Le modèle standard de la Cosmologie

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École de Gif (Marseille 2021)

Centre de Physique Théorique

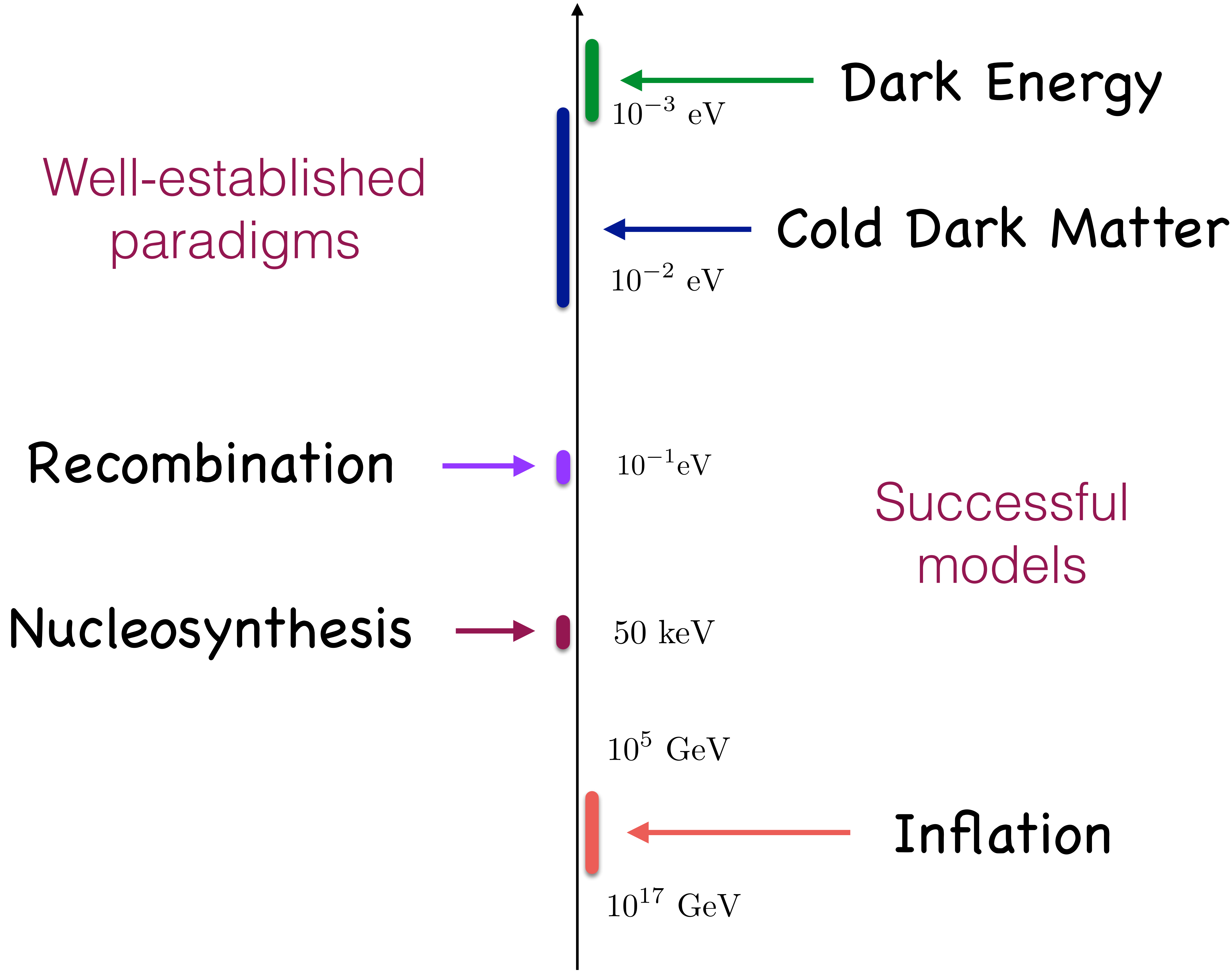
Standard model of cosmology

$$\neq$$

Standard model of particle physics

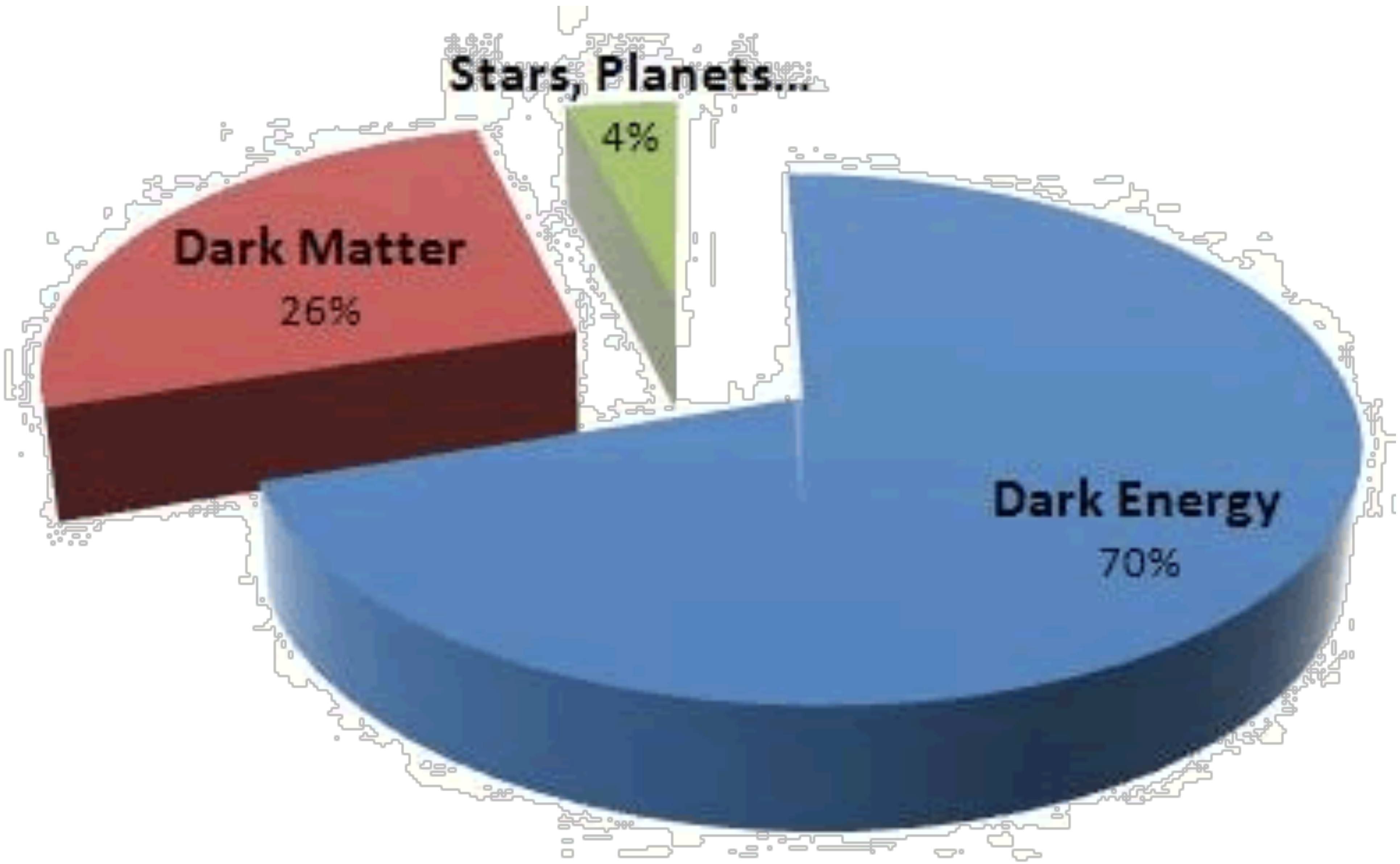
Limitations:

- Only one universe (one experiment)
- Only one point of view



Dark Energy
10^{-3} eV
Well-established paradigms
Cold Dark Matter
10^{-2} eV
Recombination
10^{-1}eV
Successful models
Nucleosynthesis
50 keV
10^5 GeV
Inflation
10^{17} GeV

ΛCDM cosmology



Natural Units

$$c = 1 \qquad [\ell] = [t]$$
$$[E] = [p] = [m]$$

$$\hbar = 1 \qquad [\ell] = [m]^{-1}$$

$$k_B = 1 \qquad [T] = [E]$$

All quantities are $[m]^{\#}$ (or $[\ell]^{-\#}$)

