Time and clock studies

Discussion LPNHE — March 25th 2021

Quantify time stability

Signal $y(t)$ (usually discrete measurements $y_i = y(t_i)$)
How to compute stability?
→ Frequency domain: **power spectral density (PSD)**

$$S_y(\omega) = \frac{|\tilde{y}(\omega)|^2}{T} = \frac{\left| \int_0^T y(t)\exp(i\omega t)\mathrm{d}t \right|^2}{T}$$

→ Time domain: **Allan variance (AV)**

$$\sigma_{\text{Allan}}(n) = \sqrt{\frac{1}{2\left(\frac{N}{n}-1\right)} \sum_{l=1}^{\frac{N}{n}-1} \left(y_{l+1}^{(n)} - y_l^{(n)}\right)^2} \quad \text{with } y_l^{(n)} = \frac{1}{n}\sum_{i=1}^{n} y_{ln+i}$$

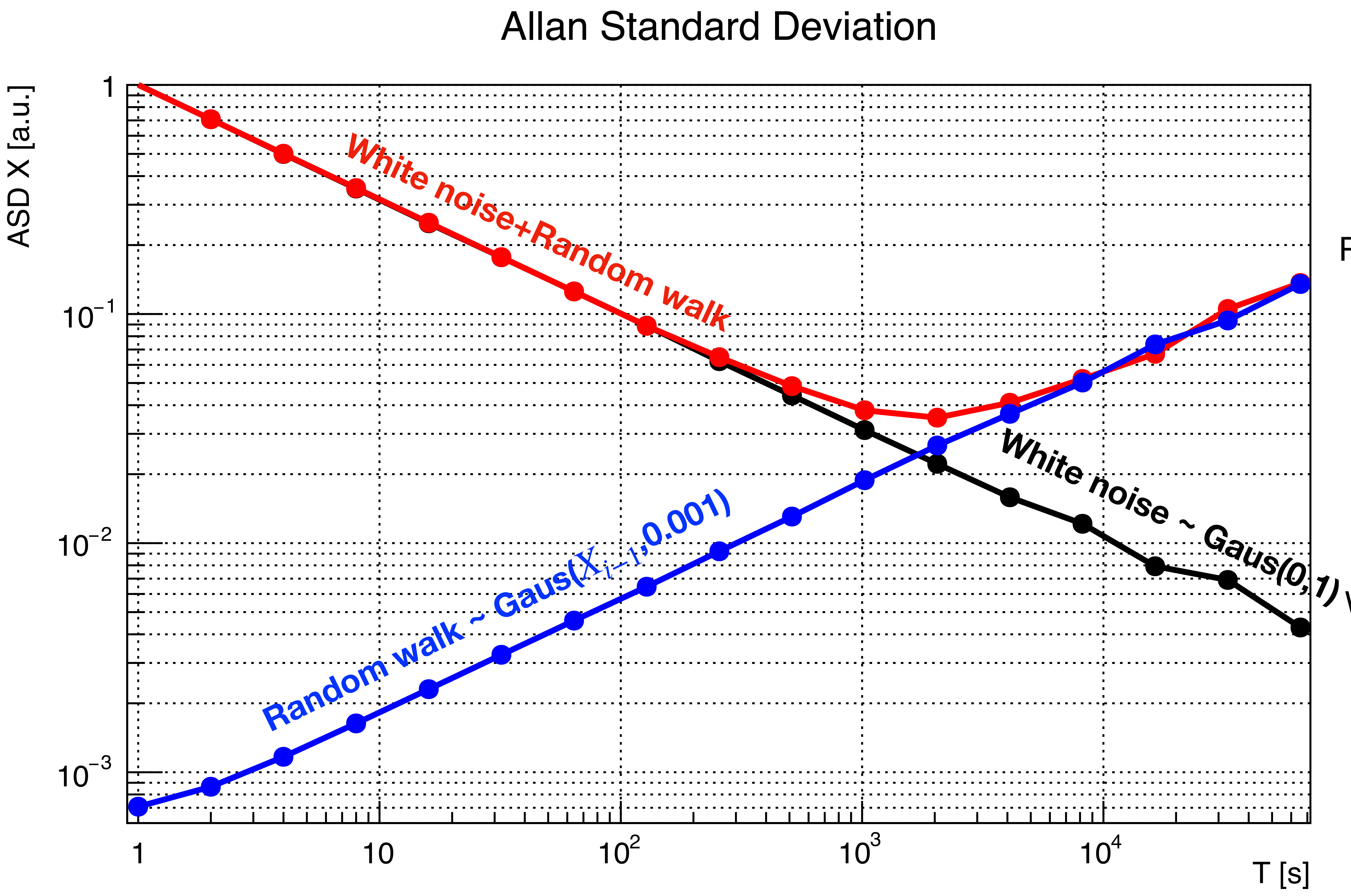
(1,5)

Many variations around this formula like **modified AV**:

$$\sigma_{\text{Mod}}(\tau) = \sqrt{\frac{1}{2n^2(N-2n+1)} \sum_{j=1}^{N-2n+1} \left(\sum_{i=j}^{j+n-1} y_{i+n}^{(n)} - y_i^{(n)} \right)^2}$$

(3,5)

Generating data and ASD



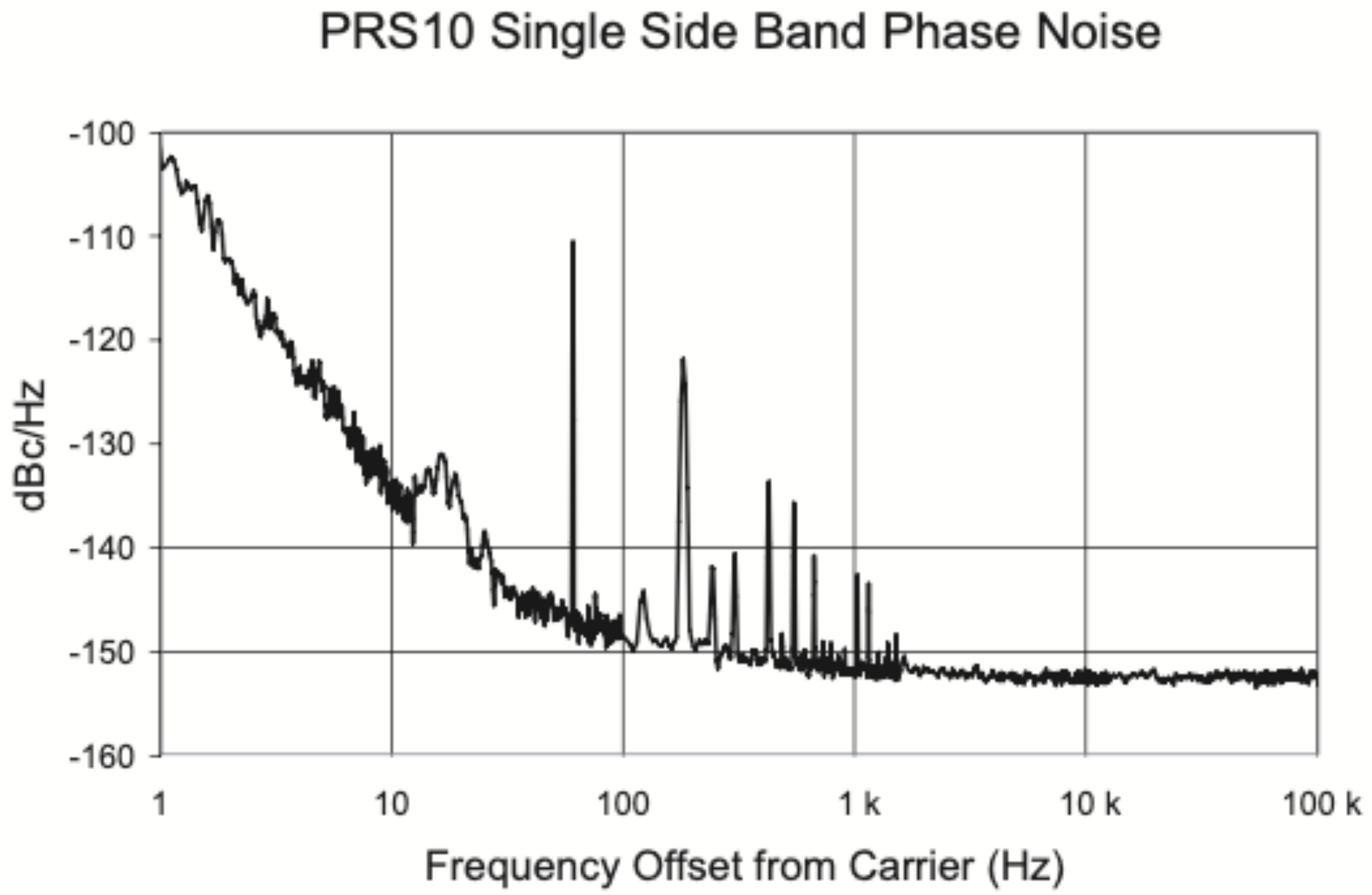
Random walk: $\sqrt{\langle X^2 \rangle} = \sqrt{D\tau}$

(Einstein equation)

White noise: $\sqrt{\langle X^2 \rangle} \approx \frac{\sigma_0}{\sqrt{\tau}}$

(Central limit theorem)

