GIF School 2021

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Sondes cosmologiques de la gravité 1

-Introduction to growth of matter fluctuations

It allows to test gravitation on cosmic scales (~ 10-100 h-1Mpc)

-Statistics of the matter field

- Two point correlation function (link the variance and PdF); implication of statistical invariance by translation (Cosmological Principle)
- Fourier basis
  Cartesian
  Spherical
- Implications of the CP on the two Fourier basis

→ $P(k)$ 3D power spectrum

→ C_ℓ angular power spectrum (or cross)

-Link to galaxy density (bias)

-Redshift space distortions → $f\sigma_8$

Sondes cosmologiques de la gravité 2

-Inference of cosmological parameters

- Bayes Theorem
- Likelihood analysis
- Definition of chi2 and related hypothesis

-Covariance matrix

- Theoretical calculation for Gaussian fields
- Theoretical calculation for Non-Gaussian fields
- Estimation of the precision matrix

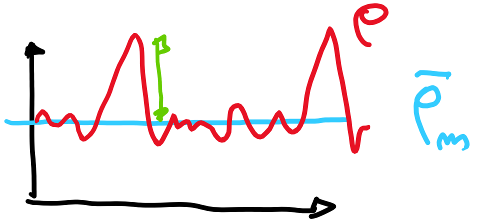
-Cross-combination with other probes

- Galaxy lensing
- 2Dx3D clustering

Introduction to matter fluctuations:

Perturbation

Describe matter as a fluid with density $\rho_m = \bar{\rho}_m (1 + \delta)$



$\delta\rho_m = \bar{\rho}_m \delta$

Simple view ⇒
- continuity equation
- Equation of motion (Newton Dynamics)
- Poisson equation for gravity

Comoving coordinate system: $\vec{r} = a\vec{x}$ a: scale factor

Note: good approximation in the Non relativistic regime:
- Scale much smaller than the Hubble Horizon
- Small velocity
- Matter domination

Continuity equation (Mass conservation) : $\frac{\partial \rho}{\partial t} + \vec{\nabla}_r \cdot (\rho \vec{V}) = 0$

Equation of motion : $\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}_r)\vec{V} = -\vec{\nabla}_r \varphi - \frac{\vec{\nabla}_r P}{\rho}$

Poisson equ. : $\Delta_r \varphi = 4\pi G \rho$

⇒ Equations for perturbations from the background FLRW :

δP. δ ; v ; ϕ ⇒

$\vec{V} = H\vec{r} + \underbrace{a\vec{v}}_{\vec{v}_p}$

$P = \bar{P} + \delta P$

$\varphi = \bar{\Phi} + \phi$

comoving coordinate and cosmic time t

$$\left\{ \begin{aligned} &\frac{\partial \delta}{\partial t} + \overset{\text{comoving}}{\vec{\nabla}} \cdot \left[(1+\delta)\vec{v}\right] = 0 \\ &\frac{\partial \vec{v}}{\partial t} + 2H\vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\vec{\nabla}\phi - \frac{\vec{\nabla}\delta P}{\rho} \\ &\Delta \phi = 4\pi G \bar{\rho} a^2 \delta \end{aligned} \right.$$

⇒ Linearize in

$$\frac{\partial^2\delta}{\partial t^2} + \boxed{2H\frac{\partial\delta}{\partial t}} - \underbrace{\frac{3}{2}\Omega_m H^2\delta} = 0$$

when $\delta P = 0$
No velocity dispersion
(CDM or Baryons)

the expansion rate of the universe decelerate the growth

matter accelerate the growth of structure

$\searrow\rightarrow$ Balance between the two

growth rate of structure $f \equiv \frac{d\ln\delta}{d\ln a}$

$$\frac{\partial}{\partial t} = H\frac{\partial}{\partial\ln a}$$

$$\dot{H} = H H'$$

$$\frac{\partial^2}{\partial t^2} = \dot{H}\frac{\partial}{\partial\ln a} + H^2\frac{\partial^2}{\partial\ln^2 a}$$