$\mathrm{GeV} \simeq m_{\mathrm{proton}}$

Natural Units

$c = 1$

$$[\ell] = [t]$$

$$[E] = [p] = [m]$$

$\hbar = 1$

$$[\ell] = [m]^{-1}$$

$k_B = 1$

$$[T] = [E]$$

Energy density:

$$[\rho] = [m]^4$$

Thermal state/radiation:

$$\rho = T^4$$

Natural Units

$$c = 1 \qquad [\ell] = [t]$$
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$$k_B = 1 \qquad [T] = [E]$$

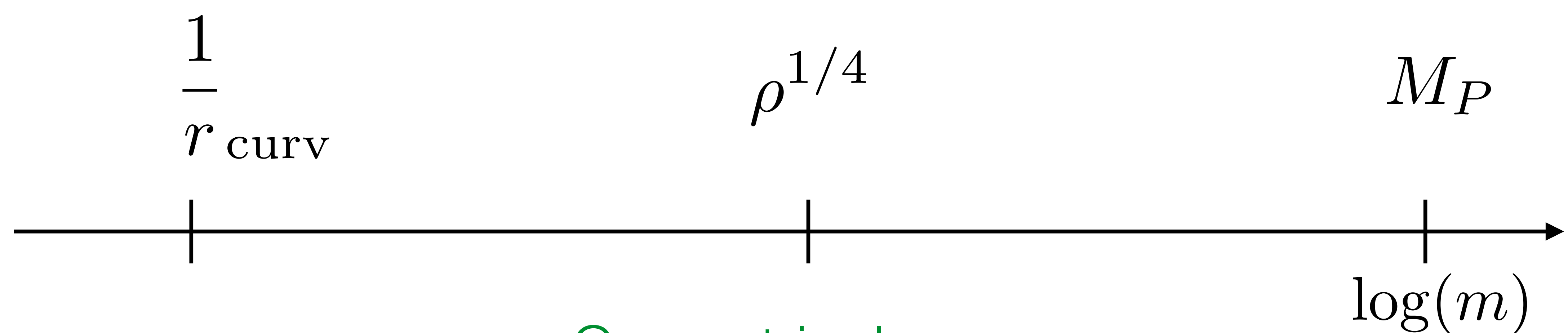
Newton's constant $\qquad G_N \equiv \frac{1}{M_P^2} \quad M_P = 10^{19}\mathrm{GeV}$

Curvature $\qquad [R] = [m]^2 \quad [K] = [m]$

$$\sim \left(\frac{1}{r_{\mathrm{curv}}}\right)^{\#}$$

One moral content of Einstein's equations

$$\rho \;=\; \left(\frac{1}{r_{\rm curv}}\right)^2 \; M_P^2$$



Geometrical mean

Fundamental hypotheses of cosmology

Homogeneity and Isotropy

Fundamental hypotheses of cosmology

Homogeneity and Isotropy

Each point equivalent to any other

Difficult to test directly:
we observe far in space AND back in time

Alternative: e.g. Bianchi-like models:
different expansion rates in different directions

Fundamental hypotheses of cosmology

Homogeneity and **Isotropy**

Each direction equivalent to any other

Good evidence from CMB and other observations

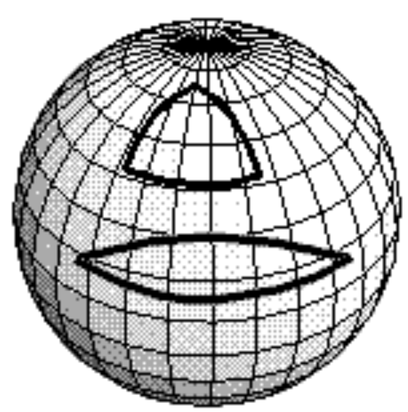
Alternative: e.g. Bondi-like models:
spherical symmetry

Fundamental hypotheses of cosmology

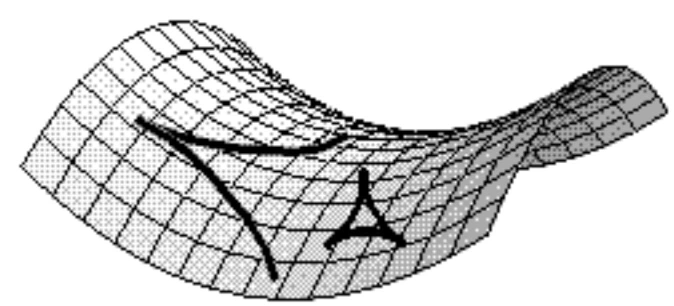
Homogeneity and Isotropy

Geometrically, it means that we can identify the t=const. surfaces with a spatial geometry that is maximally symmetric.

