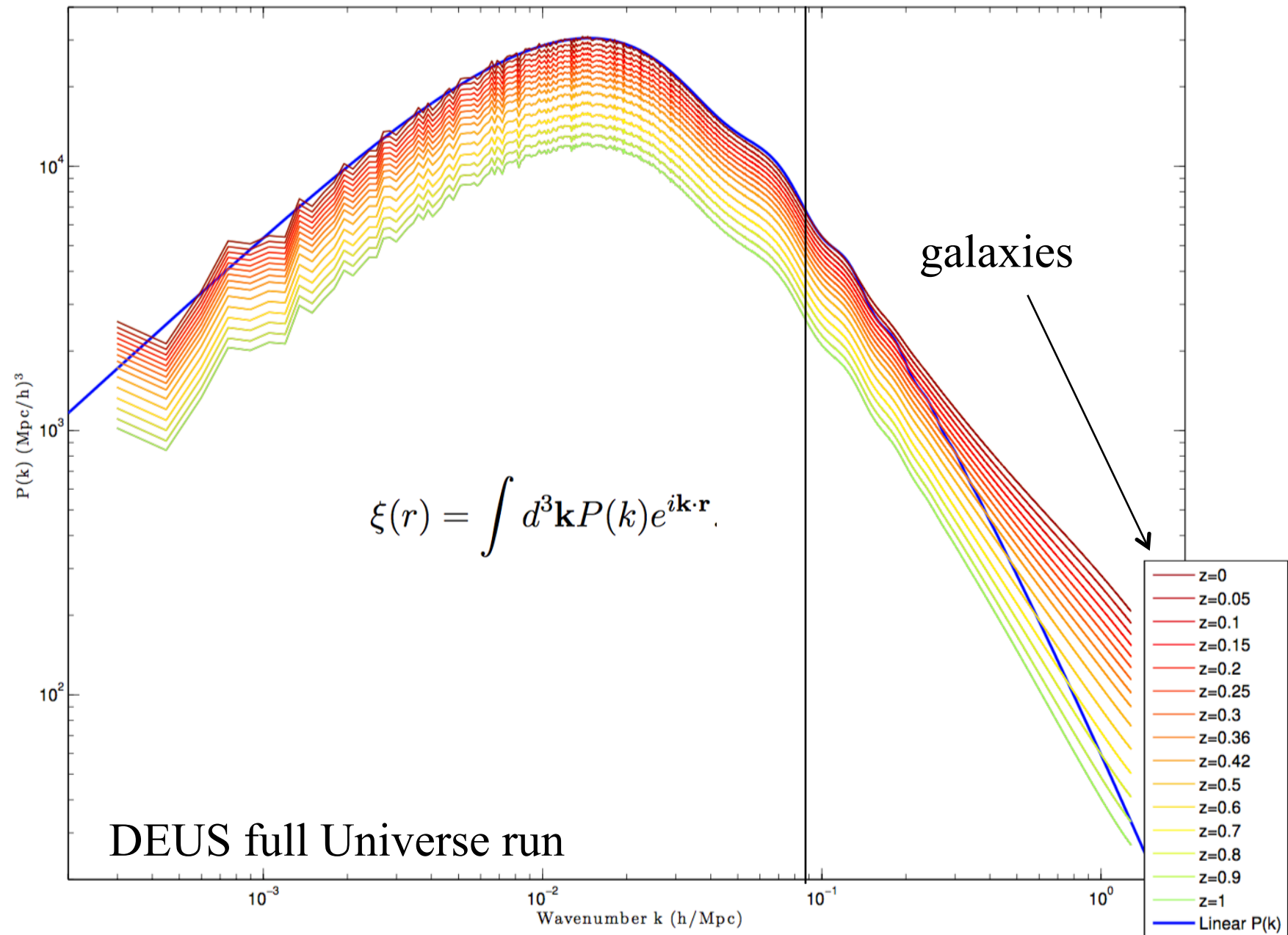




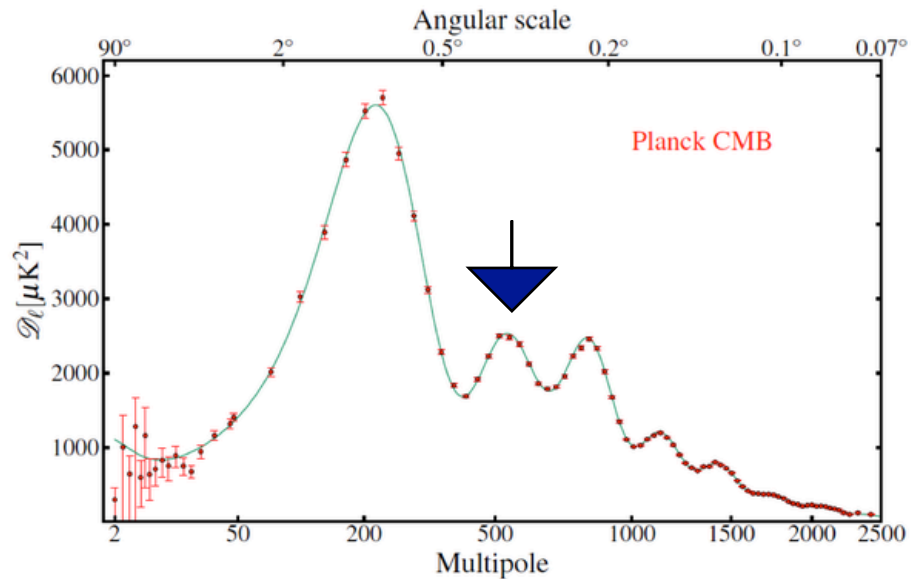
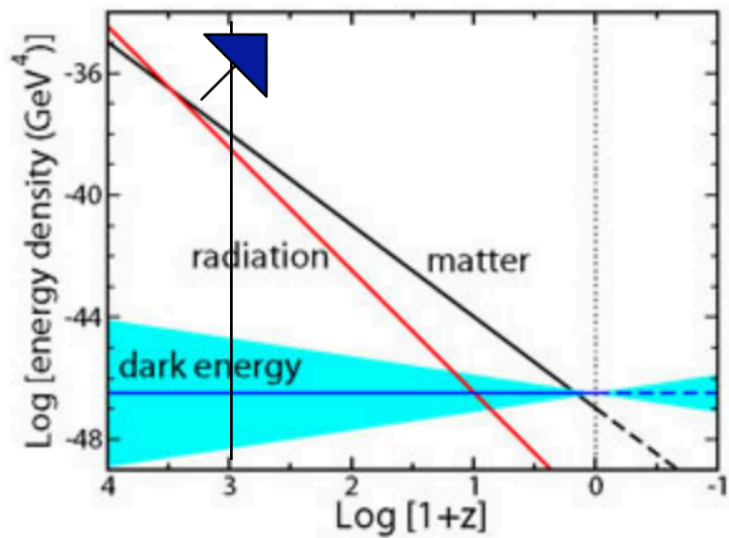
Cold dark matter and beyond: galaxy scales

Benoit Famaey

CNRS - Observatoire astronomique de Strasbourg

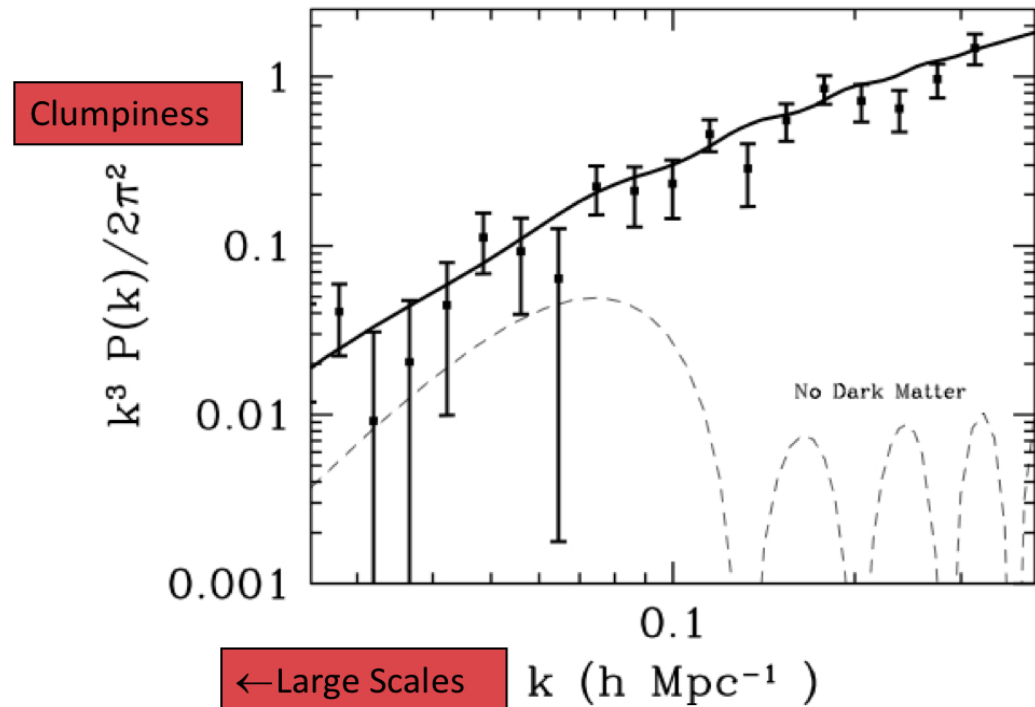


The need for Dark Matter



- CMB + other large scale probes => concordance Λ CDM model

The need for Dark Matter

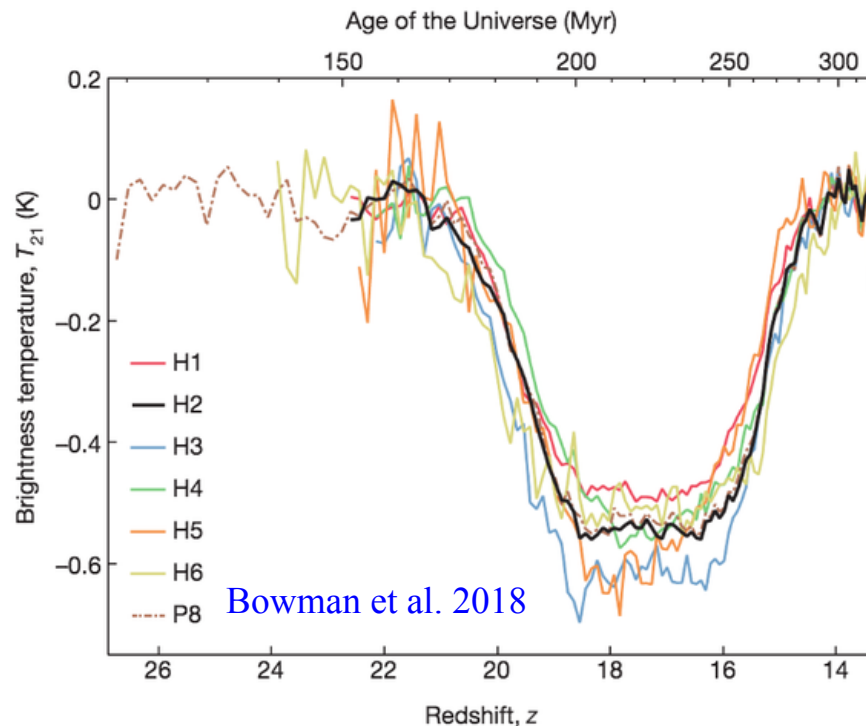


$$\xi(r) = \int d^3\mathbf{k} P(k) e^{i\mathbf{k}\cdot\mathbf{r}}$$

- CMB + other large scale probes => concordance Λ CDM model
- DM = **collisionless** and dissipationless fluid of stable elementary particles which interact with each other and with baryons (almost) entirely through gravity, & non-relativistic (**cold enough**) at matter-radiation equality to form structures down to small scales

Cosmological tensions and the nature of DM

- The **Hubble tension**? No one is really sure what is going on (e.g., Di Valentino et al. 2021)
- The **EDGES anomaly**: no one knows either, potentially a fluke? If not, might have consequences on the nature of DM



- Cosmic dawn absorption feature at $z \sim 17$
- **Factor of 2 too large**
=> fluke?
or background temp. higher at these wavelengths ?
or gas cooler ?




1. « Small-scale » challenges



« Small-scale » tensions and the nature of DM

- Galaxies in non-linear ($|\delta| \gg 1$) regime of structure formation
- It is **hard** because of the importance of baryonic physics (feedback!)
- Simulations have made **huge improvements** at forming more realistic galaxies, but some tensions persist...
- Could the problem be **fundamental**, i.e. mostly the nature of DM in the model?
- Typically two types of cosmological galaxy formation sims:
 - **Large box**: EAGLE, IllustrisTNG, HorizonAGN, ...
 - **Zoom-in**: APOSTLE, NIHAO, FIRE-2, Auriga,...




Some basics of stellar (and DM) dynamics

$$\left\{ \begin{array}{l} df/dt = 0 \Leftrightarrow \frac{\partial f}{\partial t} + [f, H] = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0, \\ \nabla^2 \Phi = 4\pi G \int d^3\mathbf{v} f \end{array} \right.$$

f for each of the stellar components and in principle also the DM component, also constrained in configuration space through Φ

Integrate Boltzmann over velocity space => **continuity equation**

$$\frac{\partial \nu}{\partial t} + \frac{\partial(\nu \bar{v}_i)}{\partial x_i} = 0$$




Some basics of stellar (and DM) dynamics

Multiply by one velocity component and integrate Boltzmann over velocity space => **Jeans equations**
(analog to Euler)

$$\nu \frac{\partial \bar{v}_j}{\partial t} + \nu \bar{v}_i \frac{\partial \bar{v}_j}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \frac{\partial(\nu \sigma_{ij}^2)}{\partial x_i}$$

$$\frac{d(\nu \bar{v}_r^2)}{dr} + 2 \frac{\beta}{r} \nu \bar{v}_r^2 = -\nu \frac{d\Phi}{dr} \quad \text{in spherical symmetry}$$

$$\sigma_\phi^2 - \bar{v}_R^2 - \frac{R}{\nu} \frac{\partial(\nu \bar{v}_R^2)}{\partial R} - R \frac{\partial(\bar{v}_R \bar{v}_z)}{\partial z} = v_c^2 - \bar{v}_\phi^2$$



Some basics of stellar (and DM) dynamics

Multiply Jeans by position x_k and integrate over all positions to get the **virial equations** and in particular the scalar virial theorem

$$2K + W = 0$$

$$K = \frac{1}{2} M \langle v^2 \rangle$$

$$W = \frac{1}{2} \int d^3 \mathbf{x} \rho(\mathbf{x}) \Phi(\mathbf{x})$$

$$r_g \equiv \frac{GM^2}{|W|}$$

King models r_h/r_g is confined to the interval $(0.4, 0.51)$



Let's go back in time

- First hint for DM came from Zwicky analyzing the velocity dispersion of 8 Coma cluster galaxies
- $\sigma = 1019 \pm 360$ km/s (not far from modern value!)

$$\langle v^2 \rangle = \frac{|W|}{M} \simeq 0.45 \frac{GM}{r_h}$$

- Used Hubble constant $H_0 = 558$ km/s/Mpc
- ⇒ Underestimated the distance and the stellar mass by a factor of ~ 8 and $64 \dots$
- + hot X-ray emitting gas not detected... However, the discrepancy hasn't gone away in clusters (factor of ~ 6)

A NUMERICAL STUDY OF THE STABILITY OF FLATTENED GALAXIES: OR, CAN COLD GALAXIES SURVIVE?*

J. P. OSTRIKER

Princeton University Observatory

AND

P. J. E. PEEBLES

Joseph Henry Laboratories, Princeton University

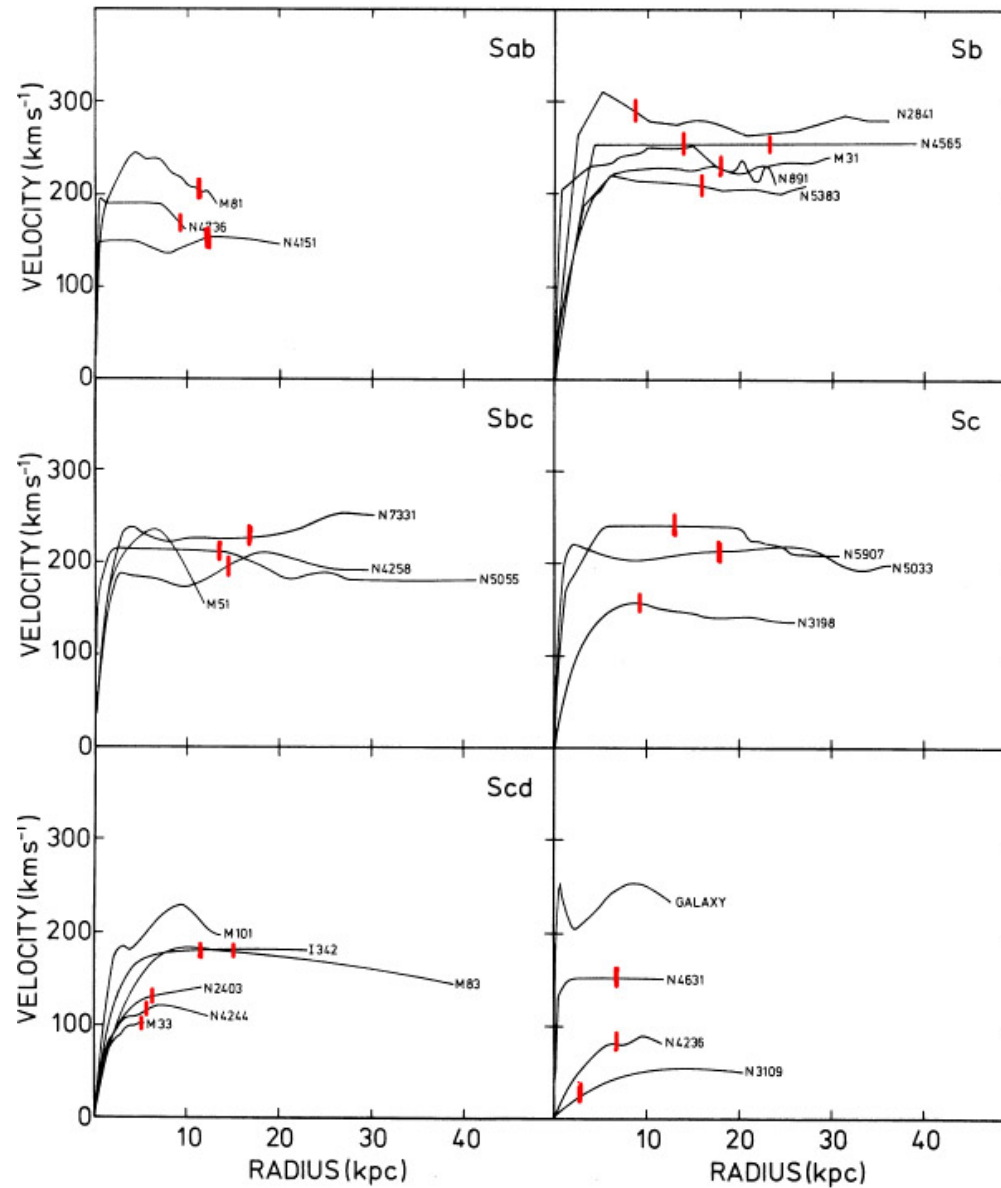
Received 1973 May 29

ABSTRACT

To study the stability of flattened galaxies, we have followed the evolution of simulated galaxies containing 150 to 500 mass points. Models which begin with characteristics similar to the disk of our Galaxy (except for increased velocity dispersion and thickness to assure local stability) were found to be rapidly and grossly unstable to barlike modes. These modes cause an increase in random kinetic energy, with approximate stability being reached when the ratio of kinetic energy of rotation to total gravitational energy, designated t , is reduced to the value of 0.14 ± 0.02 . Parameter studies indicate that the result probably is not due to inadequacies of the numerical N -body simulation method. A survey of the literature shows that a critical value for limiting stability $t \simeq 0.14$ has been found by a variety of methods.

Models with added spherical (halo) component are more stable. It appears that halo-to-disk mass ratios of 1 to $2\frac{1}{2}$, and an initial value of $t \simeq 0.14 \pm 0.03$, are required for stability. If our Galaxy (and other spirals) do not have a substantial unobserved mass in a hot disk component, then apparently the halo (spherical) mass *interior* to the disk must be comparable to the disk mass. Thus normalized, the halo masses of our Galaxy and of other spiral galaxies *exterior* to the observed disks may be extremely large.

HI galaxy rotation curves



Bosma (1978)



HI galaxy rotation curves

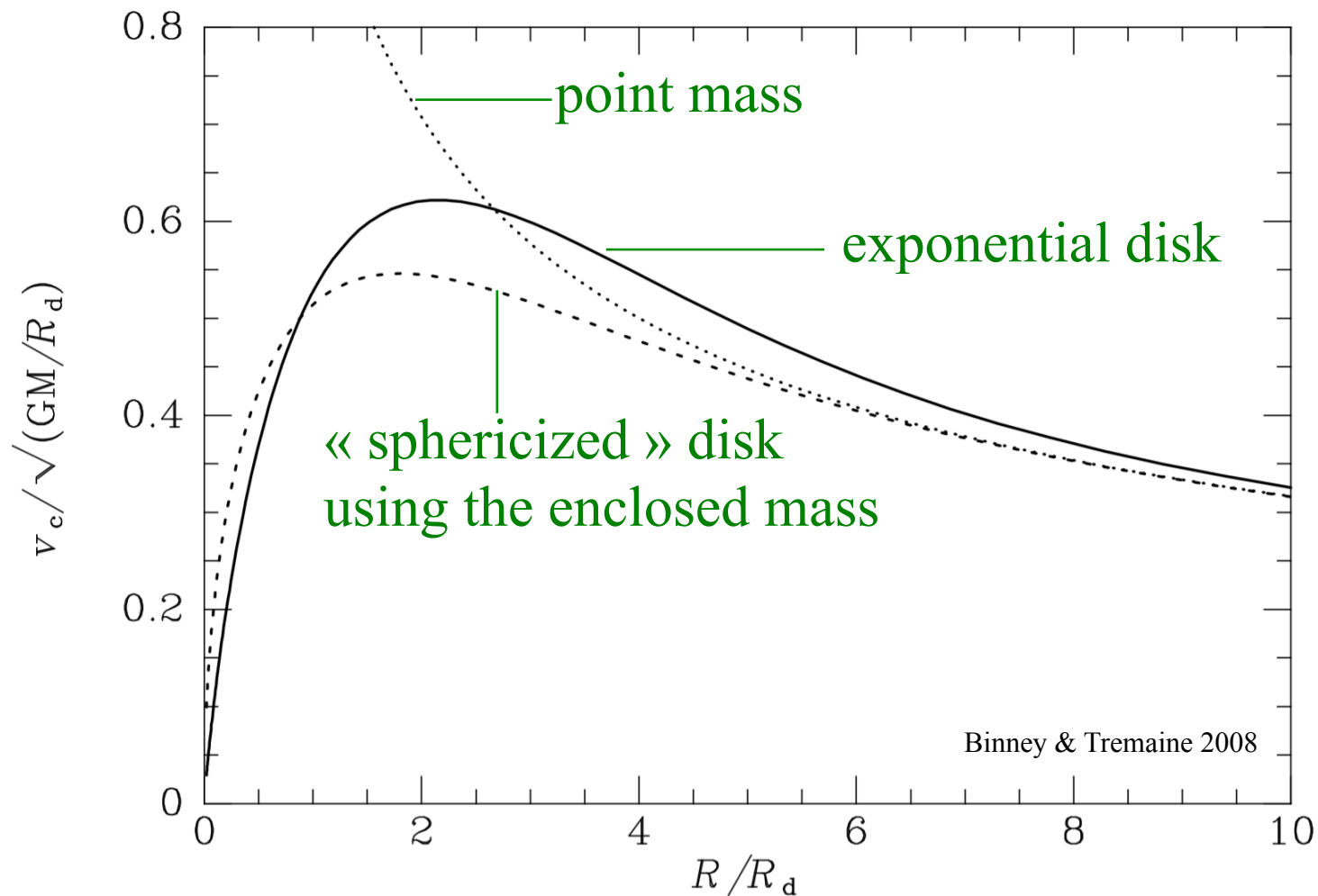
$$R_{\alpha\beta} - 1/2 R g_{\alpha\beta} + \Lambda g_{\alpha\beta} = (8\pi G/c^4) T_{\alpha\beta}$$

- Weak-field limit: $g_{00} = -1 - 2\Phi/c^2$ with $\nabla^2\Phi = 4\pi G\rho$ ($\Phi/c^2 \sim 10^{-6}$)
- Observe ρ_{bar} (needs stellar M/L) in galaxies & derive Φ_{bar}
 $(R |\partial\Phi_{\text{bar}}/\partial R|)^{1/2} = V_{\text{c bar}}$ too low in the
galactic plane compared to observed $V_c \Rightarrow$ dark matter

E.g. if exponential disk with surface density $\Sigma(R) = \Sigma_0 e^{-R/R_d}$

$$v_c^2(R) = R \frac{\partial\Phi}{\partial R} = 4\pi G \Sigma_0 R_d y^2 [I_0(y)K_0(y) - I_1(y)K_1(y)]$$

HI galaxy rotation curves



Keplerian fall-off after a few scale-lengths

HI galaxy rotation curves



Inclination i with respect to sky plane

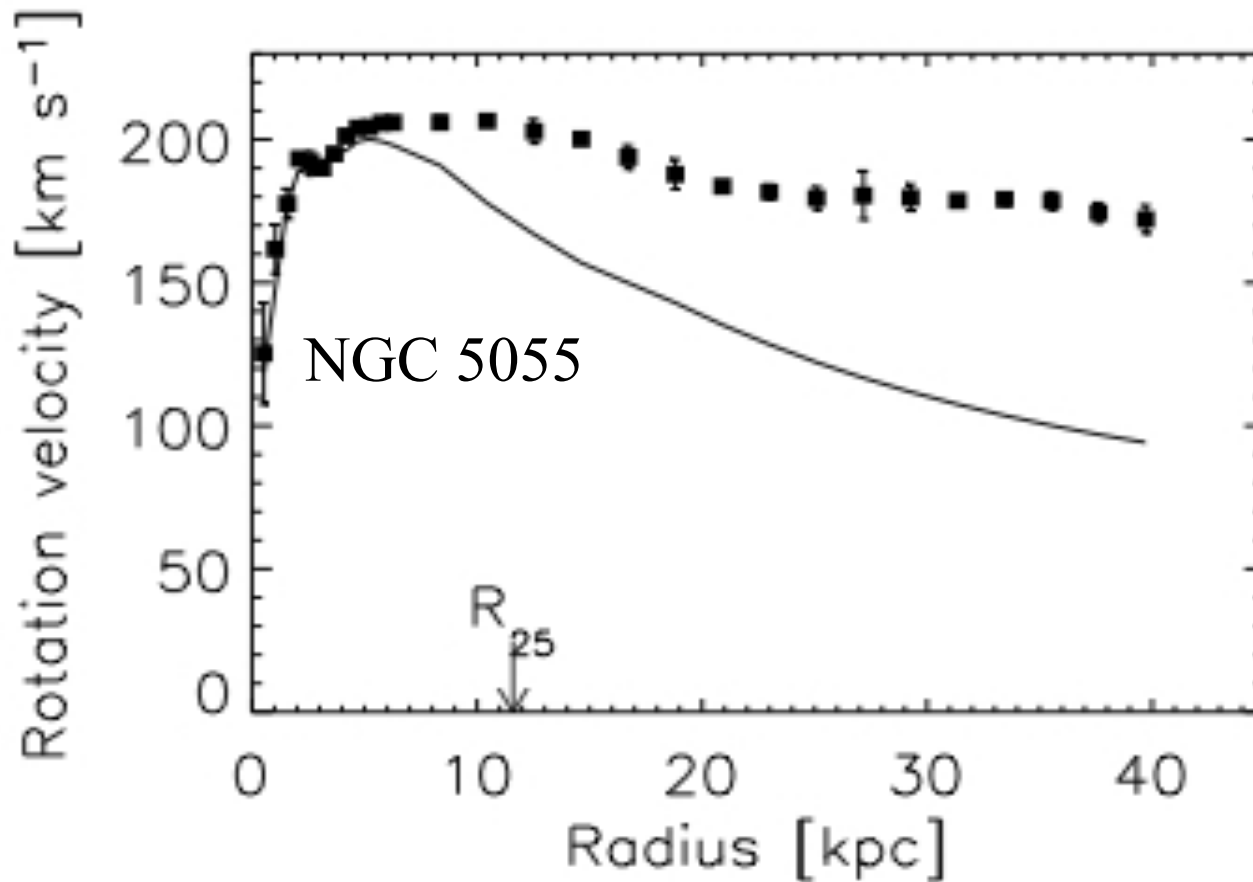
Angle ϕ from line of nodes

$\tan(\vartheta) = \tan(\phi) / \cos(i)$ = angle within the plane of the disc

$$V_{\text{los}} = V_{\text{rot}}(R) \sin(i) \cos(\vartheta) \quad (\text{fit in rings})$$

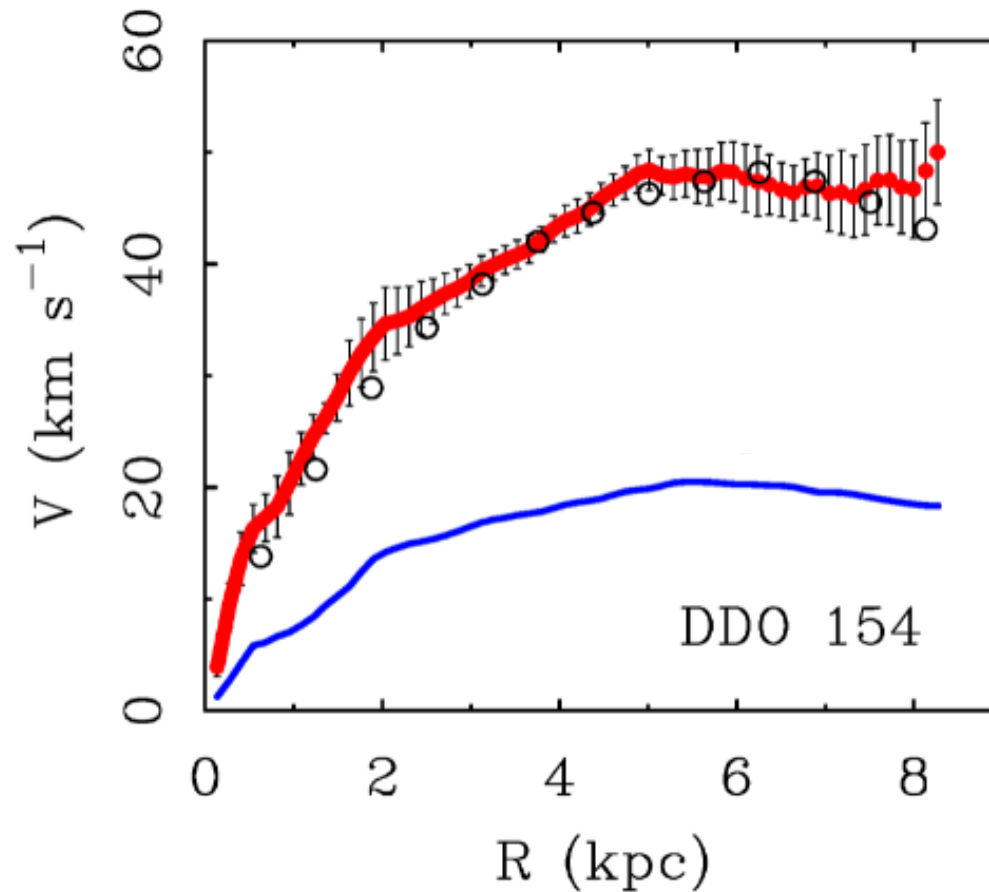
Then correct asymmetric drift with $V_c^2 - V_{\text{rot}}^2 = -\frac{R}{\rho} \frac{\partial \rho \sigma_v^2}{\partial R}$

HI galaxy rotation curves



$$\frac{dM}{dr} / \frac{dL}{dr} \neq \text{const} = \left(\frac{M}{L} \right)_\star$$

HI galaxy rotation curves



Some galaxies (typically low surface brightness) are dominated by DM all the way down to the center

HI galaxy rotation curves

When a galaxy is dominated by DM down to the center, the cored or cusped profile of DM can directly be seen in the 2D velocity field

Constant density core $\Rightarrow M(R) \sim R^3 \Rightarrow v^2/R \sim R \Rightarrow v \sim R$ (solid-body)

$$V(x, y) = \Omega R \cos \theta \sin i \quad \text{with} \quad x = R \cos \theta$$

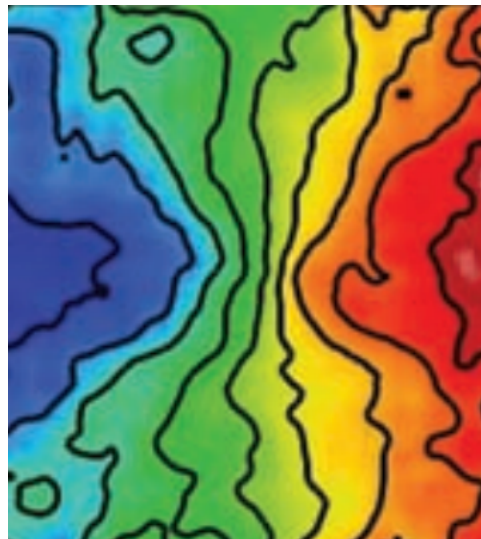
Triaxial



Spherical Cuspy

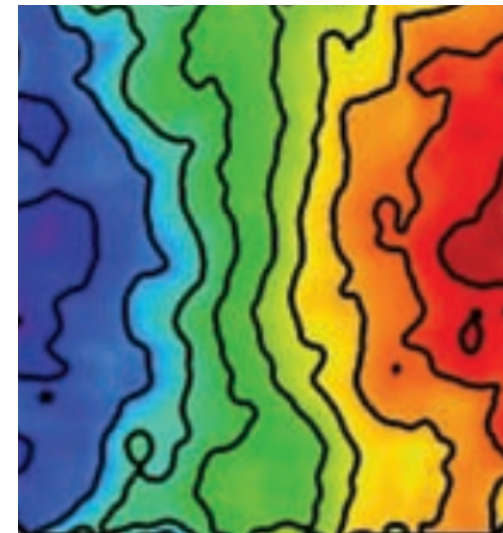


Cored



Cuspy DM halo

Kuzio de
Naray &
Kaufmann
2011



Cored DM halo



The core-cusp problem

DMO simulations predict that, if we define the virial radius as

$$R_{200} = \left(\frac{M_{200}}{(4/3)\pi 200\rho_{\text{crit}}} \right)^{1/3}$$

the universal profile of DM halos is the NFW profile :

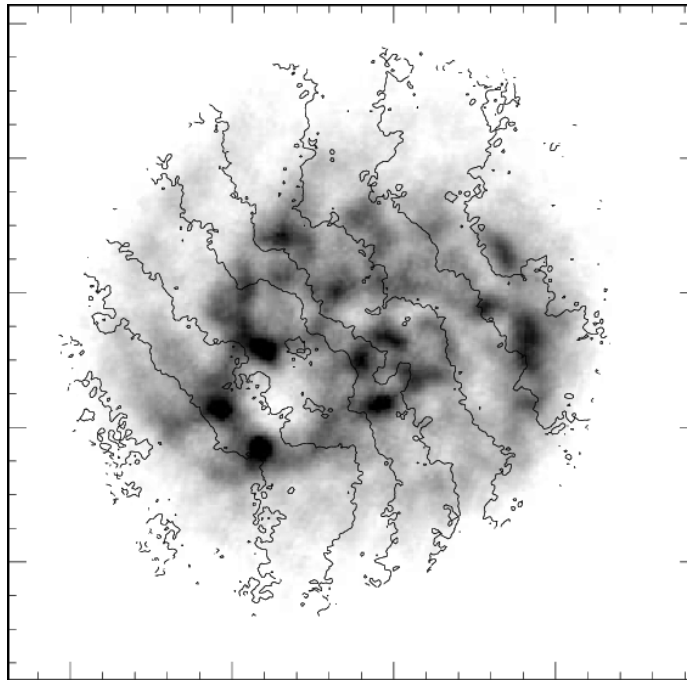
$$\rho_{\text{DM}} = \frac{200\rho_{\text{crit}}R_{200}}{3r[c^{-1} + (r/R_{200})]^2[\ln(1+c) - c/(1+c)]}$$

with an obvious $\sim r^{-1}$ cusp at the center

(in reality, modern simulations predict a very small core, and varying degrees of cuspieness, but mostly irrelevant to the rotation curves)