$$\frac{d\delta}{d\ln a} = f\,\delta \Rightarrow \frac{d^2\delta}{d\ln^2 a} = f'\delta + f^2\delta$$

$$\Rightarrow \cancel{H}\frac{H'}{H} f\cancel{\delta} + \cancel{H}^2\left(f^2\cancel{\delta} + f'\right)\cancel{\delta} + 2\cancel{H}^2 f\cancel{\delta} - \frac{3}{2}\Omega_m\cancel{H}^2\cancel{\delta} = 0$$

$$\boxed{f' + f^2 + \left(1 + \frac{H'}{H} + \nu\right) f - \frac{3}{2} g_{eff}\, \Omega_m = 0}$$

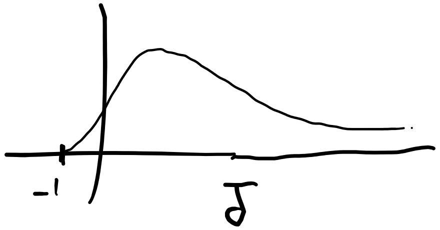
⇒ Test gravity on cosmic scales

Statistics of the matter field:

we cannot follow the evolution of a single perturbation

⇒ Need for a statistical approach

δ is a stochastic field $\delta(\vec{x})$ defined by its probability density function (PdF) $P(\delta)\,d\delta$



* Defined by its moments $\langle \delta^m \rangle$

* $P^{(N)}(\delta_1; \delta_2; \delta_3; \dots; \delta_N)\, d\delta_1 \cdots d\delta_N$ N-point PdF

* $P^{(2)}(\delta_1; \delta_2)$ depends on $\langle \delta_1 \rangle; \langle \delta_2 \rangle; \langle \delta_1^2 \rangle; \langle \delta_2^2 \rangle; \langle \delta_1 \delta_2 \rangle$

By definition $\langle \delta_1 \rangle = \langle \delta_2 \rangle = 0$ so only $\boxed{\langle \delta_1^2 \rangle = \langle \delta_2^2 \rangle}$

Cosmological principles : Translational invariance

2-Point correlation function : $\langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle$

$$\Rightarrow \boxed{\langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle = \xi(\vec{r})}$$

$\propto D^2(a) \to$ growth factor

– Information the amplitude and shape

Fourier Basis → $\delta_{\vec{k}} = \frac{1}{(2\pi)^3} \int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}\, d^3\vec{x}$

$$\Rightarrow \langle \delta_{\vec{k}}\, \delta_{\vec{k}'} \rangle = \frac{1}{(2\pi)^6} \int \underbrace{\langle \delta(\vec{x})\, \delta(\vec{x}') \rangle}_{\xi(\vec{r})} e^{-i\vec{k}\cdot\vec{x} - i\vec{k}'\cdot\vec{x}'}\, d^3\vec{x}\, d^3\vec{x}'$$

$\to \vec{x}' = \vec{x} + \vec{r}$

$d^3\vec{x}' = d^3\vec{x}$

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'} \rangle = \frac{1}{(2\pi)^3} \int e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}} \underbrace{\frac{1}{(2\pi)^3}\int \xi(\vec{r})\, e^{-i\vec{k}'\cdot\vec{r}}\, d^3\vec{r}}_{\equiv\, P(\vec{k}')}\; d^3\vec{x}$$

→ Power spectrum

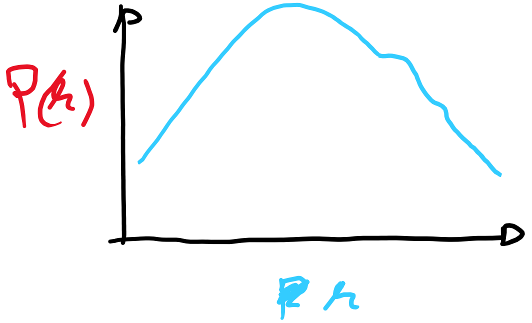
$$= \delta^{D}(\vec{k}+\vec{k}')\, P(k)$$