Positive curvature

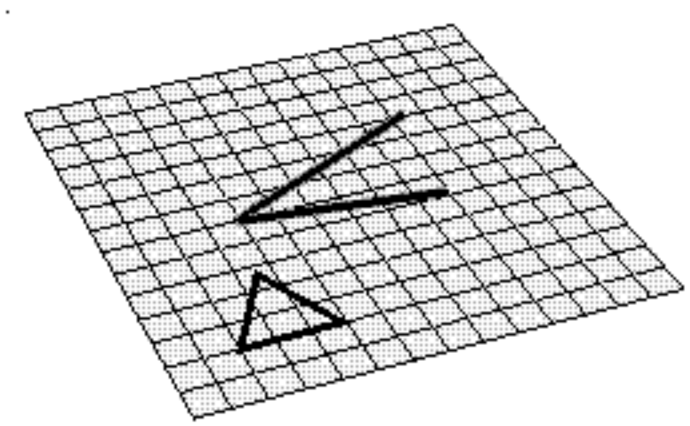


Universe with *positive* curvature. Diverging line converge at great distances. Triangle angles add to more than 180°.



Universe with *negative* curvature. Lines diverge at ever increasing angles. Triangle angles add to less than 180°.

Negative curvature



Flat

Fundamental hypotheses of cosmology

Homogeneity and Isotropy

Geometrically, it means that we can identify the t=const. surfaces with a spatial geometry that is maximally symmetric.

Broad evidences that this geometry is just flat 3d space

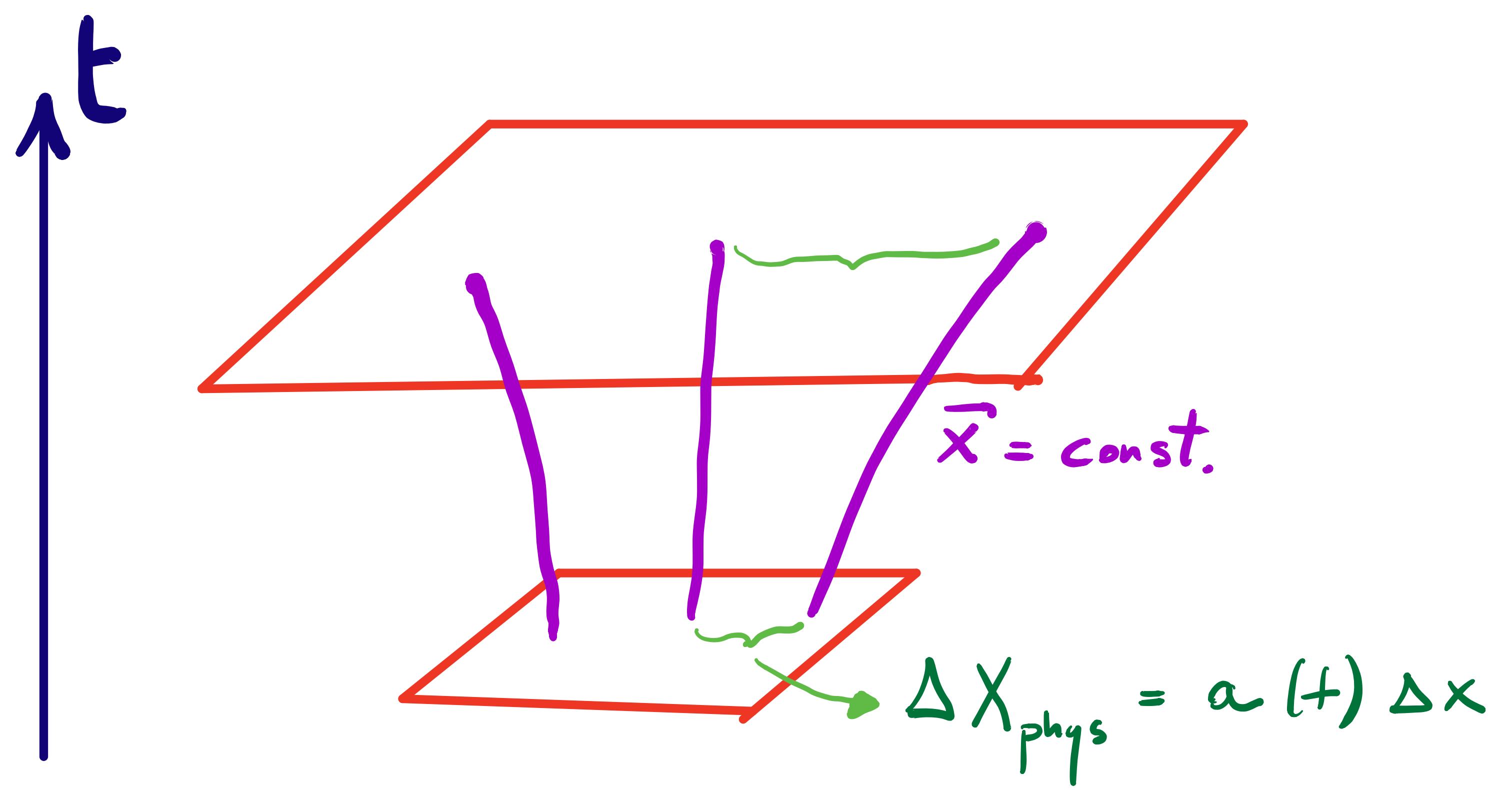
Friedmann-Lemaitre-Robertson-Walker metric