Short-term stability: jitter definition



Jitter: fluctuations on both amplitude, frequency and phase

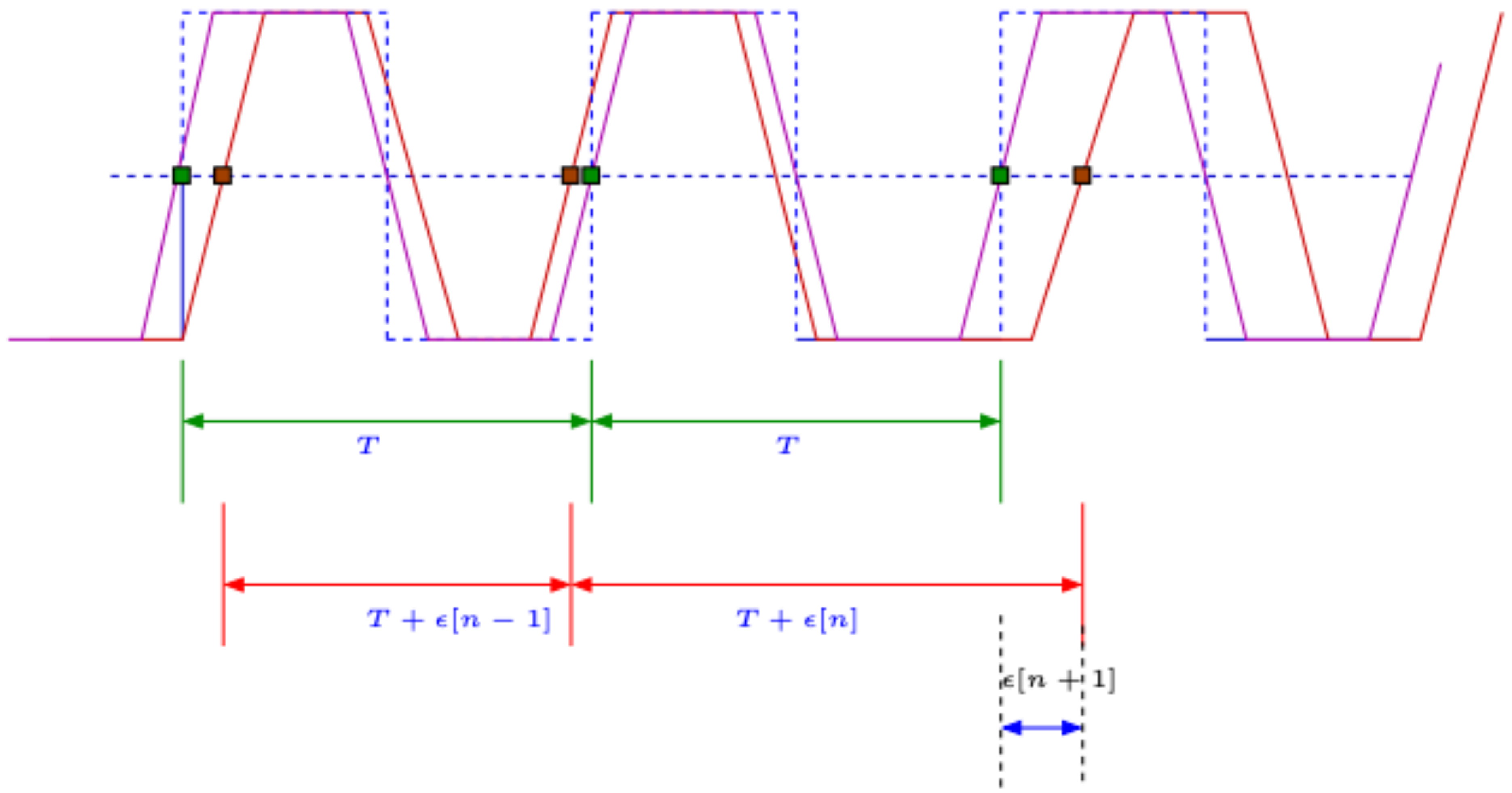


Figure 8. Sampling Clock Jitter: The time duration (period) between successive triggers vary as a result of the phase noise. Ideal clocks preserve constant period T, whereas, practical clocks vary the value randomly, leding to jitter

Phase noise:

$$\sigma_\phi^2(\tau) = \langle \bar{\phi}^2 \rangle = \frac{1}{\tau^2} \left\langle \left[\int_{t_k-\tau}^{t_k} \phi(t) dt \right]^2 \right\rangle$$

(RMS phase) jitter (@1 s):

$$J = \frac{\sigma_\phi}{2\pi f_0} = \frac{1}{2\pi f_0} \sqrt{\int_0^\infty S(f) \left(\frac{\sin \pi\tau f}{\pi\tau f} \right)^2 df}$$

https://documents.epfl.ch/users/p/pu/pulikkoo/private/report_pn_jitter_oscillator_ratna.pdf

Not quite the same as AV → is there a known relation?

Clock recommendations

Email from Michel with useful clock recommandations and price tags

1- Microsemi 5071A → 70-80 k€

2- OSA 3235B → 50 k€

3- PHM1008 → 55 k€

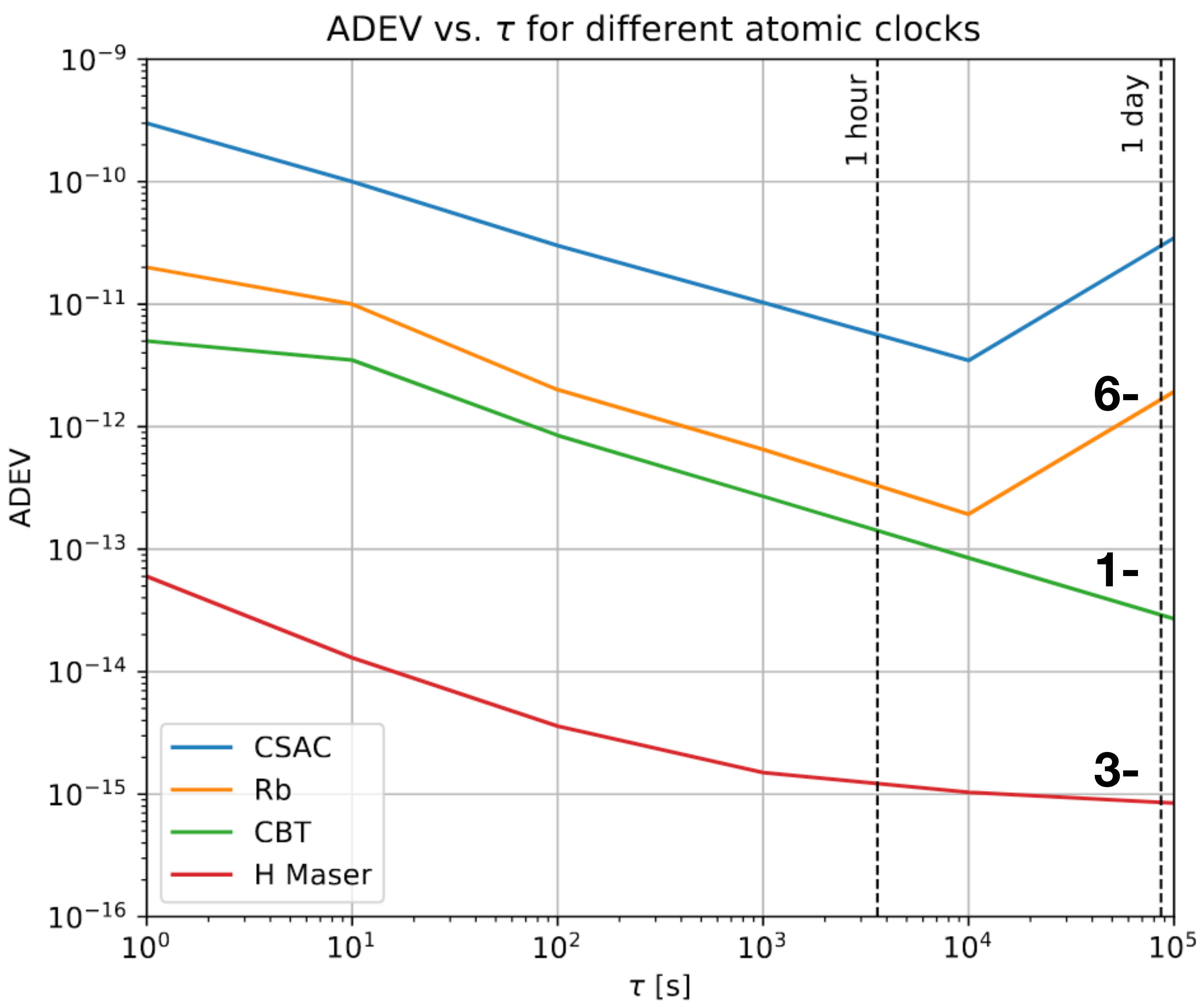
4- Cold Rb clock → >100 k€

Off-the-shelf systems:

5- SRS FS725 → 3 k€

6- SRS PRS10 → 1.5 k€

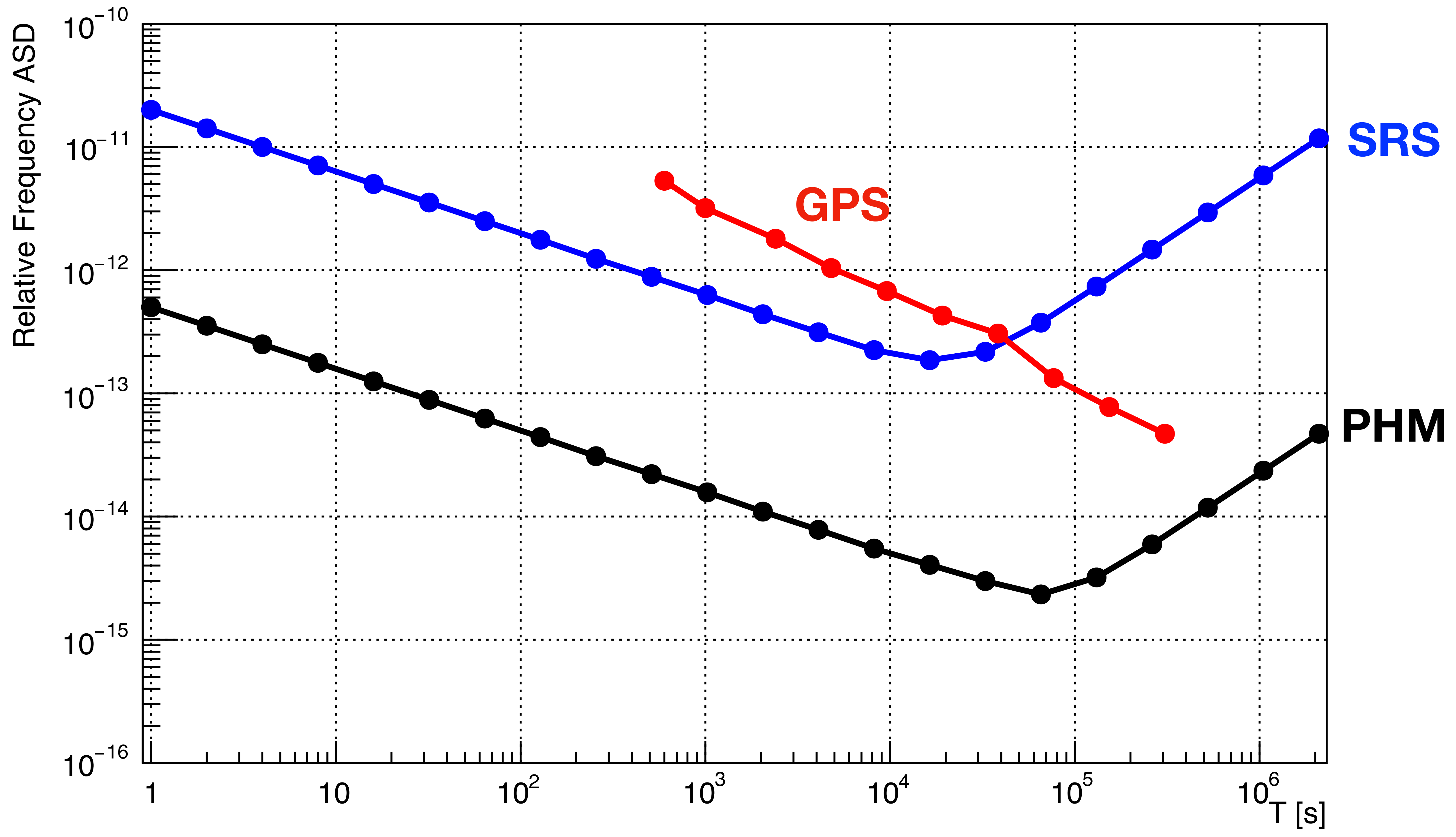
Clocks performances from litterature

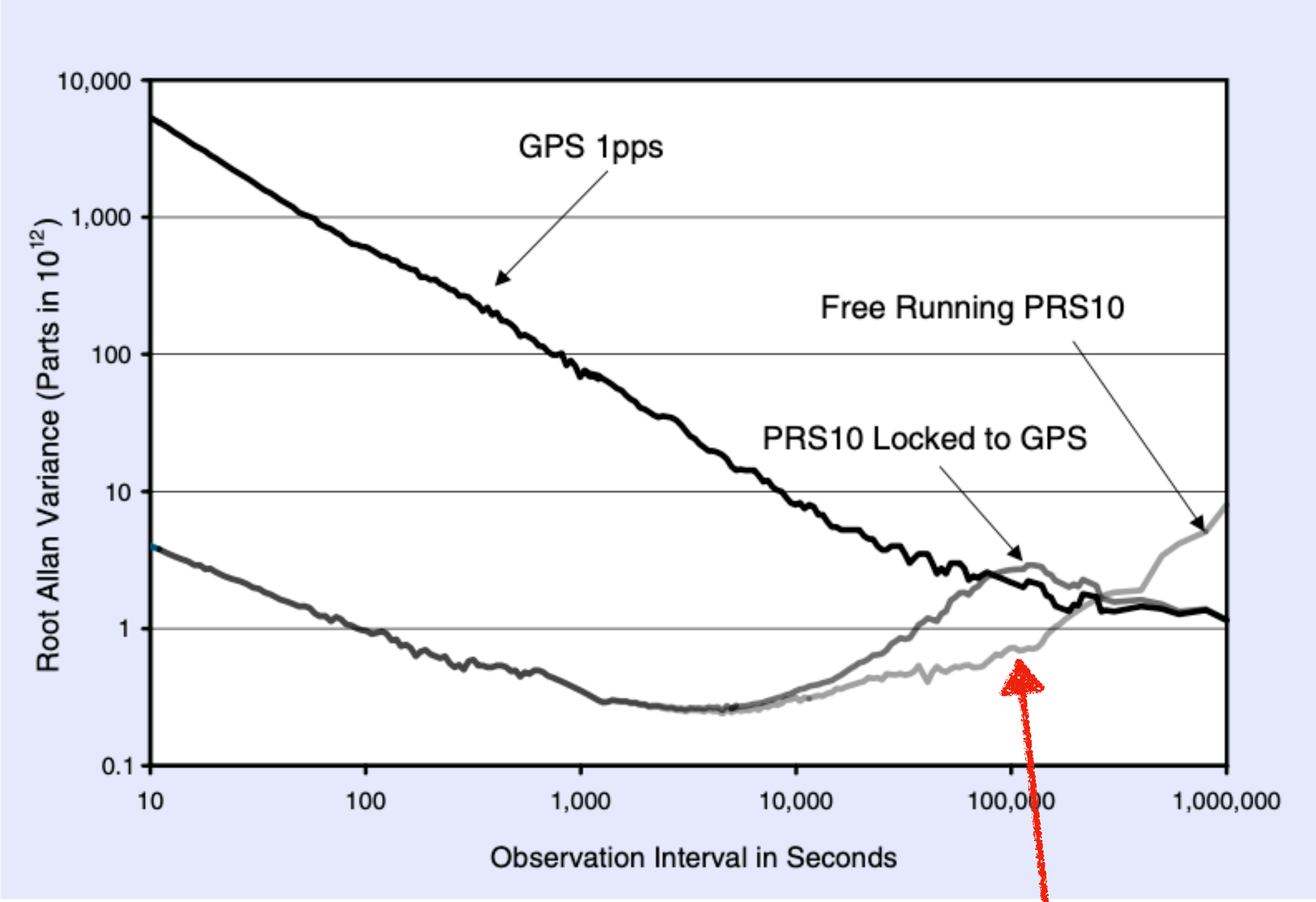


(6)

Fig. 2. Comparison of ADEV for different frequency standard technologies. The curves shown from top to bottom are the Microchip CSAC, SRS PRS10 Rb frequency standard, Microchip 5071A Cesium Beam Tube, and Vremya Active Hydrogen Maser.