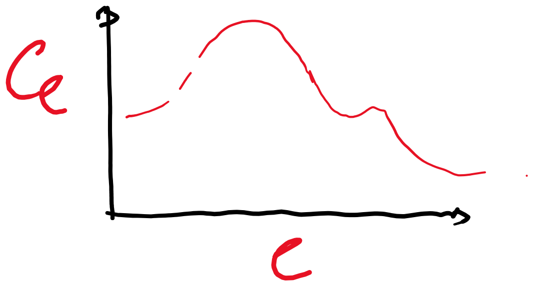
$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle = \delta^{D}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; B(\vec{k}_1, \vec{k}_2)$$ → Bispectrum

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle_c = \delta^{D}(\vec{k}_{1234})\; T(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$ – Trispectrum

Harmonic space $\delta_{\ell m}(r) \equiv \int \delta(\vec{r})\, Y_\ell^m(\theta,\phi)\, d^2\Omega$

$$\langle \delta_{\ell m}(r)\, \delta_{\ell' m'}(r') \rangle = \delta^K_{\ell\ell'}\, \delta^K_{mm'}\; C_\ell(r, r')$$





Link to galaxy density (bias):

galaxy density $\delta_g \neq$ matter density δ

There exists a mapping between the two $\delta_g = \mathcal{M}(\delta, \dots)$

Linear $\rightarrow$ $\boxed{\delta_g = b\,\delta}$ $+ b_2 \frac{\delta^2}{2} + \gamma_2\, g + \dots$

$\hookrightarrow$ tidal field

As a result
$$\begin{cases} P_g(k) = b^2\, P(k) \\ \xi_g(r) = b^2\, \xi(r) \end{cases}$$

Bias adds nuisance parameters on top of cosmological ones.

Redshift space distortions:

galaxy redshifts are distorted by Doppler effect and relativistic effects

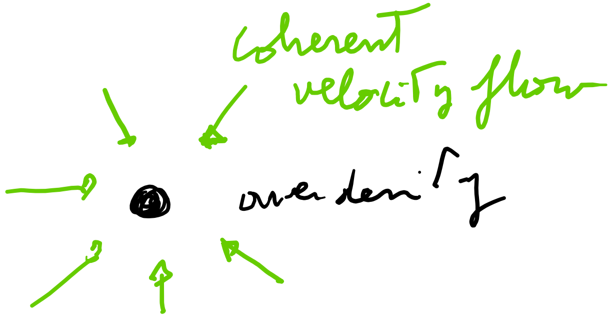
$$z_0 \simeq z + \underbrace{\frac{\vec{v}_p \cdot \vec{e}_r}{c}(1+z)}_{\Delta z} \quad ; \quad \Delta z \frac{c}{H} = u = -\frac{v_p}{aH}$$

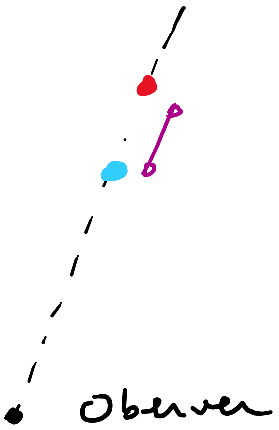
$\vec{e}_r$: line-of-sight

$$\Delta z = -\frac{v_p}{c}(1+z)$$

observed comoving position → $s = r - u$ ← comoving displacement

↳ True comoving position





Continuity equation:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v}_p = 0$$

$$f\delta - i\frac{\vec{k}\cdot\vec{v}_{p,k}}{aH} = 0$$

Potential $\theta_k \equiv \vec{\nabla}\cdot\vec{v}_p$

$$\boxed{\theta = -aHf\delta}$$

Divergence of velocity field coupled to density.