Line element: integrate if you want the length of a curve

"Scale factor"

Flat sections

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}$$



The Hubble parameter

There is no expansion velocity: the velocity depends on the relative distance.
There is an expansion rate:

$$H_0 \simeq 73 \ \frac{\mathrm{Km/s}}{\mathrm{Mpc}} \simeq \ 10^{-34}\mathrm{eV}$$

The lowest
interesting mass scale!

H_0 $\qquad$ $\rho^{1/4} \simeq \Lambda^{1/4}$ $\qquad$ M_P

10^{-34} $\qquad$ 10^{-3} $\qquad$ 10^{28} $\quad \log(m)$

$\simeq \mathrm{mm}^{-1}$

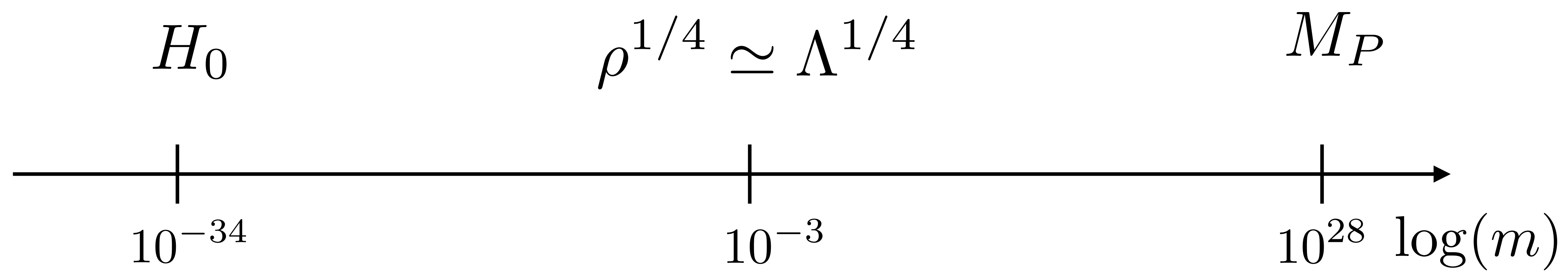
The Hubble parameter

In FLRW : $H = \dfrac{\dot{a}}{a}$

$H_0 \simeq$ curvature scale

$H_0^{-1} \simeq$ age of the universe $\simeq 10^{10}$ years

$H_0^{-1} \simeq$ size of the observable universe



The equations

$$H^2 = \frac{1}{3M_P^2}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}(\rho + 3p)$$

Gravity is an attractive force...
well it depends on the sign of p!