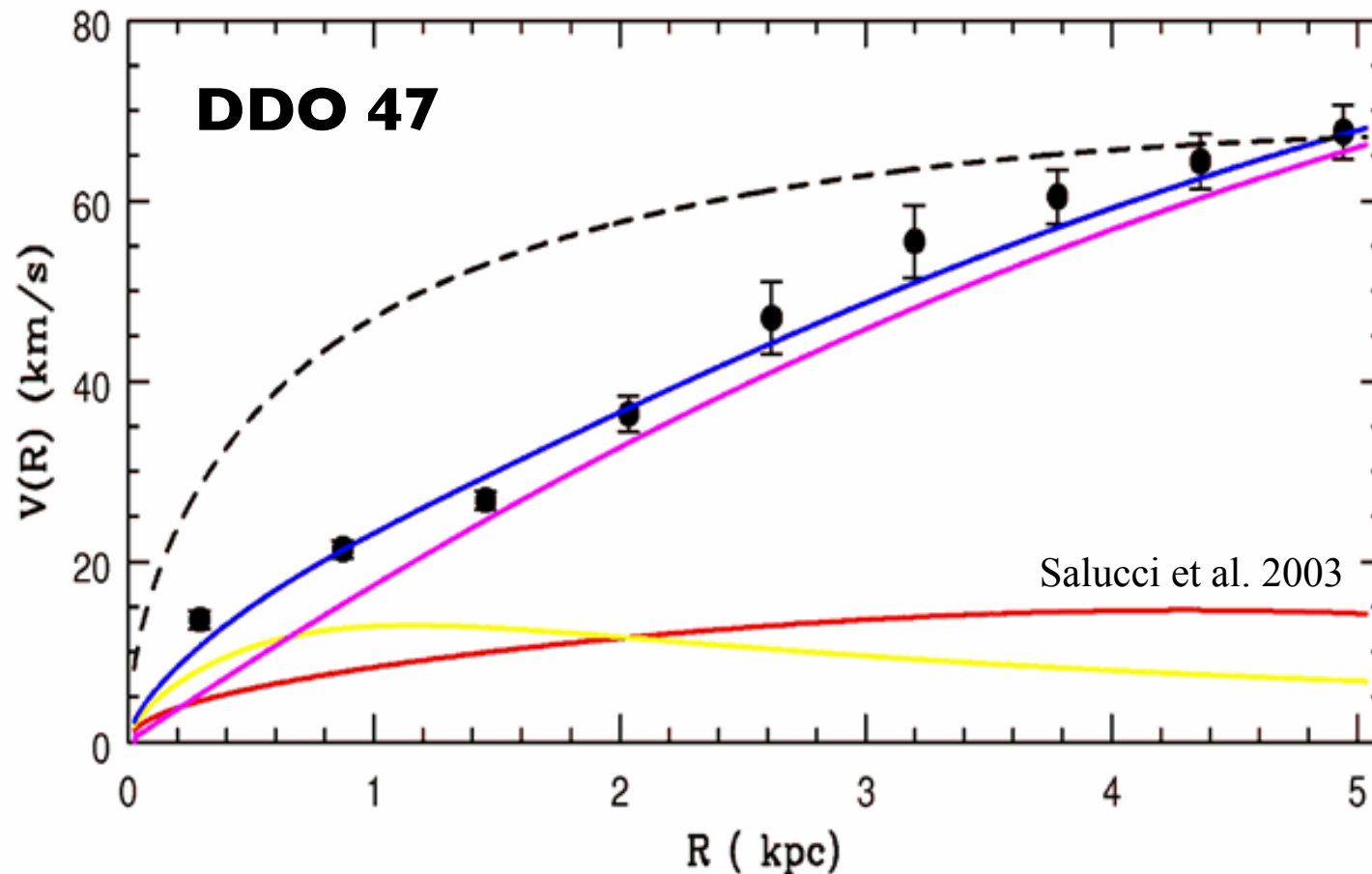
The core-cusp problem

DDO 47



Many galaxies (but not all!) dwarf galaxies have cored DM halos immediately visible from the 2D velocity field with no signs of DM halo triaxiality

The core-cusp problem



Many galaxies (but not all!) dwarf galaxies have cored DM halos immediately visible from the 2D velocity field with no signs of DM halo triaxiality



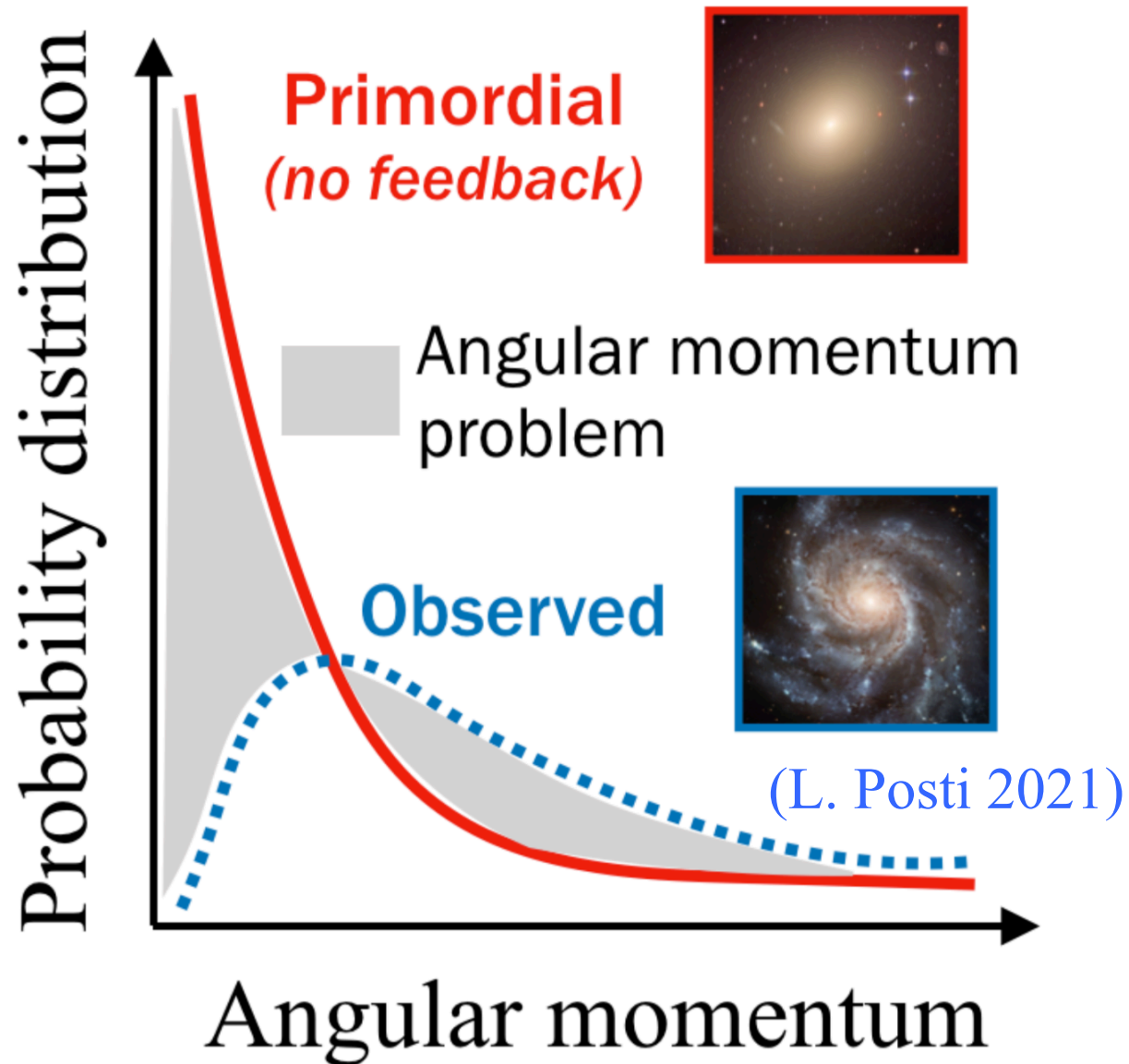
The core-cusp problem

This problem has been a motivation for exploring alternatives to CDM for 30 years

However, it can *in principle* be solved by **feedback** in hydrodynamical simulations of galaxy formation

And feedback (mostly SN and/or AGN) is actually necessary to avoid the *angular momentum catastrophe*

The core-cusp problem





The core-cusp problem

While feedback primarily redistributes the angular momentum of baryons, this redistribution of baryons can also in principle act on the DM distribution, especially if it is bursty, with many recurring episodes

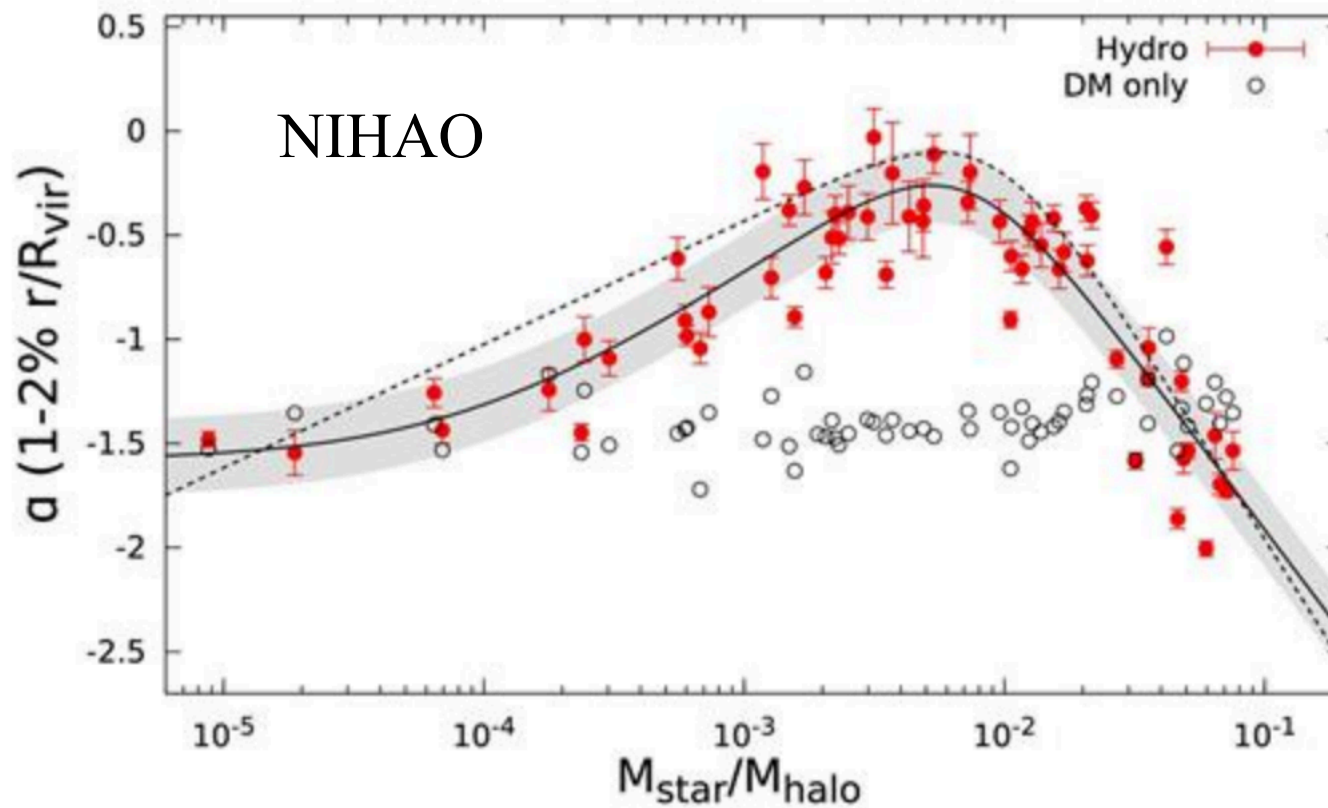
Fluctuation-dissipation theorem \Rightarrow potential fluctuations reorganize the DM distribution

Highly dependent on subgrid recipes! (e.g., high gas density for SF threshold)

EAGLE/APOSTLE \Rightarrow almost **no** core formation !

NIHAO \Rightarrow **all** cores at $z=0$ for M^*/M_h in appropriate range
(actually *too many* cores in this range!)

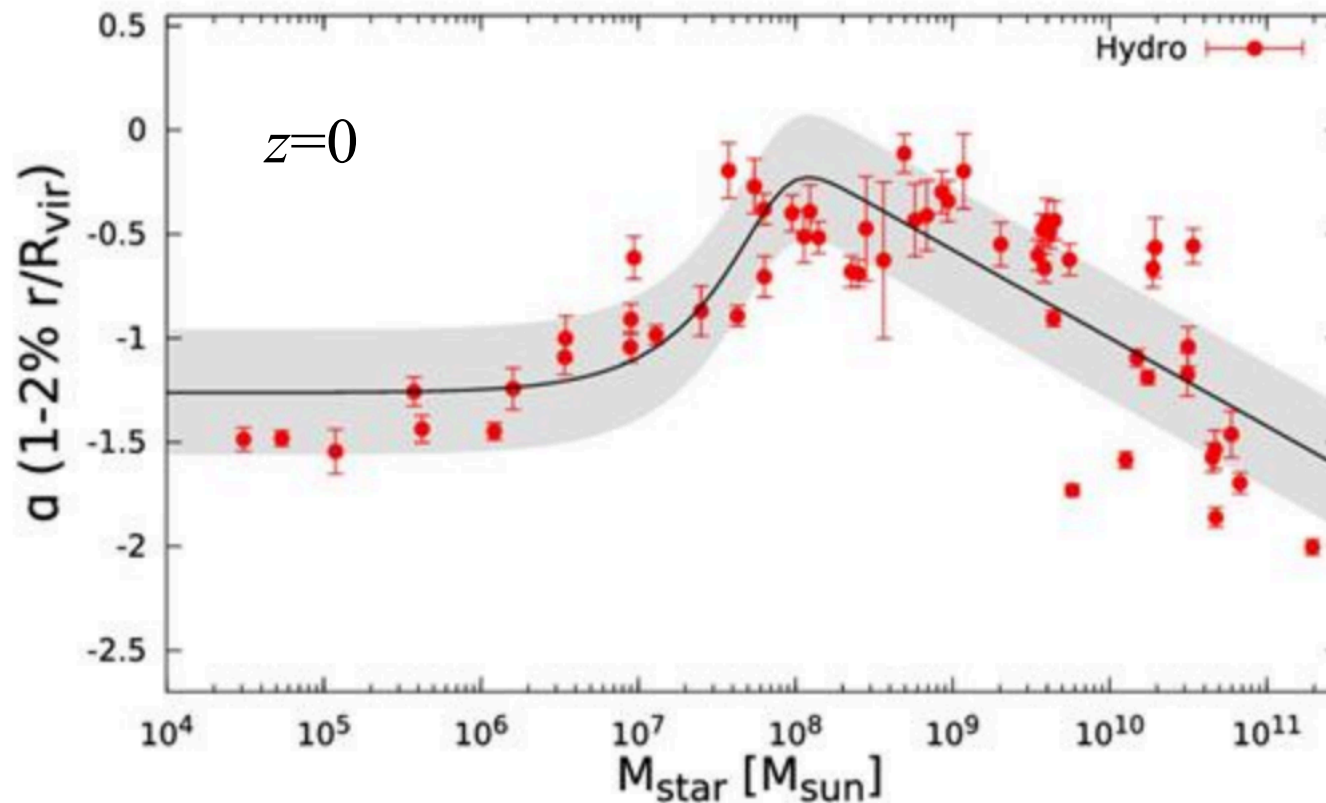
The core-cusp problem



$$z = 0$$

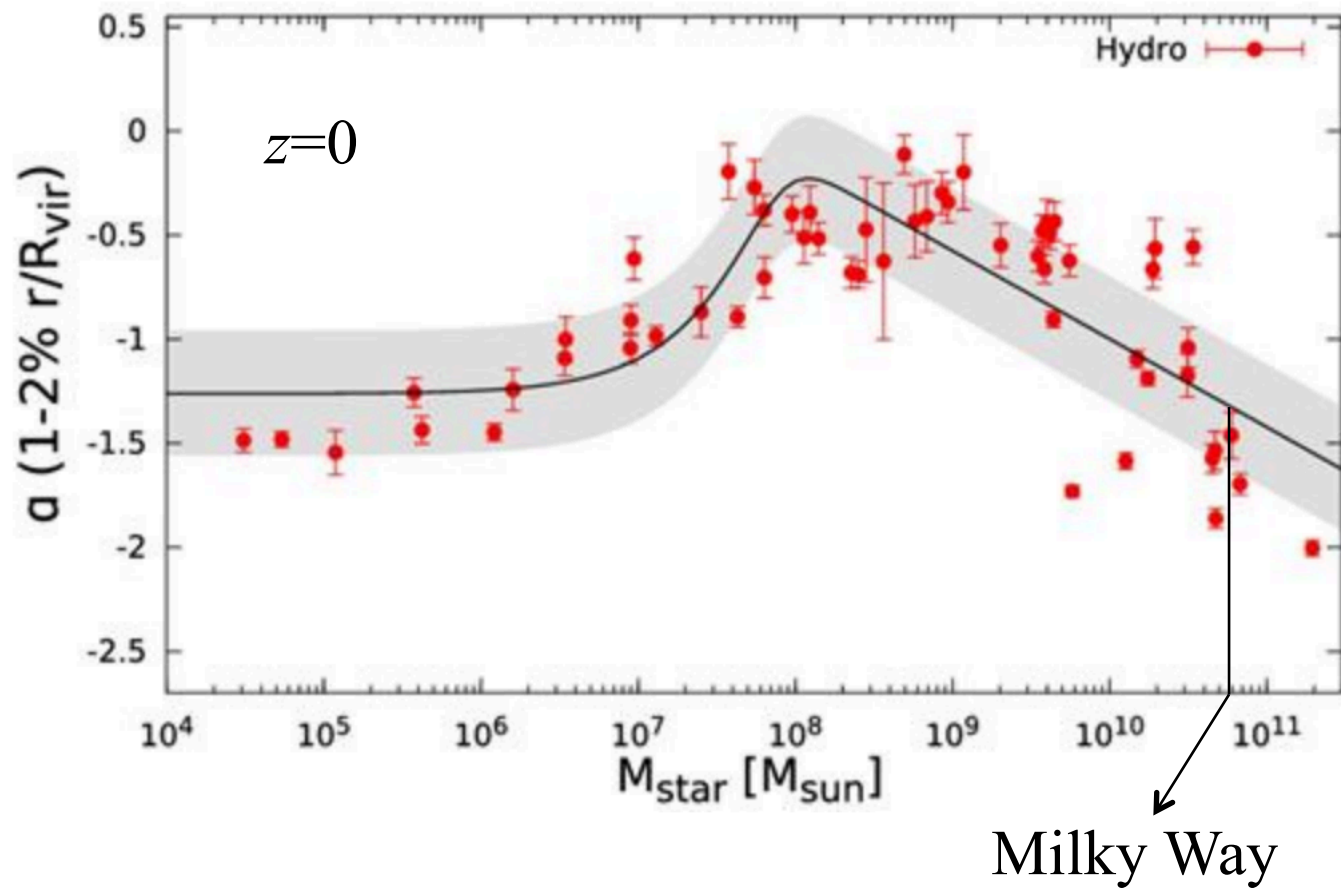
Tollet et al. 2016

The core-cusp problem

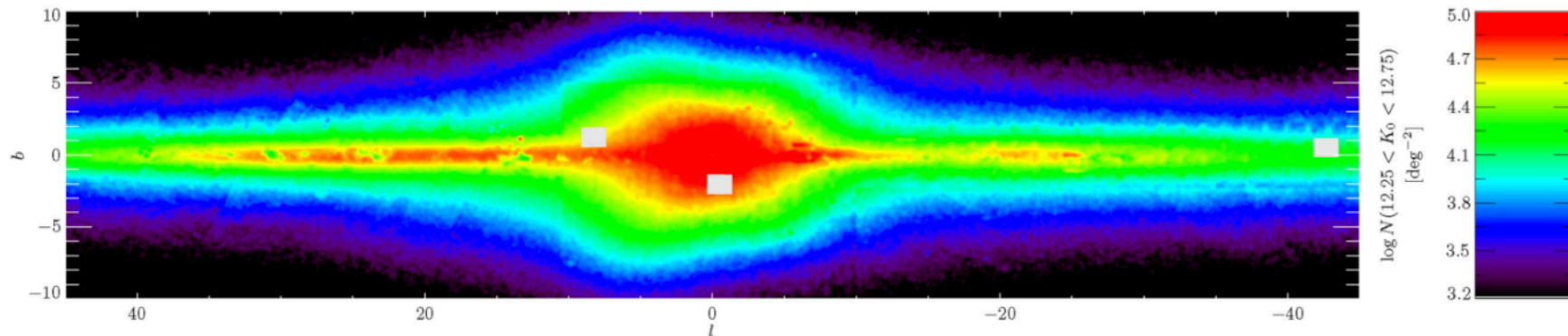


Genzel et al. (2020): H α and CO RCs at $z=0.65-2.5$ show large DM cores in massive halos, not predicted by sims (Dekel et al. 2021 invoke mergers and dynamical friction)

The core-cusp problem



Modelling the MW bar



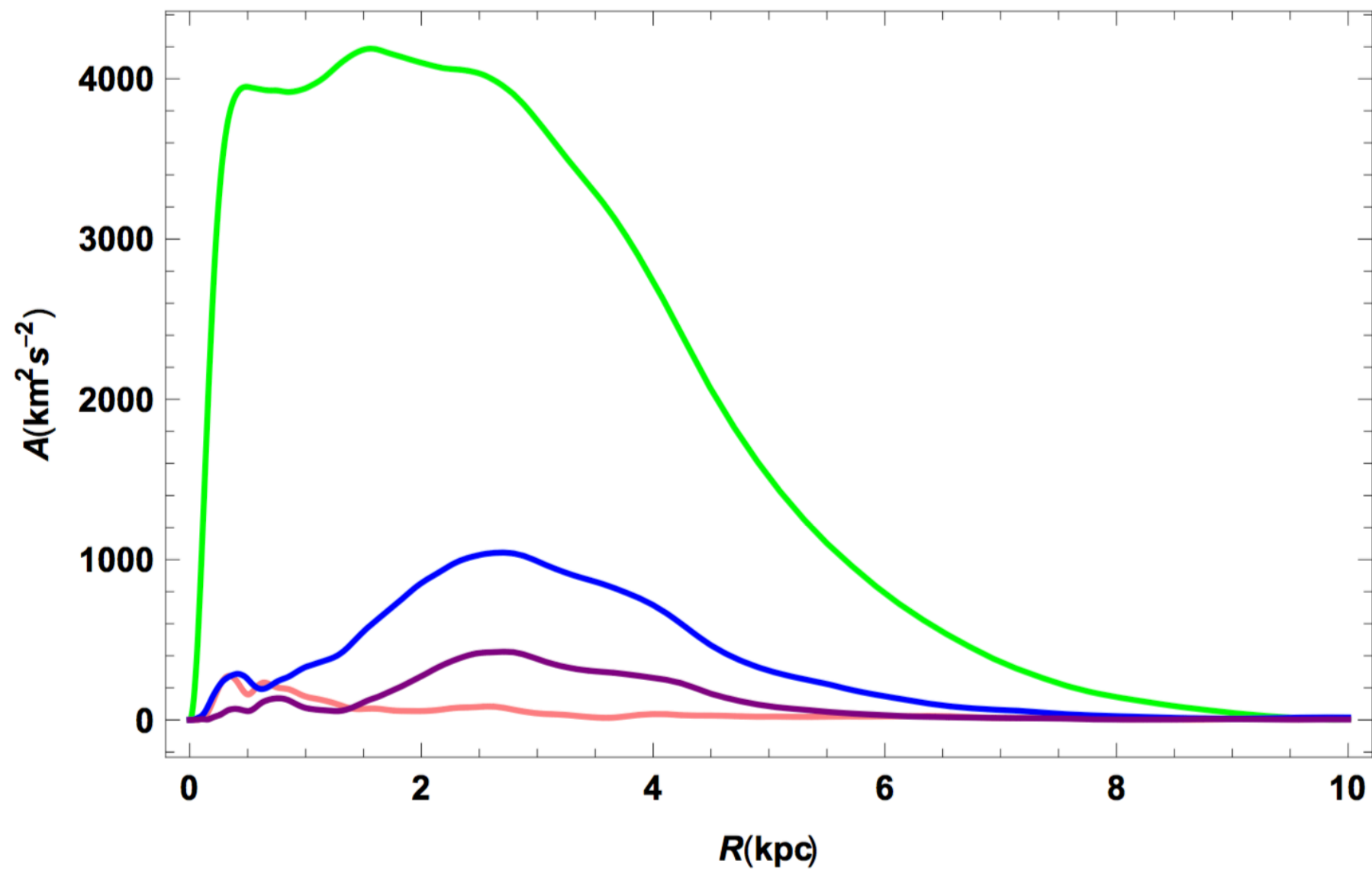
Wegg C., Gerhard O., Portail M., 2015, MNRAS, 450, 4050

- Millions of RC stars from VVV survey + 2MASS+ UKIDSS + GLIMPSE
- \Rightarrow long flat ($h_z < 50$ pc) extension of the bar out to 5 kpc from the center ($l > 30^\circ$)

- Fit to BRAVA (central 10° in long.)
- +ARGOS (28000 stars $-30^\circ < l < 30^\circ$ and $-10^\circ < b < -5^\circ$)

$\Rightarrow \Omega_b = 40 \text{ km/s/kpc} \sim 1.35 \Omega_0$ (Portail et al. 2016)

\Rightarrow Corotation at ~ 6 kpc and OLR beyond 10 kpc !



Modelling the MW bar

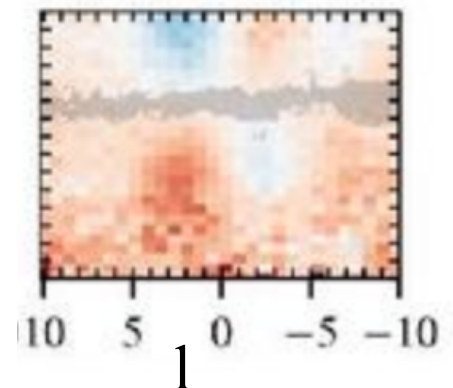
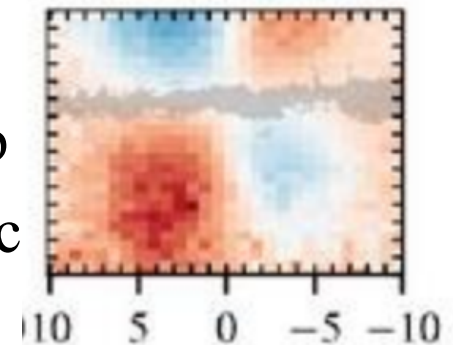
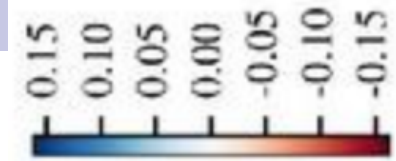
1.75x10⁸ PMs (!!!) at
-10°<l<10°, -10°<b<5°
in the VVV Infrared
Astrometric Catalogue
(VIRAC), calibrated on
Gaia DR2 (Clarke et al. 2019)

See also Sanders et al. (2019)
Tremaine-Weinberg method

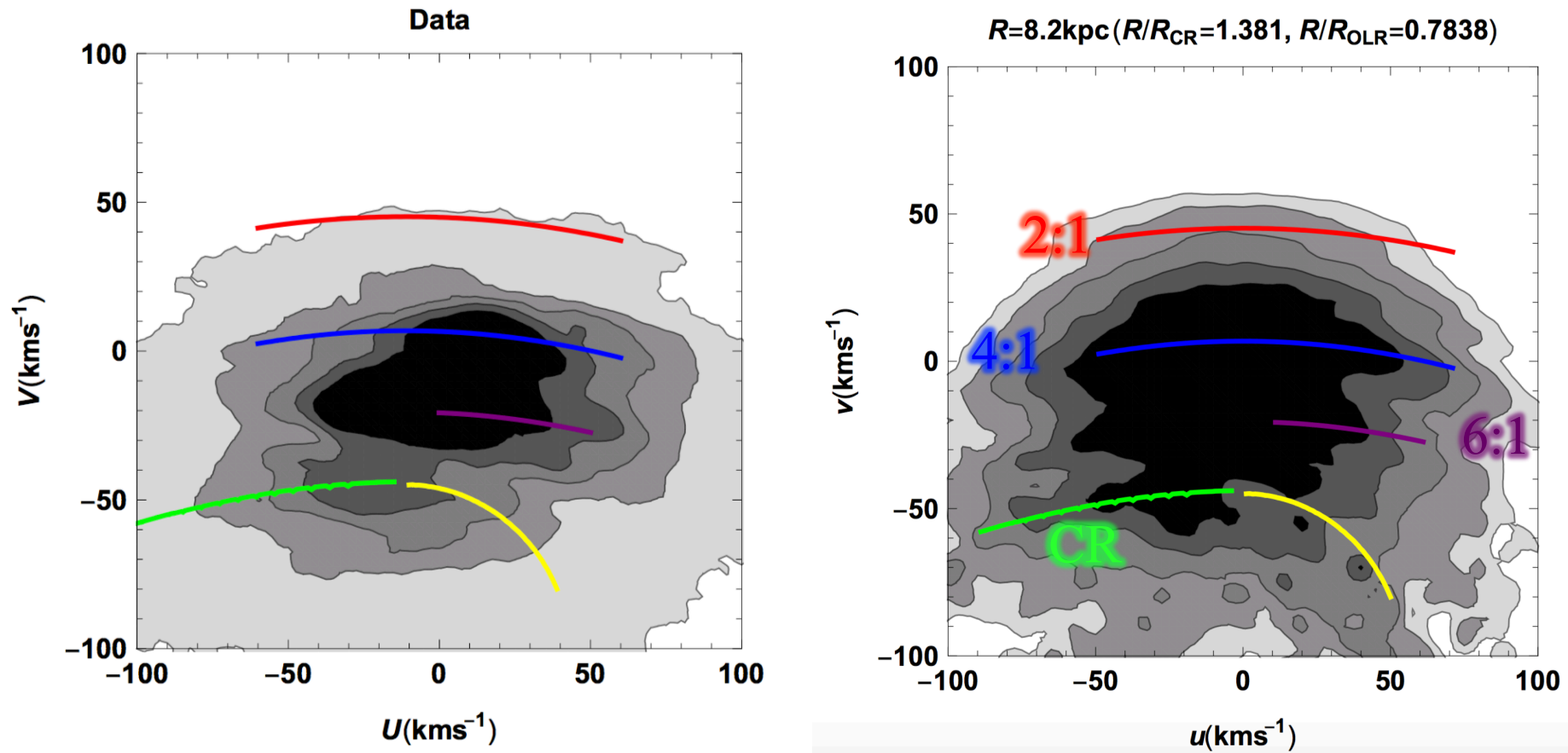
$$\frac{\sigma_{lb}^2}{\sigma_l \sigma_b}, \text{ obs.}$$

b
37.5 km/s/kpc

50 km/s/kpc



The local velocity field from Gaia

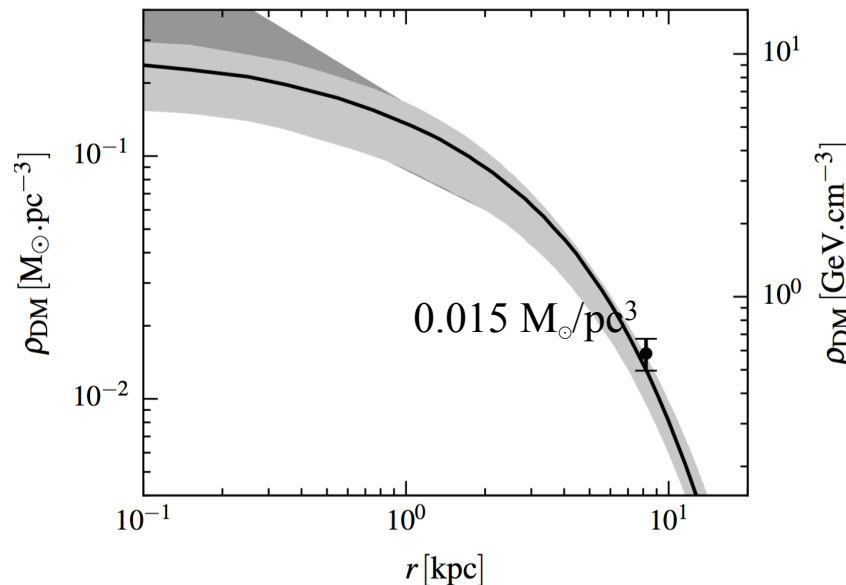


Monari et al. (2019)

$V_{\odot} = 0$ km/s, declining RC allows to get a more realistic $V_{\odot} = 8$ km/s

A cored DM halo in the MW?

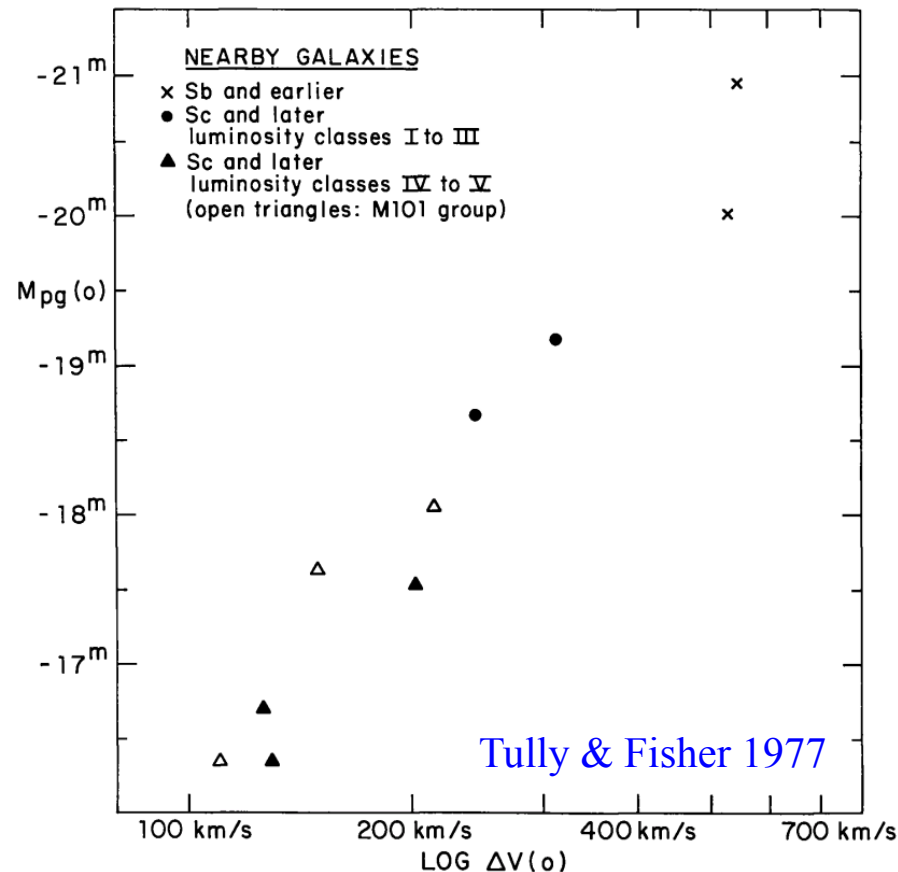
- Bulge mass (2.2 kpc, 1.4 kpc, 1.2 kpc): $1.85 \times 10^{10} M_{\odot}$
 - Stellar mass: $1.32 \times 10^{10} M_{\odot}$
 - Additional nuclear disk: $2 \times 10^9 M_{\odot}$
 - Dark matter mass: $3.2 \times 10^9 M_{\odot}$



Portail et al. (2017)

Sharp falloff to keep the RC constant between 6 kpc and 8 kpc => **cored profile at the center**

Regularities in the dynamics of galaxies: let's go back in time again



$$L \propto \Delta V^\alpha$$

$$\alpha = 2.5 - 4$$

(slope of 6.25-10
in mag)

Half of the velocity width at 20% of the peak flux = proxy for rotational velocity



Regularities in the dynamics of galaxies: let's go back in time again

Armed with the following knowledge at the beginning of the 80's,
Milgrom proposed his MOND paradigm, or just *Milgrom's relation*:

- If observed RCs are flat, then gravity must effectively fall like $1/r$
- The discrepancy sets in at different radii in different galaxies, so a more relevant scale is the centripetal acceleration

$$g = g_N$$

$$g = (g_N a_0)^{1/2}$$

$$\text{if } g \gg a_0$$

$$\text{if } g \ll a_0$$

MOND

Milgrom 1983

$$a_0 \sim 10^{-10} \text{ m/s}^2$$



Spherical approximation:

$$V^2 / r = (ga_0)^{1/2} = (GMa_0)^{1/2} / r$$

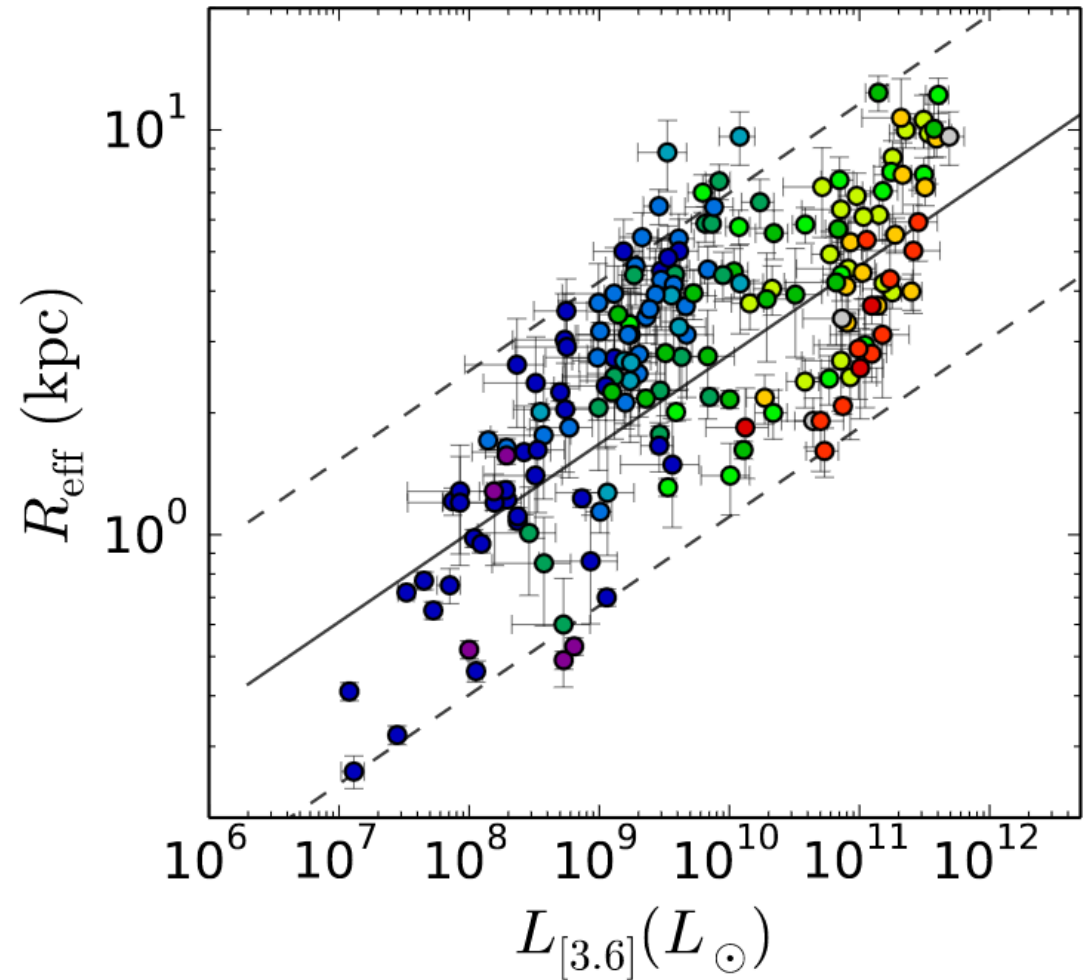
$$V = \text{constant} = (GMa_0)^{1/4}$$

⇒ Velocity predicted to be flat, and Tully-Fisher relation predicted to be a relation between the **total baryonic mass** of galaxies and the **asymptotic circular velocity**, with a **slope of 4**

Very strong and unintuitive predictions at the time!