* Galaxies follow dark matter velocity field.

$$\tilde{P}(\vec{k}) = \left(b^2 + 2f\mu_k^2 b + f^2\mu_k^4\right)P(k), \quad \mu_k \equiv \frac{\vec{k}\cdot\vec{e}_r}{k}$$

* Normalisation of $P(k)$

$$\sigma_8^2(z) = \int P(k)\, W^2(kR)\, d^3\vec{k}$$

Be $\hat{P} \equiv \dfrac{P(k)}{\sigma_8^2}$

$$\boxed{\tilde{P}(\vec{k}) = \left\{ \underline{b^2\sigma_8^2} + 2\, b\sigma_8 \mu_k^2\, f\sigma_8 + (f\underline{\sigma_8})^2 \mu_k^4 \right\} \hat{P}(k)}$$

$b\sigma_8$ → Nuisance

$f\sigma_8$ → growth of structure

or velocity growth factor

multipole expansion:

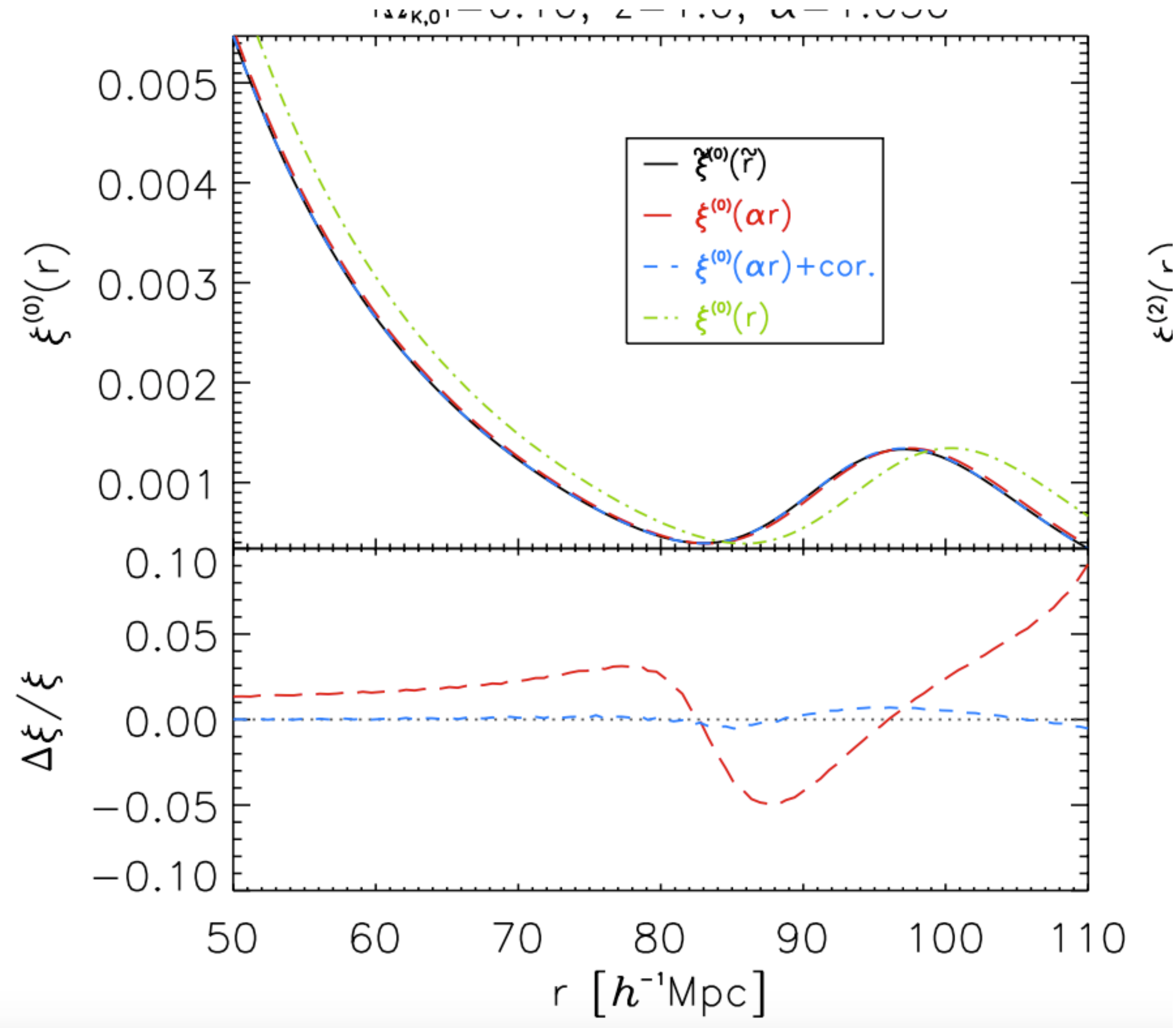
$$\tilde{P}^{(m)}(k) \equiv \frac{2}{2m+1} \int_{-1}^{1} \tilde{P}(\vec{k})\, L_m(\mu_k)\, d\mu_k$$