Locking clock PPS on GPS



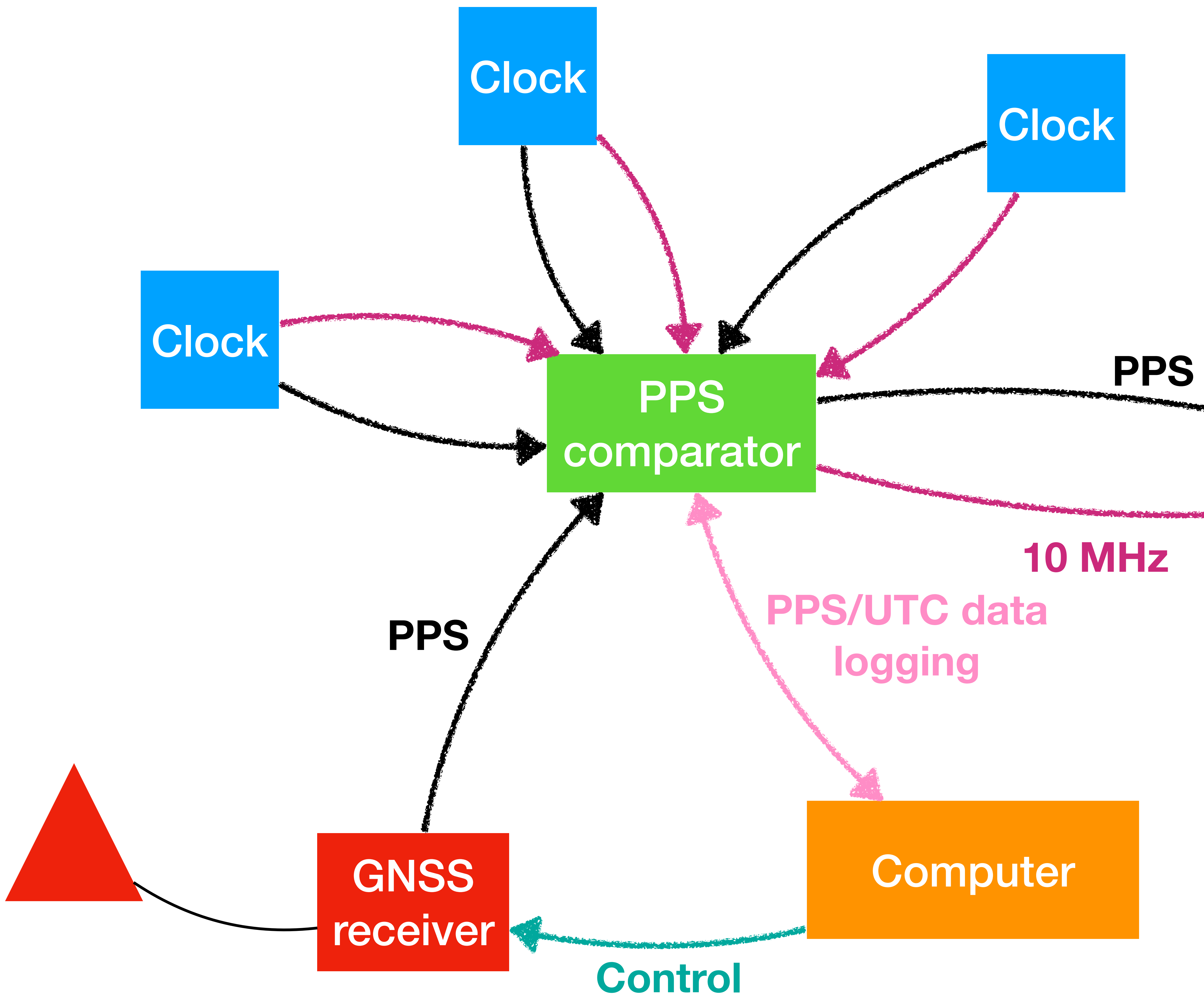


PRS10 Allan variance plot

Locking doesn't seem to be the best of both → why?

SYRTE recommended post-treatment time corrections

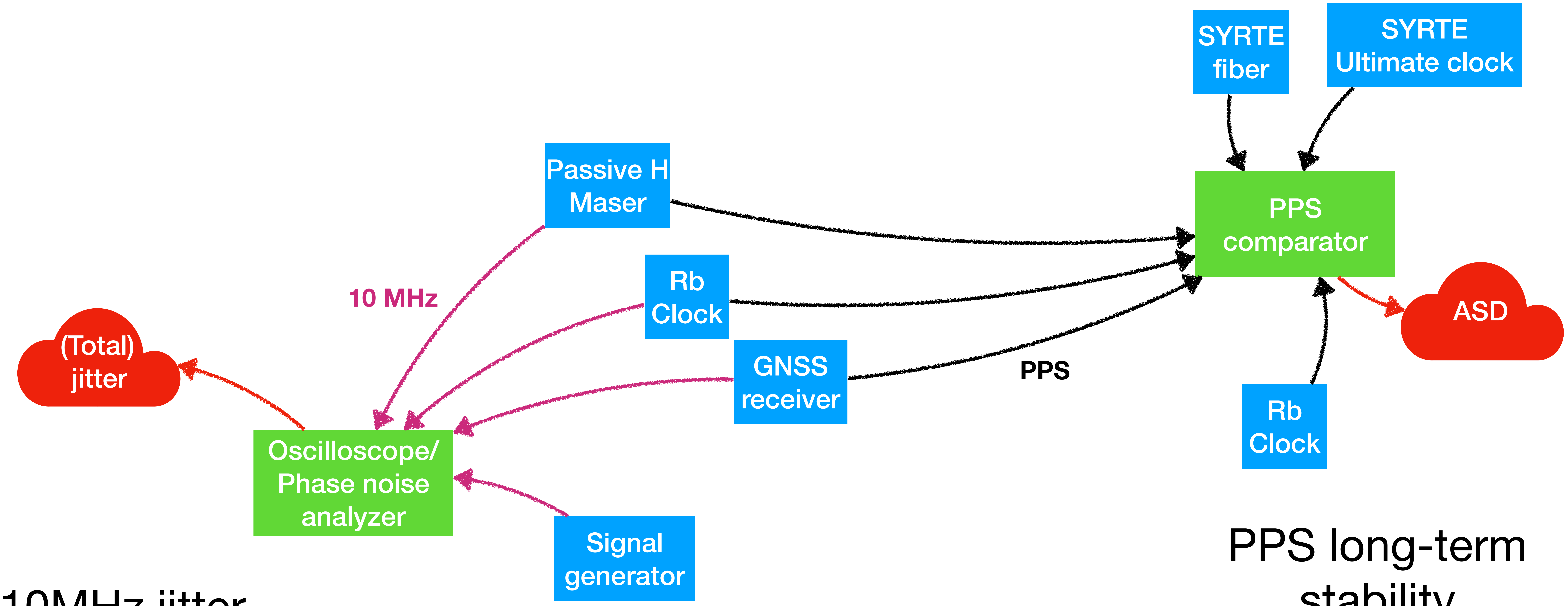
Overall system for HK



Once every week/month(?):
- Compare GNSS PSS with clock's
- Measure phase difference
- Tag Clock PPS with UTC
- Only use clocks PPS/10MHz

PPS (not 10MHz?) comparator monitors clocks instabilities

List of measurements to do



10MHz jitter measurements

PPS long-term stability

How to measure time stability of clocks?

Jitter@1sec (“wander”):
- Standard oscilloscope (Tektronic MSO Series 5?)
- Phase noise analyzer (Integrated PSD/jitter)

Long-term ASD:
- Is this useful or should we just trust specs?
- How are the parameters (frequencies?) computed?
- Are these oscilloscopes/analyzers stable enough > 1day?
- Use a more stable clock as comparison?
- Impact of temperature and humidity drifts on clock?
- Maintenance?

GNSS: what's next?

- Install and test software interfaces

→ Backend API necessary/available?

- Install antenna on Jussieu roof (in the coming months)
- Test long-term stability of GNSS 1pps with and without atomic clock

→ what equipment?

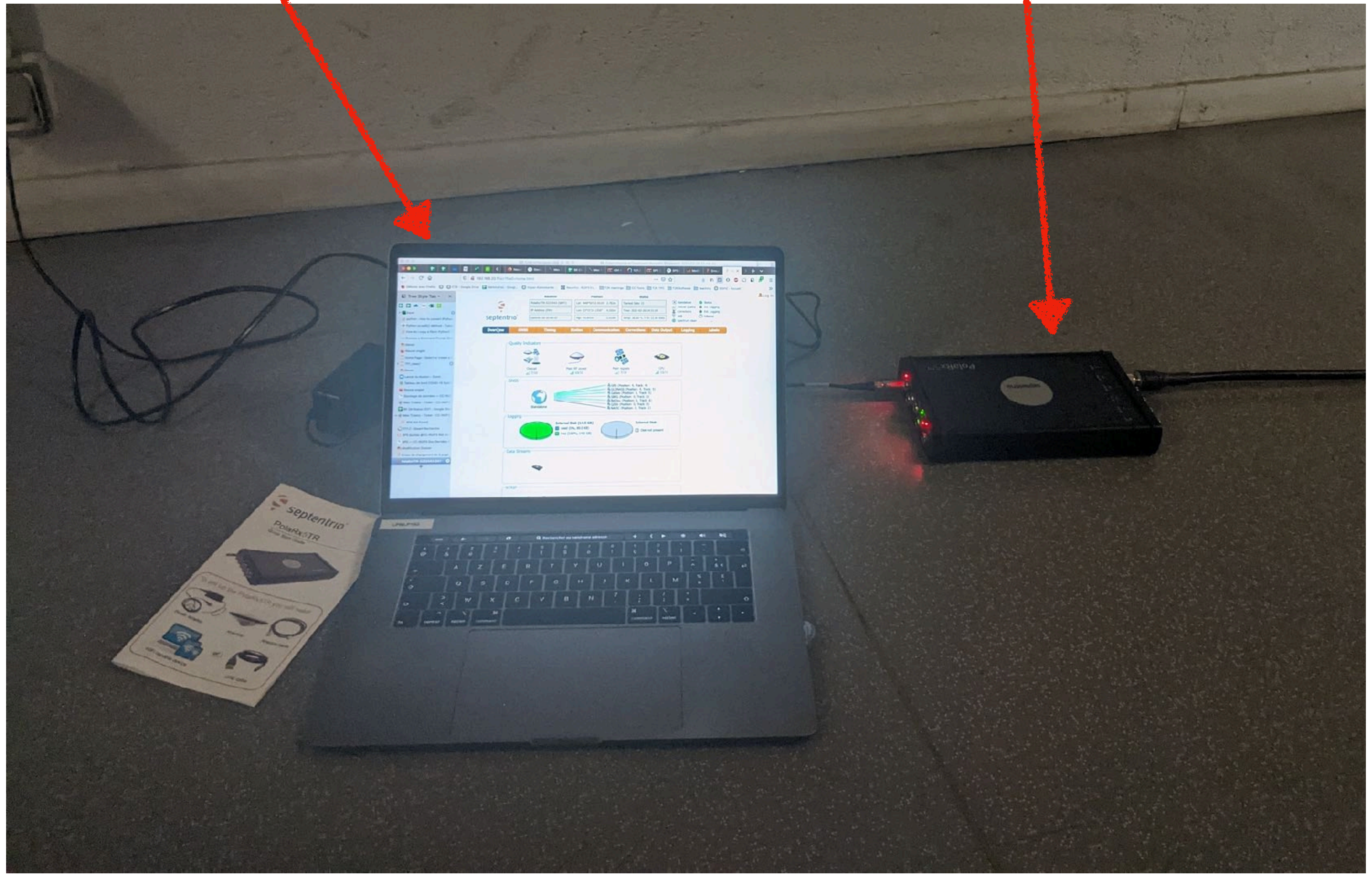
- Test locking method
- Calibrate cables and antenna at SYRTE