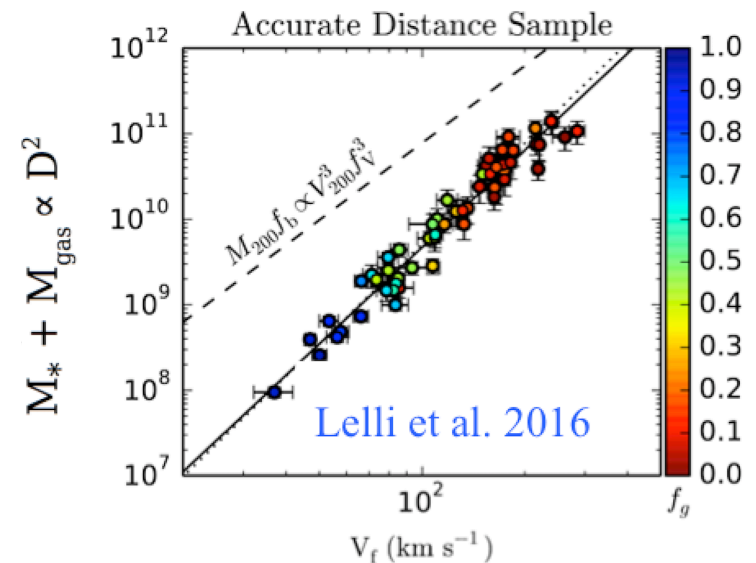
HI galaxy rotation curves

- SPARC (Lelli et al.)
- 175 galaxies with high quality HI RCs
- Homogeneous Spitzer photometry at $3.6\mu\text{m}$
- M_*/L known to be roughly constant (0.5-0.7) in the NIR



Baryonic Tully-Fisher

- $\text{Log } M_b = \alpha \log V - \log \beta$
- $\alpha = 3.9 \pm 0.4$
- Zero-point defines an acceleration constant $a_0 \approx V^4/(GM_b) \approx 10^{-10} \text{ m/s}^2$ such that $\beta = Ga_0$

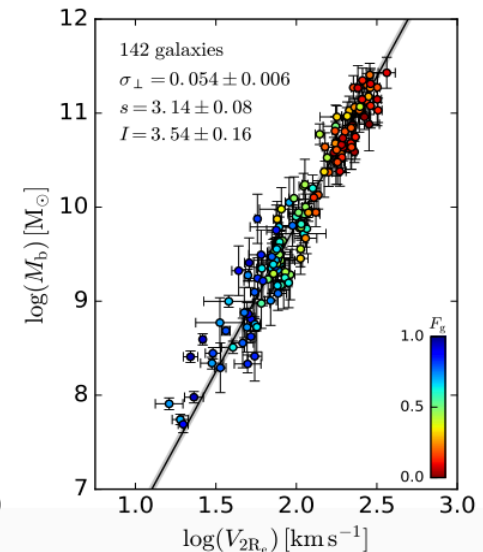
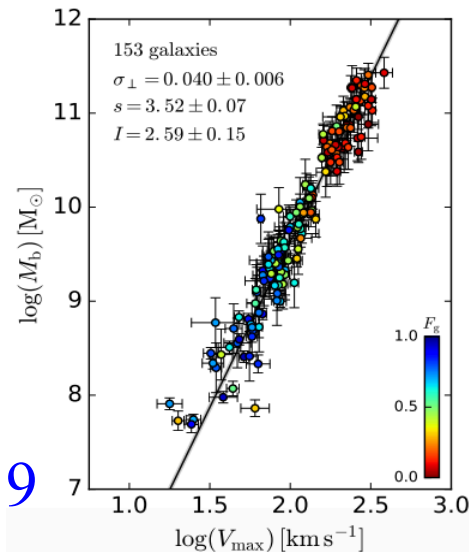
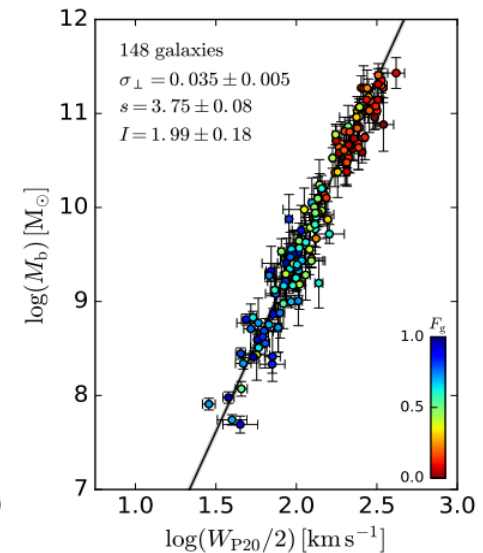
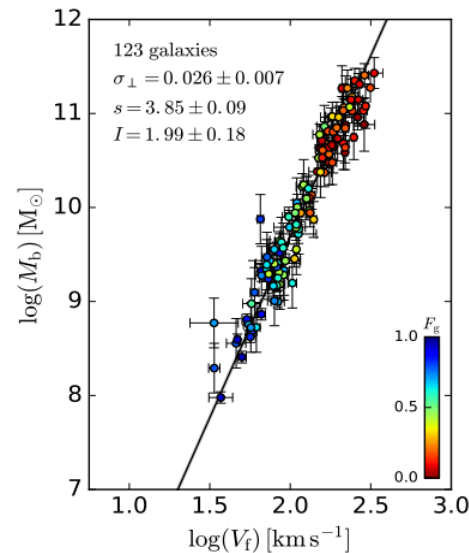


Baryonic Tully-Fisher

- $\log M_b = \alpha \log V_f - \log \beta$
- $\alpha \approx 3.9$

- Intrinsic scatter
 ~ 0.025 dex

Lelli et al. 2019





Unintuitive because:

First of all, if galaxies were representative of the overall cosmic baryon-to-DM ratio, the expected slope would be ~ 3

$$R_{\text{vir}} \text{ (at any multiple of the critical density)} \propto M_{\text{vir}}^{1/3}$$

$$V_{\text{vir}}^2 \approx GM_{\text{vir}}/R_{\text{vir}} \propto M_{\text{vir}}^{2/3}$$

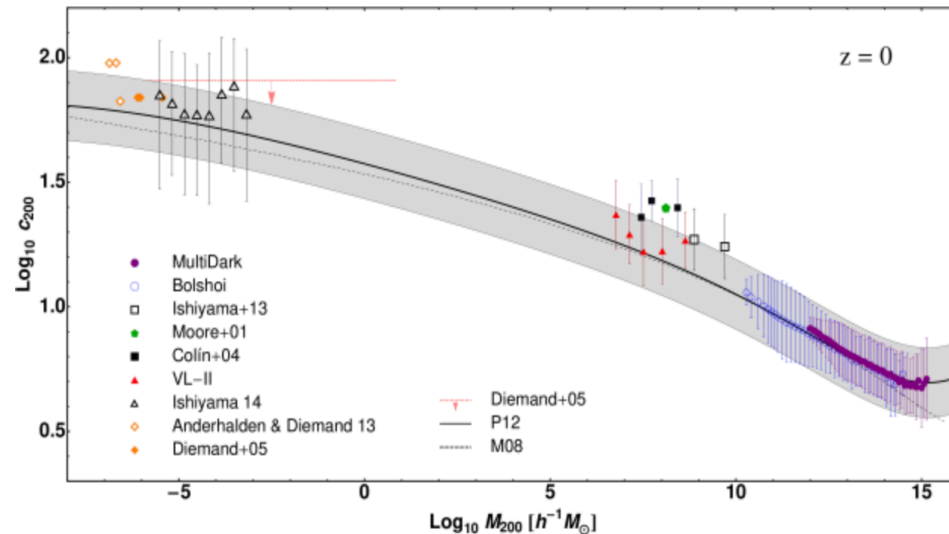
If constant baryon fraction, expectation would be $M \propto V^3$

To get a slope of 4, one needs **baryon fraction to go down with mass**

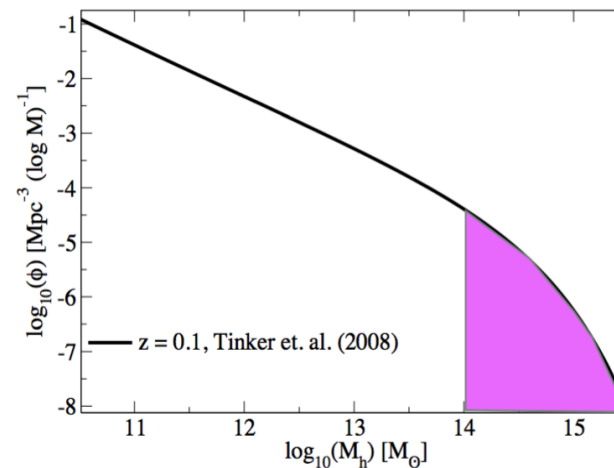
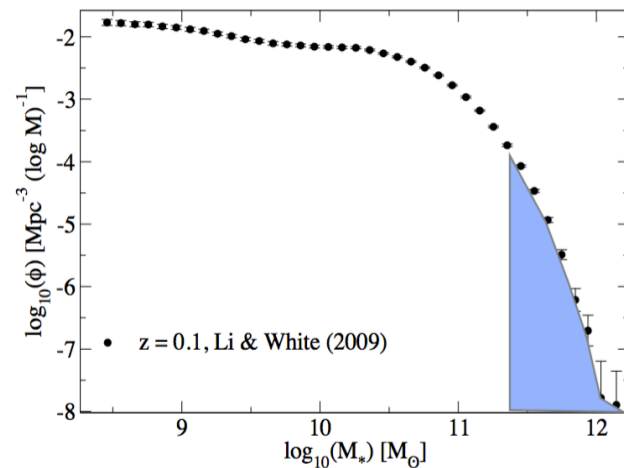
Luckily (for LCDM), this must happen in LCDM too !

... but the scatter is still not right

Halo scaling relations and abundance matching

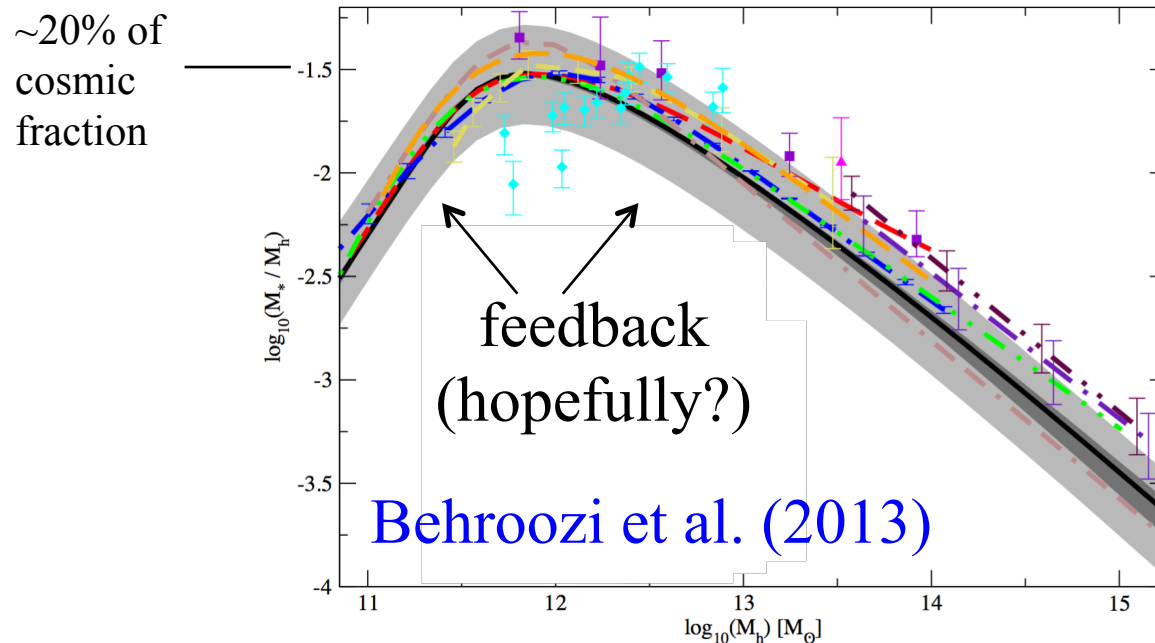


Halo mass-concentration relation
(with some scatter of ~ 0.1 dex)



Match stellar mass
function to halo
mass function by
assigning $n(>M^*)$
to $n(>M_h)$

Stellar-to-halo mass relation (SHMR)



Typical scatter ~ 0.15 dex

⇒ Adding the gas, the intrinsic BTFR scatter **cannot go below 0.05 dex**

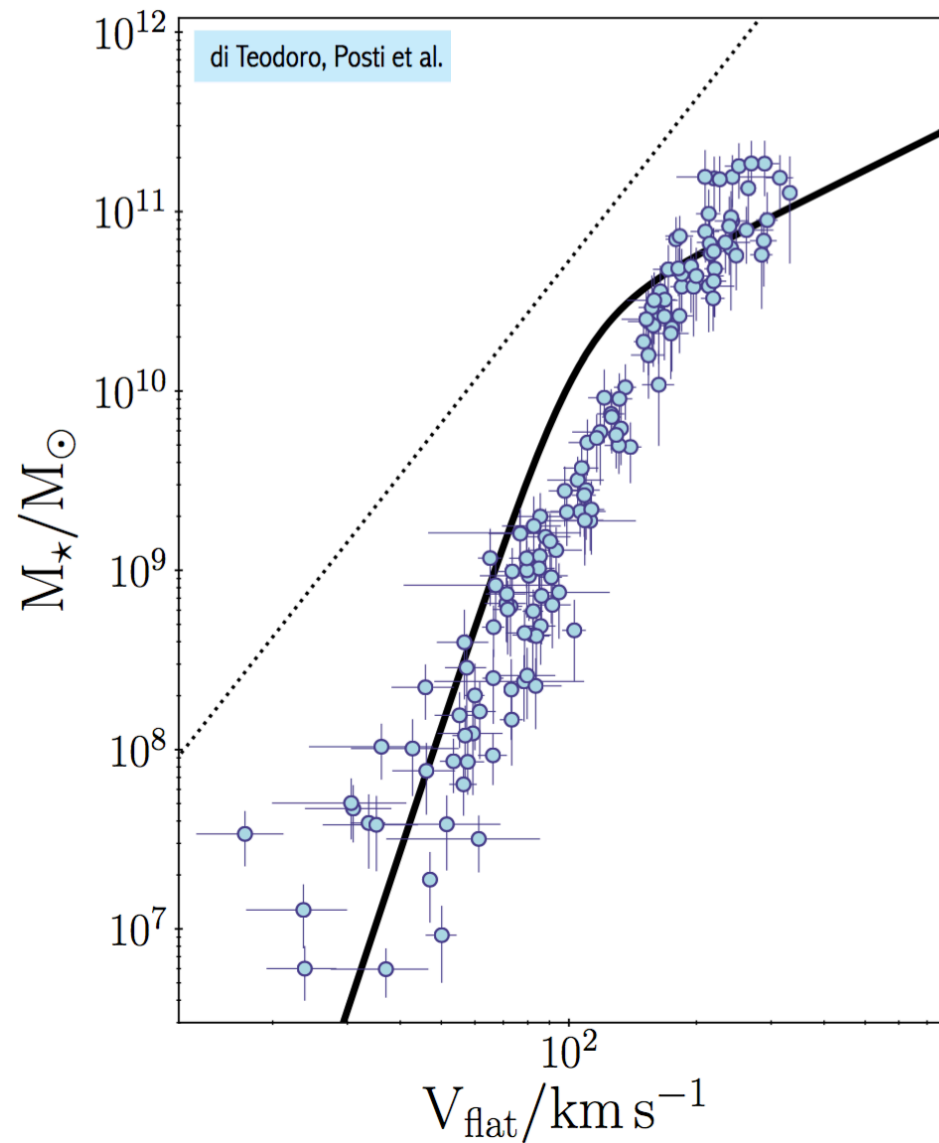
Twice too high!

The scatter, residual correlations and curvature of the SPARC baryonic Tully–Fisher relation

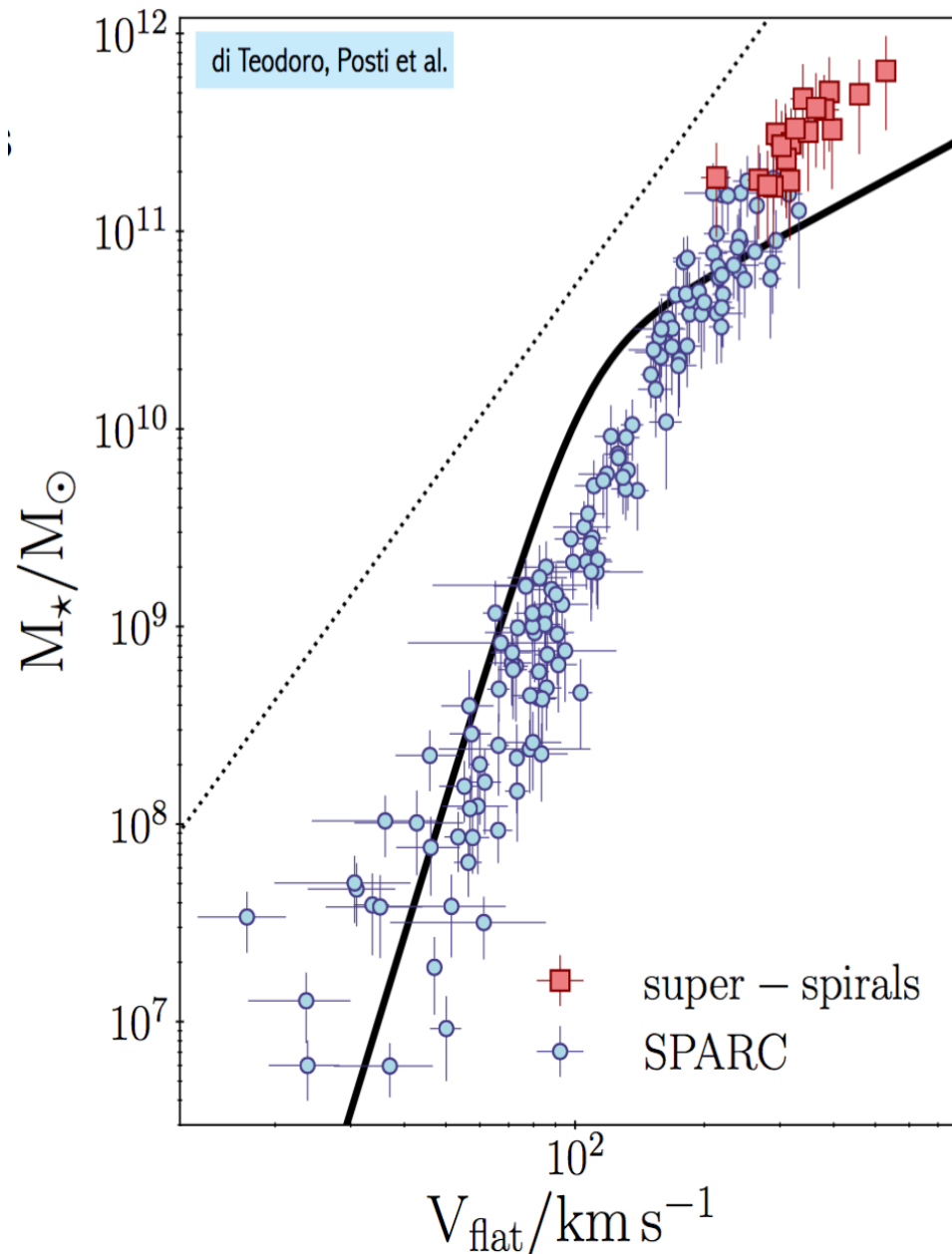
Harry Desmond^{1,2*} (2017)

¹Kavli Institute for Particle Astrophysics and Cosmology, Physics Department, Stanford University, Stanford, CA 94305, USA

calculate the statistical significance of these results in the framework of halo abundance matching, which imposes a canonical galaxy–halo connection. Taking full account of sample variance among SPARC-like realisations of the parent halo population, we find the scatter in the predicted BTFR to be **3.6 σ too high.**



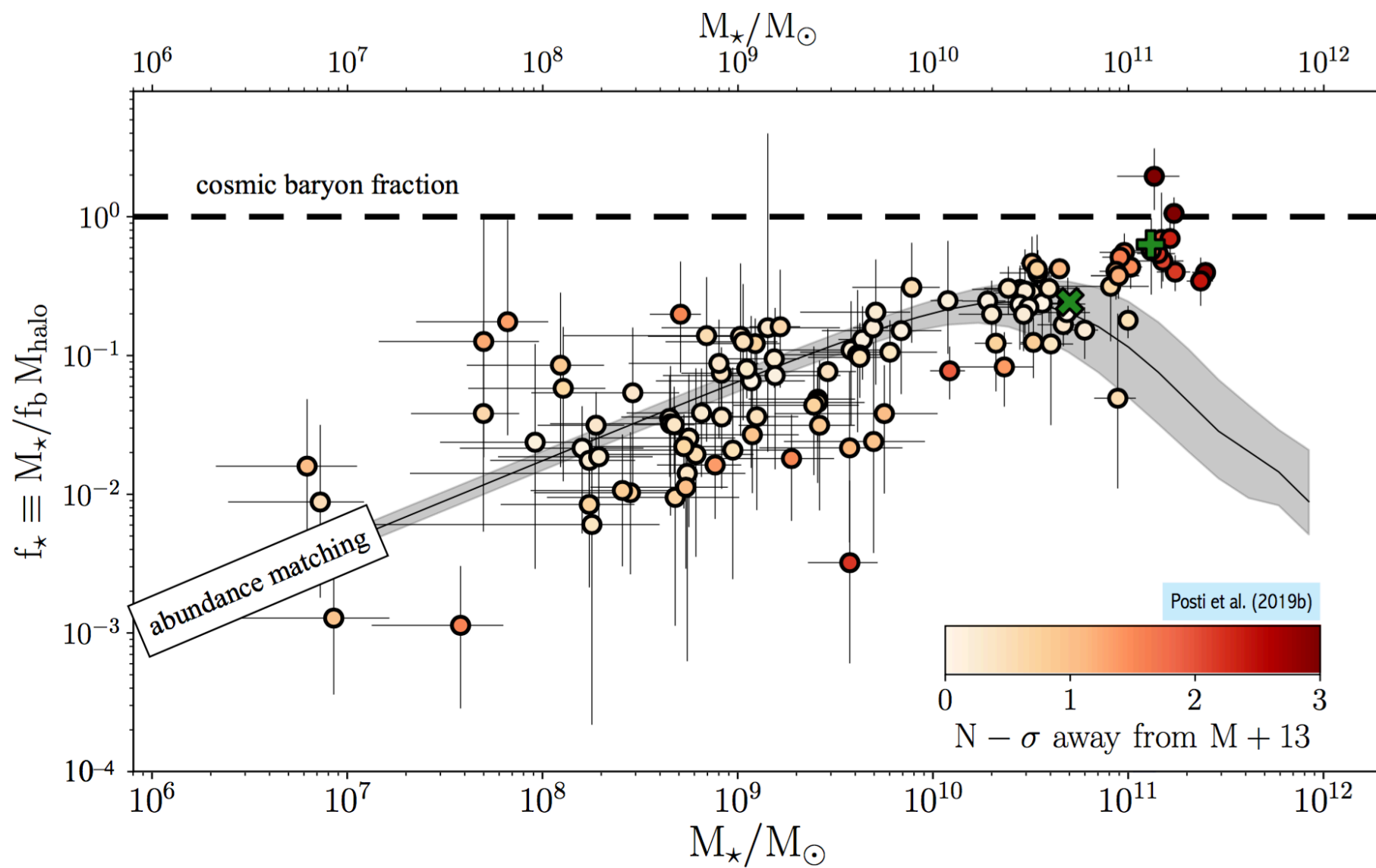
Roughly ok at low masses but AM predicts a tilt of the stellar TF relation (too large V_f at large masses)



Roughly ok at low masses but AM predicts a tilt of the stellar TF relation (too large V_f at large masses)

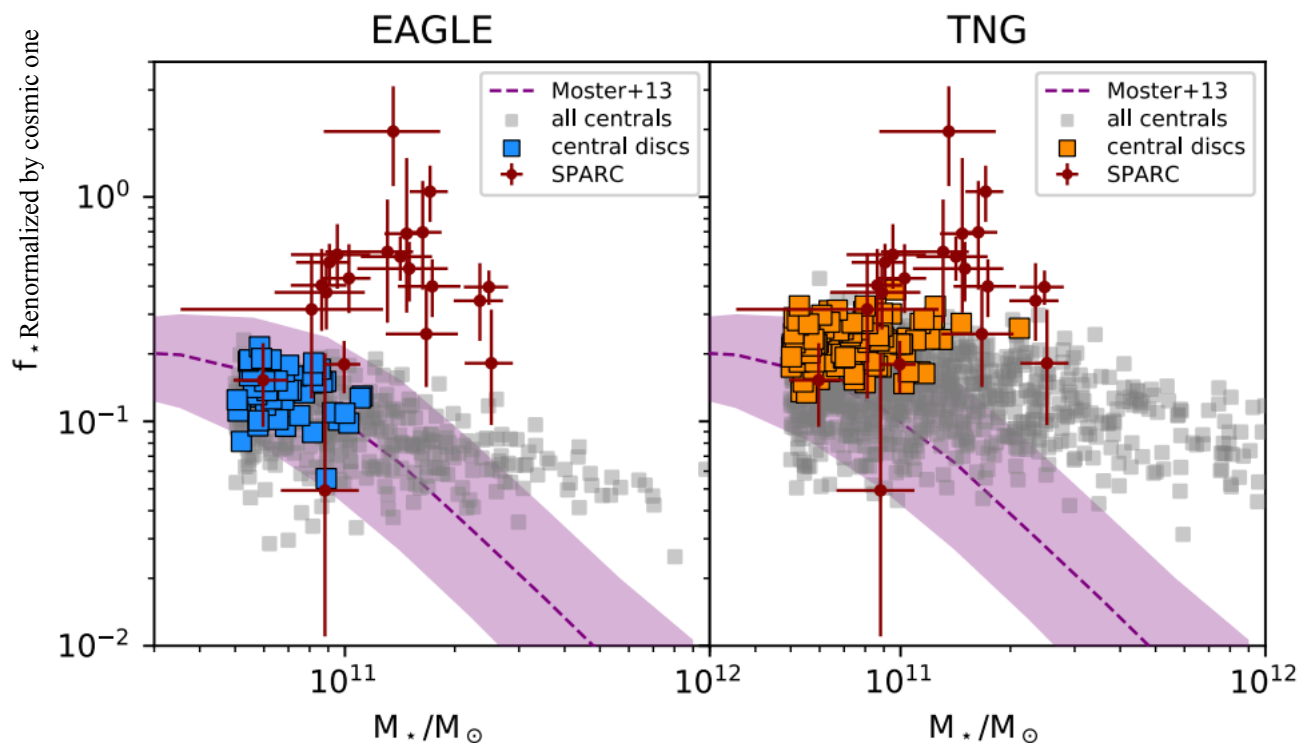
Even true when adding the newly discovered « super-spirals » (after a re-analysis of the RCs)

=> AM predicts massive disk galaxies to be too DM dominated



The failed feedback problem

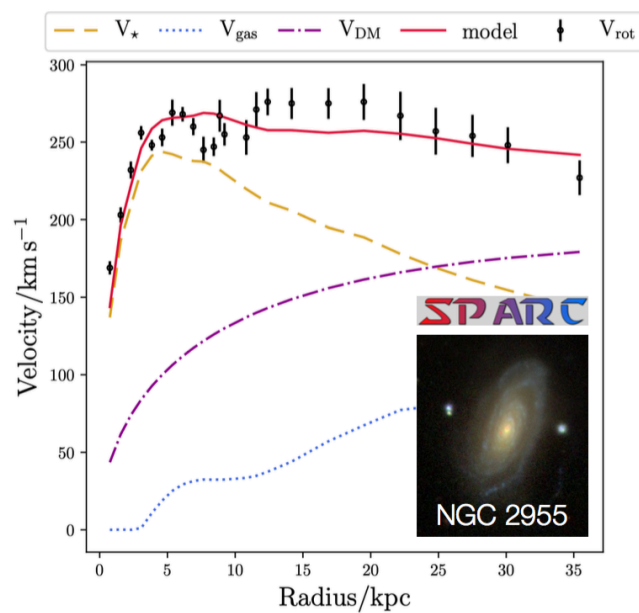
Particle DM mass resolution $< 10^7 M_{\text{sun}}$, **EAGLE** and **Illustris TNG100** allow for a fair evaluation of the behavior of massive disks in simulations



Marasco, Posti, Oman, Famaey, Cresci & Fraternali 2020

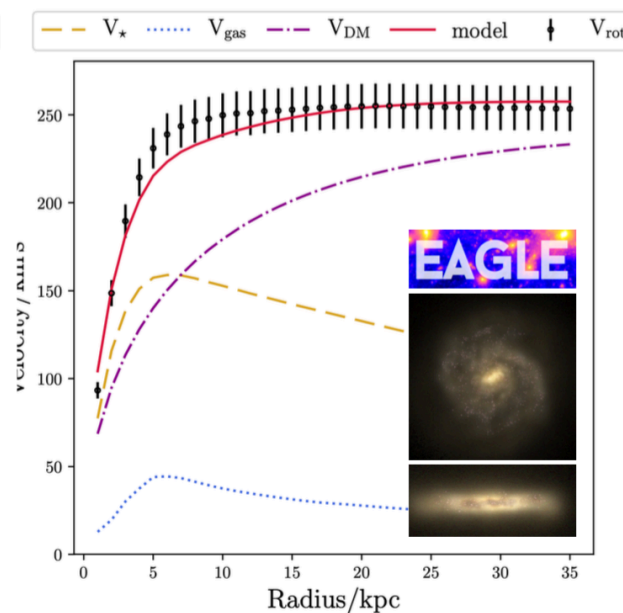
The failed feedback problem

Marasco, Posti et al. (2020)



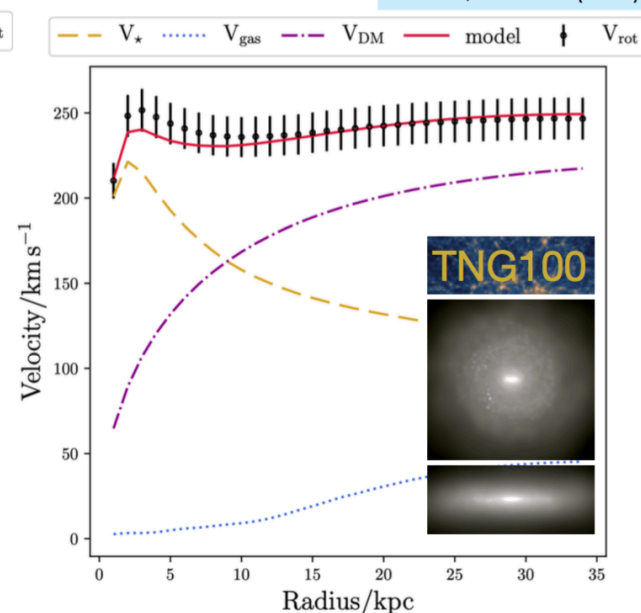
$$M_{\star} \sim 10^{11}$$

$$M_h \sim 10^{12}$$



$$M_{\star} \sim 7 \times 10^{10}$$

$$M_h \sim 2 \times 10^{12}$$

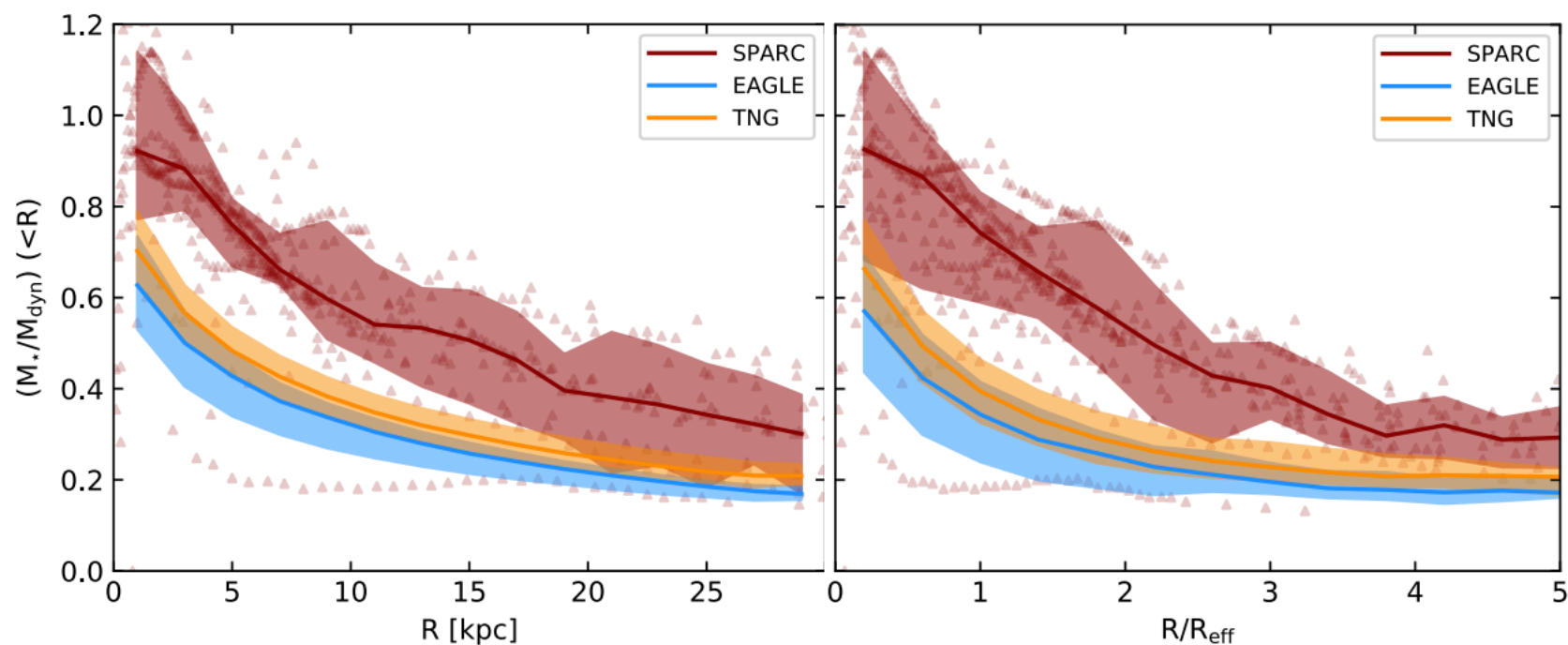


$$M_{\star} \sim 10^{11}$$

$$M_h \sim 2 \times 10^{12}$$

The failed feedback problem

Simulated halos hosting massive disks are too inefficient at converting their baryons into stars, through **too efficient feedback**, AND they have undergone halo contraction because of apparently **not efficient enough feedback**...



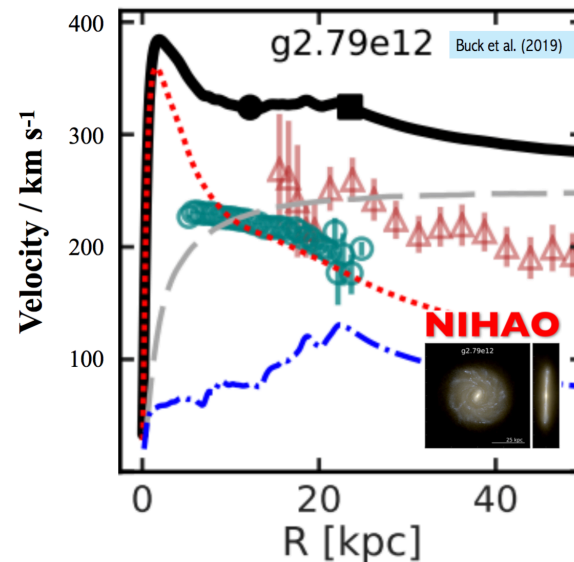


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Turning off AGN feedback can allow more baryons to cool down, but hard to avoid an overcooled bulge (back to some degree of angular momentum « catastrophe ») and to get the right gas fraction

The failed feedback problem



Turning off AGN feedback can allow more baryons to cool down, but hard to avoid an overcooled bulge (back to some degree of angular momentum « catastrophe ») and to get the right gas fraction

Auriga simulations seem to manage this, although at the expense of overly massive stellar halos



In summary:

Baryonic Tully-Fisher relation between baryonic mass of spiral galaxies and asymptotic velocity is captured by Milgrom's relation

Abundance Matching helps explaining the slope,

but

Problem at the high-mass end (failed feedback problem)

AM-predicted scatter at least twice too high

And... there is more to Milgrom's relation: the **shape** of RCs



“We predict a correlation between the value of the average surface density of a galaxy and the steepness with which the rotational velocity rises to its asymptotic value.”