$$\tilde{\xi}^{(n)}(\tilde{r}) = \frac{2n+1}{2} \int_{-1}^{1} \tilde{\xi}(\tilde{r}, \tilde{\mu}) L_n(\tilde{\mu}) \mathrm{d}\tilde{\mu}.$$

AP correction:

$$\left\{ \begin{array}{rcl} r_{\|} E(z) & = & \tilde{r}_{\|} \tilde{E}(z) \\ \frac{r_{\perp}}{D_A(z)} & = & \frac{\tilde{r}_{\perp}}{\tilde{D}_A(z)}. \end{array} \right.$$

$$\alpha_{\|} = \frac{\tilde{E}(z)}{E(z)} \quad \alpha_{\perp} = \frac{D_A(z)}{\tilde{D}_A(z)}.$$



$\tilde{\xi}^{(0)}(\tilde{r})$
$\xi^{(0)}(\alpha r)$
$\xi^{(0)}(\alpha r)$+cor.
$\xi^{(0)}(r)$
$\xi^{(0)}(r)$
0.005
0.004
0.003
0.002
0.001
$\Delta\xi/\xi$
0.10
0.05
0.00
−0.05
−0.10
50
60
70
80
90
100
110
r $[h^{-1}$Mpc]

Inference of cosmological parameters:

Be $\vec{\theta}$ a vector containing parameters of a model

Be $\vec{x}$ an observable

$L(\vec{x} \mid \vec{\theta})$ likelihood

$P(\vec{\theta} \mid \vec{x})$? posterior

$$P(\vec{\theta} \mid \vec{x}) = \frac{L(\vec{x} \mid \vec{\theta})\, P(\vec{\theta})}{P(\vec{x})}$$

→ prior

Usual hypothesis: The likelihood is Gaussian

$$L(\vec{x}\,|\,\vec{\theta}) = \frac{1}{(2\pi)^{\frac{k}{2}}\sqrt{|G|}}\, \exp\left\{-\frac{1}{2}(\vec{x}-\vec{\mu})^T G^{-1}(\vec{x}-\vec{\mu})\right\}$$

where $\vec{\mu} = \vec{\mu}(\vec{\theta})$ is the expectation value given the model

G is the covariance matrix $(k \times k)$

minimisation of

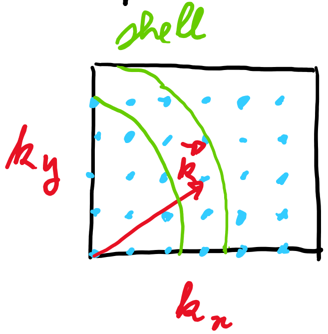
$$-2\ln\left[P(\vec{\theta}\,|\,\vec{x})\right] = \underbrace{\left[\vec{x}-\vec{\mu}(\vec{\theta})\right]^T G^{-1}\left[\vec{x}-\vec{\mu}(\vec{\theta})\right]}_{\equiv\ \chi^2} + \ln|G| - 2\ln P(\theta) + A$$

precision matrix

Covariance matrix:

* Define an estimator for the observable (Assume ergodicity)

\+ compute its covariance matrix



$$\hat{P}(k) = \frac{k_F^3}{N_k}\sum |\delta_{\vec{k}_m}|^2$$

$$k_F = \frac{2\pi}{L}$$ fundamental mode

$$\langle \hat{P}(k_i)\hat{P}(k_j)\rangle = \frac{k_F^6}{N_{k_i}N_{k_j}}\sum_{n,m}\overbrace{\langle \delta_{\vec{k}_n}\delta_{-\vec{k}_n}\delta_{\vec{k}_m}\delta_{-\vec{k}_m}\rangle}^{\text{4-Point moment}}$$

$$\delta^{K}_{n,m}\frac{P(k_n)}{k_F^3}$$

$$\langle\delta_{k_n}\delta_{-k_n}\rangle\langle\delta_{k_m}\delta_{-k_m}\rangle + \langle\delta_{k_n}\delta_{k_m}\rangle\langle\delta_{-k_n}\delta_{-k_m}\rangle + \langle\delta_{k_n}\delta_{-k_m}\rangle\langle\delta_{-k_n}\delta_{k_m}\rangle$$

$$+ \langle\delta_{k_n}\delta_{-k_n}\delta_{k_m}\delta_{-k_m}\rangle_c$$

$$= \langle\hat{P}(k_i)\rangle\langle\hat{P}(k_j)\rangle + \underbrace{2\,\frac{\delta^K_{ij}}{N_{k_i}N_{k_j}}P^2(k_i)\,N_{k_i} + \frac{k_F^6}{N_{k_i}N_{k_j}}\sum_{n,m}\langle\quad\rangle_c}_{C[\hat{P}(k_i),\hat{P}(k_j)]}$$

gaussian

Non gaussian

$$\langle \delta_{k_m} \delta_{-k_m} \delta_{k_m} \delta_{-k_m} \rangle_c = \frac{1}{k_F^3} T(k_m, -k_m, k_m) \; ; \; \bar{T}(k_i, k_j) \equiv \int_{k_i} \int_{k_j} T(\quad) \frac{d^3\vec{k}_m}{V_{k_i}} \frac{d^3 k_m}{V_{k_j}}$$

Cross combination with other probes:

-angular galaxy clustering

$$\delta_{lm}(r) = \int_S \delta(r, \theta, \phi) Y^*_{l,m}(\theta, \phi) \mathrm{d}^2\Omega,$$

$$\delta_{lm}(r) = 4\pi i^l \int \delta_k Y^*_{l,m}(\theta_k, \phi_k) j_l(kr) \mathrm{d}^3\vec{k},$$

$$C_l(z, z') = (4\pi)^2 \int_0^\infty k^2 P(k) j_l(kr) j_l(kr') \mathrm{d}k,$$

Define estimators:

$$\hat{C}_l(z_1, z_2) = \frac{1}{2l+1} \sum_{m=-l}^{l} \tilde{\delta}(z_1)\tilde{\delta}^*(z_2).$$

$$\hat{P}^{(n)}(k) = \frac{k_F^3}{N_k}(2n+1) \sum_{i=1}^{N_k} |\delta_{\vec{k}_i}|^2 \mathcal{L}_n(\mu_{\vec{k}_i}),$$

Compute the expectation value of their product:

$$\langle \hat{C}_l \hat{P}^{(n)} \rangle$$

-Galaxy-galaxy lensing:

SIMULATIONS ARE NEEDED

The example of the cosmic microwave background (CMB):

