slower calculations, more EFT parameters, etc. Higher accuracy required.

Scale independent MG model:

- Above standard methods (with new PT kernels) can be applied to Horndeski/EFT of DE models. **Symmetries are the same!**
- New phenomenology (yet to explore fully) in broken time-dep. translation symmetry (beyond Horndeski).
- Challenge: Many DE/MG (time-dependent) parameters. How to parametrise them? Degeneracies with EFT of LSS parameters.

Scale dependent MG model:

- Above methods can be extended to scale-dep. models. Work in progress.
- Challenge: How to include scale-dependence in loop integrals in a fast way (FFTLog)?
- Challenge: How to include scale-dependence in bias expansion? Are higher-order bias term sufficient? How scale dependence affect EFT of LSS parameters and their scale-dependence?