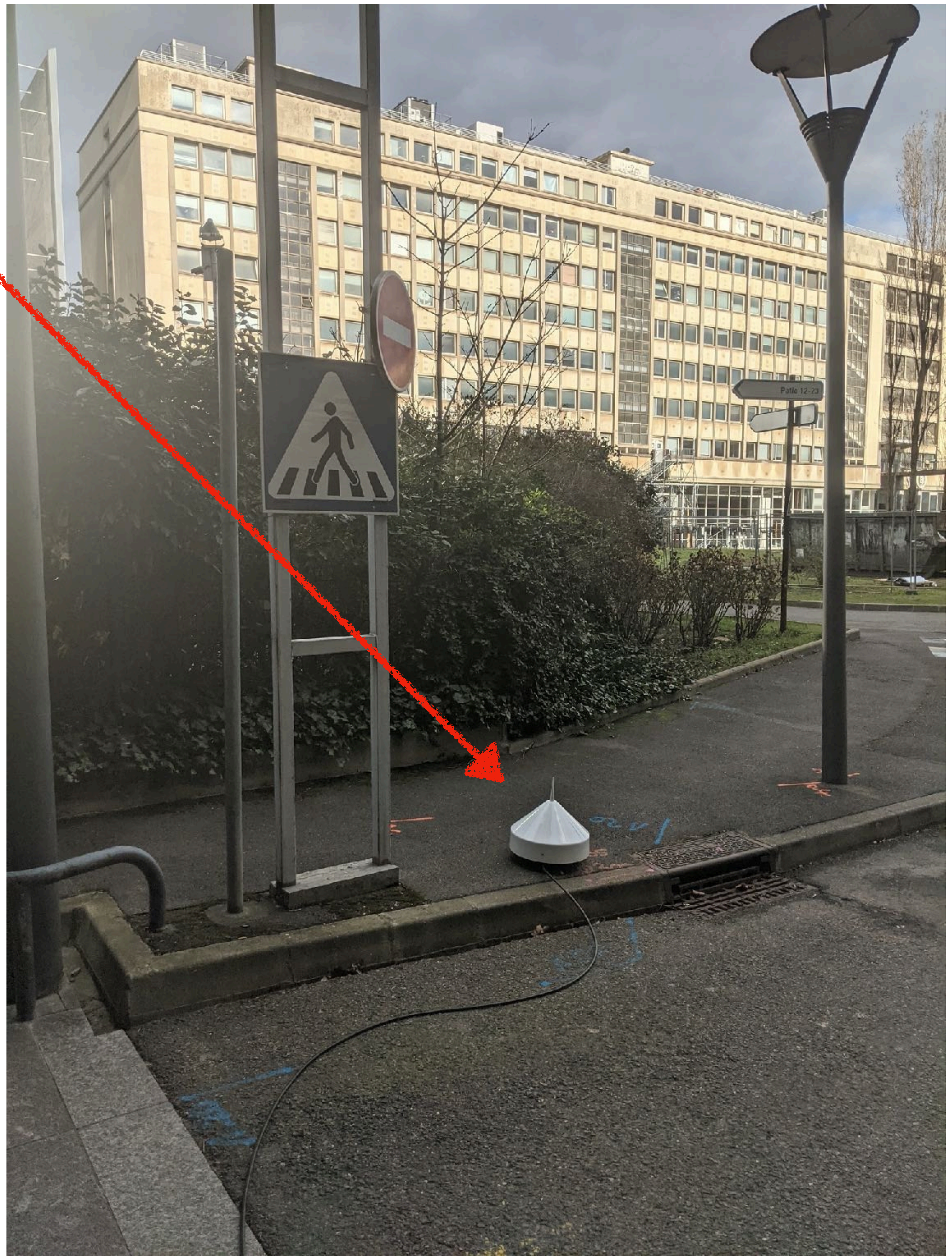
First test of GNSS antenna

Computer
connected by Wifi

Septentrio module

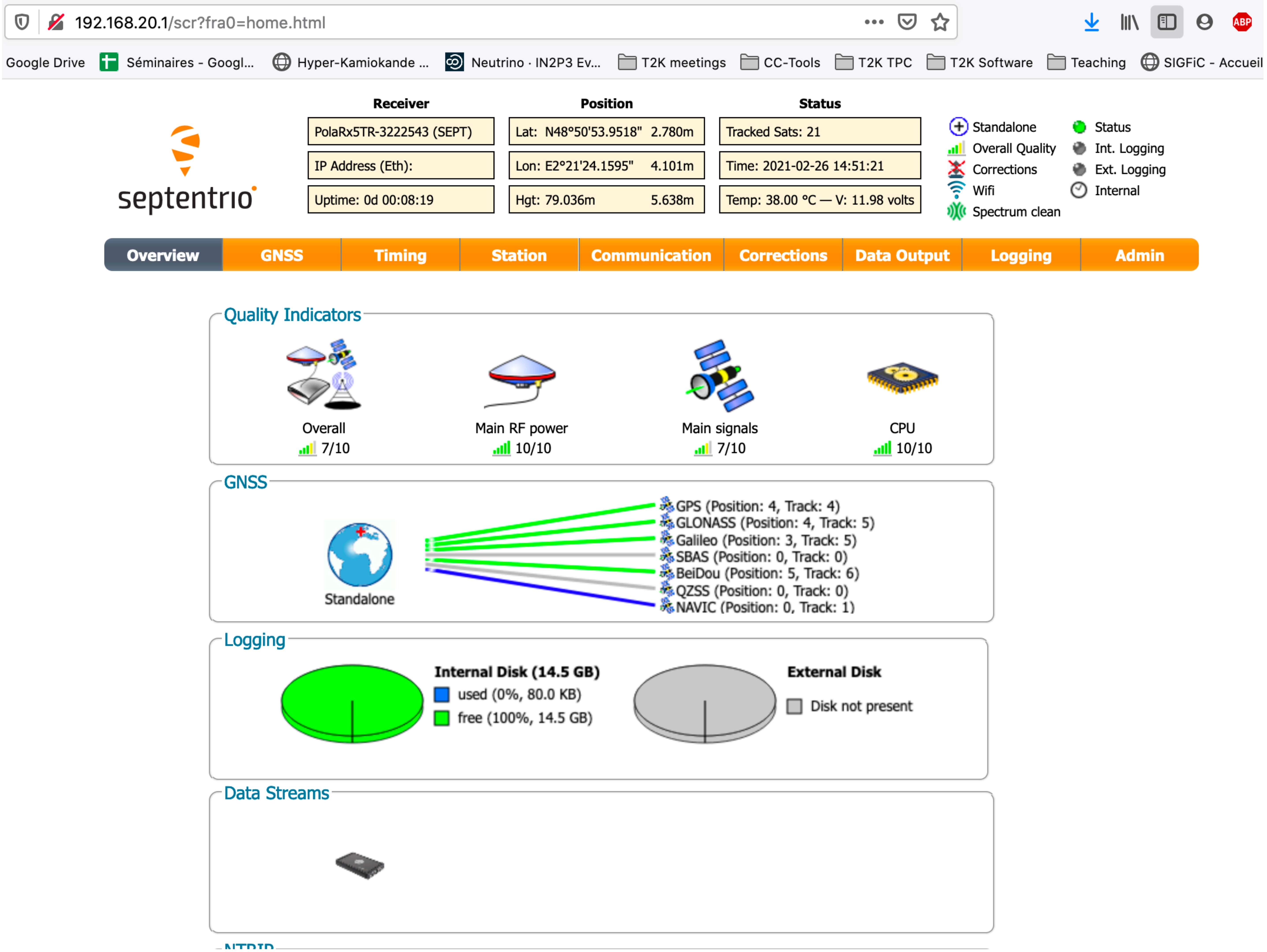
Antenna

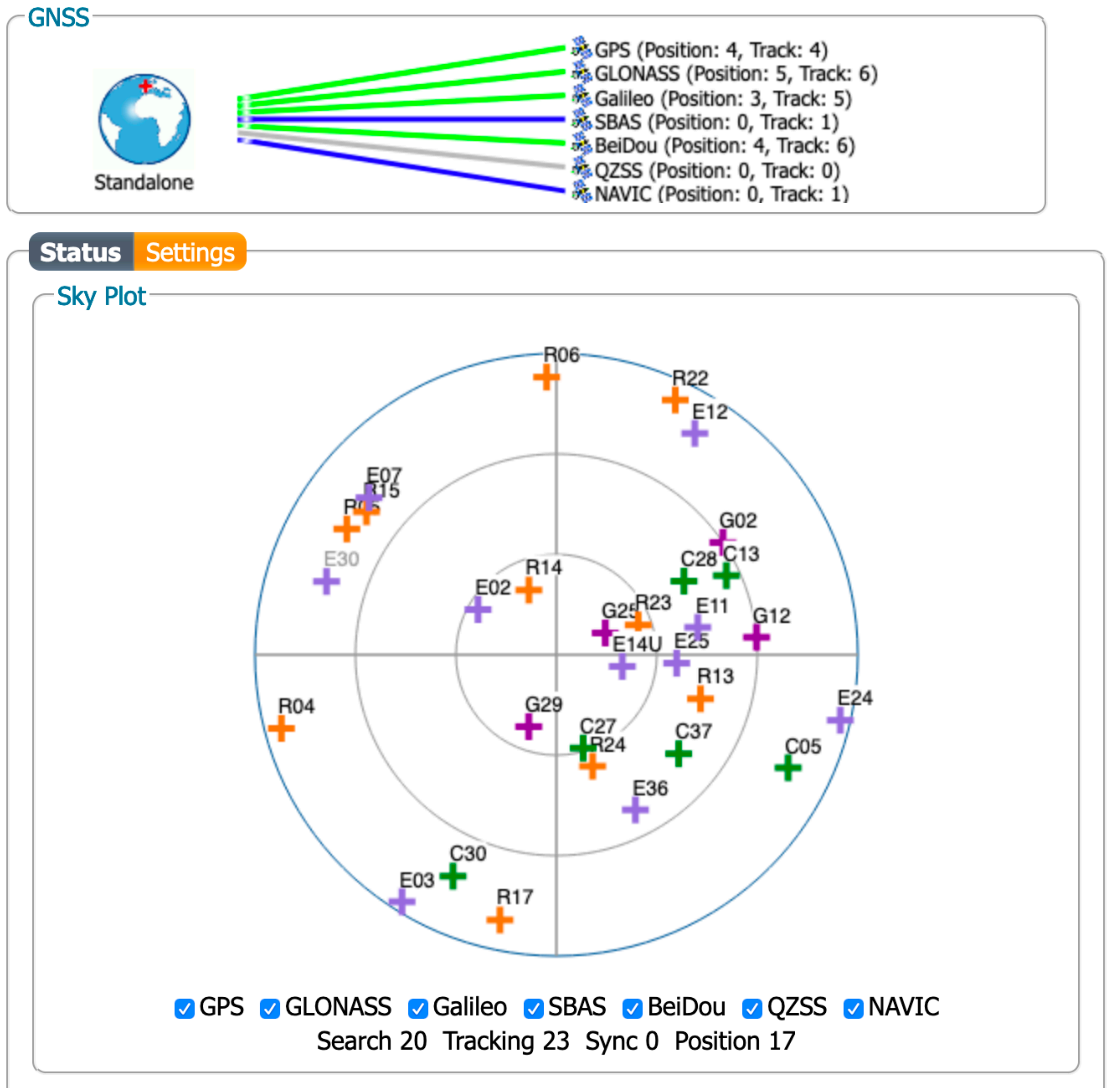




Friday February 26th

First results of GNSS antenna





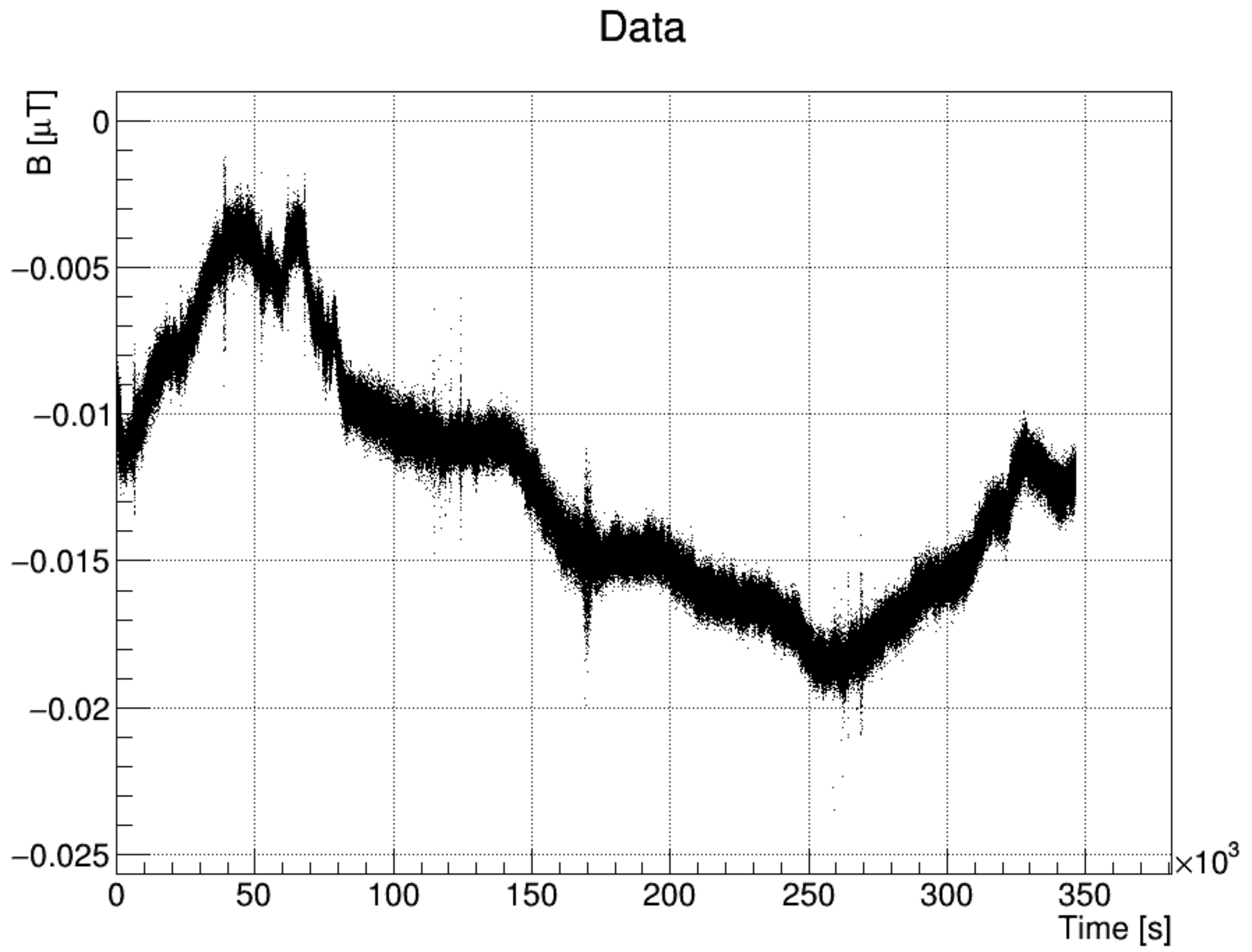
It works (but only outside…)!

Biblio

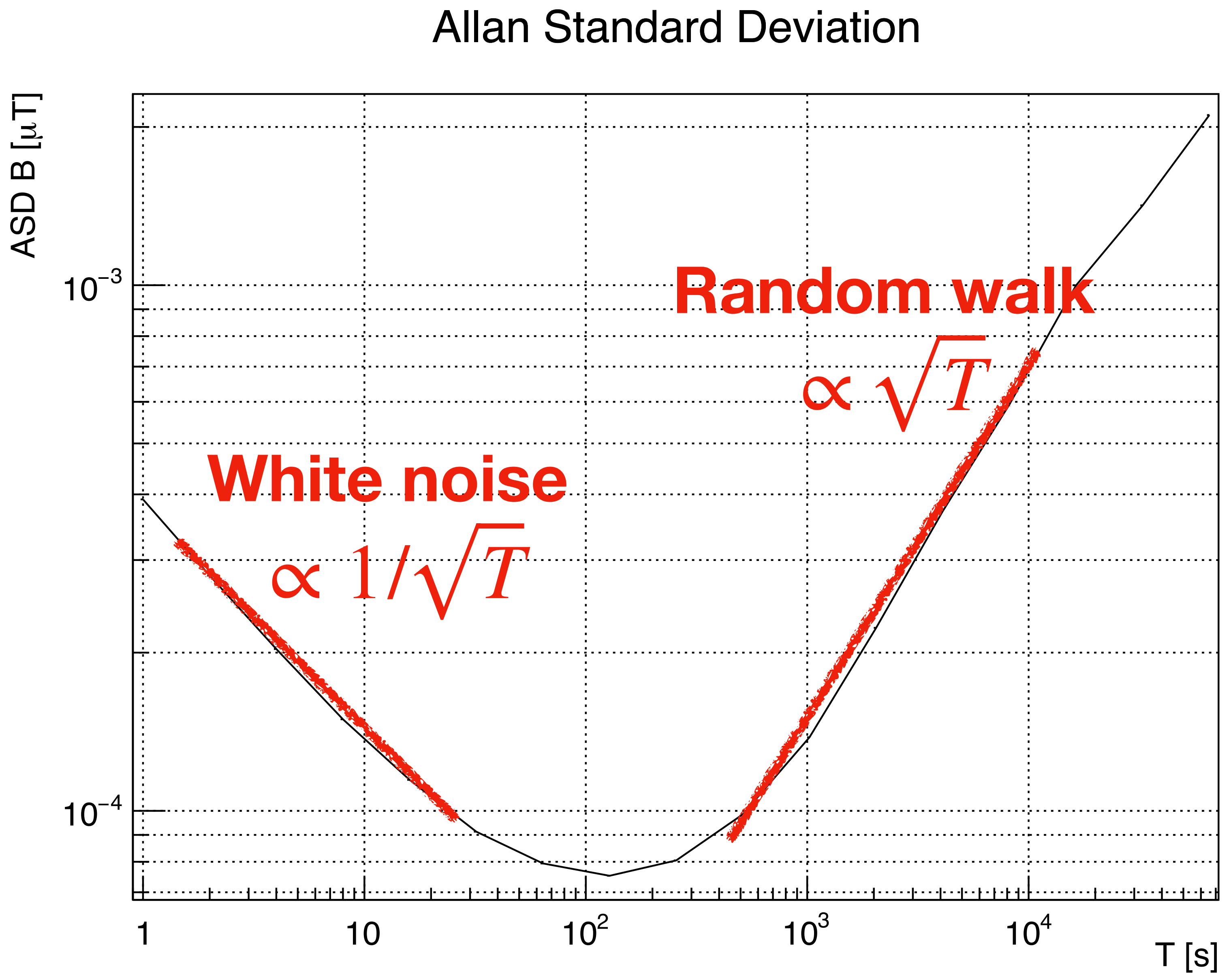
(1) *Statistics of Atomic Frequency Standards*, D. W. Allan, Proceedings of the IEEE, 54 **2** 221-230, Feb. 1966, doi: 10.1109/PROC.1966.4634.
(2) *Characterization of frequency stability*, J. A. Barnes, A. R. Chi, et al., IEEE Trans. Instrum. Meas. IM-20 **2** 105 (1971).
(3) Long-range time transfer with optical fiber links and cross comparisons with satellite-based methods, N. Kaur, PhD thesis 2019.
(4) T4Science pH Maser 1008 Specifications sheet
(5) *Handbook of Frequency Stability Analysis*, W.J. Riley, here
(6) *A Review of Contemporary Atomic Frequency Standards*, B.L. Schmittberger, D.R. Scherer, arXiv:2004.09987
(7) cRb-Clock Preliminary Data sheet, SpectraDynamics, here
(8) *Time and Frequency Measurements Using the Global Positioning System*, M.A. Lombardi, L.M. Nelson, A.N. Novick, V.S. Zhang Cal Lab 8. 26-33
(9) *Direct comparisons of European primary and secondary frequency standards via satellite techniques*, F. Riedel et al 2020 Metrologia **57** 045005