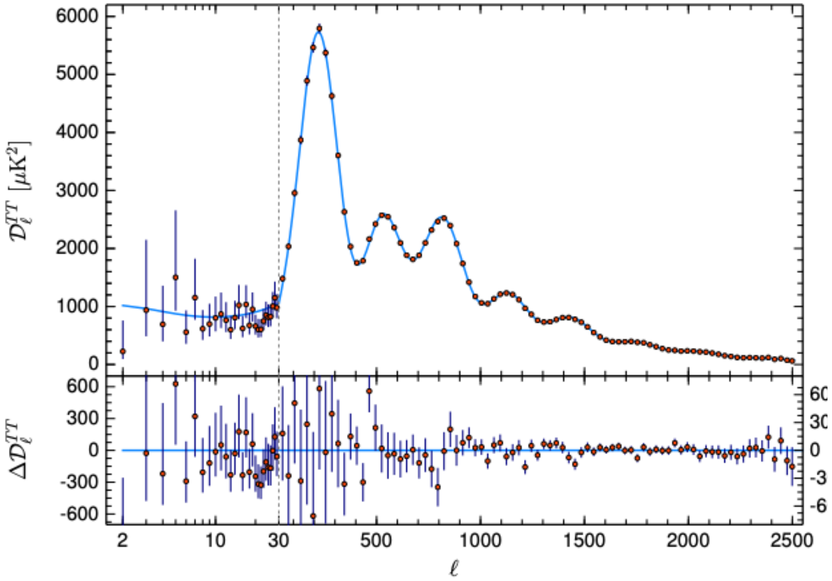
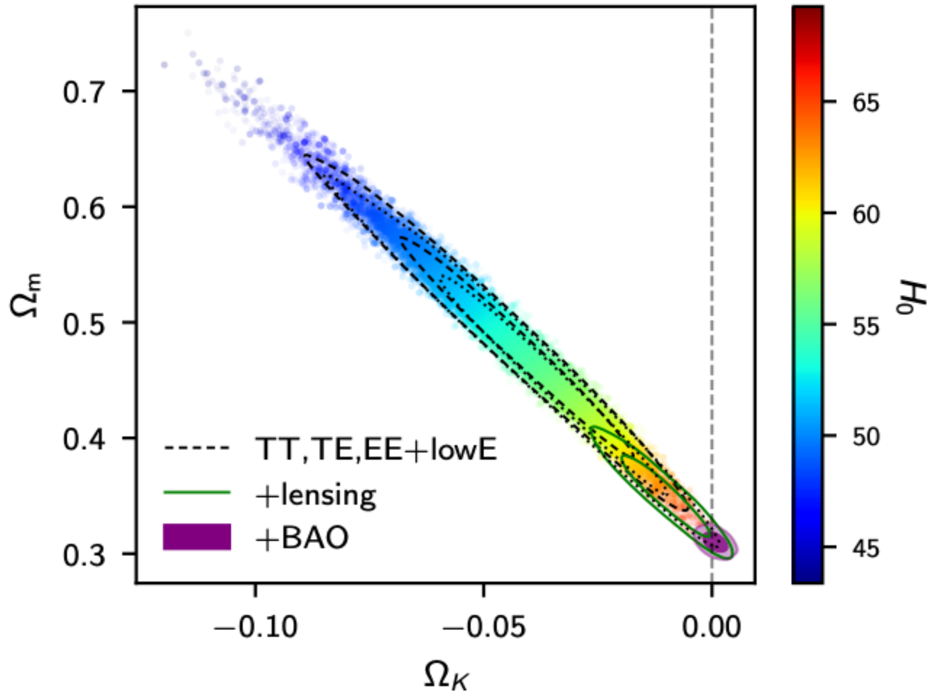


Fig. 1. *Planck* 2018 temperature power spectrum. At multipoles $\ell \geq 30$ we show the frequency-coadded temperature computed from the `Plik` cross-half-mission likelihood, with foreground and other nuisance parameters fixed to a best fit the base-ΛCDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the Co component-separation algorithm, computed over 86 % of the sky. The base-ΛCDM theoretical spectrum best fit to th TT,TE,EE+lowE+lensing likelihoods is plotted in light blue in the upper panel. Residuals with respect to this model are the lower panel. The error bars show ±1 σ diagonal uncertainties, including cosmic variance (approximated as Gaussian, including uncertainties in the foreground model at $\ell \geq 30$. Note that the vertical scale changes at $\ell = 30$, where the horizontal axis switches from logarithmic to linear.