Milgrom (1983)

Illustration:

Consider two fully dark matter dominated exponential disks of the same mass in the low ($<a_0$) acceleration regime

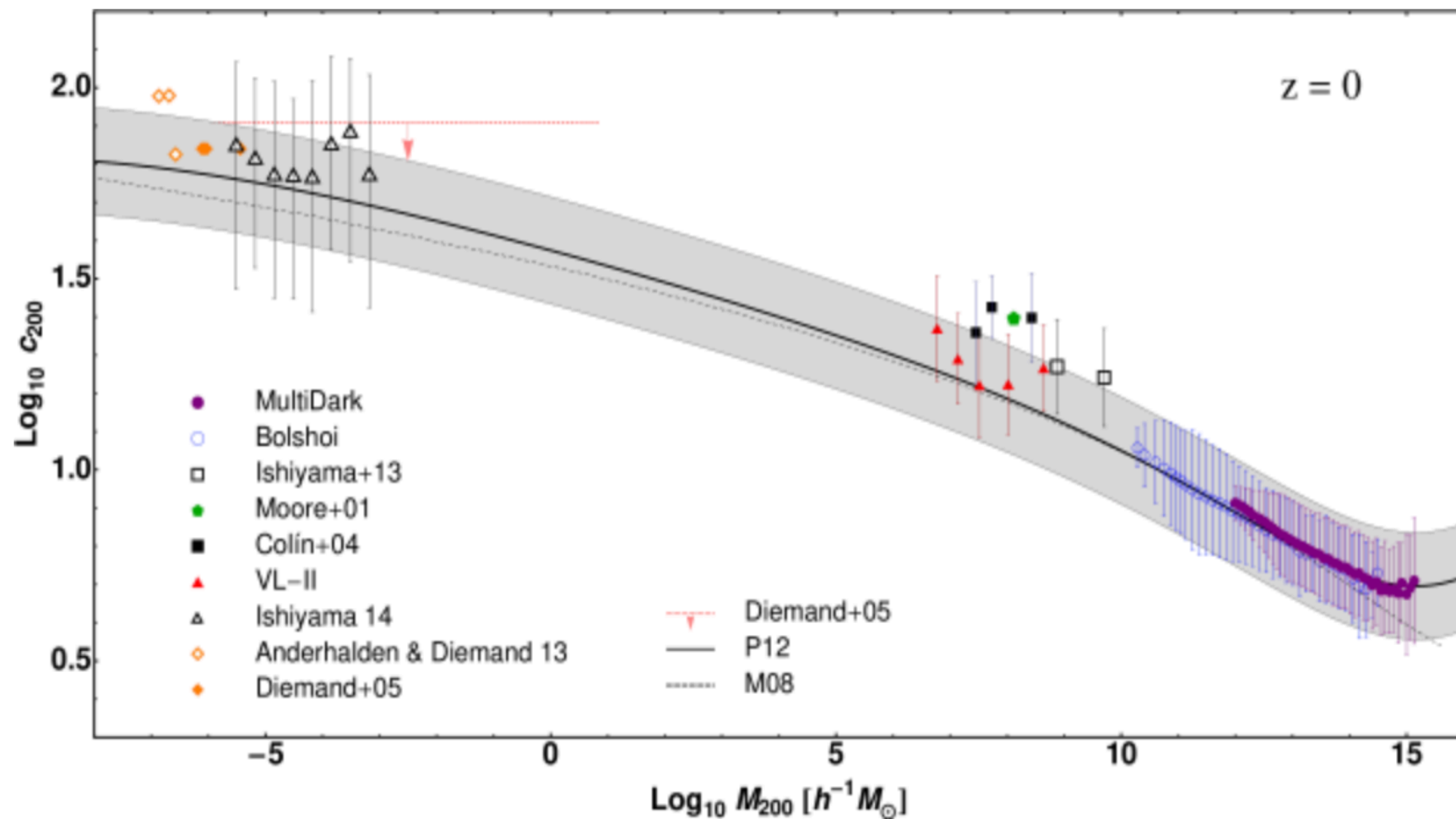
They are BTFR « twins », but is the rotation curve shape always the same?

$$g = (g_n a_0)^{1/2} = g_n (a_0/g_n)^{1/2}$$

- 
- Two exponential disks of **same** baryonic mass M_b in the low acceleration regime but **different** scale-length L
(central surface density = $M_b/2\pi L^2$)
 - $M_b(\lambda L)$ **identical**
 - $g_n(\lambda L) \sim G M_b(\lambda L) / (\lambda L)^2 \sim (\lambda L)^{-2}$
 - $V_{cb}^2(\lambda L) \sim G M_b(\lambda L) / \lambda L \sim (\lambda L)^{-1}$
 - If boost of gravity due to DM at $R=\lambda L$ is prop. to $1/\sqrt{g_n}$
(hence prop. to λL)

then $V_c(\lambda L)$ **identical** $\Rightarrow V_1(R) = V_2((L_2/L_1)R)$

Not a priori expected in LCDM



Dark matter halos are (almost) a one-parameter family (driven by mass)

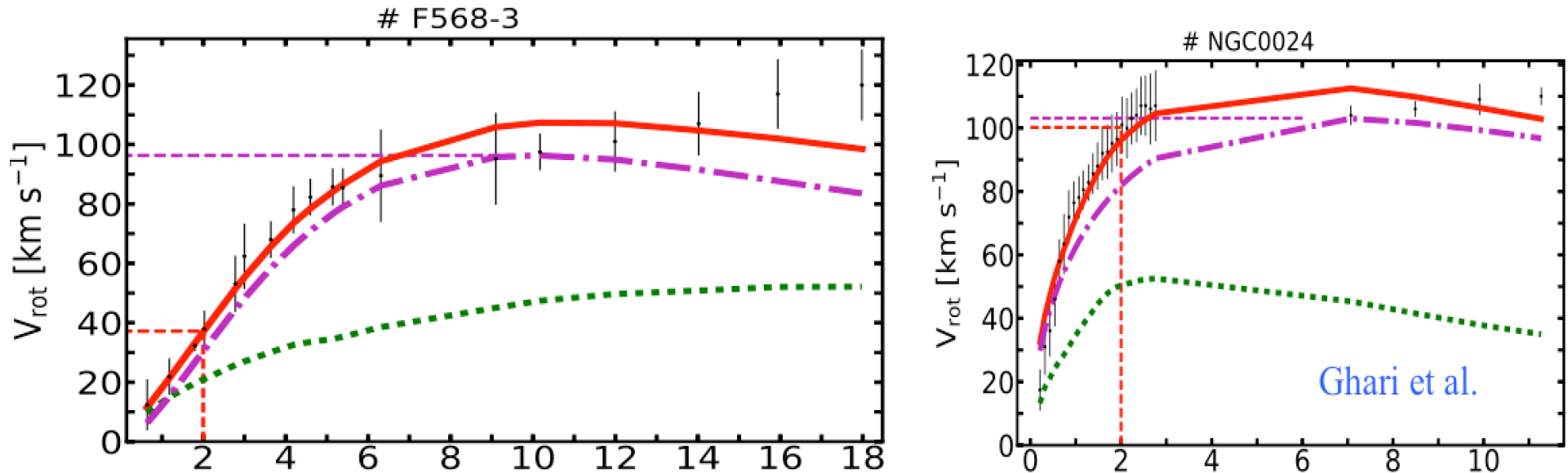


The rotation curves shapes of late-type dwarf galaxies

R. A. Swaters^{1,2,★}, R. Sancisi^{3,4}, T. S. van Albada³, and J. M. van der Hulst³ (2009)

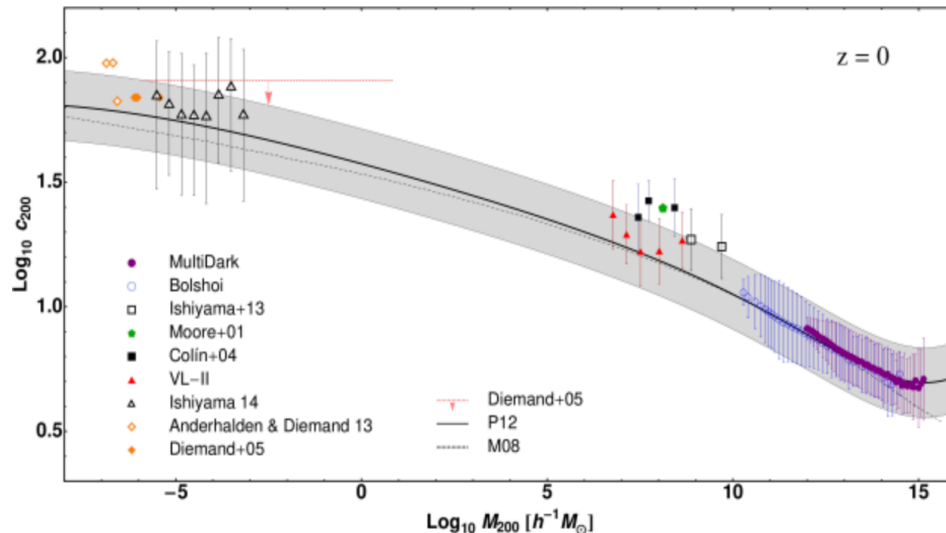
HI observations for a sample of 62 galaxies [...] procedure takes the rotation curve shape, the HI distribution, the inclination, and the size of the beam into account, and makes it possible to correct for the effects of beam smearing.

In spiral galaxies and even in the central regions of late-type dwarf galaxies, the shape of the central distribution of light and the inner rise of the rotation curve are related. This implies that galaxies with stronger central concentrations of light also have higher central mass densities, and it suggests that the luminous mass dominates the gravitational potential in the central regions, even in low surface brightness dwarf galaxies (NB: dominated by... dark matter?!)



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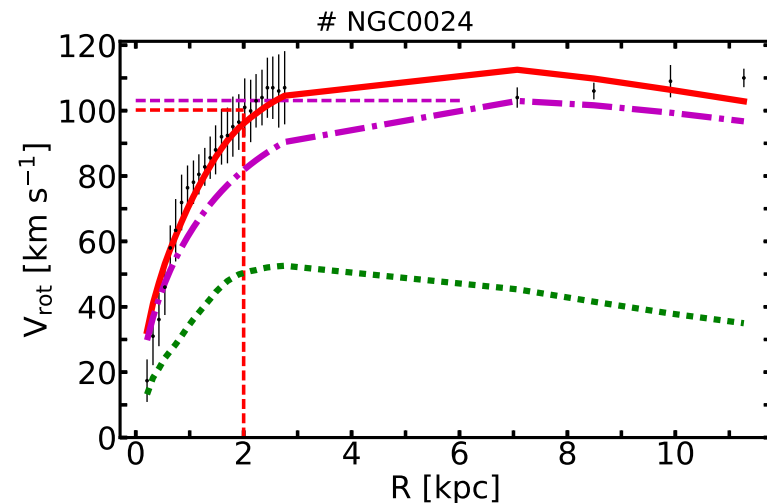
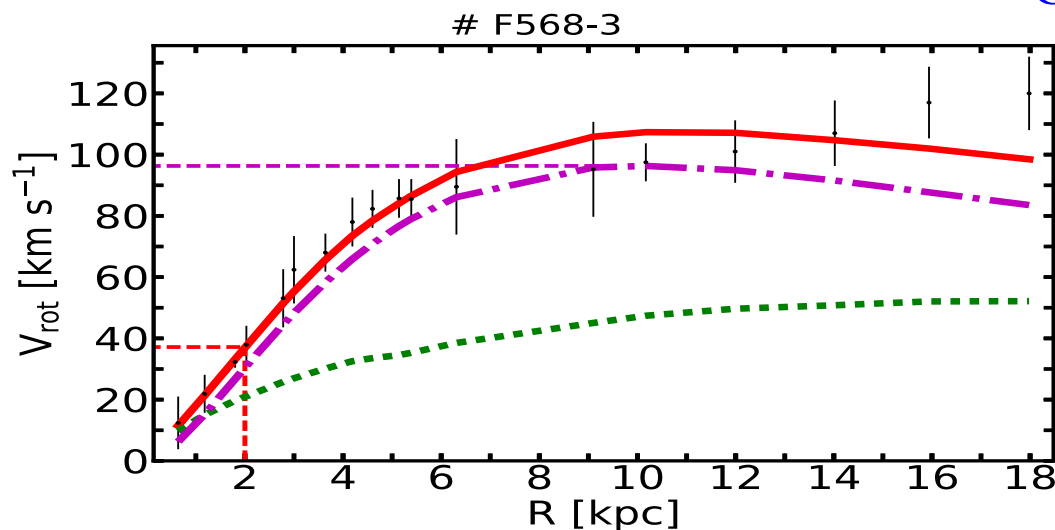
The BTFR twin paradox of LCDM

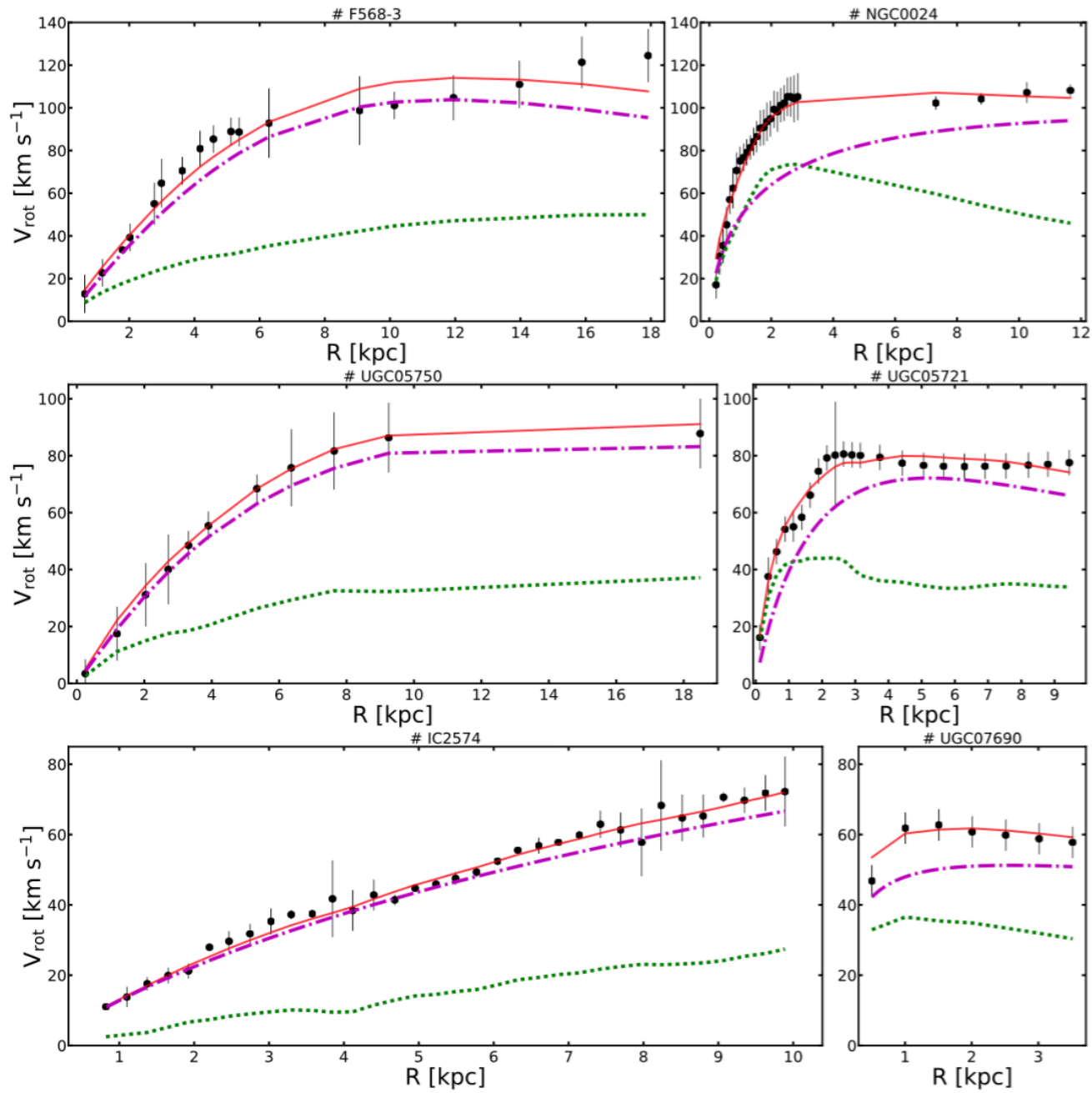


Dark matter halos are (almost) a one-parameter family (driven by mass)

=> At the same V_{flat} , why so different profiles??

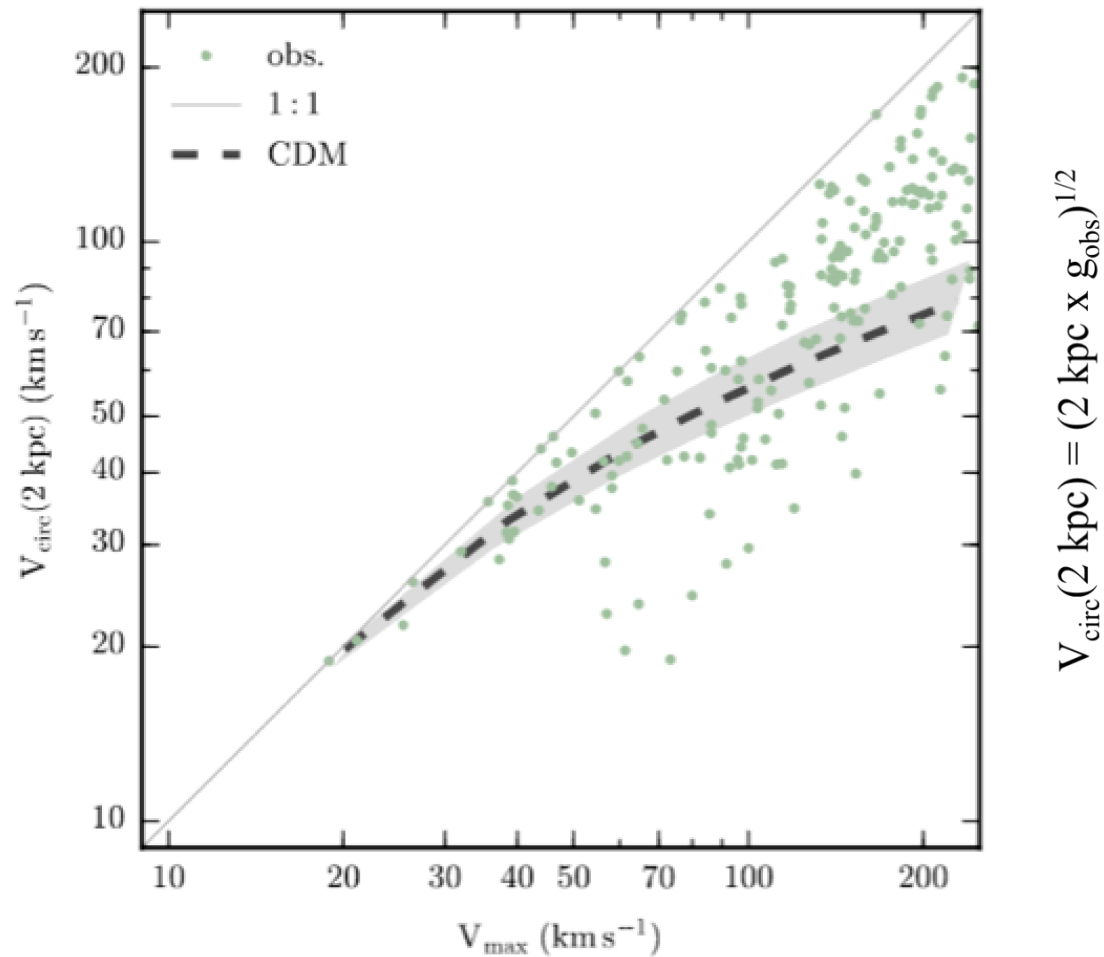
Ghari, Famaey, Laporte & Haghi 2019





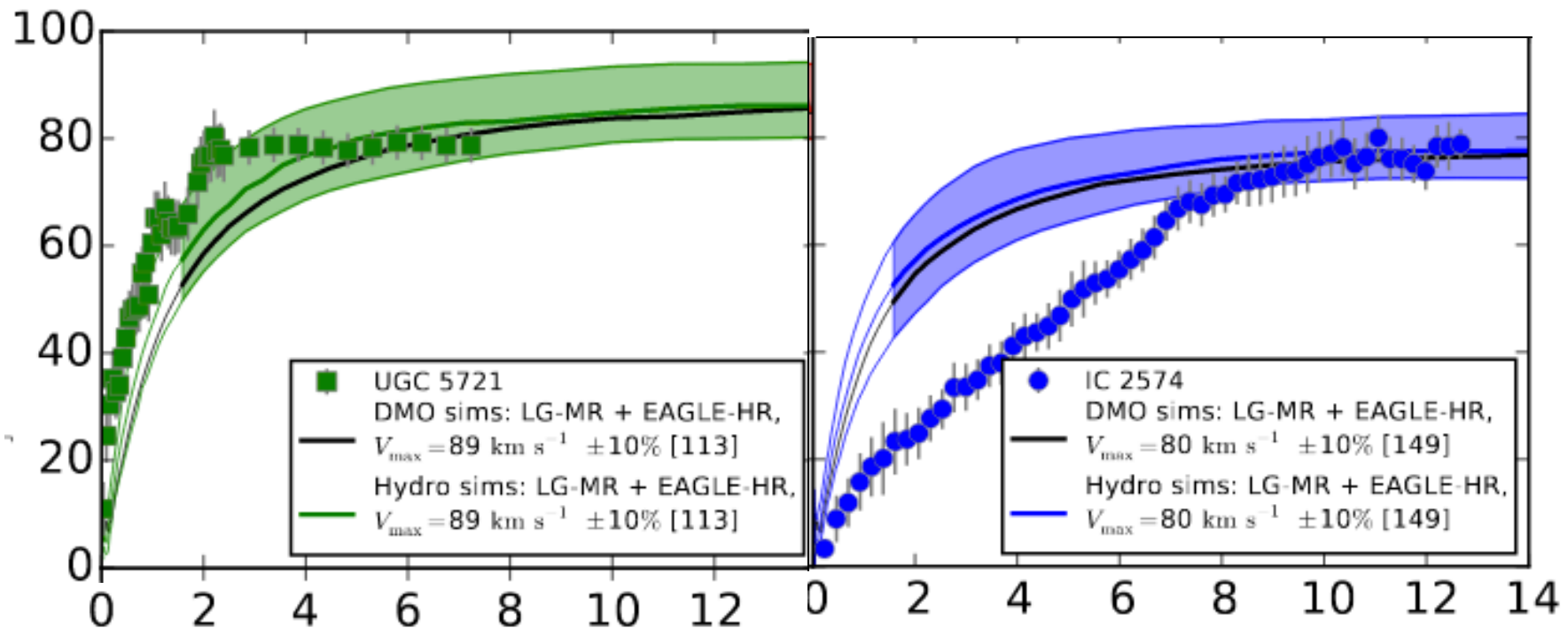
Ghari et al.
(2019)

Also called the diversity problem



Oman et al. 2015, Bullock & Boylan-Kolchin 2017

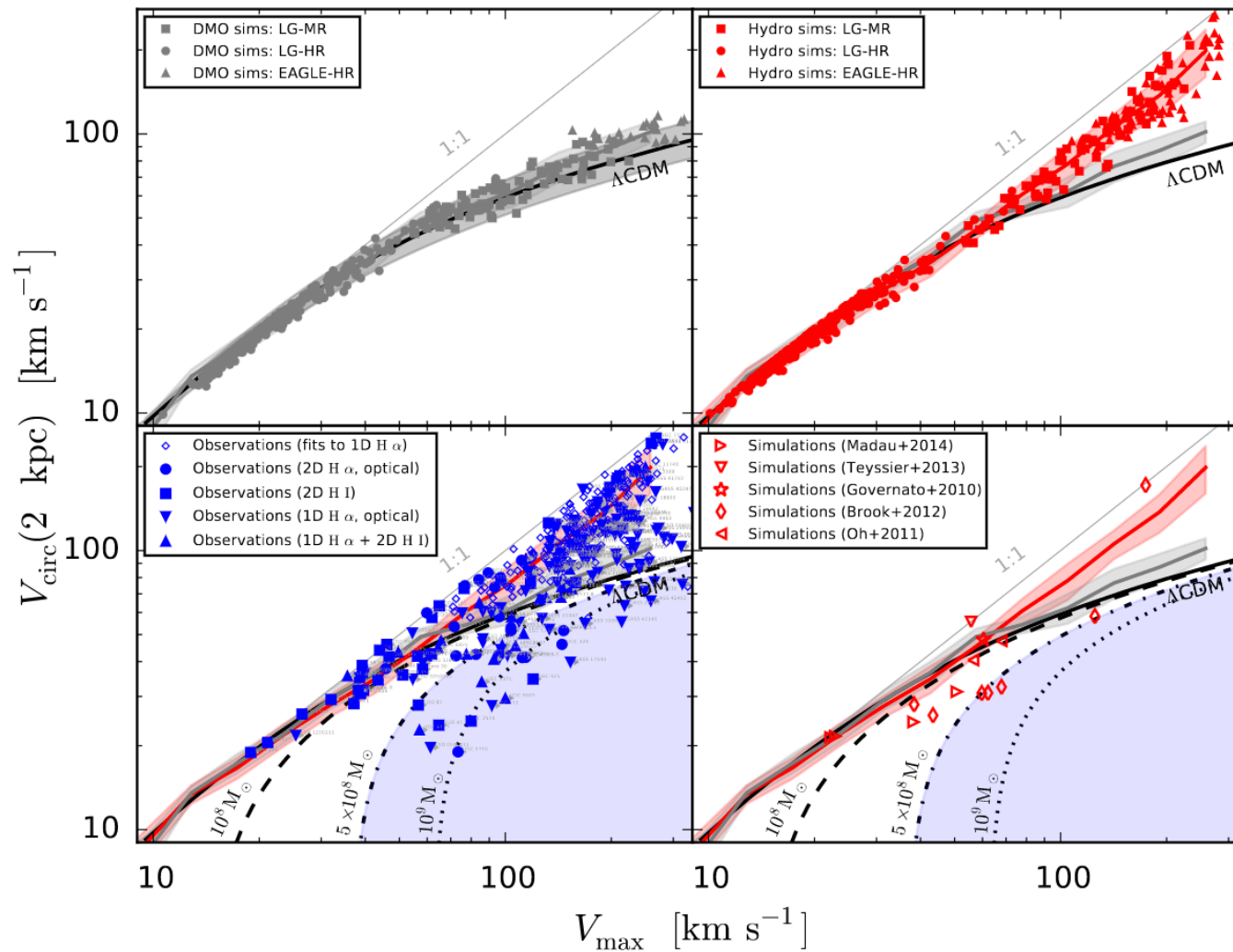
The diversity problem or the modern core-cusp problem

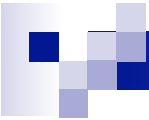


APOSTLE/EAGLE simulations
=> cannot form large cores

Oman et al. 2015

Diversity of RC profiles at given V_{\max} scale





The unexpected diversity of dwarf galaxy rotation curves

Kyle A. Oman^{1,*}, Julio F. Navarro^{1,2}, Azadeh Fattahi¹, Carlos S. Frenk³,
Till Sawala³, Simon D. M. White⁴, Richard Bower³, Robert A. Crain⁵,
Michelle Furlong³, Matthieu Schaller³, Joop Schaye⁶, Tom Theuns³

¹ Department of Physics & Astronomy, University of Victoria, Victoria, BC, V8P 5C2, Canada

² Senior CIFAR Fellow

³ Institute for Computational Cosmology, Department of Physics, University of Durham, South Road, Durham DH1 3LE, United Kingdom

⁴ Max-Planck Institute for Astrophysics, Garching, Germany

⁵ Astrophysics Research Institute, Liverpool John Moores University, IC2, Liverpool Science Park, 146 Brownlow Hill, Liverpool, L3 5RF, United Kingdom

⁶ Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, the Netherlands

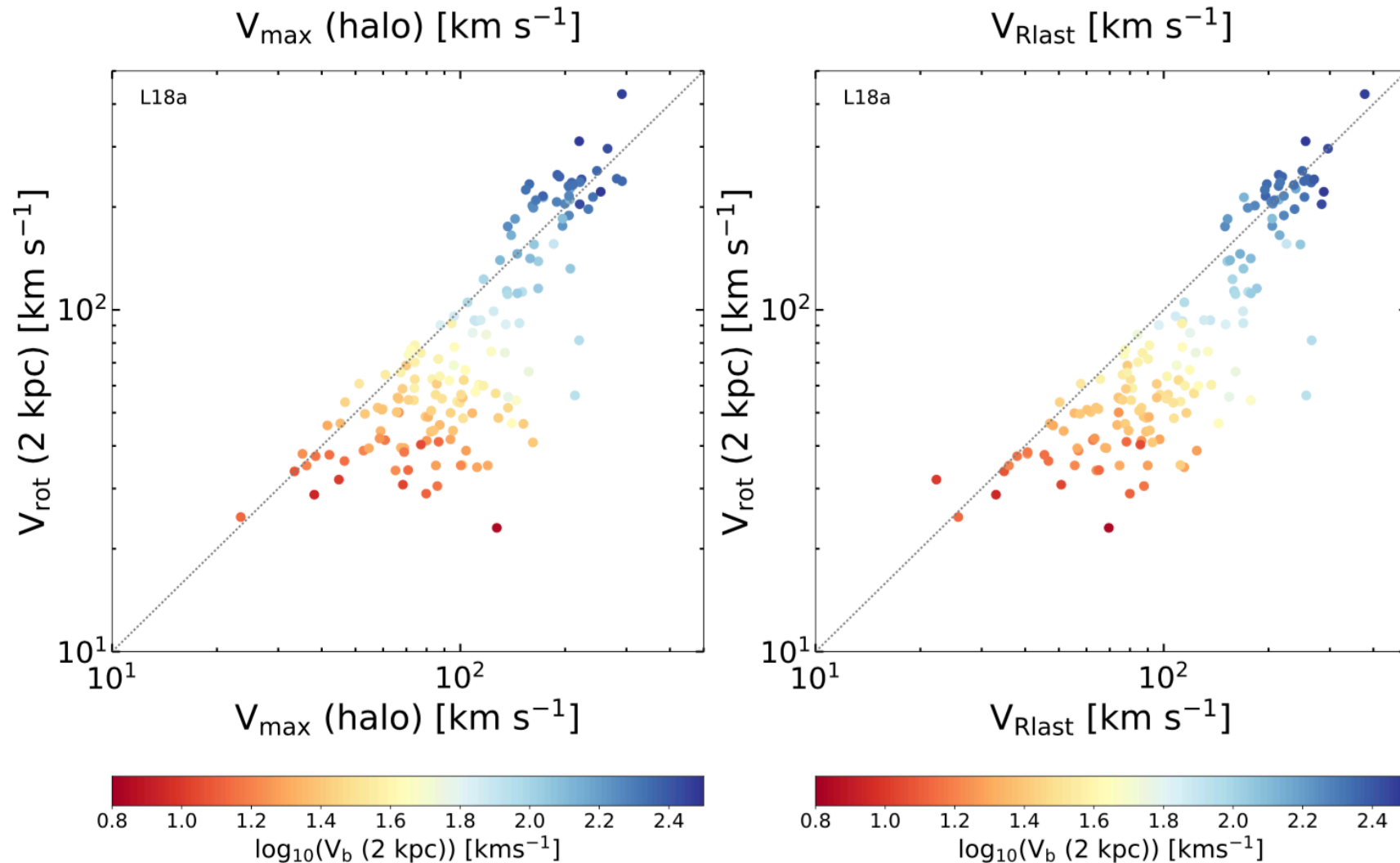
We conclude that one or more of the following statements must be true: (i) the dark matter is more complex than envisaged by any current model; (ii) current simulations fail to reproduce the diversity in the effects of baryons on the inner regions of dwarf galaxies; and/or (iii) the mass profiles of “inner mass deficit” galaxies inferred from kinematic data are incorrect.

...ing as a function of galaxy mass, but shows remarkable diversity at fixed maximum circular velocity. This is especially true for low-mass dark matter-dominated systems, reflecting the expected similarity of the underlying cold dark matter haloes. This is at odds with observed dwarf galaxies, which show a large diversity of rotation curve shapes, even at fixed maximum rotation speed. Some dwarfs have rotation curves that agree well with simulations, others do not. The latter are systems where the inferred mass enclosed in the inner regions is much lower than expected for cold dark matter haloes and include many galaxies where previous work claims the presence of a constant density “core”. The “cusp vs core” issue is thus better characterized as an “inner mass deficit” problem than as a density slope mismatch. For several galaxies the magnitude of this inner mass deficit is well in excess of that reported in recent simulations where cores result from baryon-induced fluctuations in the gravitational potential.

We conclude that one or more of the following statements must be true: (i) the dark matter is more complex than envisaged by any current model; (ii) current simulations fail to reproduce the diversity in the effects of baryons on the inner regions of dwarf galaxies; and/or (iii) the mass profiles of “inner mass deficit” galaxies inferred from kinematic data are incorrect.

Key words: dark matter, galaxies: structure, galaxies: haloes

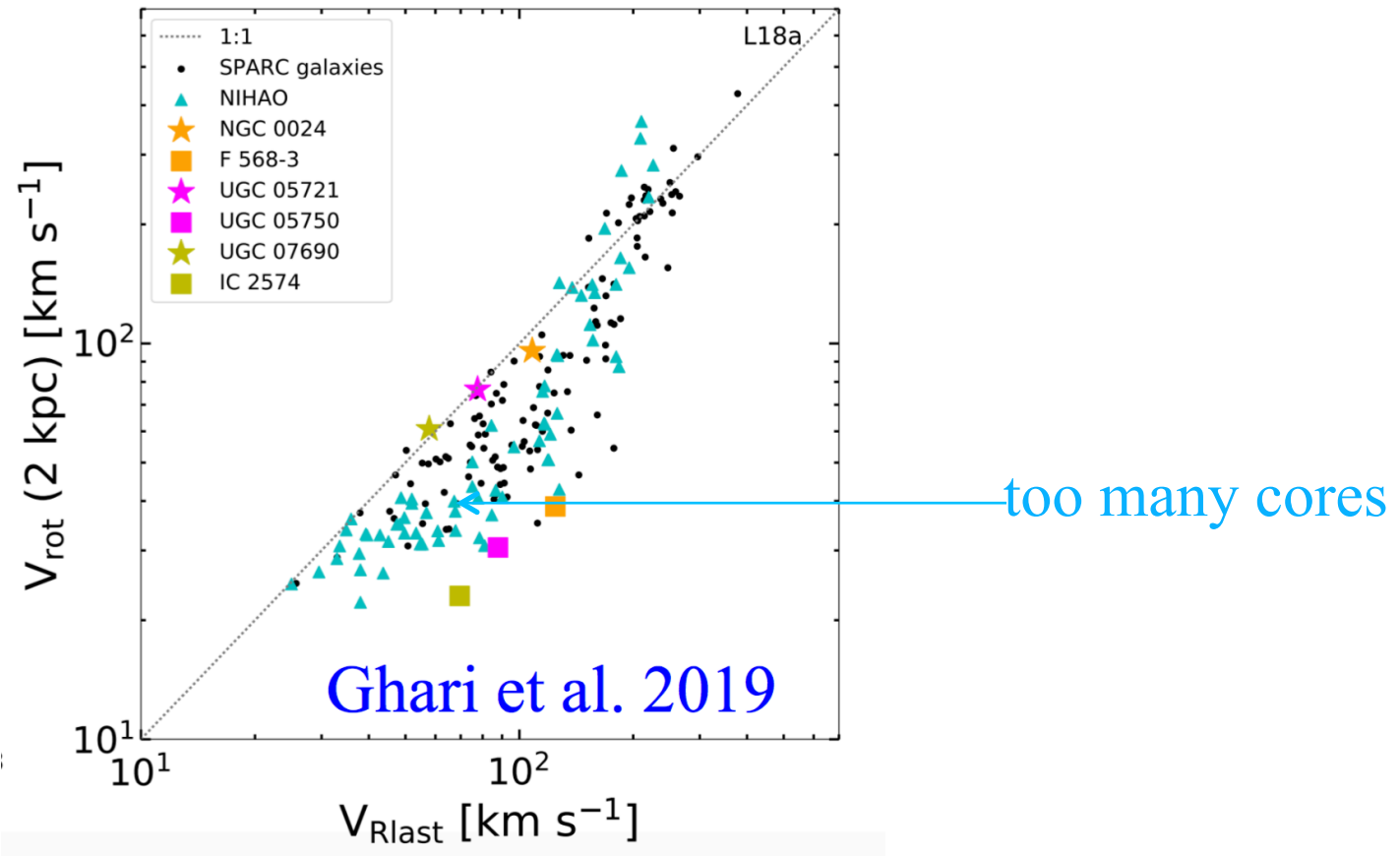
Diversity driven by the baryons



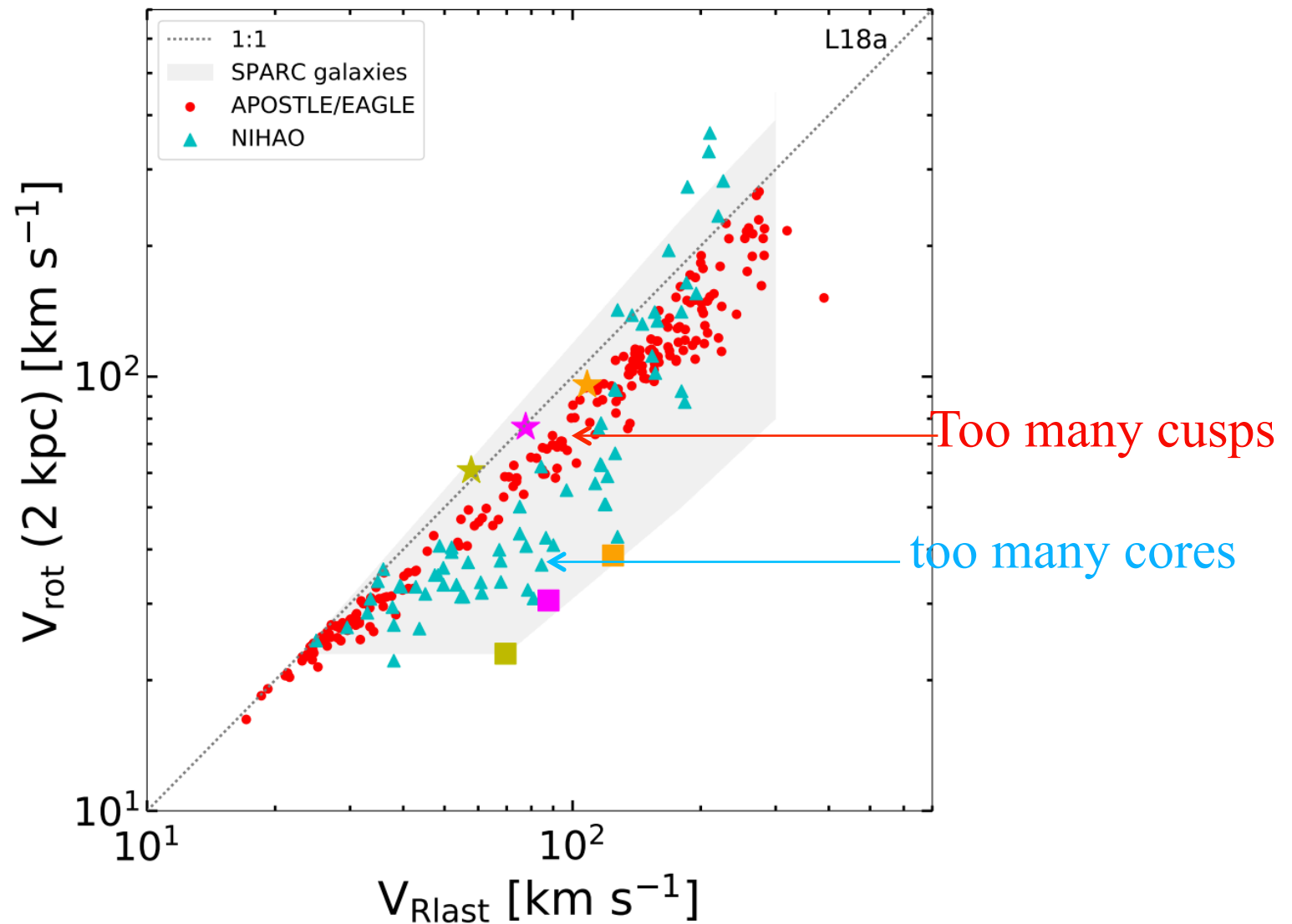
Ghari, Famaey, Laporte & Haghi (2019)

Does core creation solve the diversity problem?