Backup

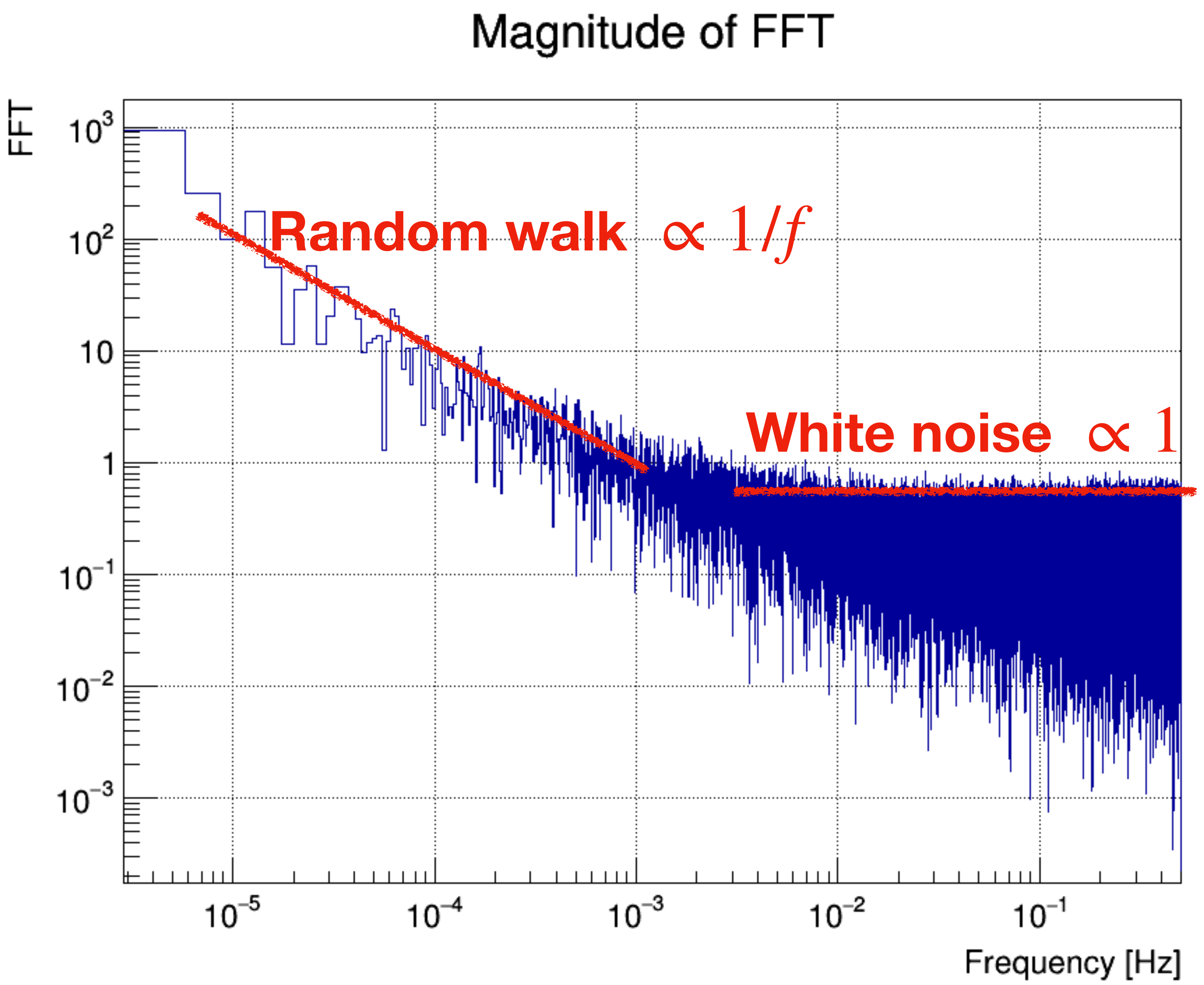
Visualizing things



Simpler representation less sensible to stats.



More structures visibles e.g. peaks = periodic signals



Some pieces of code: https://gitlab.in2p3.fr/hk/clocks/asd

Relation between PSD and AV

Similarly to the usual variance is related to the Discrete Fourier transform (via Parseval-Plancherel theorem), we have:

$$\sigma_{\text{Allan}}(n) = \frac{2}{\pi n \tau} \int_0^{\infty} \mathrm{d}u S_y\left(\frac{u}{\pi n \tau}\right) \frac{\sin^4 u}{u^2} \quad (5)$$

(Averaging over all noise frequency scales gives time stability)

Why phase noise instead of frequency noise?

For comparing two clocks (especially digital clocks), easier to look at phase differences defined by the up-front (or down-front) instead of looking at the instantaneous period/frequency of the signal.

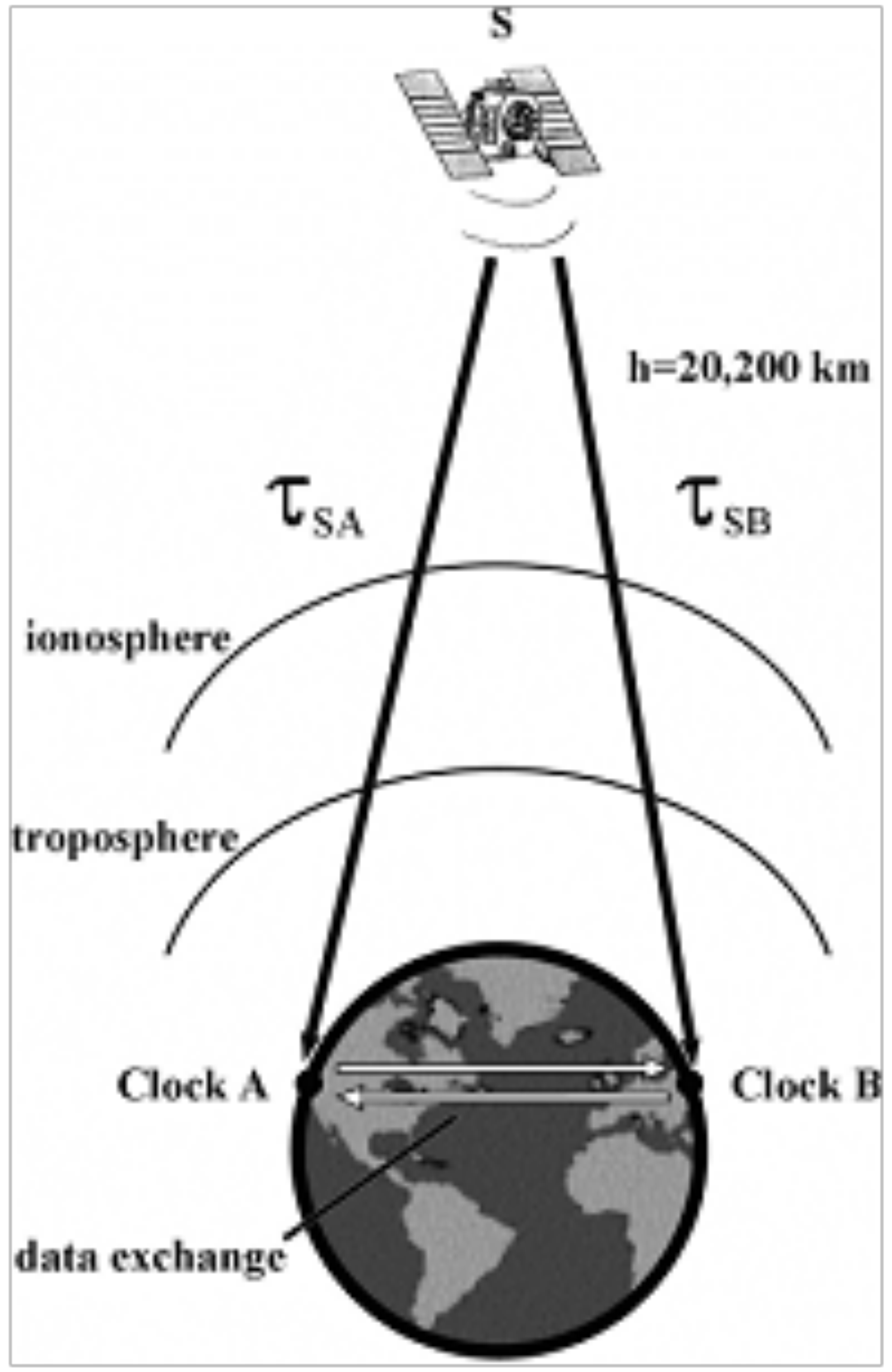
Relations between frequency and phase:

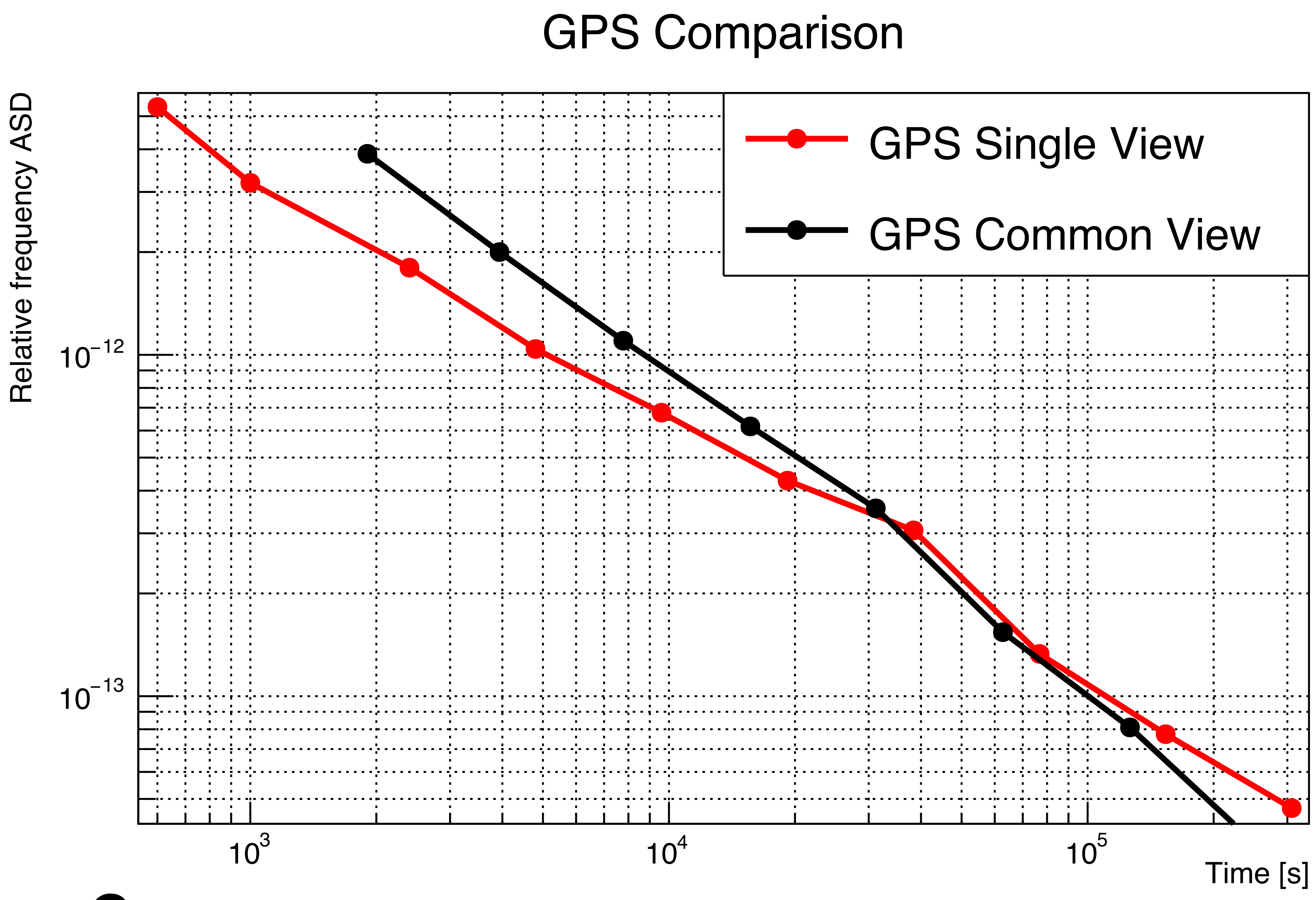
$$S_{\dot{\phi}}(\omega) = \omega^2 S_{\phi}(\omega)$$

$$\sigma_{\phi}^2(\tau) = \left(\frac{\tau^2}{3}\right) \cdot \mathrm{Mod}\,\sigma_{\dot{\phi}}^2(\tau) \quad (5)$$

GPS frequency stability and time uncertainty

Technique	Timing Uncertainty, 24 h, 2σ	Frequency Uncertainty, 24 h, 2σ
One-Way	< 20 ns	$< 2 \times 10^{-13}$
Common View	≈ 10 ns	$\approx 1 \times 10^{-13}$
Melting pot	< 5 ns	$< 5 \times 10^{-14}$
Carrier-Phase	< 500 ps	$< 5 \times 10^{-15}$





(8)

Not quite $1/\sqrt{T}$ dependence…?

Old paper: how about now? Galileo?

White Rabbit architecture

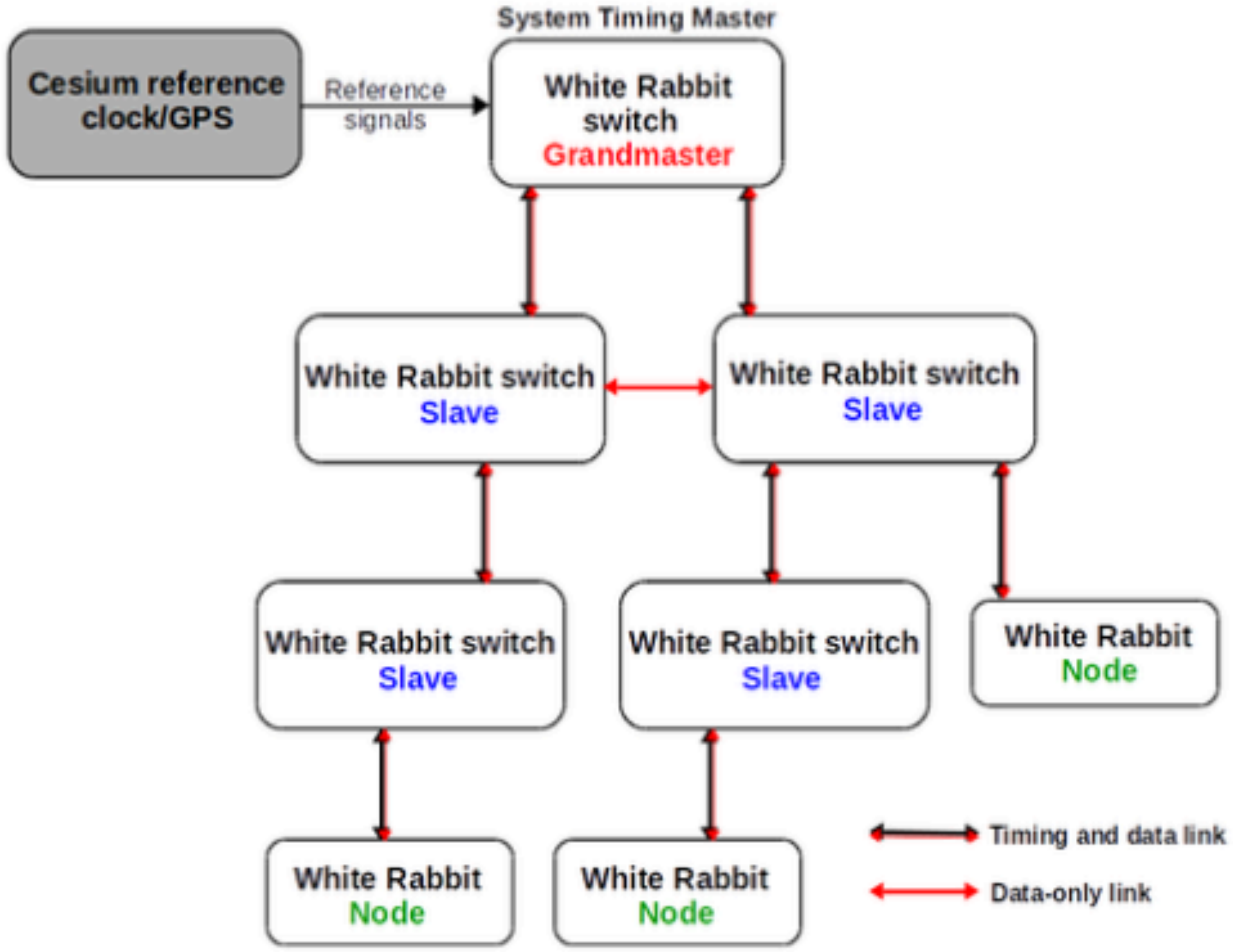


FIGURE 2.5: A typical White Rabbit Network [42]. (3)