$$\dot{\rho} + 3H(\rho + p) = 0$$

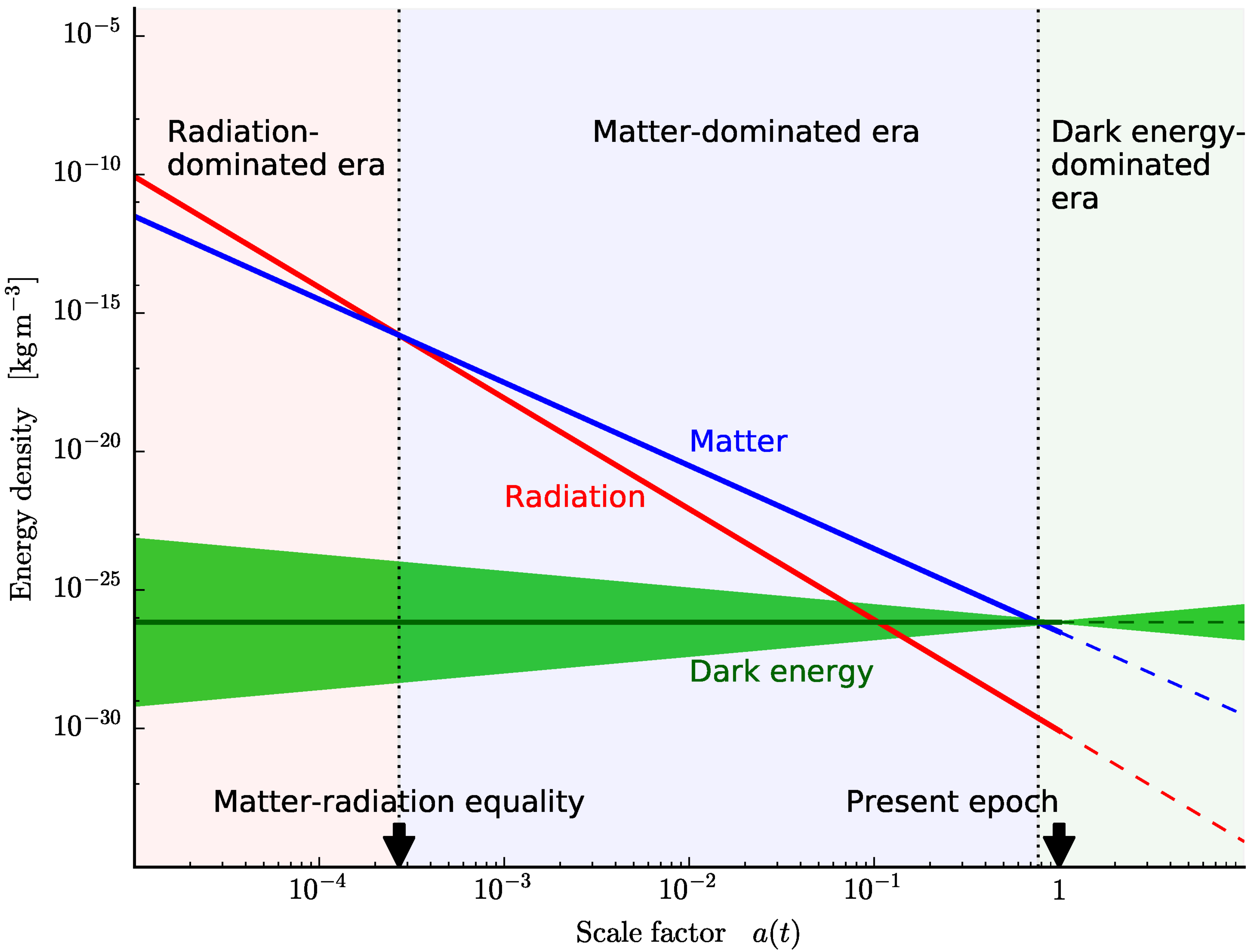
$$w = \frac{\text{pressure}}{\text{energy density}}$$