NIHAO has a rather extreme feedback recipe, leading to too many **cores** at low masses :



More than just the old core-cusp





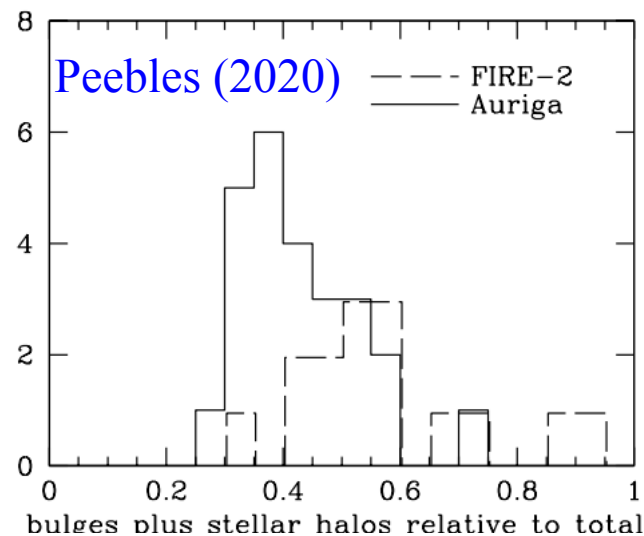
In summary:

Baryonic Tully-Fisher relation between baryonic mass of spiral galaxies and asymptotic velocity is captured by Milgrom's relation, while the high-end slope and the scatter remain challenging

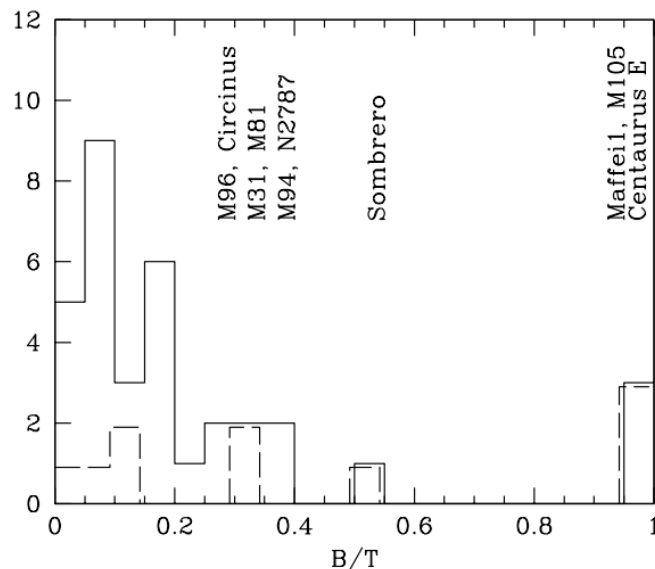
The diversity of RC shapes driven by the surface density of baryons is also captured by Milgrom's relation, and remains challenging for simulations that either produce too few or too many cores

Let's now move to other challenges more independent of Milgrom's relation

Do simulated galaxies look like real ones? The hot orbits problem



- Most local disk galaxies are nearly **bulgeless with light stellar halos**
- The only zoom simulations avoiding the formation of too massive bulges do so at the expense of overly massive stellar halos
- Typically almost half of the orbits have $L_z/L_c < 0.7$ in simulations



Partly due to too much substructures falling onto the galaxy while it forms (too many mergers) but also dynamical friction



Do simulated galaxies look like real ones? The bar problem

- Most local disk galaxies are nearly **bulgeless** with light stellar halos
- Moreover, **70% are barred** at $M_* \sim 10^{10} M_{\text{sun}}$ (Erwin 2018)

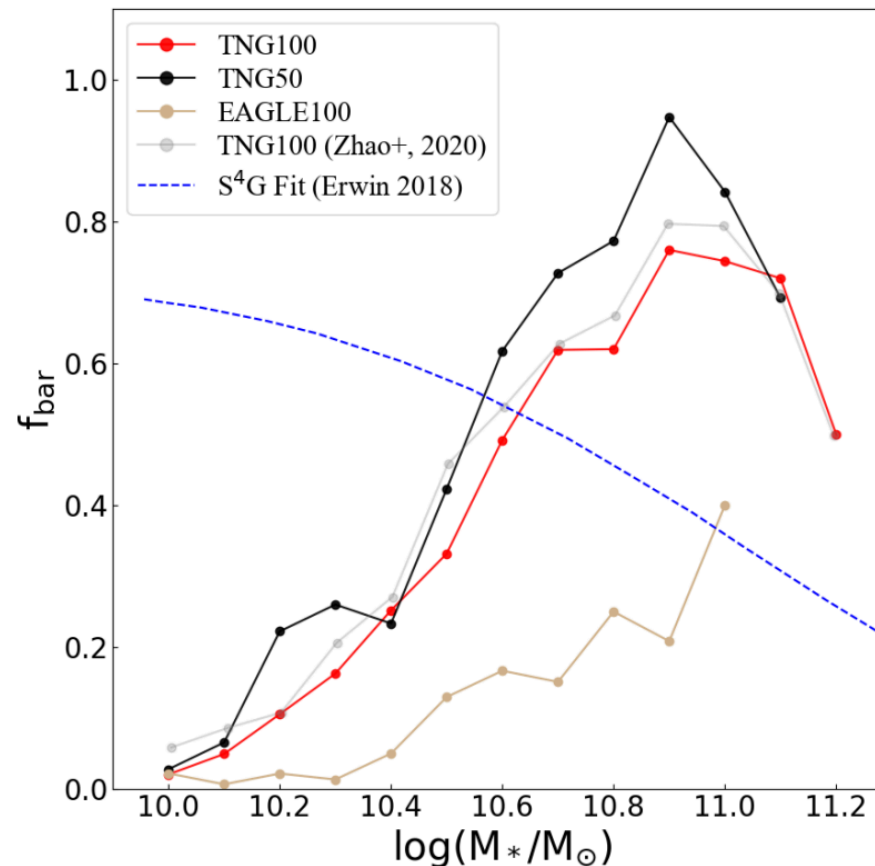
However, all large-box cosmological simulations with high spatial resolution of the order of 100 pc fail to form enough bars.

E.g., TNG50: softening length of 288 pc ($m_{\text{baryon}} = 8.5 \times 10^4 M_{\text{sun}}$), NewHorizons: maximum resolution of 34 pc ($m_{\text{star}} = 1.3 \times 10^4 M_{\text{sun}}$)

Galaxy unbarred if $A_{2\text{max}} < 0.2$ in Fourier decomposition

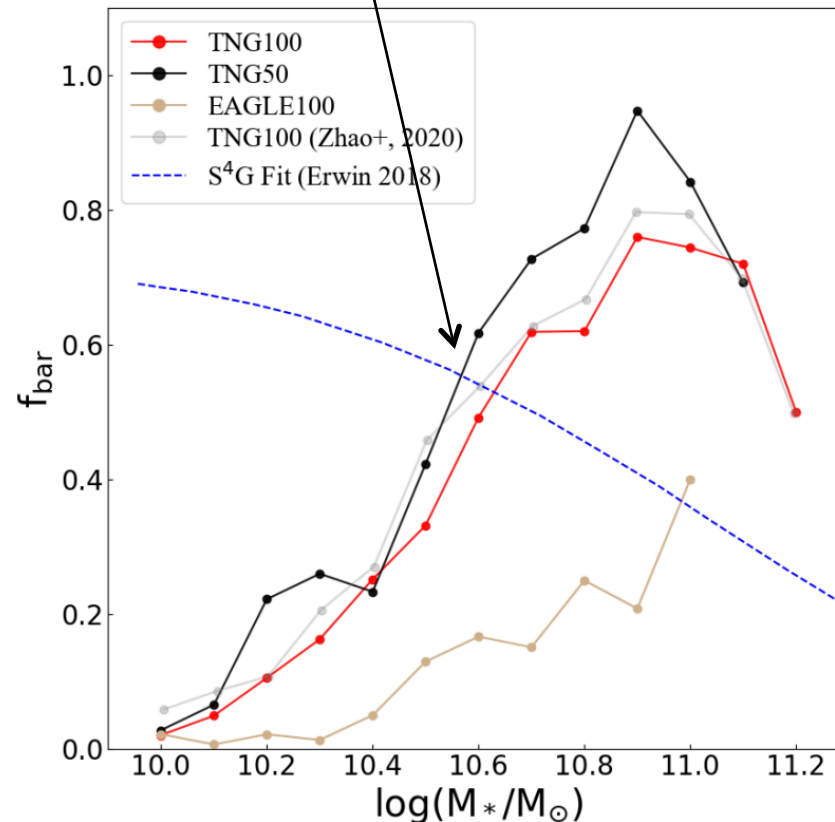
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Tremaine-Weinberg method:

$$\frac{\partial \Sigma(x, y, t)}{\partial t} + \frac{\partial}{\partial x} [\Sigma(x, y, t) v_x(x, y, t)] + \frac{\partial}{\partial y} [\Sigma(x, y, t) v_y(x, y, t)] = 0,$$



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Tremaine-Weinberg method:

$$\Sigma(x, y, t) = \tilde{\Sigma}(r, \phi - \Omega_p t)$$

$$\frac{\partial \Sigma}{\partial t} = -\Omega_p \frac{\partial \tilde{\Sigma}}{\partial \phi} = \Omega_p \left(y \frac{\partial \Sigma}{\partial x} - x \frac{\partial \Sigma}{\partial y} \right)$$

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Tremaine-Weinberg method:

$$\underbrace{\Omega_p y \int_{-\infty}^{\infty} \frac{\partial \Sigma}{\partial x} dx - \Omega_p \int_{-\infty}^{\infty} x \frac{\partial \Sigma}{\partial y} dx}_{=0} + \underbrace{\int_{-\infty}^{\infty} \frac{\partial(\Sigma v_x)}{\partial x} dx}_{=0} + \int_{-\infty}^{\infty} \frac{\partial(\Sigma v_y)}{\partial y} dx = 0.$$



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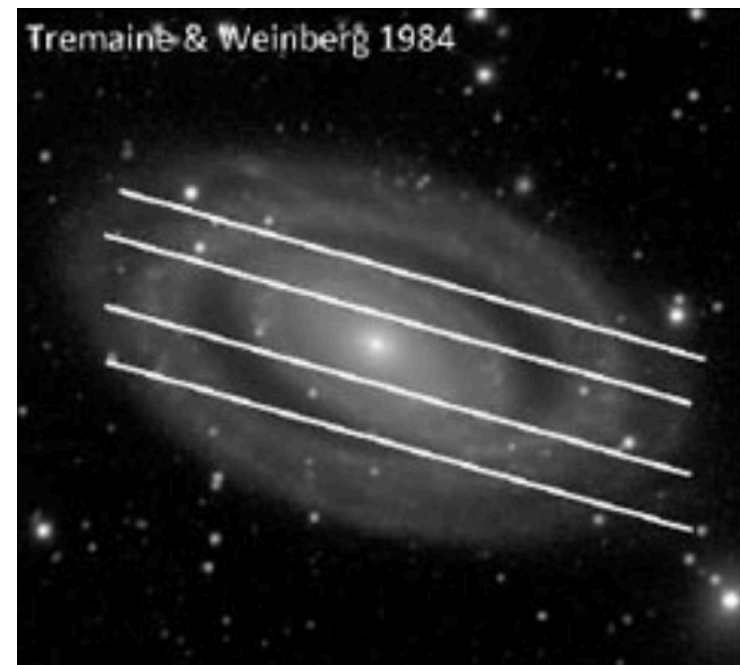
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Tremaine-Weinberg method:

$$\langle V \rangle \equiv \frac{\int V_{\text{LOS}} \Sigma dX}{\int \Sigma dX}$$

$$\langle X \rangle \equiv \frac{\int X \Sigma dX}{\int \Sigma dX}$$

$$\boxed{\Omega_p \sin i = \langle V \rangle / \langle X \rangle}$$





Do simulated galaxies look like real ones? The bar problem

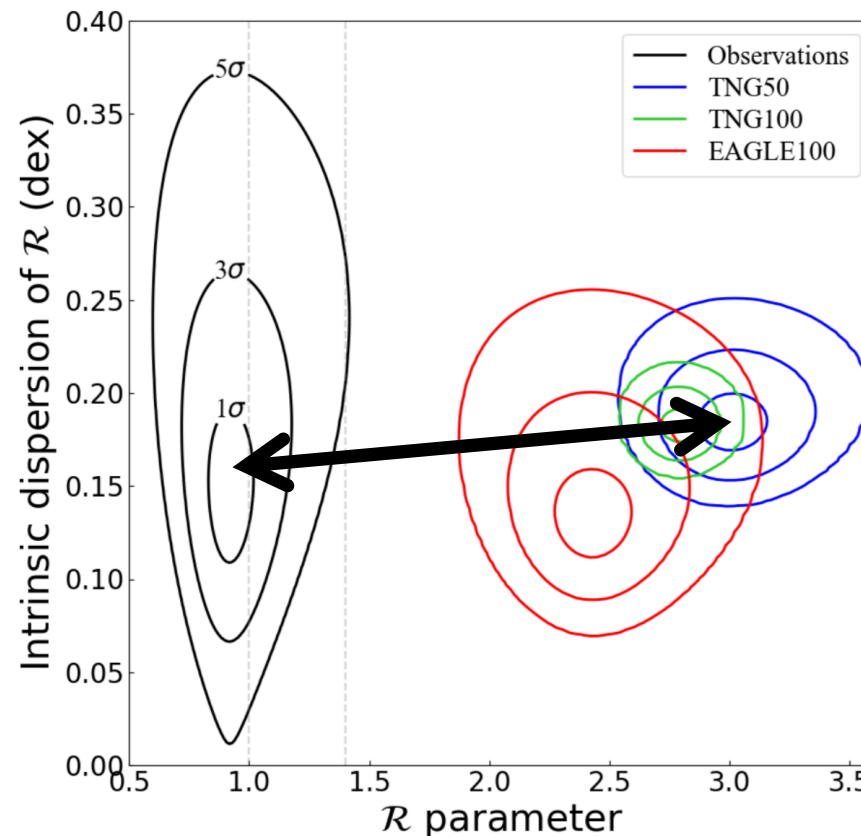
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More than 100 galaxies from (long-slit or IFU) spectroscopy analyzed with Tremaine-Weinberg method (Cuomo et al. 2020):

All consistent with being fast

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Roshan et al. 2021



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The only simulations with fast bars are the zoom-in simulations in Auriga, avoiding too heavy bulges (at the expense of overly massive stellar halos) and with stellar disk masses well above abundance matching (Fragkoudi et al. 2021)

Total sample of 30 galaxies, 16 barred, difficult to assess the consequences on galaxy statistics such as luminosity function etc., difficult to hold results at lower galaxy masses



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In summary:

The heavy bulges and DM halos in high-resolution large-box simulations tend to prevent bar formation in the right amount

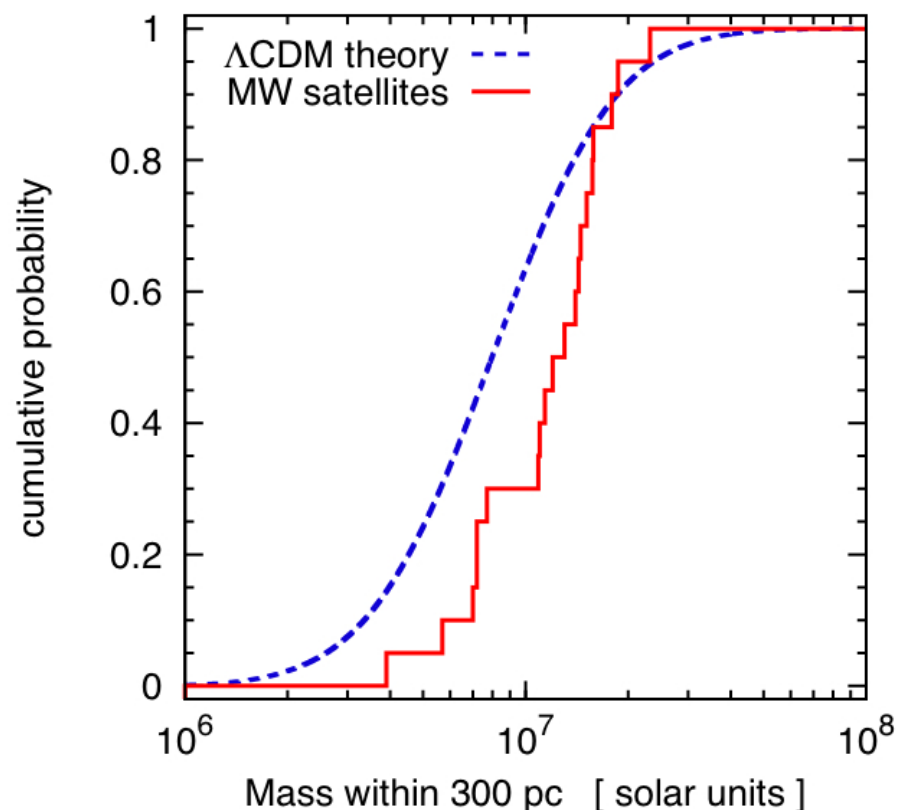
When bars form, their pattern speeds are generally too low when compared to observed ones, owing to dynamical friction with the DM halo

Reducing dynamical friction by either reducing the DM fraction (failed feedback+core-cusp) or reducing dynamical friction itself (through the nature of DM) is the way to solve this

Simulations that solve the problem have too heavy stellar halos

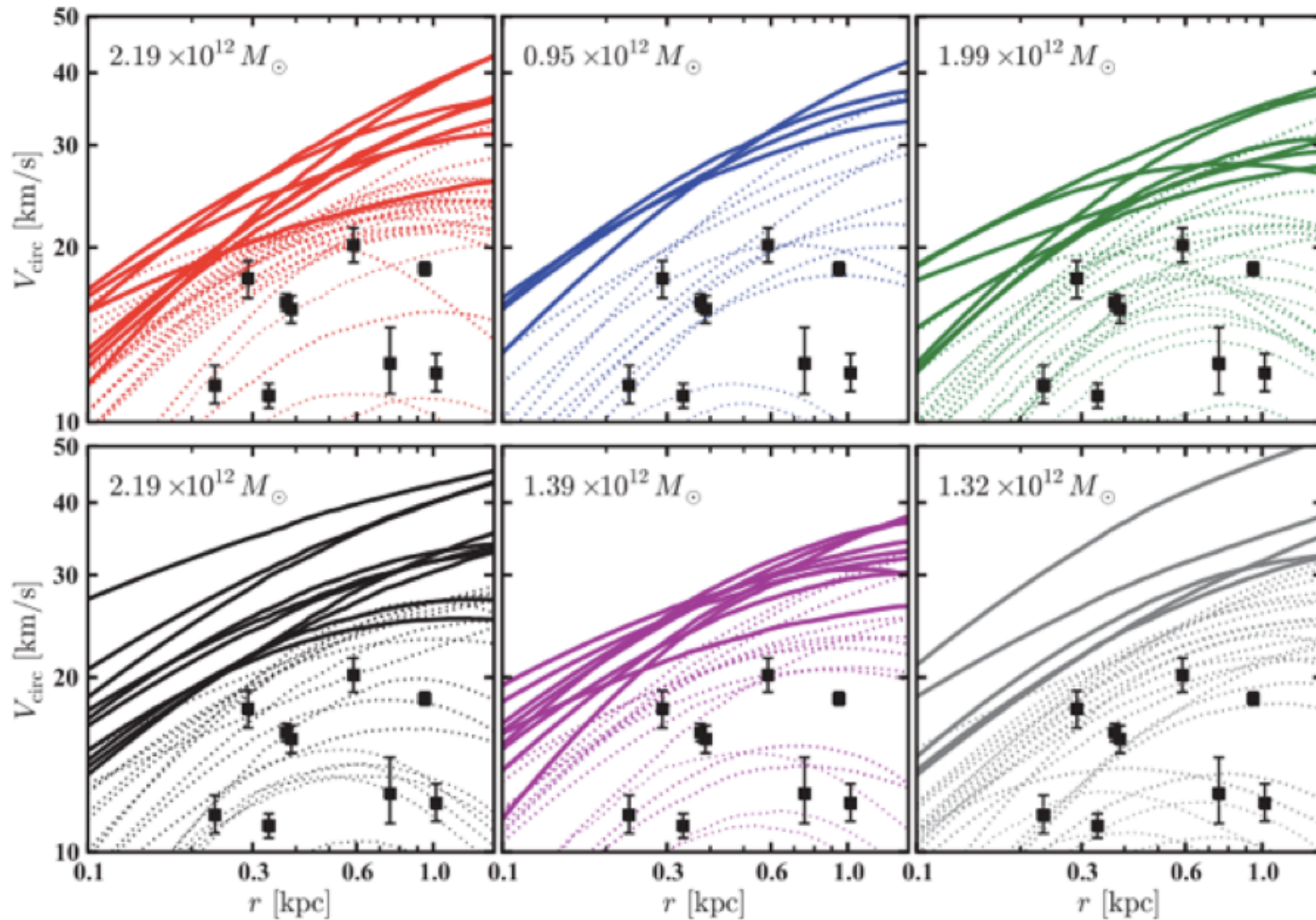
Dwarf spheroidals: missing satellites?

- This has never really been a problem, as Λ CDM already indicates that low-mass halos are increasingly unlikely to form stars
- Reionization likely suppresses gas accretion below $10^9 M_{\text{sun}}$
- However, the most **massive** ones seem to be missing



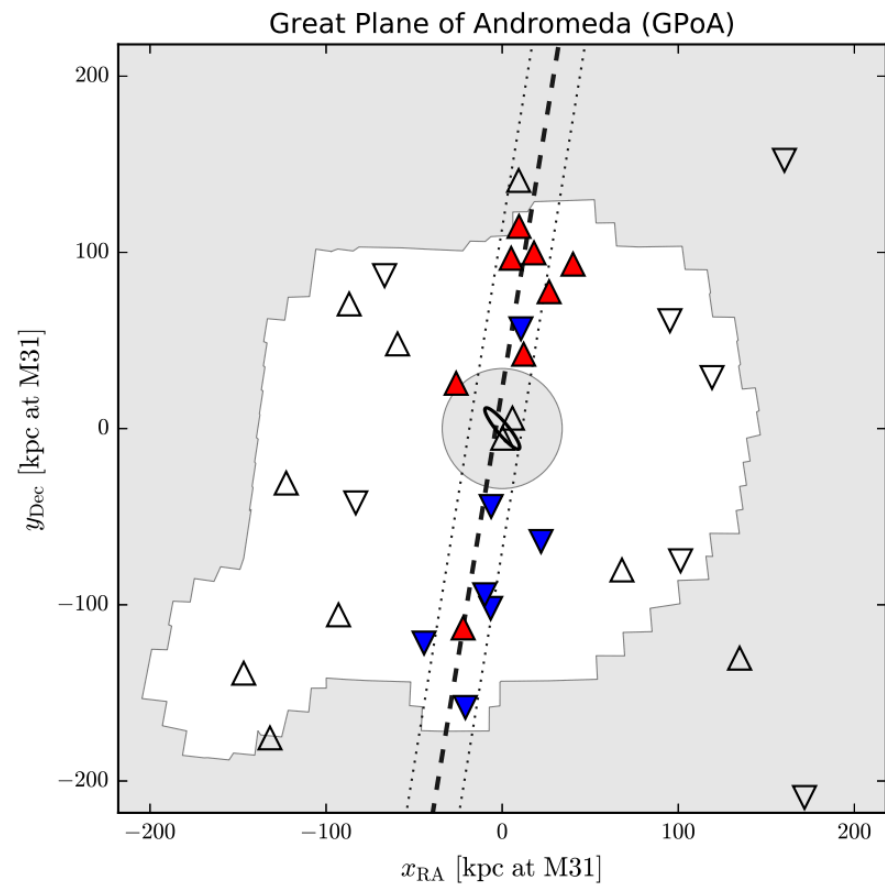
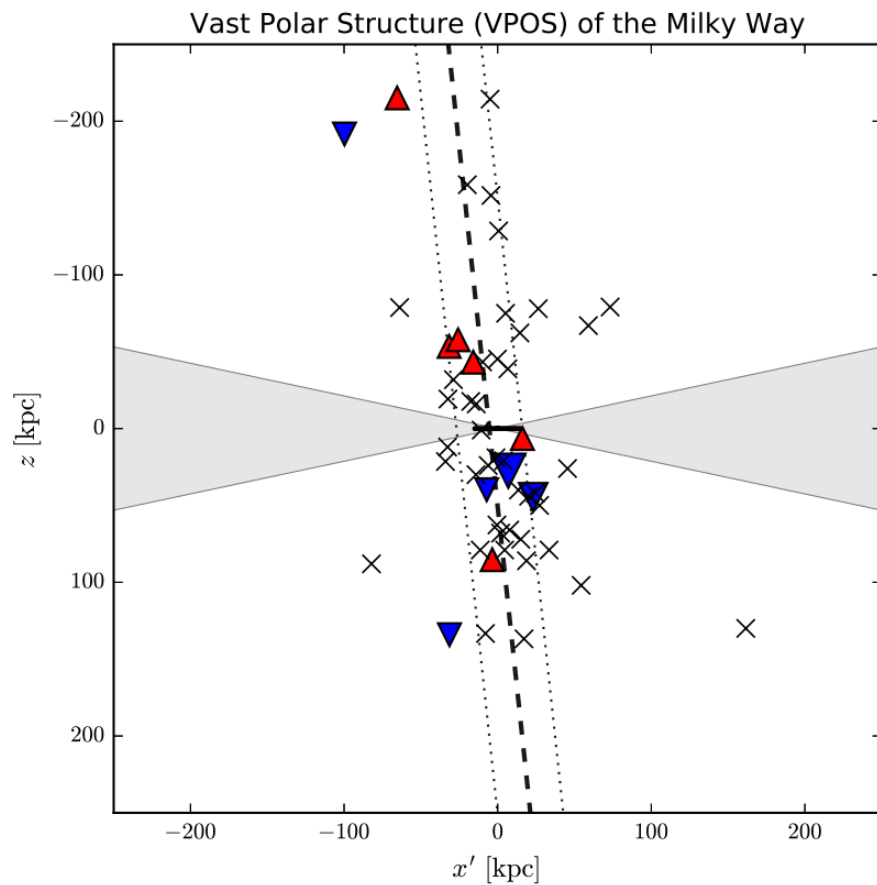
Kroupa et al. (2010)

Dwarf spheroidal galaxies: Too-big-to-fail



Boylan-Kolchin et al. 2012

The satellites phase-space correlation problem



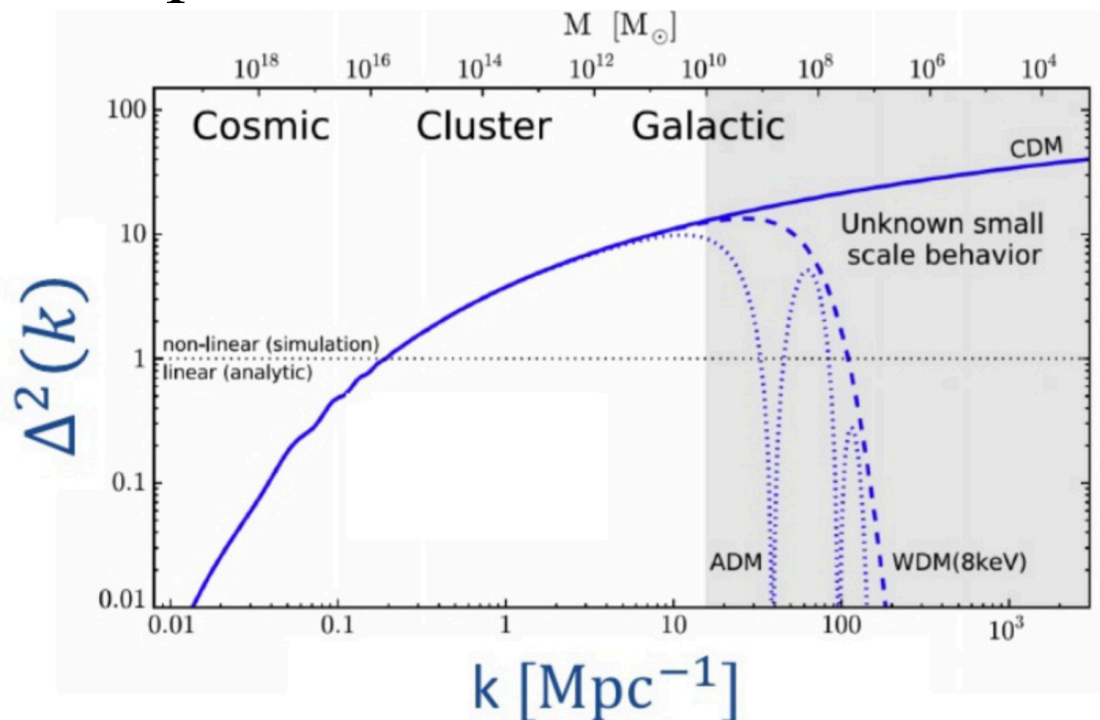
Pawlowski (2018)



2. Alternative DM solutions to small-scale problems?

A plethora of alternatives to CDM

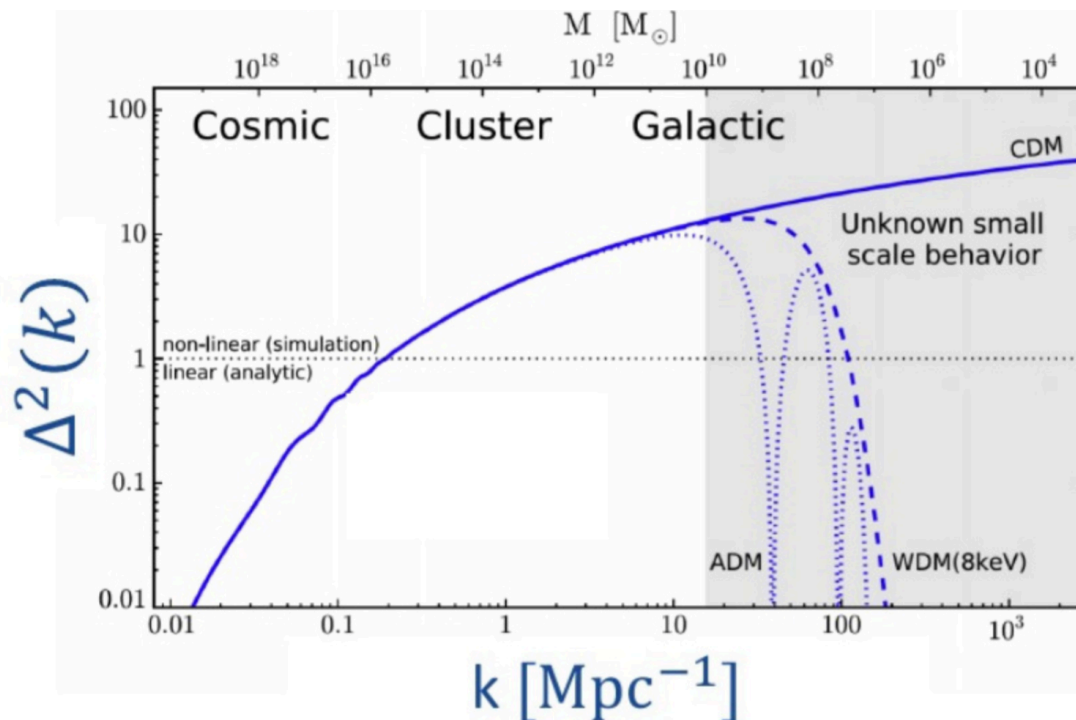
- All interesting in their own right: who knows what DM might be, how it is produced, etc.
- However, most of them **mostly** affect the matter power spectrum



M. Kuhlen et al. 2012

A plethora of alternatives to CDM

- This can be due to free-streaming from overdense to underdense regions in the case of warm DM or to collisional damping when interactions with photons or neutrinos are considered (interacting DM)



M. Kuhlen et al. 2012



Warm dark matter?

The simplest ‘modification’ of DM: **does it really have to be cold?**
CDM often assumed to be fermions of a few GeV to a few TeV

What about sterile neutrinos or thermally produced DM of **a few keV**?



Warm dark matter?

Gaussian random field as usual:

$$P(\delta|R)d\delta = \frac{1}{\sqrt{2\pi\sigma^2(R)}} \exp\left(-\frac{\delta^2}{2\sigma^2(R)}\right) d\delta$$

fully characterized by the power-spectrum:

$$\sigma^2(R) = \langle \delta(\mathbf{x}, R)^2 \rangle = \frac{1}{(2\pi^3)} \int d^3\mathbf{k} P(k) W^2(k; R)$$

$$P(k) \propto k^{n_s} T^2(k)$$

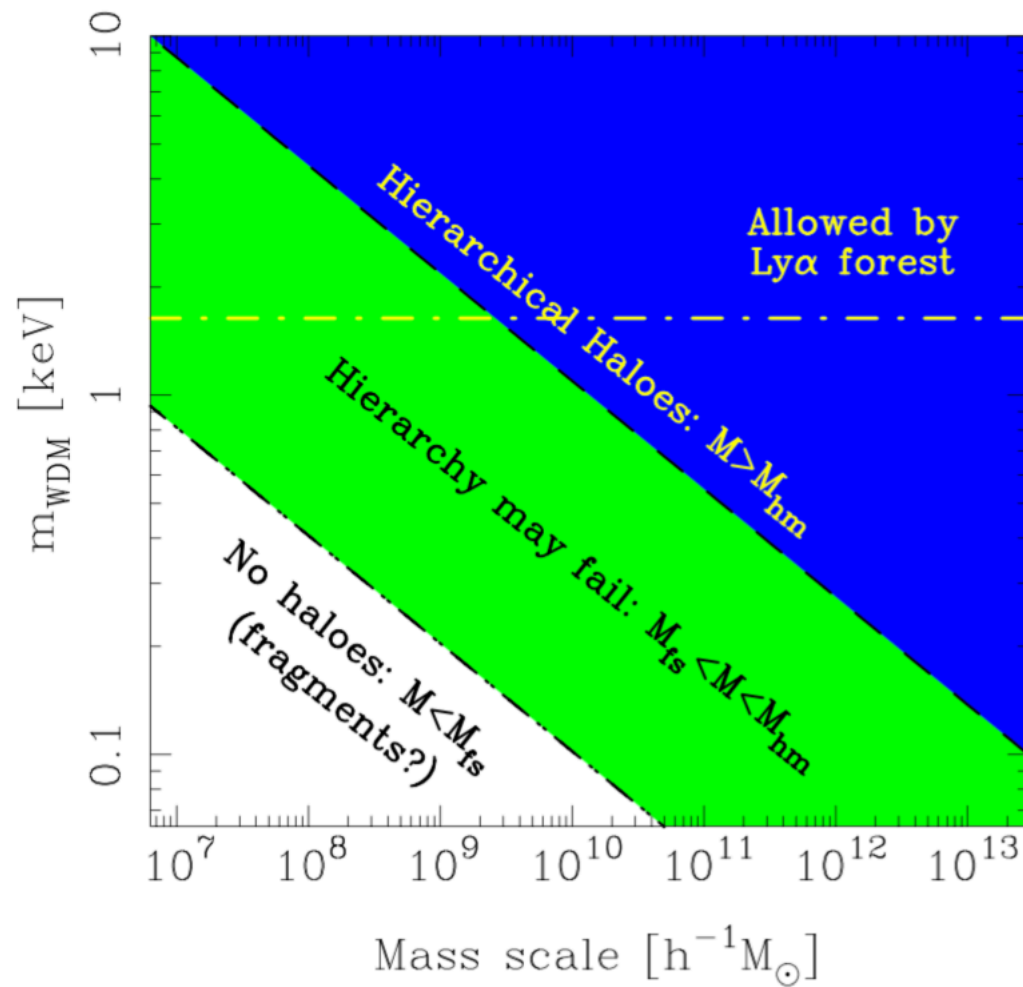
with a cutoff in the transfer function:

$$T_{\text{WDM}}(k) = [1 + (\alpha k)^{2\mu}]^{-5/\nu}$$

where $\nu = 1.12$ and

$$\alpha = 0.049 \left[\frac{m_{\text{WDM}}}{\text{keV}} \right]^{-1.11} \left[\frac{\Omega_{\text{WDM}}}{0.25} \right]^{0.11} \left[\frac{h}{0.7} \right]^{1.22} \text{Mpc/h.}$$

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CDM often assumed to be fermions of a few GeV to a few TeV

What about sterile neutrinos or thermally produced DM of **a few keV**?