Planck (2018)

Problem: Combination of LSS probes

e.g. galaxy-galaxy lensing and redshift space galaxy clustering

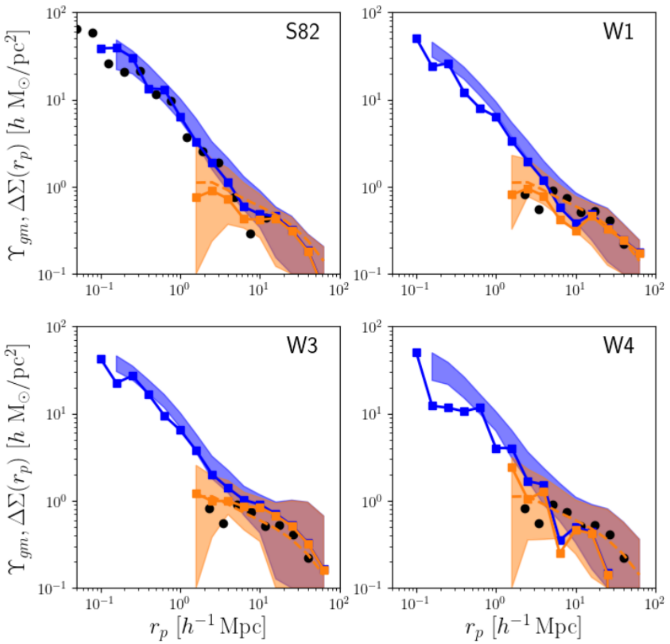


Fig. 9. Filtered $\Upsilon_{\rm gm}$ and non-filtered $\Delta\Sigma$ GGL measurements with mocks (shaded regions), $\Delta\Sigma$ and Υ data (blue and cyan points respectively), and theory with a linear bias parameter $b_1 = 1.8$ (dashed line). Black dots in S82 panel represent $\Delta\Sigma$ measurements from L16, and $\Upsilon_{\rm gm}$ measurements from Alam et al. (2016) in CFHTLS panels.

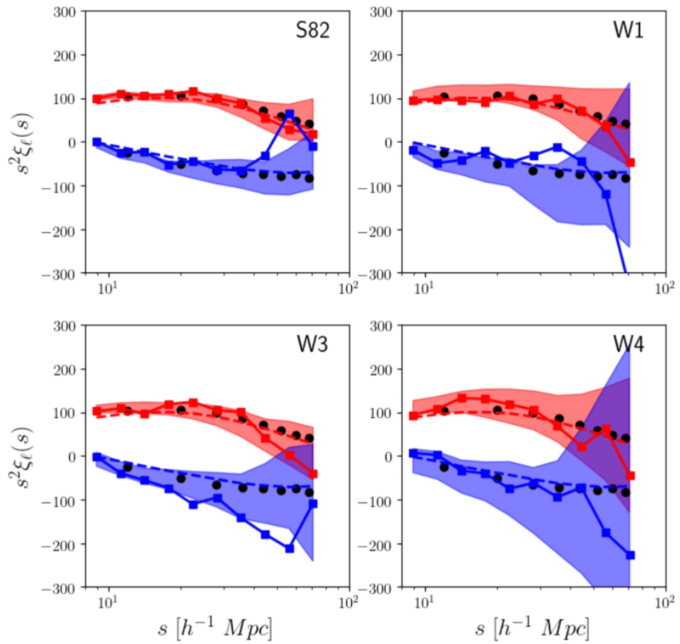


Fig. 8. Monopole (red) and quadrupole (blue) measurements with mock catalogs (shaded region), real data (solid lines) and theoretical predictions with a linear bias parameter $b_1 = 1.8$ (dashed lines). Black dots represent pre-reconstruction measurements with the full DR12v5 CMASS sample from Cuesta et al. (2016).

Jullo et al. (2019)