- Damps structure formation close to the free-streaming scale
(1 keV \sim 100 kpc) \Rightarrow constraints from Lyman-alpha forest > 1.9 keV
- lower concentration than CDM halos \Rightarrow **helps TBTf**
- To create a core of ~ 1 kpc, needs 0.1 keV, which prevents the formation of the dwarf gal. altogether \Rightarrow **doesn't help diversity**

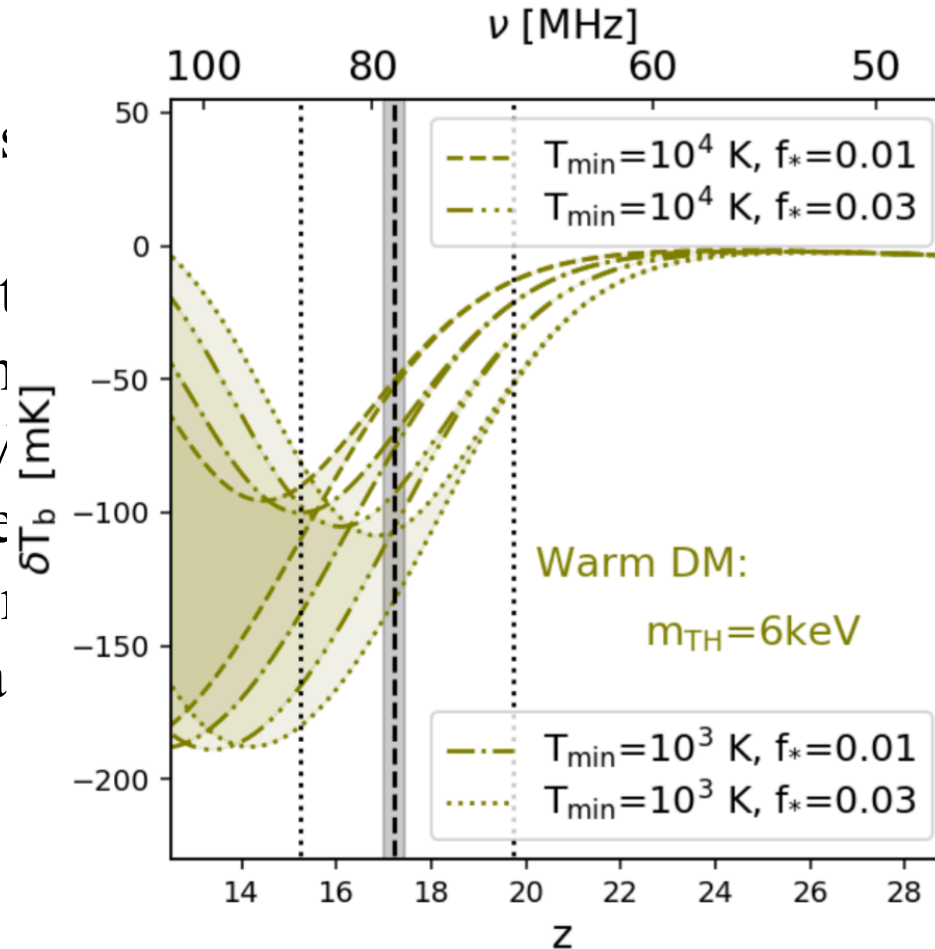
Schneider (2018): delayed formation of small-scale halos in contradiction with EDGES timing for **$m < 7$ keV** (but at higher masses, **cannot solve any small-scale tension !**)

Warm dark matter?

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Warm dark matter with non-gaussianities on small scales?

Peebles (2020) suggests a more radical alternative combining WDM with non-gaussianities on small-scales close to the free-streaming scale (a few 100 kpc). Not clear what the constraints are on such small scales:

$$\delta(x) = \frac{\delta_G(x) + F\delta_G(x)^3 / \langle \delta_G^2 \rangle}{(1 + 6F + 15F^2)^{1/2}}$$

⇒ Increases density fluctuations above 2σ but decreases them below 2σ , hence avoiding too much substructuring

⇒ More isolated protogalaxies that could avoid the hot orbits problem? (Peebles notes that the Local Void at $d < 8$ Mpc might be too empty with just 3 galaxies instead of ~ 20 , pointing in the same direction)



A plethora of alternatives to CDM

- All interesting in their own right: who knows what DM might be, how it is produced, etc.
- However, most of them **mostly** affect the matter power spectrum
- The most interesting alternatives regarding the small-scale challenges are those that affect the internal structure of DM halos
- This is the case for, e.g., fuzzy dark matter and self-interacting dark matter



Fuzzy dark matter?

An idea that gained traction after [Hui, Ostriker, Tremaine & Witten \(2017\)](#) that DM might be composed of **ultra-light bosons** w/ de Broglie wavelength:

$$\lambda = h/(m_b v) \simeq 1.20 \text{ kpc} (10^{-22} \text{ eV}/m_b)(100 \text{ km s}^{-1}/v)$$

- Above that scale, behaves **like CDM**, below it it is **different**
- **Damps** formation of halos lighter than $10^{10} (m/10^{-22} \text{ eV})^{-4/3} M_\odot$
- Creates **central cores** w/ **reduced dynamical friction** by one order of magnitude (plus spike at the center + large-scale fluctuations)
- These two effects help solving **TBTF**, **fast bar** problem, maybe **hot orbits** problem, ... [nothing to say on BTFR tightness](#)
- [Not clear it can help anyhow to solve the diversity problem](#)

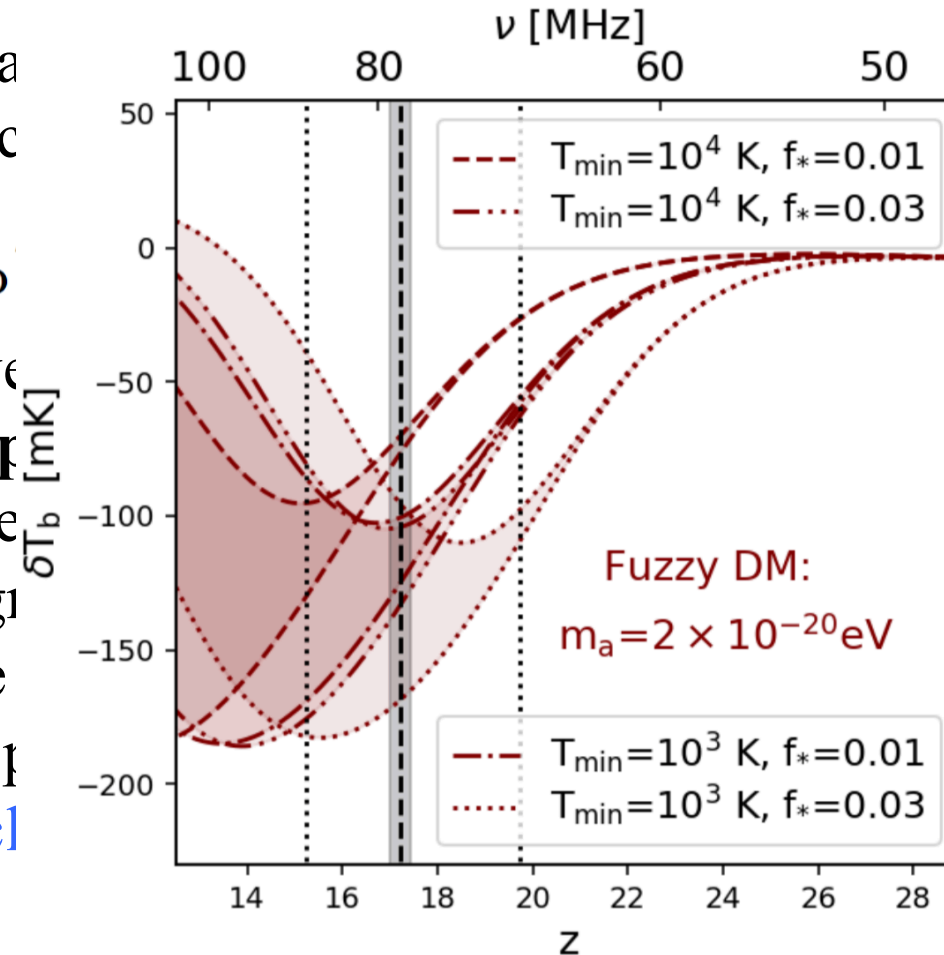
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An idea that ga
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$$\lambda = h/(m_b$$

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Witten (2017) that
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Schneider (2018): delayed formation of small-scale halos in
contradiction with EDGES timing for $m < 10^{-20} \text{ eV}$ (but at higher
masses, **cannot solve any small-scale tension !**)



Self-interacting dark matter?

The 2nd simplest modif. of DM: **does it really have to be collisionless?**
Self-interactions have little effect on the matter power spectrum, but can drastically change the DM profiles in relaxed halos!

Collisional Boltzmann equation instead of Vlasov:

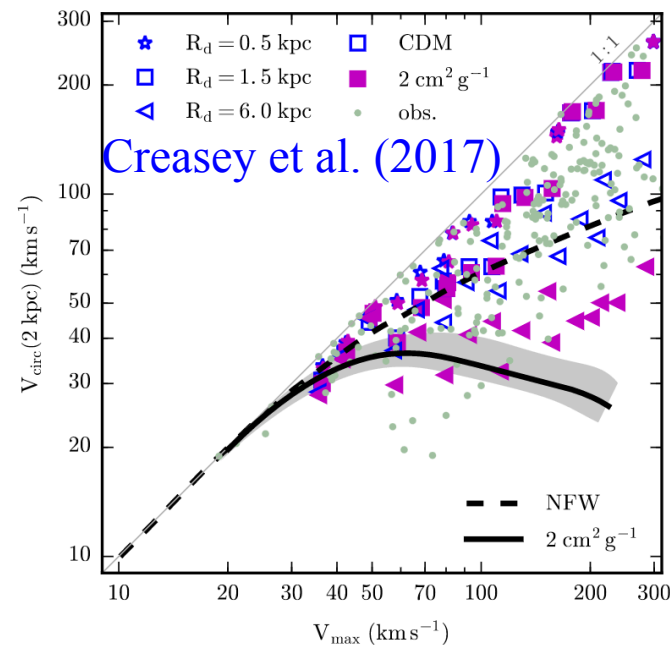
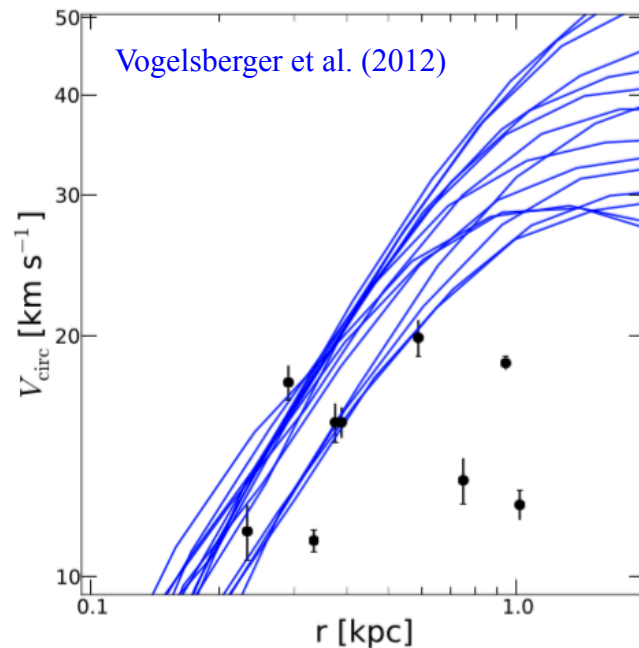
Scattering rate (T^{-1}) goes like $\rho \times \sigma/m \times v$

Include in simulation code: Discretize phase-space, compute scattering prob. when two phase-space patches overlap, repopulate phase-space with Monte-Carlo and replace the old particles by the new ones

Self-interacting dark matter?

The 2nd simplest modif. of DM: **does it really have to be collisionless?**
Self-interactions have little effect on the matter power spectrum, but can drastically change the DM profiles in relaxed halos!

Self-interacting cross-sections $\sigma/m = 1-10 \text{ cm}^2/\text{g}$ can have a drastic effect on halo profiles \Rightarrow can solve **TBTF, diversity, and (perhaps) fast bar**



Nothing on hot orbits, and might make **FFP worse!** (Sameie et al. 2021)

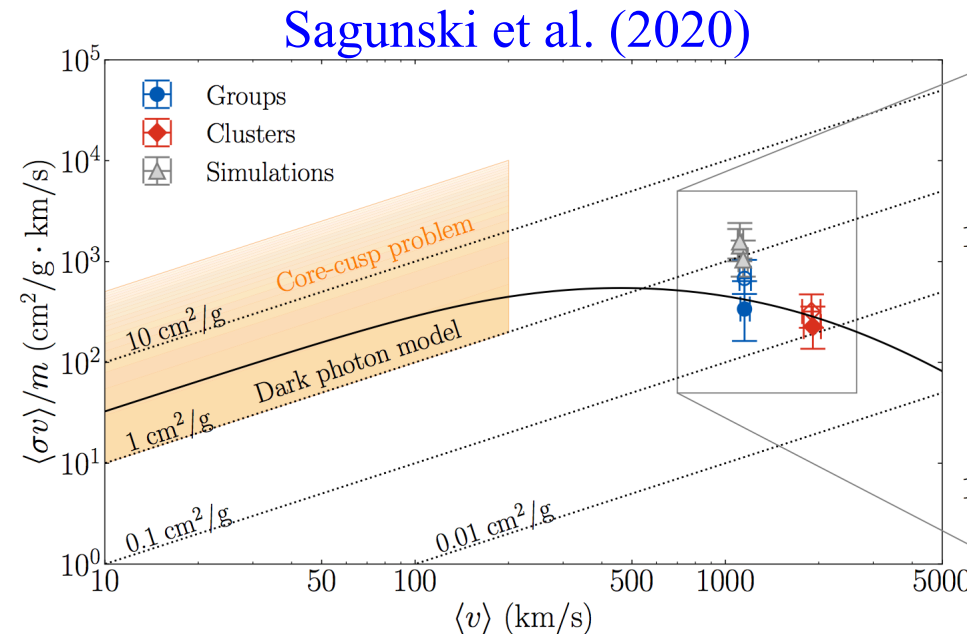
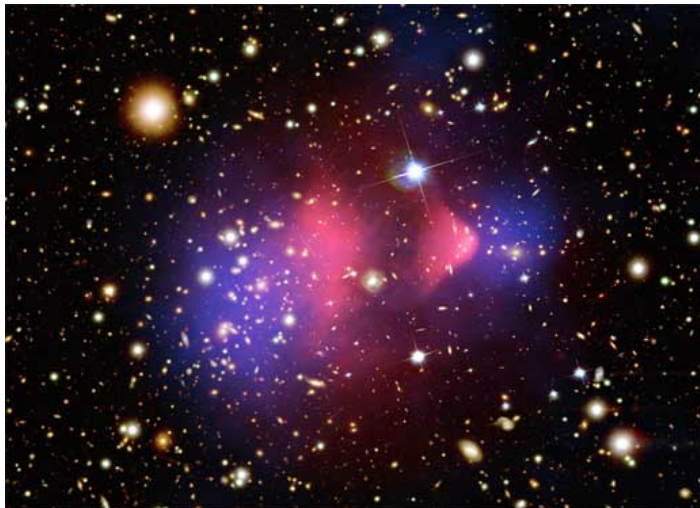
Self-interacting dark matter?

Conflicting constraints with galaxies coming from galaxy clusters:

Colliding clusters (bullet) $\Rightarrow \sigma/m < 0.7 \text{ cm}^2/\text{g}$ (Randall et al. 2008)

Strong lensing of cluster centers $\Rightarrow \sigma/m < 0.065 \text{ cm}^2/\text{g}$ (Andrade et al. 2021)

Cannot solve any tension on galaxy scales with such cross-sections
 \Rightarrow **velocity-dependent cross-section needed**





Self-interacting dark matter?

In summary:

SIDM with velocity-dependent cross-section very promising at alleviating small-scale problems

Although:

- No explanation for the tightness of BTFR
- Can lead to too steep DM profiles in MW-like and massive spirals (core collapse)

Still the most interesting ‘classical’ alternative regarding small-scale problems



3. Modified gravity?

Modifying gravity?

$$\begin{aligned} g &= g_N \\ g &= (g_N a_0)^{1/2} \end{aligned}$$

$$\begin{aligned} &\text{if } g \gg a_0 \\ &\text{if } g \ll a_0 \end{aligned}$$

MOND
Milgrom 1983

A characteristic **acceleration scale** present in the BTFR and diversity

$$\nabla \cdot [\mu(|\nabla\Phi|/a_0) \nabla\Phi] = 4\pi G \rho_{\text{bar}} \quad \text{AQUAL: Bekenstein \& M (1984)}$$

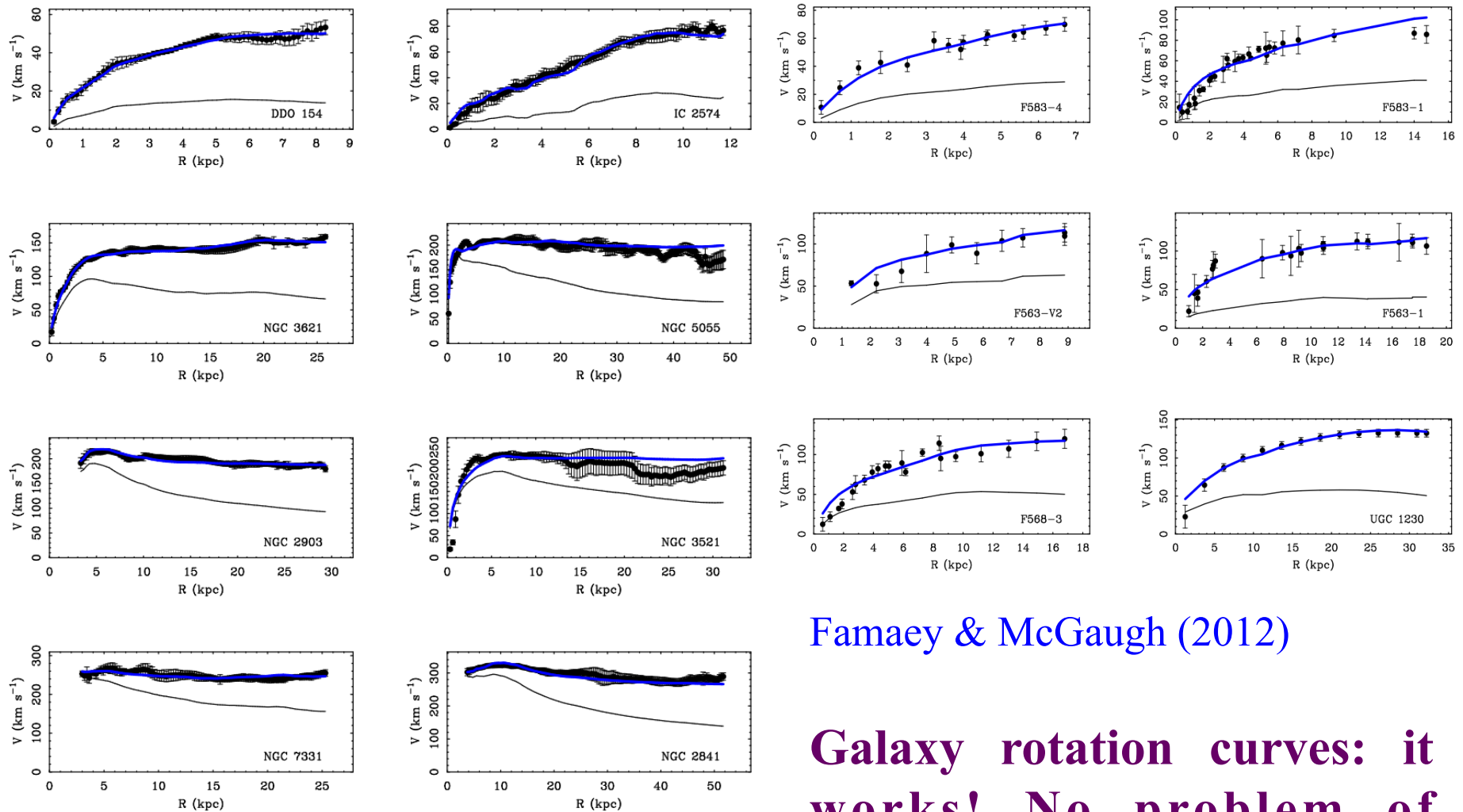
or

$$\nabla^2 \Phi = \nabla \cdot [\nu(|\nabla\Phi_N|/a_0) \nabla\Phi_N] \quad \text{QUMOND: Milgrom (2010)}$$

⇒ Getting a **tight and straight BTFR**, solving the **failed feedback** problem and the **diversity** for free

+ no dynamical friction with DM

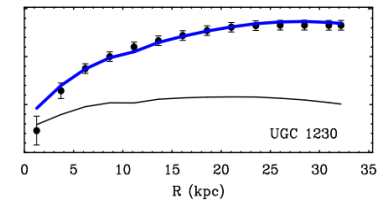
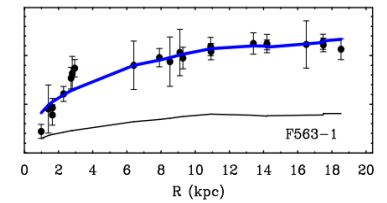
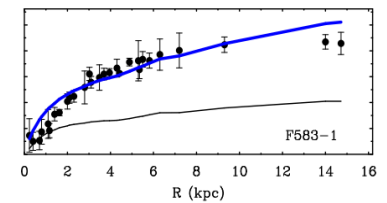
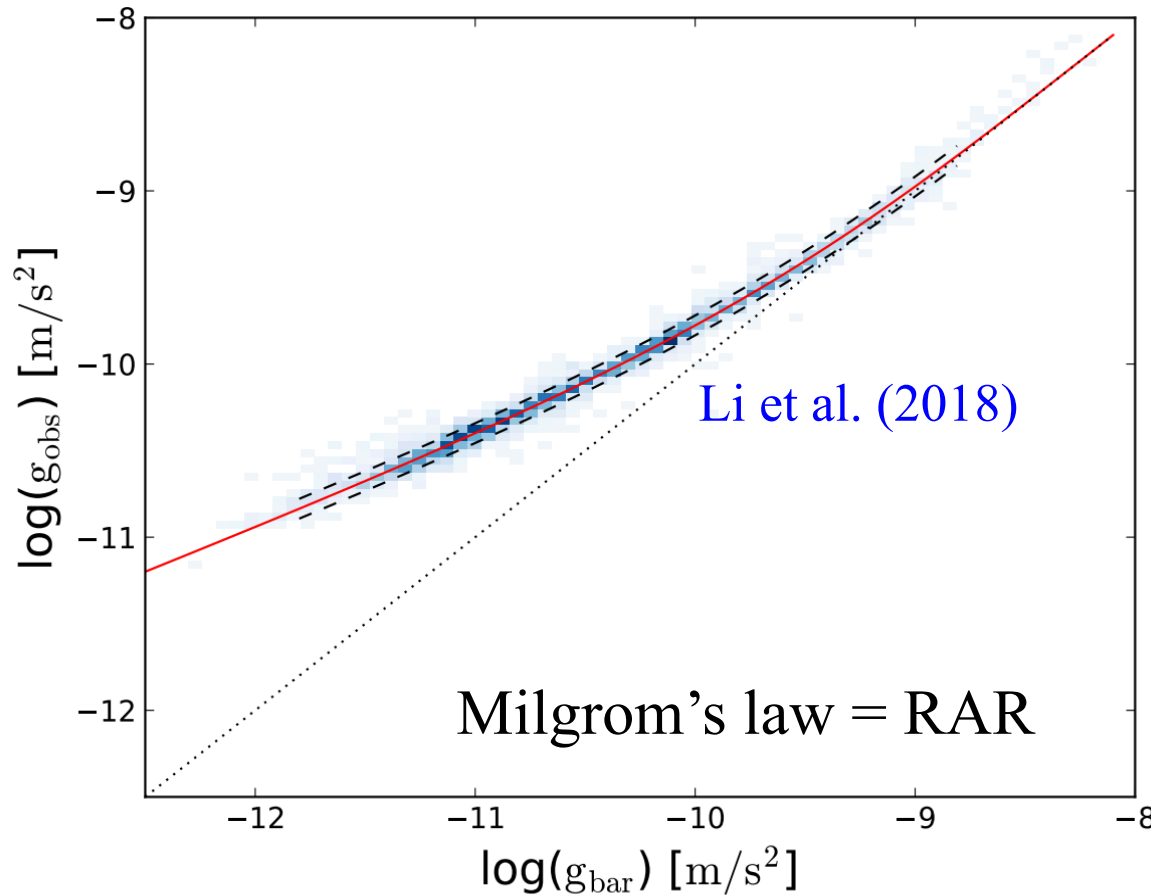
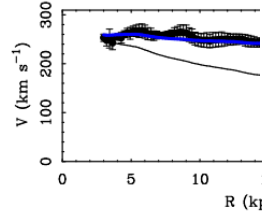
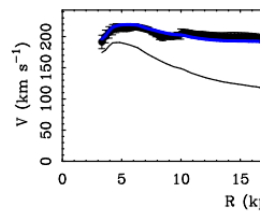
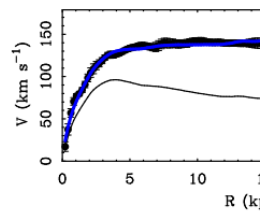
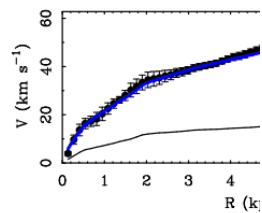
Modifying gravity?



Famaey & McGaugh (2012)

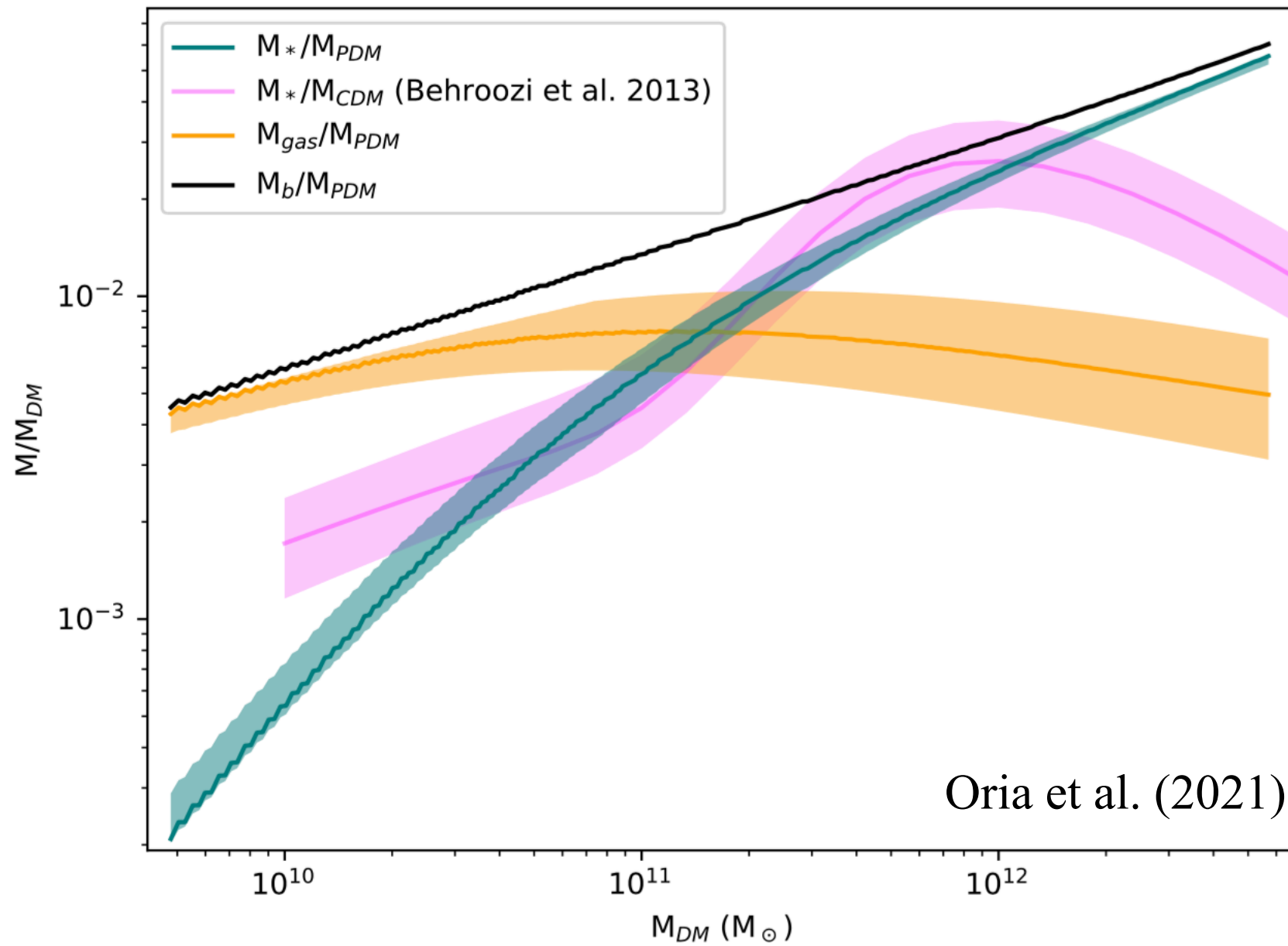
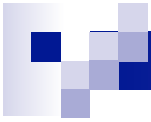
Galaxy rotation curves: it works! No problem of diversity or BTFR tightness

Modifying gravity?



(2012)

Galaxy rotation curves: it works! No problem of diversity or BTFR tightness



Be careful with the Solar System

Hees, Famaey et al. (2016):

Strong constraints on
modified gravity versions
of MOND from Cassini

⇒ *But « just » needs to tune
the interpolating function*

$$\nu_n(y) = \left[\frac{1 + (1 + 4y^{-n})^{1/2}}{2} \right]^{1/n},$$

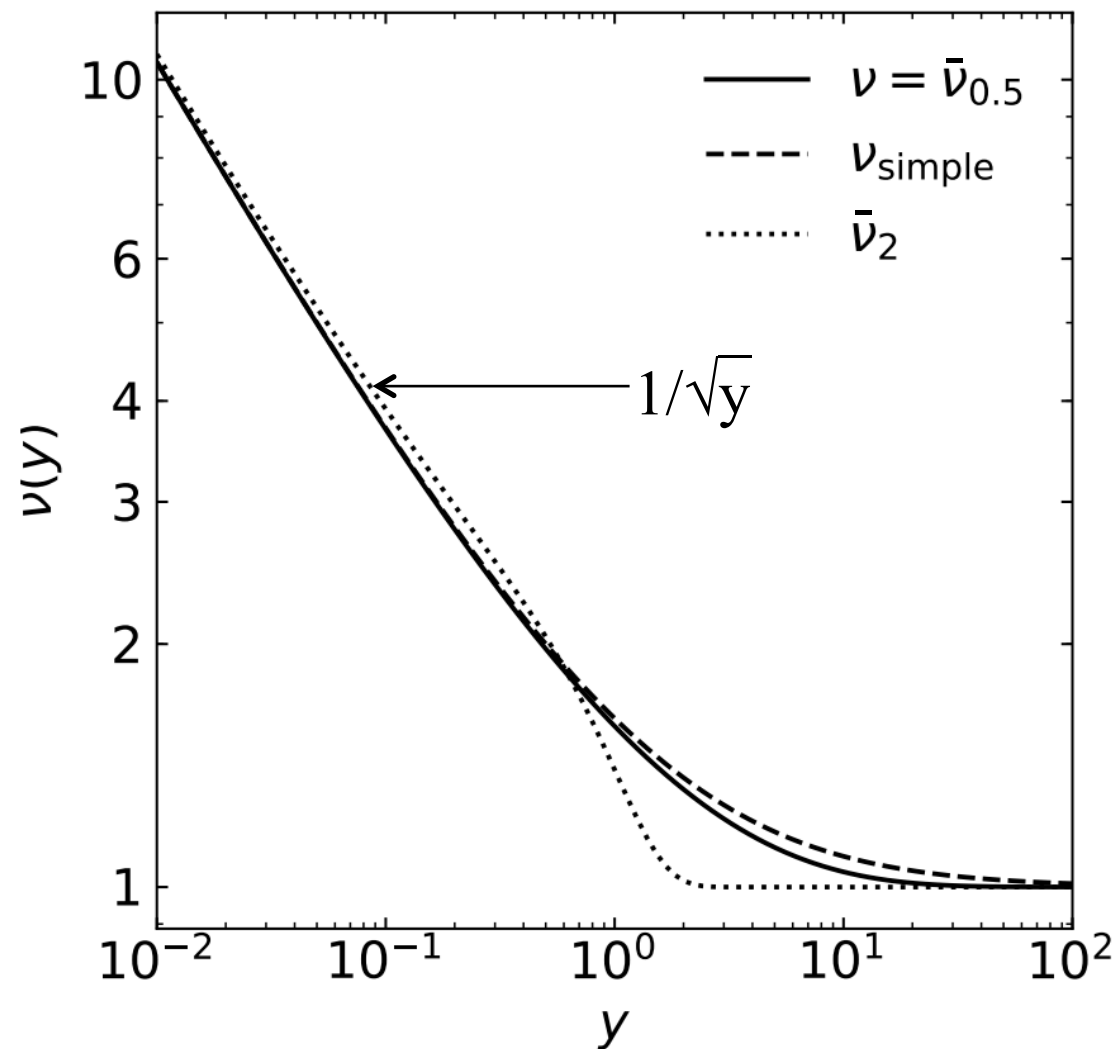
$$\tilde{\nu}_\alpha(y) = (1 - e^{-y})^{-1/2} + \alpha e^{-y},$$

$$\bar{\nu}_\alpha(y) = (1 - e^{-y^\alpha})^{-1/2\alpha} + (1 - 1/2\alpha) e^{-y^\alpha},$$

$$\hat{\nu}_\alpha(y) = (1 - e^{-y^{\alpha/2}})^{-1/\alpha}.$$

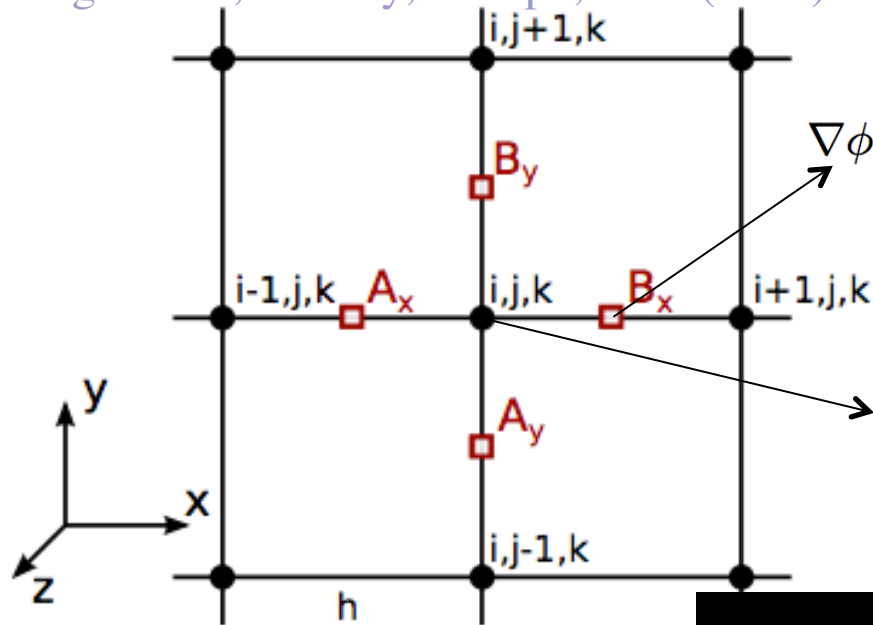
	a_0 10^{-10} [m/s ²]	χ^2_{red}	$g_e = g_{\text{emin}}$			$g_e = g_{\text{emax}}$		
			η	$-q$ 10^{-2}	Q_2 10^{-27} [s ⁻²]	η	$-q$ 10^{-2}	Q_2 10^{-27} [s ⁻²]
ν_2	1.60	2.02	1.20	10.	26.	1.50	11.3	30.
ν_3	1.55	1.97	1.20	8.29	21.	1.50	7.82	20.
ν_4	1.51	1.94	1.30	5.76	16.	1.60	5.34	13.
ν_5	1.49	1.93	1.30	5.51	13.	1.60	3.71	8.7
ν_6	1.46	1.92	1.30	4.55	11.	1.60	2.67	6.2
ν_7	1.45	1.92	1.30	3.82	8.7	1.70	2.01	4.6
ν_8	1.44	1.92	1.30	3.27	7.3	1.70	1.58	3.5
$\tilde{\nu}_{0.5}$	1.48	2.16	1.30	14.8	35.	1.60	18.5	44.
$\tilde{\nu}_1$	1.38	2.12	1.40	18.3	38.	1.70	25.	53.
$\tilde{\nu}_{1.5}$	1.18	2.16	1.60	24.1	40.	2.00	34.2	57.
$\tilde{\nu}_2$	0.815	2.24	2.30	44.8	43.	2.90	47.9	46.
$\tilde{\nu}_{2.5}$	0.977	2.23	1.90	38.1	42.	2.50	51.7	65.
$\tilde{\nu}_3$	0.743	1.07	2.60	58.8	47.	3.20	65.5	55.
$\tilde{\nu}_4$	0.723	2.01	2.60	54.8	44.	3.30	85.9	69.
$\tilde{\nu}_5$	0.715	1.97	2.70	48.1	38.	3.40	94.7	75.
$\bar{\nu}_1$	1.38	2.12	1.40	16.1	34.	1.70	19.5	41.
$\bar{\nu}_{1.5}$	1.18	2.16	1.60	19.3	32.	2.00	15.8	26.5
$\bar{\nu}_2$	0.815	2.24	2.30	6.2	5.9	2.90	2.63	2.52
$\bar{\nu}_3$	0.743	2.07	2.60	1.9	1.6	3.20	0.82	0.68
$\bar{\nu}_4$	0.723	2.01	2.60	1.3	1.	3.30	0.56	0.45
$\bar{\nu}_5$	0.715	1.97	2.70	1.08	0.85	3.40	0.	0.
$\bar{\nu}_6$	0.713	1.95	2.70	1.02	0.8	3.40	0.	0.
$\bar{\nu}_7$	0.729	1.95	2.60	1.07	0.87	3.30	0.	0.
$\hat{\nu}_1$	1.48	2.15	1.30	13.1	31.	1.60	17.5	41.
$\hat{\nu}_2$	1.59	2.01	1.20	10.2	27.	1.50	11.4	30.
$\hat{\nu}_3$	1.55	1.96	1.20	8.32	21.	1.60	7.49	19.
$\hat{\nu}_4$	1.51	1.94	1.30	6.66	16.	1.60	4.79	12.
$\hat{\nu}_5$	1.48	1.93	1.30	5.34	13.	1.60	3.1	7.3
$\hat{\nu}_6$	1.46	1.92	1.30	4.31	9.9	1.60	2.11	4.9
$\hat{\nu}_7$	1.45	1.92	1.30	3.55	8.	1.70	1.55	3.5

Be careful with the Solar System



Phantom of Ramses

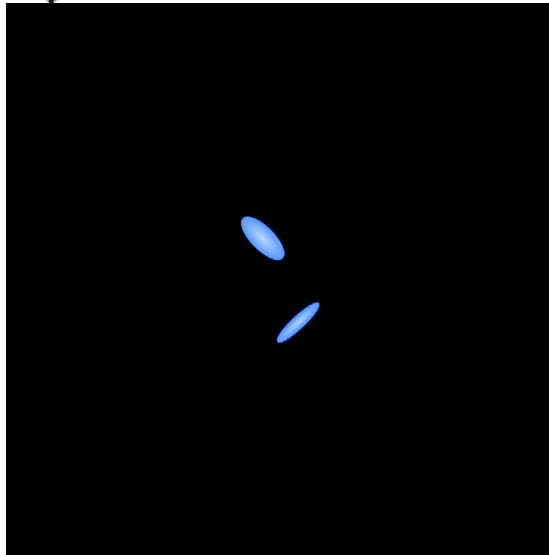
Lüghausen, Famaey, Kroupa, et al. (2013)



$$\nabla\phi = \frac{1}{4h} \begin{pmatrix} 4(\phi^{i+1,j,k} - \phi^{i,j,k}) \\ \phi^{i+1,j+1,k} - \phi^{i+1,j-1,k} + \phi^{i,j+1,k} - \phi^{i,j-1,k} \\ \phi^{i,j,k+1} - \phi^{i,j,k-1} + \phi^{i+1,j,k+1} - \phi^{i+1,j,k-1} \end{pmatrix}$$

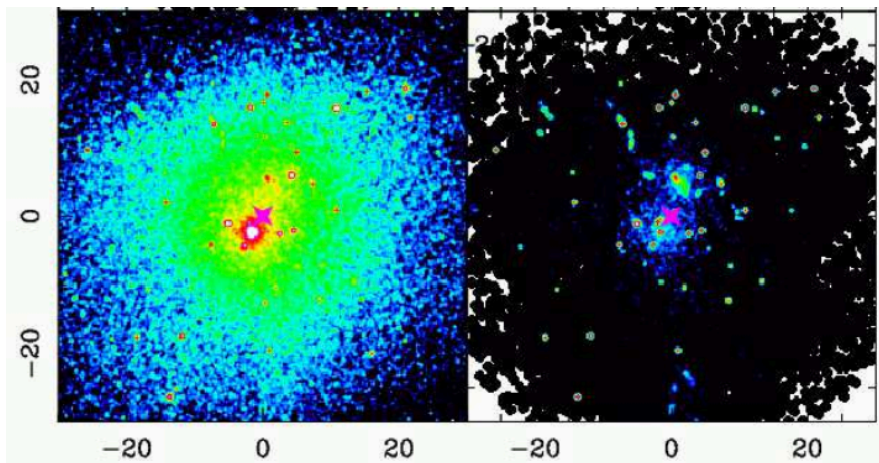
$$\rho_{\text{ph}}^{i,j,k} = \frac{1}{4\pi G} \frac{1}{h^2} \left[\begin{aligned} &(\phi^{i+1,j,k} - \phi^{i,j,k}) \nu_{Bx} \\ &- (\phi^{i,j,k} - \phi^{i-1,j,k}) \nu_{Ax} \\ &+ (\phi^{i,j+1,k} - \phi^{i,j,k}) \nu_{By} \\ &- (\phi^{i,j,k} - \phi^{i,j-1,k}) \nu_{Ay} \\ &+ (\phi^{i,j,k+1} - \phi^{i,j,k}) \nu_{Bz} \\ &- (\phi^{i,j,k} - \phi^{i,j,k-1}) \nu_{Az} \end{aligned} \right]$$

Renaud, Famaey, Kroupa (2016)

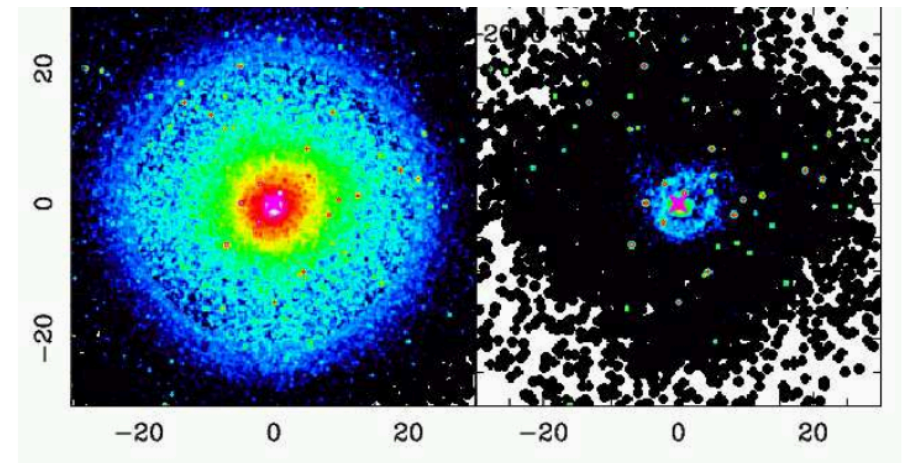


Solving the hot orbits and fast bar problems?

Too many mergers & clumps at high- z spiral-in to form **bulges**: might be solved in MOND by less mergers and **decreased dynamical friction for massive clumps** in high- z clumpy disks



MOND



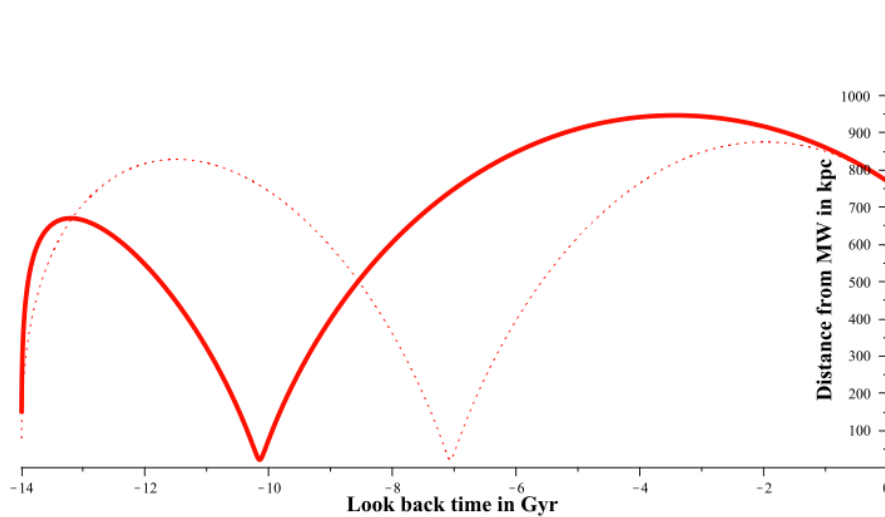
Newton+DM

Same clumpy disk ICs: 2 Gyr of evolution ([Combes 2014](#))

Less dynamical friction imply **faster bars**: [Tiret & Combes \(2007, 2008\)](#), [Roshan et al. 2021](#)

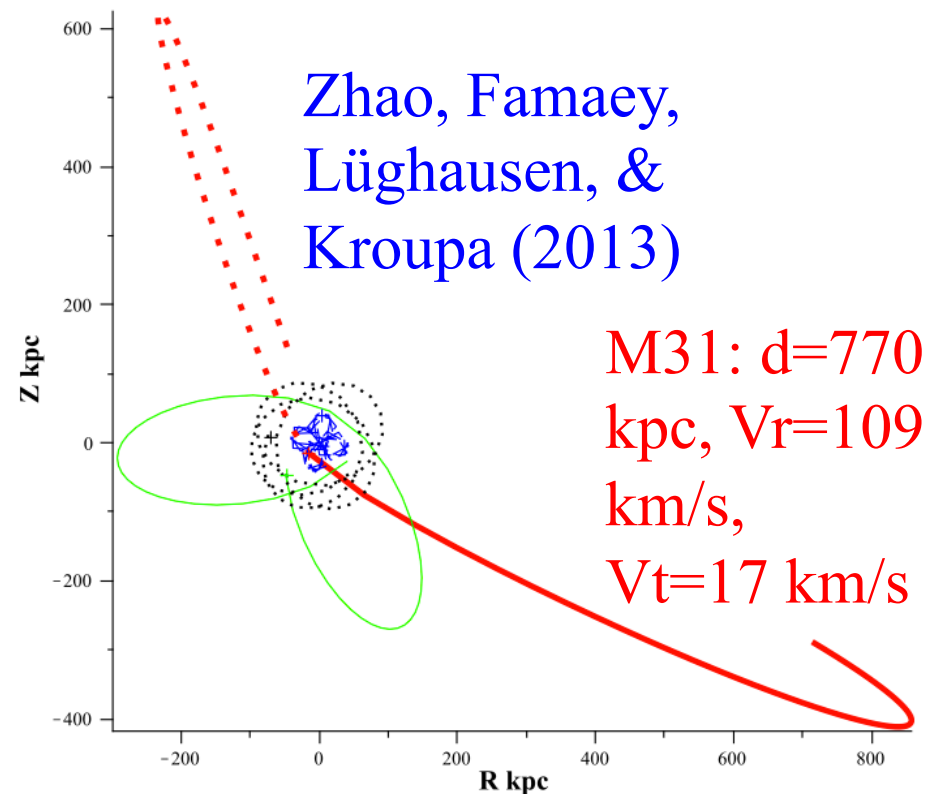
An explanation for the satellite planes?

$$F_{2\text{body}} = \frac{2}{3} \left[(m_1 + m_2)^{3/2} - m_1^{3/2} - m_2^{3/2} \right] \frac{\sqrt{Ga_0}}{r}, \quad \frac{d^2}{dt^2} \mathbf{r}_{12} = K \mathbf{r}_{12} - \frac{m_1 + m_2}{m_1} \left[\frac{\mathbf{F}_{12}}{m_2} \right], \quad K \equiv \frac{d^2 a}{a dt^2}$$



$$F_{12} \approx \frac{\tilde{G} m_1 m_2}{r_{12}^2}, \quad \tilde{G} \equiv G \left[1 + \left(y + \frac{g_{\text{ext}}^2}{a_0^2} \right)^{-\alpha} \right]^{\frac{1}{2\alpha}}$$

$$y \equiv \left[\frac{\sqrt{G(m_1 + m_2)a_0}}{r_{12} Q a_0} \right]^2, \quad Q \equiv \frac{2(1 - q_1^{3/2} - q_2^{3/2})}{3q_1 q_2} \quad \text{and} \quad q_1 \equiv 1 - q_2 \equiv \frac{m_1}{m_1 + m_2}$$



Weak lensing

Reminder:

IF the weak-field metric can be written (at 1PN) as:

$$g_{00} = -e^{2\Phi/c^2}, \quad g_{ij} = e^{2\Psi/c^2} \delta_{ij}$$

AND $\boxed{\Psi = -\Phi}$ (we'll get back to MOND model building later)

$$\theta = \beta + \frac{D_{ls}}{D_s} \alpha, \quad \text{with} \quad \alpha = \frac{2}{c^2} \int_{-\infty}^{\infty} \nabla_{\perp} \Phi dz$$

Observed angular position

Original (unlensed) position

Weak lensing

Inverse magnification matrix:

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}, \text{ where } \mathcal{A}_{11} = 1 - \kappa - \gamma_1, \mathcal{A}_{12} = \mathcal{A}_{21} = -\gamma_2, \mathcal{A}_{22} = 1 - \kappa + \gamma_1$$

$$\kappa = \frac{1}{2} \nabla^2 \Upsilon$$

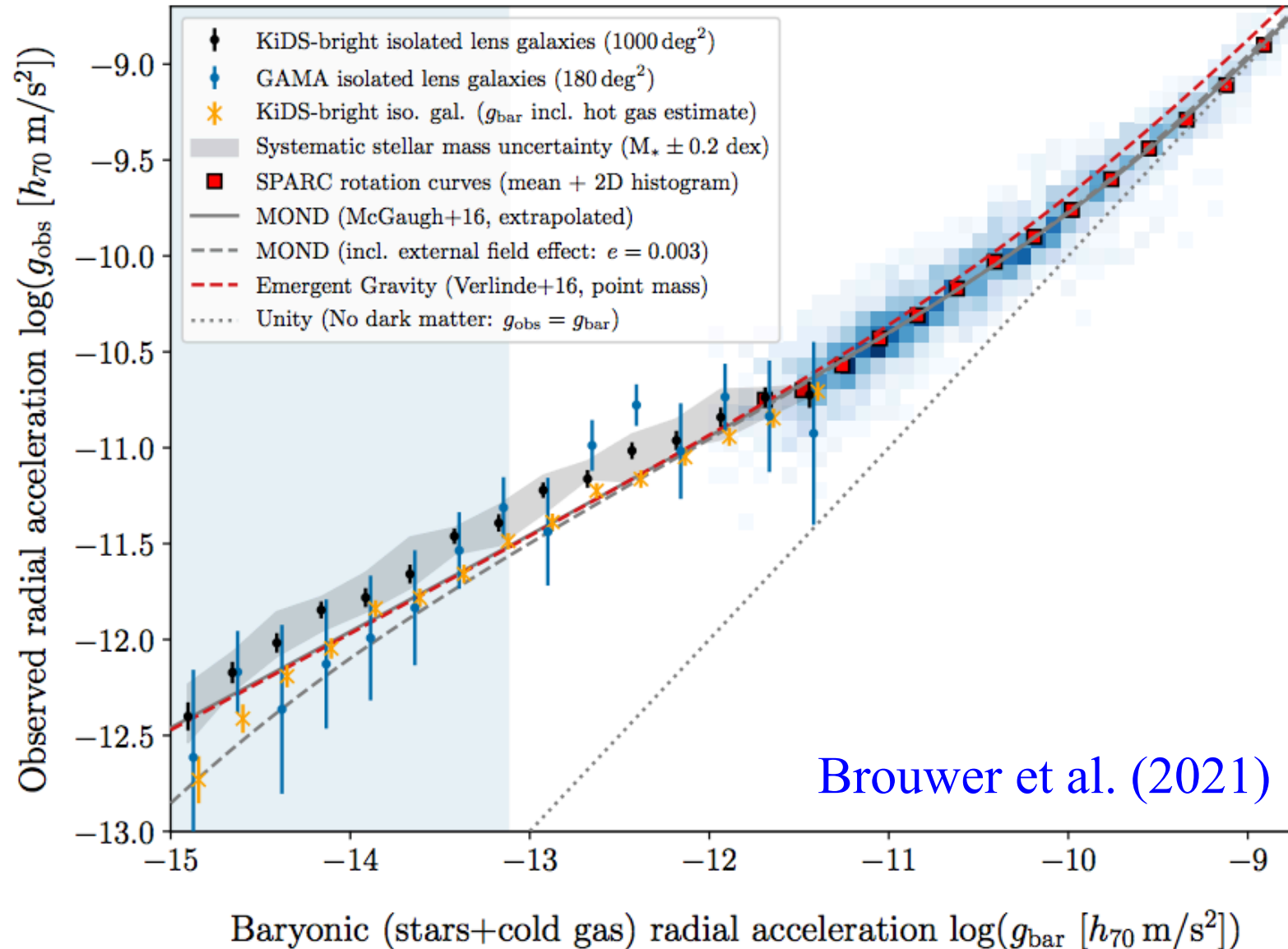
$$\gamma_1 = \frac{1}{2} \left(\frac{\partial^2 \Upsilon}{\partial \theta_1^2} - \frac{\partial^2 \Upsilon}{\partial \theta_2^2} \right), \quad \gamma_2 = \frac{\partial^2 \Upsilon}{\partial \theta_1 \partial \theta_2}$$

Computed from ellipticity of the images

$$\Upsilon(\boldsymbol{\theta}) = \frac{2D_{ls}}{c^2 D_s D_l} \int_{-\infty}^{\infty} \Phi(D_l \boldsymbol{\theta}, z) dz,$$

Gravitational potential of the lens

Weak lensing

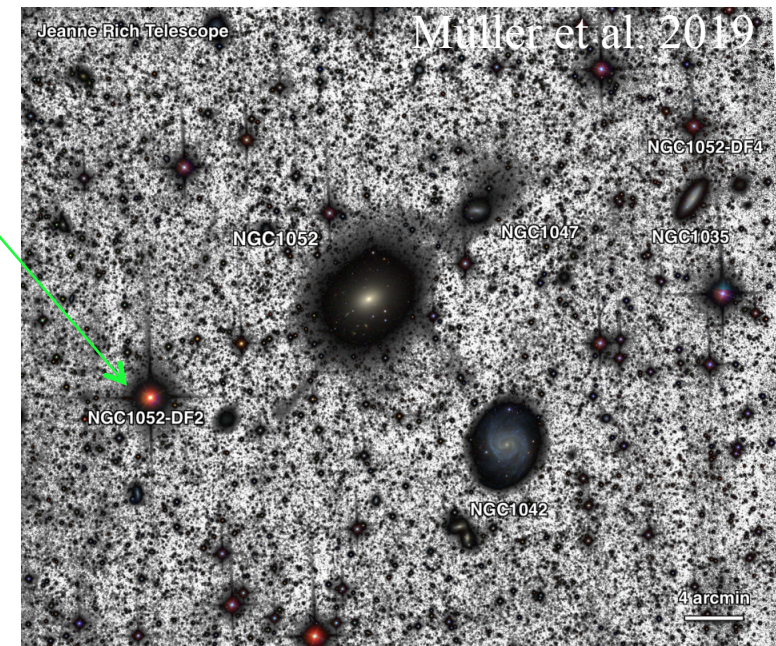


Low mass discrepancy? The external field effect to the rescue!

Ultra-diffuse galaxy with low DM content
Isolated mond predicted velocity dispersion:
 $\sigma_{\text{MOND}} \approx 20 \text{ km/s}$ but measured at $\sim 10 \text{ km/s}$

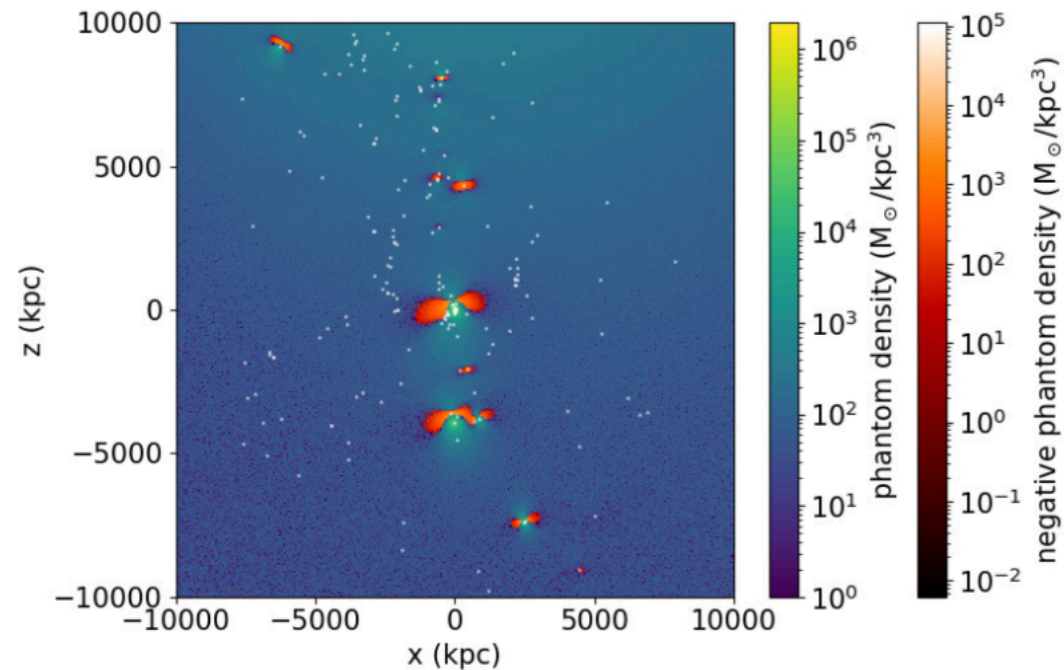
But $g_{\text{ext}} \sim g_{\text{int}} \sim 0.15 a_0$

$\Rightarrow \sigma_{\text{MOND}}$ ranges from ~ 9 to 19 km/s depending on int. function, stellar M/L, & 3D distance to the host (Famaey, McGaugh & Milgrom 2018)

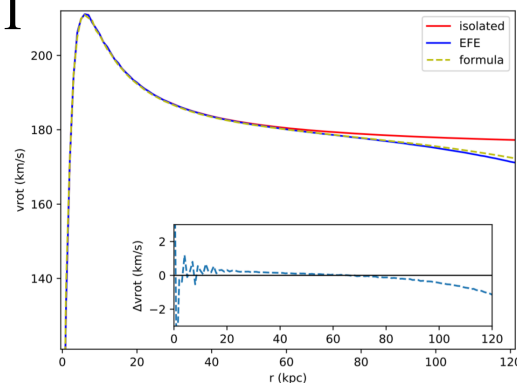


M31 dwarfs: McGaugh & Milgrom (2013) **a priori** predictions compared to Collins et al. (2013) and Tollerud et al. (2013): correct for And XVII, **And XIX**, And XX, **And XXI**, And XXIII, **And XXV**, And XXVIII & And XIX
 \Rightarrow large dSphs with low σ because EFE

Negative convergence: a smoking gun?



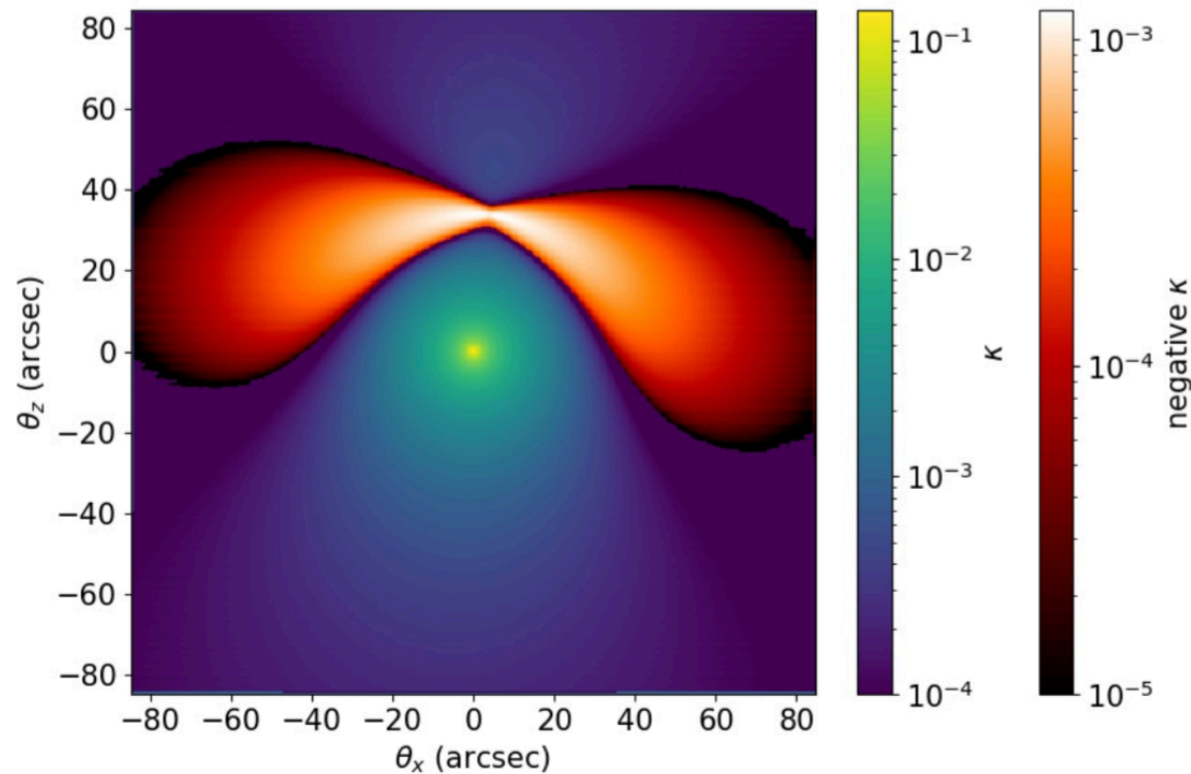
Oria et al. 2021



NGC 5055 galaxy of
the Local Volume
under strong influence
from the Virgo cluster

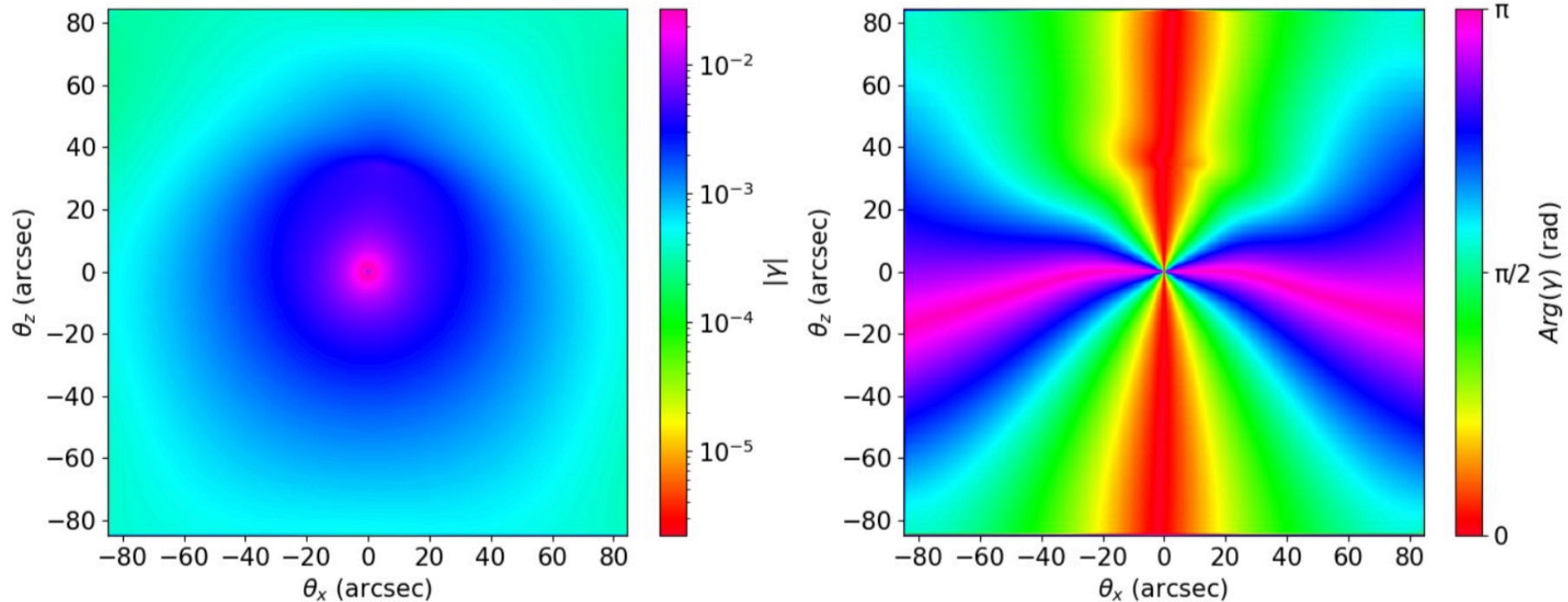
Negative convergence: a smoking gun?

Artificially place NGC 5055 at $z=0.3$ for sources at $z=5$



Negative convergence: a smoking gun?

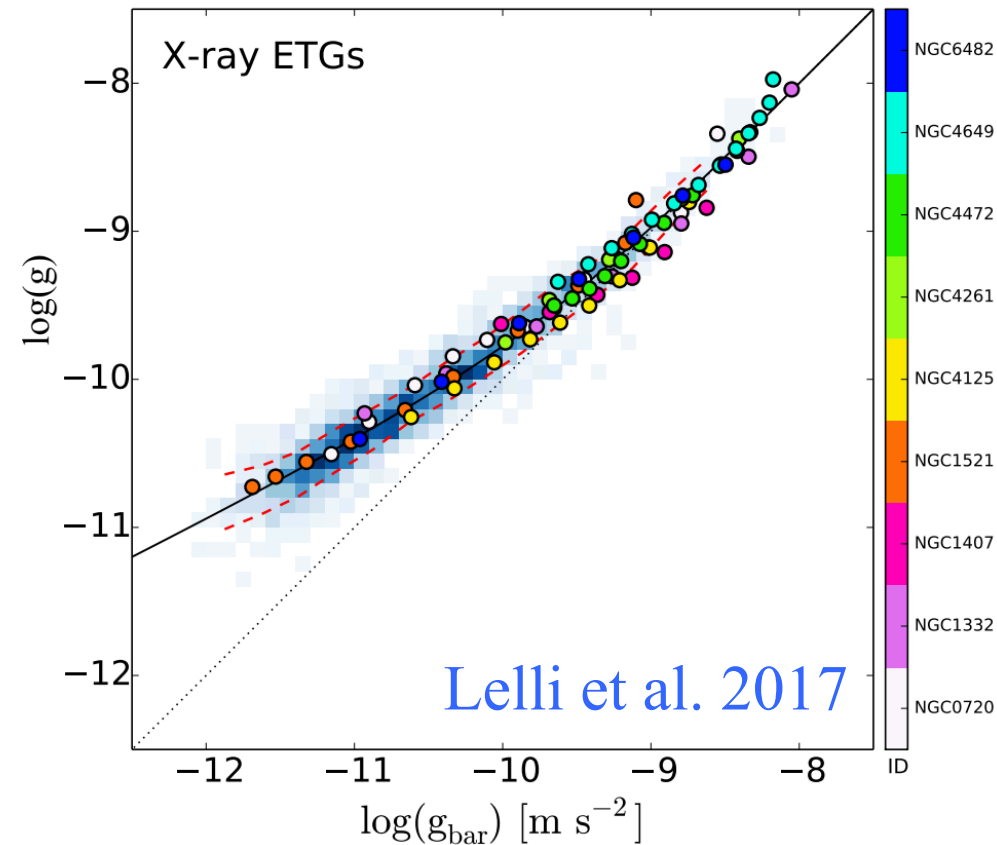
Artificially place NGC 5055 at $z=0.3$ for sources at $z=5$



Elliptical galaxies

Hydrostatic equilibrium for X-ray gas temperature profile:

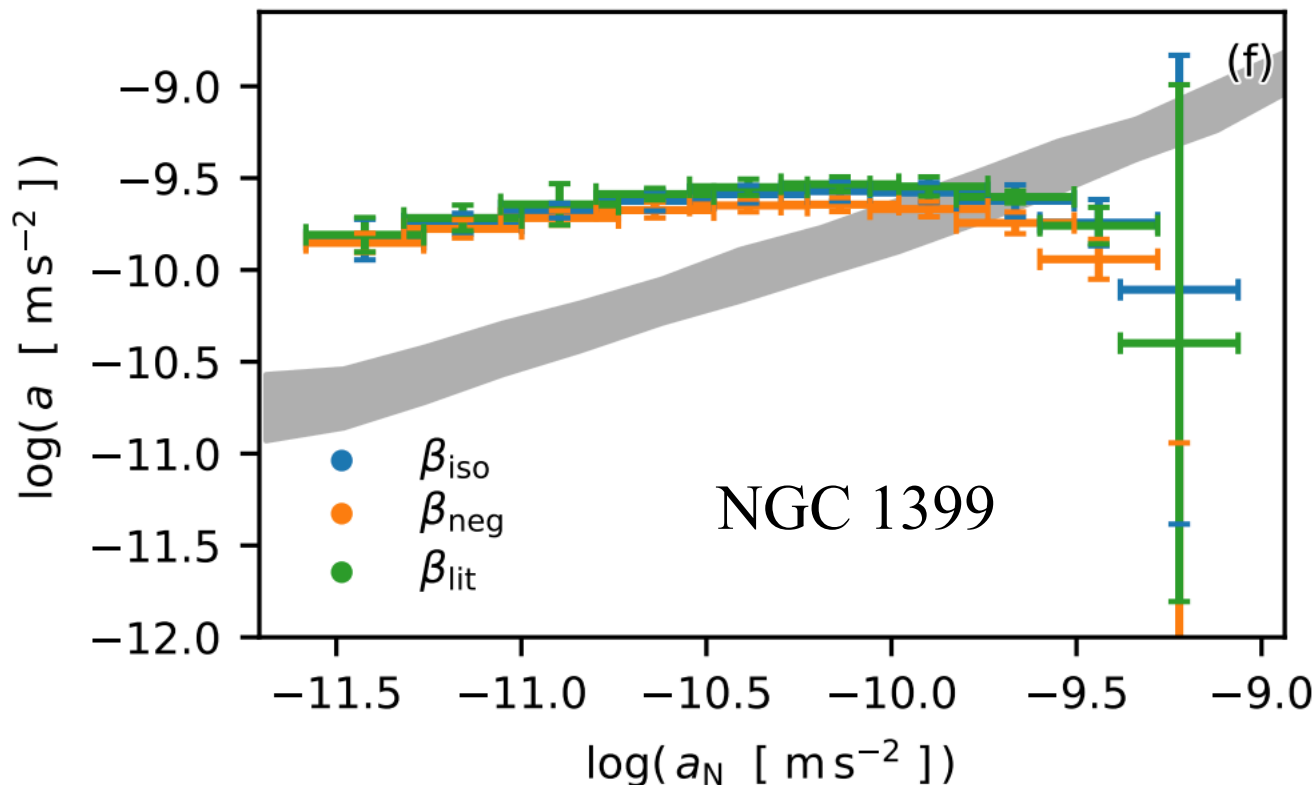
$$g = [-kT(r)/(r\langle m \rangle)] \times [\ln \rho_x / \ln r + \ln T / \ln r]$$



Elliptical galaxies

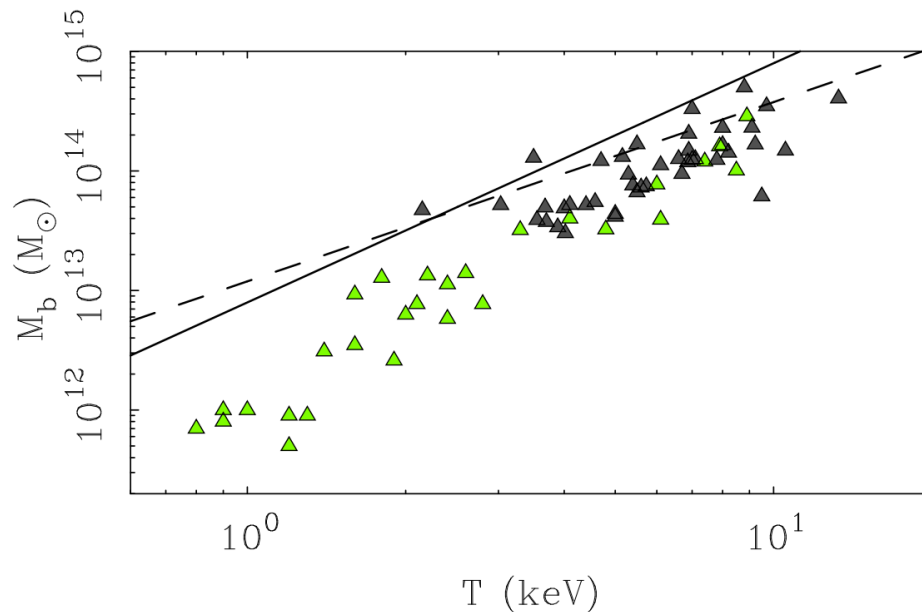
Jeans modelling of globular cluster systems:

[Bilek et al. 2019](#): *Most galaxies can be fitted by the MOND models successfully, but for some of the galaxies, especially those in centers of galaxy clusters, the observed GCs velocity dispersions are too high*



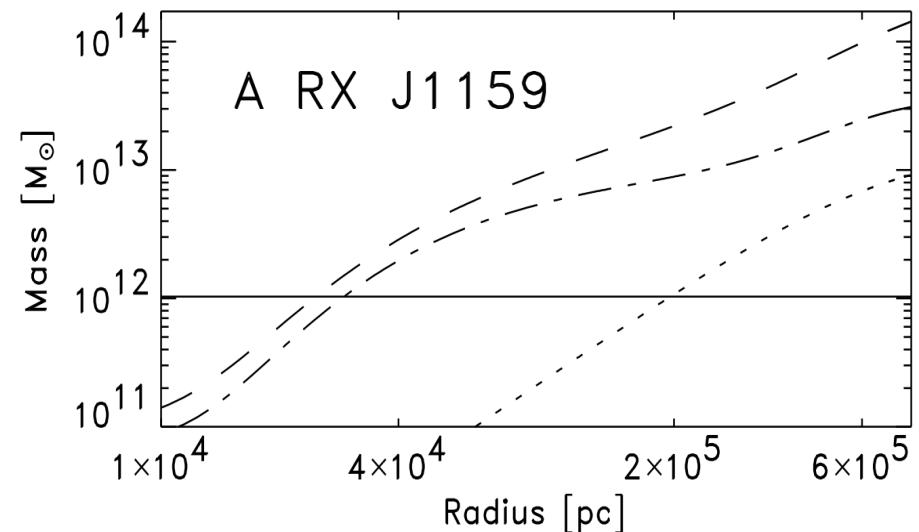
Galaxy clusters: where it all breaks down...

Temperature profiles of X-ray emitting gas in clusters:



Famaey & McGaugh (2012)

Globally, a **factor of 2** of residual missing mass



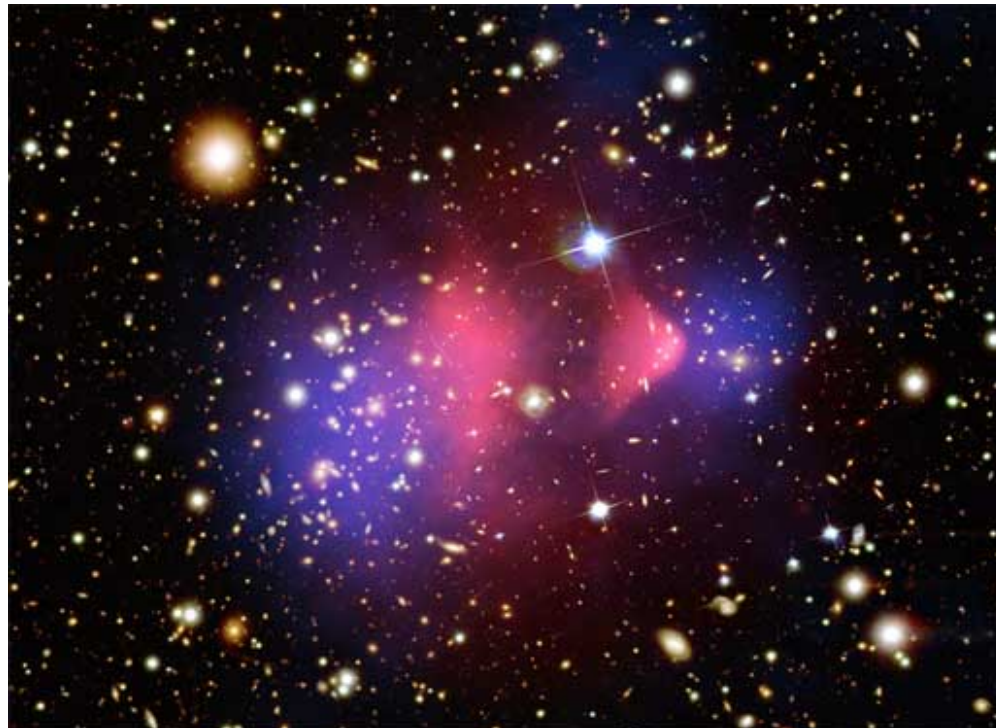
Angus, Famaey & Diaferio (2010)

Can easily reach a **factor of 10** in central parts

Galaxy clusters: where it all breaks down...

The discrepancy seems to be related with the depth of the potential well => EMOND ([Zhao & Famaey 2012](#)) where a_0 becomes $a_0(\phi)$

BUT then hard to also make the « residual mass » collisionless !!



Angus, Shan, Zhao & Famaey 2007:

- Take parametric logarithmic potential $\Phi(r)$

$$\Phi_i(r) = 1/2 v_i^2 \ln[1+(r/r_i)^2]$$

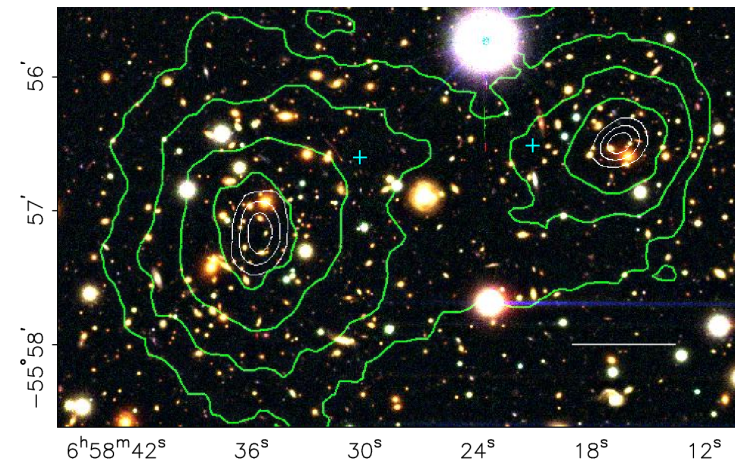
- Use $\Phi_1, \Phi_2, \Phi_3, \Phi_4$ for the 4 mass components of the bullet cluster

\Rightarrow Parametric convergence $\kappa(R)$

- χ^2 fitting the 8 parameters on 233 points of the original convergence map

- With $\mu(x) = 1$ (\rightarrow GR), or e.g. $\mu(x) = x/(1+x)$, get enclosed $M(r)$:

$$4\pi GM(r) = \int \mu(|\nabla\Phi|/a_0) \partial\Phi/\partial r dA$$



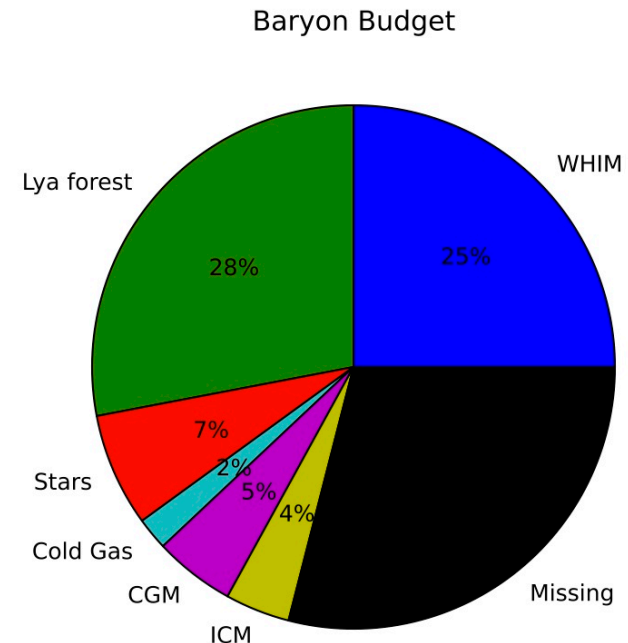


- ⇒ Large amount of missing mass around the (gasless) galaxy centers of the Bullet cluster
- ⇒ Density relatively low: $10^{-3} \text{ Msun/pc}^3$ compatible with a hot DM component

Galaxy clusters: where it all breaks down...

What remains:

- Hot dark matter (HDM, e.g., sterile neutrinos, [Angus 2009](#))
- Cluster baryonic dark matter (CBDM, [Milgrom 2008](#)), cold dense H₂ clouds
- New d.o.f. behaving like DM in clusters, see, e.g., [Dai, Matsuo & Starkman \(2008\)](#) ... **but not in galaxies** (like HDM)



Model building: classical action

$$S_N = S_{\text{kin}} + S_{\text{in}} + S_{\text{grav}} = \int \frac{\rho \mathbf{v}^2}{2} d^3x dt - \int \rho \Phi_N d^3x dt - \int \frac{|\nabla \Phi_N|^2}{8\pi G} d^3x dt.$$

$$\Rightarrow S_{\text{grav BM}} \equiv - \int \frac{a_0^2 F(|\nabla \Phi|^2/a_0^2)}{8\pi G} d^3x dt,$$

$$F(z) \rightarrow z \text{ for } z \gg 1 \text{ and } F(z) \rightarrow \frac{2}{3} z^{3/2} \text{ for } z \ll 1$$

→ The hallmark of
MOND-like actions

$$\Rightarrow \nabla \cdot \left[\mu \left(\frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right] = 4\pi G \rho$$

Other version (QUMOND):

$$\nabla^2 \Phi = \nabla \cdot [\nu (|\nabla \Phi| / a_0) \nabla \Phi] \quad \text{with } \nu(x) \sim x^{-1/2} \text{ for } x \ll 1$$

Model building: modifying GR ?

Quite a few ideas around: for instance, based on the Coincident formulation of GR based on non-metricity, made non-linear $f(Q)$ with the usual 3/2 exponent

(D'Ambrosio, Garg & Heisenberg 2020)

More « classical » attempts: start from EH action and add fields with their own actions:

$$S = \frac{c^4}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x + S_{\text{matiere}}[\psi; g_{\mu\nu}] - \int c^2 \rho_{\text{matnoire}} \sqrt{-g} d^4x$$

$$+ S_\varphi + S_{U_\mu} + \dots$$

Coupling

$$+ \int \left[c^2 \rho_{\text{matnoire}} u_\mu \dot{\xi}^\mu - V(|\rho \xi^\mu|_\perp) \right] \sqrt{-g} d^4x$$

$$\tilde{g}_{\mu\nu} = f(\varphi, U_\mu, \dots, g_{\mu\nu})$$

- dissipationless

Recovering lensing

Einstein equations relate metric to stress-energy tensor just like Poisson equation relates potential to density. In **weak-field**:

$$g_{00} = -e^{2\Phi} = -(1 + 2\Phi)$$
$$g_{ij} = e^{2\Psi} \delta_{ij} = (1 + 2\Psi) \delta_{ij}$$

$\Phi = -\Psi = \Phi_N$ in GR ($\Phi \Rightarrow$ dynamics, $\Phi - \Psi \Rightarrow$ lensing)

Idea: replace GR with a theory reducing to the SAME weak-field metric but replacing Φ_N by Φ obeying MOND

Needs $\boxed{\Psi = -\Phi}$

k-essence scalar field

- Make the modification act only on an additional scalar field ϕ such that in the weak-field: $\Phi = \Phi_N + \phi$
- Matter fields couple to: $g_{\mu\nu} \equiv e^{2\phi} \tilde{g}_{\mu\nu}$

$$S_\phi \equiv -\frac{c^4}{2k^2 l^2 G} \int d^4x \sqrt{-\tilde{g}} f(X) \quad X = kl^2 \tilde{g}^{\mu\nu} \phi_{,\mu} \phi_{,\nu}.$$

- Problem for lensing: $g_{ij} = e^{-2\Phi_N + 2\phi} \delta_{ij}$
- What we need is an *action invariant under disformal transformations* of the type :

$$[\Phi \rightarrow \Phi + \beta(r); \Psi \rightarrow \Psi - \beta(r)]$$



Vector fields

- TeVeS (Bekenstein 2004): introduce unit-norm vector field and

$$g_{\mu\nu} \equiv e^{-2\phi} \tilde{g}_{\mu\nu} - 2\sinh(2\phi) U_\mu U_\nu$$

- But then GW and photons don't follow same path
=> different Shapiro delay
- Kilonova GW170817 excludes it !

Vector fields

- Possible to recast TeVeS as single metric theory with vector field B such that

$$B_\mu = e^{-\phi} \hat{A}_\mu \text{ and } \hat{A}_\mu \hat{A}^\mu = -1$$

⇒ Speed of light and GWs equal (Skordis & Zlosnik 2019)

How to reproduce the CMB? (Skordis & Zlosnik 2020)

Basically needs to make the scalar field gravitate (i.e., become a form of DM) in time-dependent situations, and act as a modification of gravity in quasi-static limit

The SZ action for relativistic MOND

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}} \left[R - \frac{K_B}{2} \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} + 2(2 - K_B) \hat{J}^\mu \nabla_\mu \phi - (2 - K_B) \mathcal{Y} - \mathcal{F}(\mathcal{Y}, \mathcal{Q}) - \lambda(\hat{A}^\mu \hat{A}_\mu + 1) \right] + S_m[g]$$

$$\hat{F}_{\mu\nu} = 2\nabla_{[\mu} \hat{A}_{\nu]}, \quad \hat{J}_\mu = \hat{A}^\alpha \nabla_\alpha \hat{A}_\mu$$

$$\begin{aligned} \mathcal{Q} = A^\mu \nabla_\mu \phi &\longrightarrow \dot{\phi} \\ \mathcal{Y} = \mathcal{Q}^2 + (\nabla\phi)^2 &\longrightarrow |\vec{\nabla}\varphi|^2 \end{aligned}$$

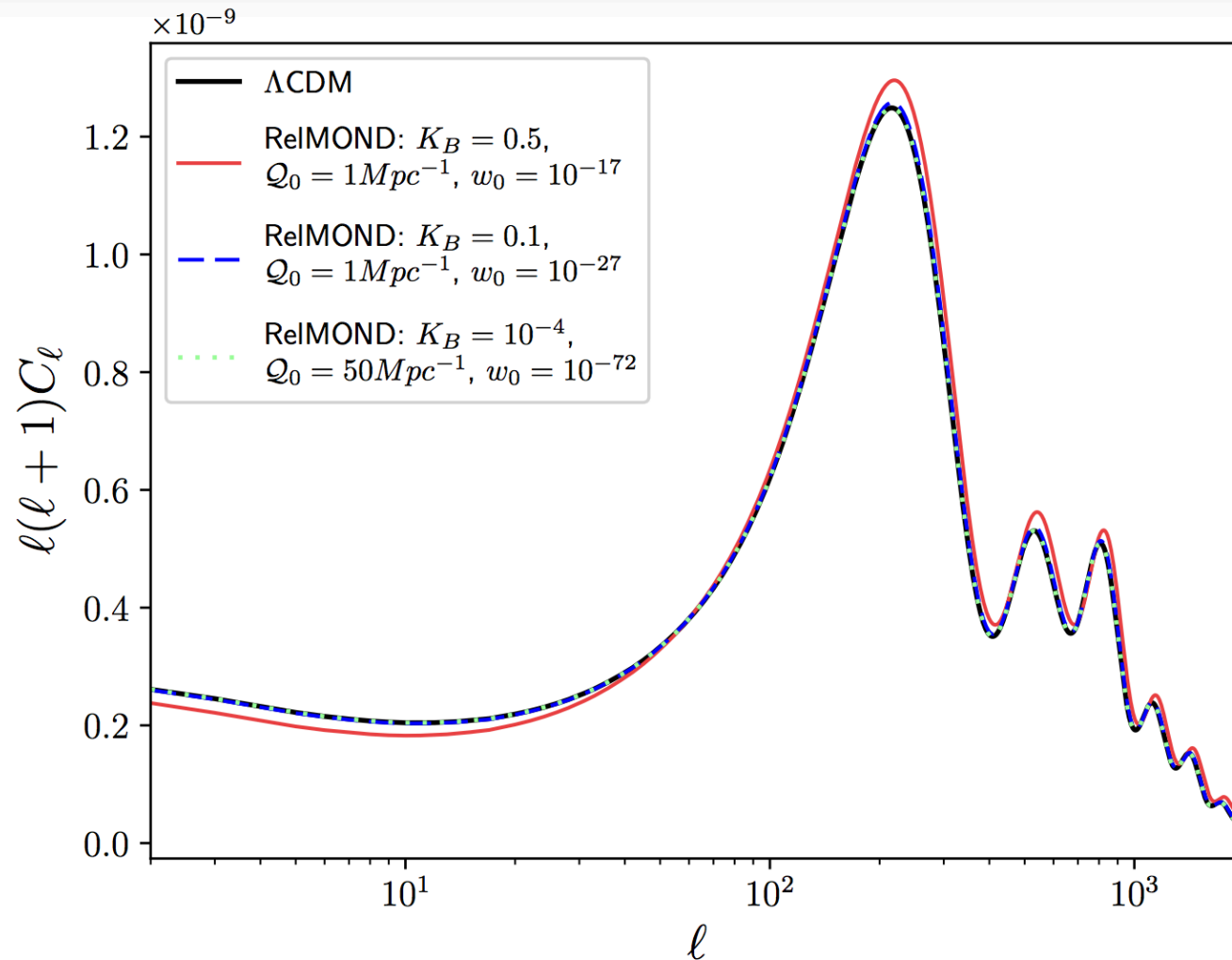
$$\mathcal{F} = -2\mathcal{K}_2 (\mathcal{Q} - \mathcal{Q}_0)^2 + (2 - K_B) \mathcal{Y} + \frac{2(2 - K_B)}{3a_0} \mathcal{Y}^{3/2} + \dots$$

"dust" cosmology

Mixing

MOND

The CMB in relativistic MOND



Skordis & Zlosnik

Modifying gravity?

$$g = g_N$$

$$g = (g_N a_0)^{1/2}$$

$$\text{if } g \gg a_0$$

$$\text{if } g \ll a_0$$

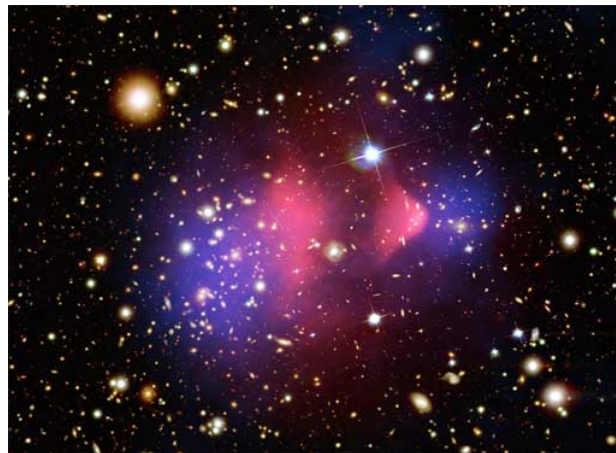
MOND

Milgrom 1983

⇒ **Convolut ed relativistic theory, needs a field behaving like DM in cosmology, but real challenge: non-linear regime and galaxy clusters!**

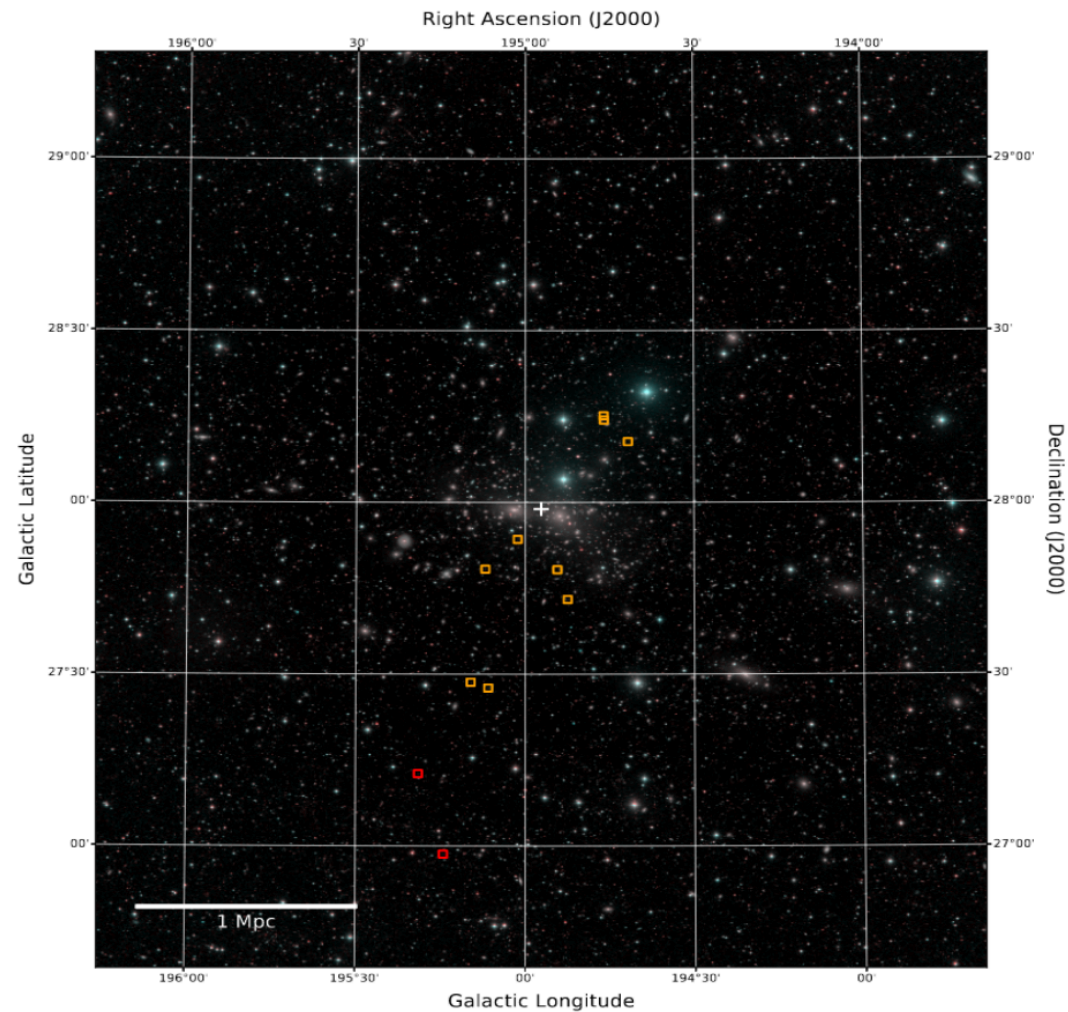
Intermediate regime of barely virialized systems??

Ultra-diffuse galaxies in clusters immune to the EFE?



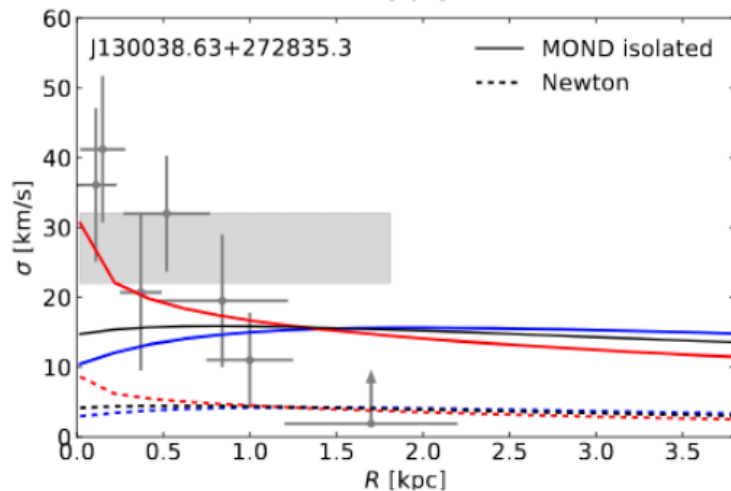
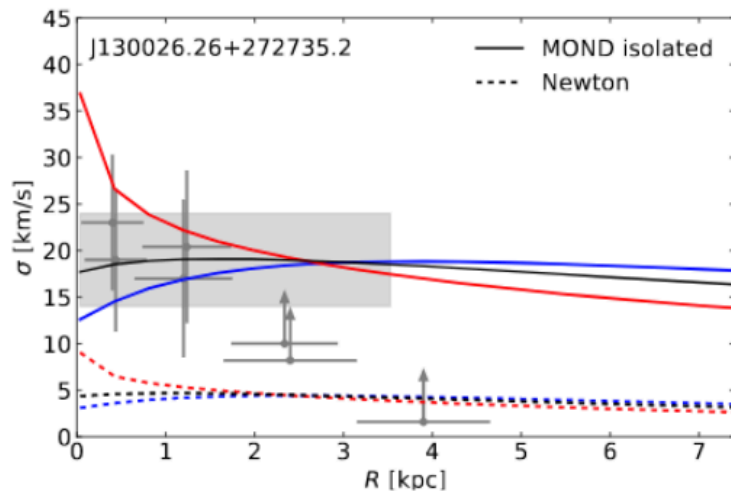
??

Clues from ultra-diffuse galaxies in the Coma cluster



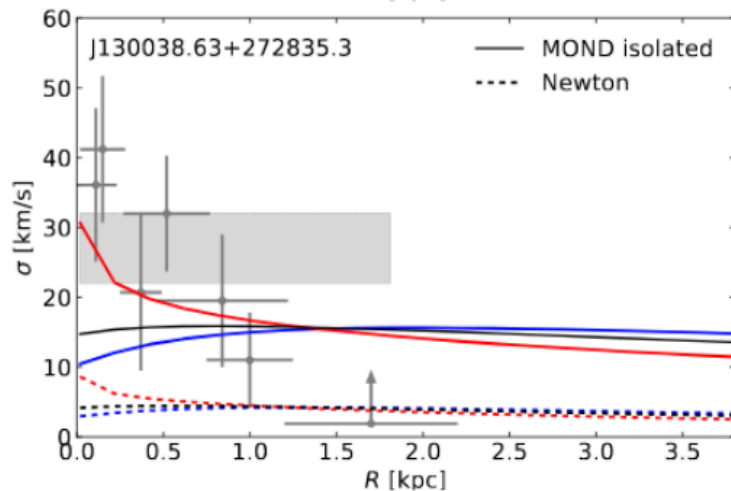
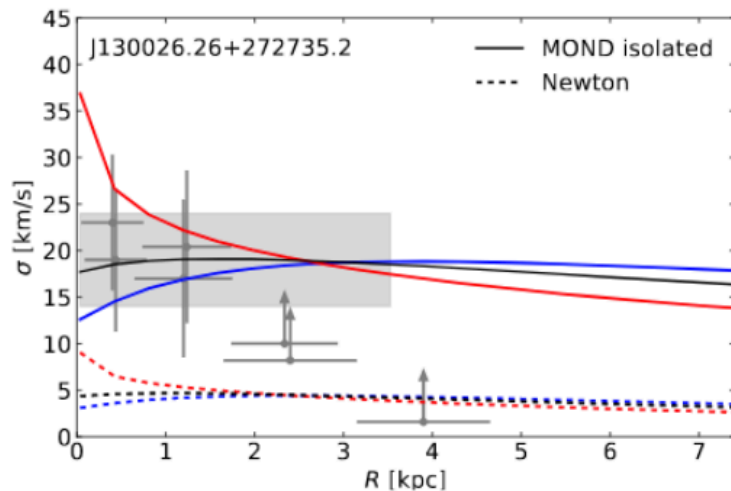
Work with J. Freundlich, P.-A. Oria, M. Bilek

Clues from ultra-diffuse galaxies in the Coma cluster



- The agreement of the velocity dispersions with MOND are impressive !
- But the **EFE** ruins the agreement if $d < 5\text{Mpc}$ ($d > 5\text{Mpc}$ would require a very peculiar observer-dependent bias in spatial distribution)
- Difficult to understand if HDM makes up the residual missing mass... can't cluster in the UDGs

Clues from ultra-diffuse galaxies in the Coma cluster



- If CBDM makes up the missing mass in the cluster, it could also make up the missing mass in the UDGs, **but why then such a good agreement with isolated MOND ?**


- ‘*Last-hope*’ hypothesis: the new d.o.f. making up the residual missing mass (same as sourcing structure in ‘SZ-MOND’ ?) does not couple to the field generating MOND in the UDGs

=> decoupling kills the EFE in clusters (?)



Conclusions on « small-scale » tensions and the nature of DM

- WDM: good for TBTF, not so much for the other challenges, **above ~ 10 keV, does not really solve any challenge**. Perhaps hot orbits if coupled with non-gaussianities
- FDM: good for TBTF and reducing dynamical friction, not so much other challenges such as diversity of RC, **above $\sim 10^{-20}$ eV, does not really solve any challenge**
- SIDM: very promising for diversity! **could make failed feedback at the high mass end worse**, velocity-dependence tightly constrained by galaxy clusters
- MOND: solves quite a few challenges at galaxy scales! But also creates new ones (convoluted relativistic theory, **missing mass in clusters, UDGs in clusters,...**)
- BIDM: not explored very much yet...



Q: Can the MOND phenomenology result from a quasi-equilibrium configuration linked to baryon-DM particle **collisions**?
(with high cross sections $> 10^{-30} \text{ cm}^2 \Rightarrow$ not WIMPS)

A: NO

Reason:

Baryons are clumped into stars (especialy in HSB galaxies), and time to encounter a star would be several millions of Hubble times even with such a large interaction cross-section

However, it could (perhaps?) work with a fluid-like scenario where baryons would heat the fluid through collective excitations, or with baryons emitting some form of ‘dark radiation’ in the presence of DM...

Let's proceed under such assumptions...

Baryon-interacting dark matter?

Change from CBE to BTE with two fluids through some long-range interaction (Famaey et al. 2018, 2020)

⇒ second order moments then give a **heat equation** which can resemble the MOND equation if roughly assuming $T \propto \Phi$

$$\frac{3}{2} \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \right) \frac{T}{m} + \frac{1}{\rho} P^{ij} \partial_i u_j + \frac{1}{\rho} \vec{\nabla} \cdot \vec{q} = \frac{\dot{\mathcal{E}}}{m}$$

Spherical symmetry+isotropy+no spin+equilibrium (no t dependence) for halo:

$$\vec{\nabla} \cdot \left(m \kappa \vec{\nabla} v^2 \right) = -\rho \frac{\dot{\mathcal{E}}}{m}$$

Two things to fix: **thermal conductivity** and **heating rate**



Thermal conductivity :

$$\kappa = \frac{3}{2} \frac{\rho v^2 t_{\text{relax}}}{m} \quad \text{through some sort of DM self-interactions}$$

Needs a relatively short relaxation time, let's take: $t_{\text{relax}} = \frac{\mathcal{N}}{\sqrt{G\rho}}$

Heating rate :

We want a_0 in the denominator on the l.h.s., hence should be prop. to a_0 , simplest is to take $a_0 v$, and dimensionless dependence on ρ and ρ_b

$$\frac{\dot{\mathcal{E}}}{m} = C a_0 v \frac{\rho_b}{\rho} \quad \Rightarrow \text{little interaction for CMB, just the right energy exchange for EDGES... (simply by putting } a_0 \text{ scale)}$$



Let's recap all equation for DM (continuity, Jeans, Heat, Poisson):

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0;$$

$$\rho \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \right) u^i + \partial^i (\rho v^2) = \rho g^i;$$

$$\frac{3}{2} \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \right) v^2 + v^2 \vec{\nabla} \cdot \vec{u} - \frac{1}{\rho} \vec{\nabla} \cdot \left(\frac{3}{2} \mathcal{N} \sqrt{\frac{\rho}{G}} v^2 \vec{\nabla} v^2 \right) = C a_0 v \frac{\rho_b}{\rho}$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G (\rho + \rho_b) .$$

In the DM dominated regime $\rho_b \ll \rho$ in Poisson, equations invariant under:

$$\vec{x} \rightarrow \lambda \vec{x};$$

$$t \rightarrow \lambda^y t;$$

$$v \rightarrow \lambda^{1-y} v;$$

$$\vec{u} \rightarrow \lambda^{1-y} \vec{u};$$

$$\vec{g} \rightarrow \lambda^{1-2y} \vec{g};$$

$$\rho \rightarrow \lambda^{-2y} \rho;$$

$$\rho_b \rightarrow \lambda^{1-4y} \rho_b$$

so if scale-lengths $L_2 = \lambda L_1$ then

$$M_{b,2} = \lambda^{4-4y} M_{b,1},$$

$$\vec{V}_2(\lambda \vec{x}) = \lambda^{1-y} \vec{V}_1(\vec{x})$$

\Rightarrow

$$\boxed{V_1(R)/M_{b1}^{1/4} = V_2(L_2 R/L_1)/M_{b2}^{1/4}}$$



Superfluid dark matter

Idea of [Berezhiani & Khoury](#): DM could have **strong self-interactions** and enter a **superfluid** phase when

- cold enough (i.e; their de Broglie wavelength $\lambda \sim 1/(mv)$ is large
- dense enough (i.e. the interparticle separation is smaller than λ)

⇒ Superfluid core (~ 50 - 100 kpc in MW) where collective excitations (phonons) are the only relevant degree of freedom (represented by a scalar field in EFT) and can couple to baryons and mediate a long-range force + NFW-like « normal » atmosphere outside of the core

Parameters of the theory (or rather, of the toy-model theory):

- DM particle mass m ($\sim \text{eV}$)
 - Self-interaction cross-section σ ($\sigma/m \ll 1 \text{ cm}^2/\text{g}$)
 - Self-interaction « strength » Λ ($\sim 0.05 \text{ meV}$)
 - Coupling constant of the scalar field to baryons α
 - Parameter accounting for non-zero temperature effects β (will be fixed)
- } combination of Λ^2 and α^3 related to a_0

Superfluid dark matter

Transition radius R_T when inverse of self-interaction rate of the order of dynamical time:

$$\Gamma = \frac{\sigma}{m} \mathcal{N} v \rho = t_{\text{dyn}}^{-1}$$

EFT Lagrangian for the phonons:

$$\mathcal{L} = \frac{2\Lambda(2m)^{3/2}}{3} X \sqrt{|X - \beta Y|} - \alpha \frac{\Lambda}{M_{\text{Pl}}} \phi \rho_b$$

$$\text{where } X = \hat{\mu}(\Phi) - (\vec{\nabla} \phi)^2 / 2m$$

=> Varying w.r.t. to the scalar field gives the phonon equation of motion and varying w.r.t. grav. potential gives the superfluid density

Phonon-mediated force: simple case

$$\vec{a}_\phi = \alpha \frac{\Lambda}{M_{\text{Pl}}} \vec{\nabla} \phi$$

Static profile + ignore finite-temperature term:

$$\vec{\nabla} \cdot \left(\frac{(\vec{\nabla} \phi)^2 - 2m\hat{\mu}}{\sqrt{(\vec{\nabla} \phi)^2 - 2m\hat{\mu}}} \vec{\nabla} \phi \right) = \frac{\alpha \rho_b}{2M_{\text{Pl}}}$$

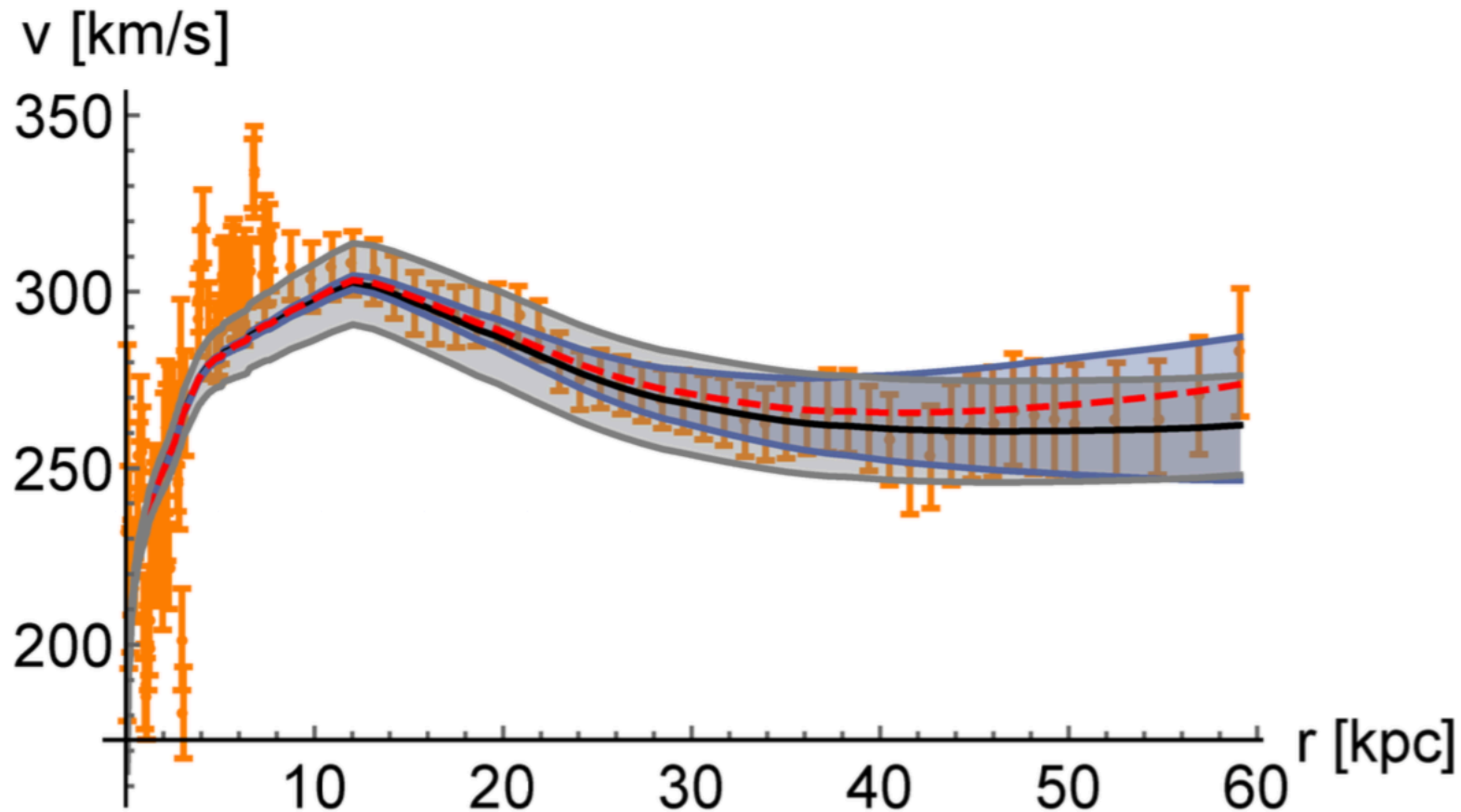
$$\Rightarrow |\vec{\nabla} \phi| \vec{\nabla} \phi \simeq \alpha M_{\text{Pl}} \vec{a}_b$$

$$\Rightarrow a_\phi = \sqrt{\frac{\alpha^3 \Lambda^2}{M_{\text{Pl}}}} a_b \quad \Rightarrow \quad a_0 = \frac{\alpha^3 \Lambda^2}{M_{\text{Pl}}}$$

General case

Spherical symmetry (next step: Kuzmin disks and then numerical solution for general disk configuration):

- 1) Solve
$$\frac{(\vec{\nabla}\phi)^2 + 2m\left(\frac{2\beta}{3} - 1\right)\hat{\mu}}{\sqrt{(\vec{\nabla}\phi)^2 + 2m(\beta - 1)\hat{\mu}}} \vec{\nabla}\phi = \alpha M_{\text{Pl}} \vec{a}_b$$
- 2) Insert $(\vec{\nabla}\phi)^2$ in
$$\rho_{\text{SF}} = \frac{2\sqrt{2}m^{5/2}\Lambda \left(3(\beta - 1)\hat{\mu} + (3 - \beta)\frac{(\vec{\nabla}\phi)^2}{2m}\right)}{3\sqrt{(\beta - 1)\hat{\mu} + \frac{(\vec{\nabla}\phi)^2}{2m}}}$$
- 3) Solve Poisson
$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi}{dr} \right) = 4\pi G_N \left(\tilde{\rho}_b(r) + \rho_{\text{SF}}(\Phi(r), a_b) \right)$$
- 4) Match density and pressure of NFW profile at R_{NFW}
=> get virial mass M_{200} (only free parameter, start again with different central values of potential to get different M_{200})



Berezhiani, Famaey, Khoury 2018

UGC 2953 (sphericized profile, $a_0 \sim 0.9 \times 10^{-10} \text{ m/s}^2$)

Black : $M_{\text{DM}} = 1.6 \times 10^{12} M_{\text{sun}}$ ($R_{\text{T}} = 82 \text{ kpc}$, $R_{\text{NFW}} = 76 \text{ kpc}$)

Red-dashed: $M_{\text{DM}} = 10^{13} M_{\text{sun}}$ ($R_{\text{T}} = 129 \text{ kpc}$, $R_{\text{NFW}} = 95 \text{ kpc}$)



System	Behavior
Rotating Systems	
Solar system	Newtonian
Galaxy rotation curve shapes	MOND (+ small DM component)
Baryonic Tully–Fisher Relation	MOND for RCs (but particle DM for lensing)
Bars and spiral structure in galaxies	MOND
Interacting Galaxies	
Dynamical friction	Absent in superfluid core
Tidal dwarf galaxies	Newtonian when outside of superfluid core
Spheroidal Systems	
Star clusters	MOND with EFE inside galaxy host core - Newton outside of core
Dwarf Spheroidals	MOND with EFE inside galaxy host core - MOND+DM outside of core
Clusters of Galaxies	particle DM
Ultra-diffuse galaxies	MOND without EFE outside of cluster core

Next step: model stellar streams