Acceleration: $w < -\frac{1}{3}$

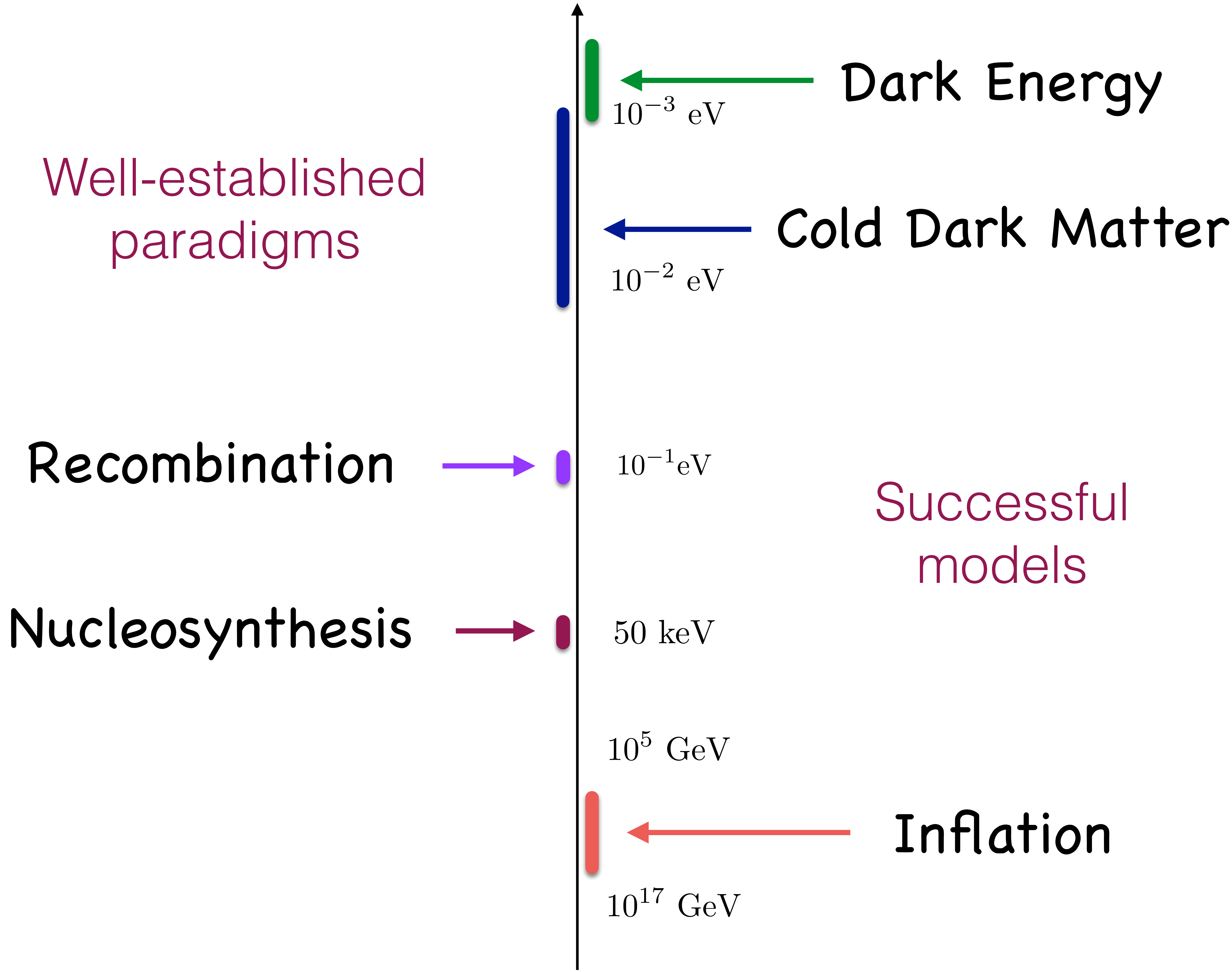
Types of matter

Non relativistic	$v \ll 1, \quad p \simeq 0$	$\rho \propto a^{-3}$
Radiation	$v = 1, \quad p = \rho/3$	$\rho \propto a^{-4}$
Spatial curvature	$r_{\rm curv} \propto a$	$\rho \propto a^{-2}$
Scalar field	$w = \dfrac{\frac{\dot\phi^2}{2} - V}{\frac{\dot\phi^2}{2} + V}$	$\rho \propto a^{-3(1+w)}$

Epochs



Radiation-dominated era
Matter-dominated era
Dark energy-dominated era
Matter
Radiation
Dark energy
Matter-radiation equality
Present epoch
Energy density [kg m^{-3}]
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
Scale factor $a(t)$



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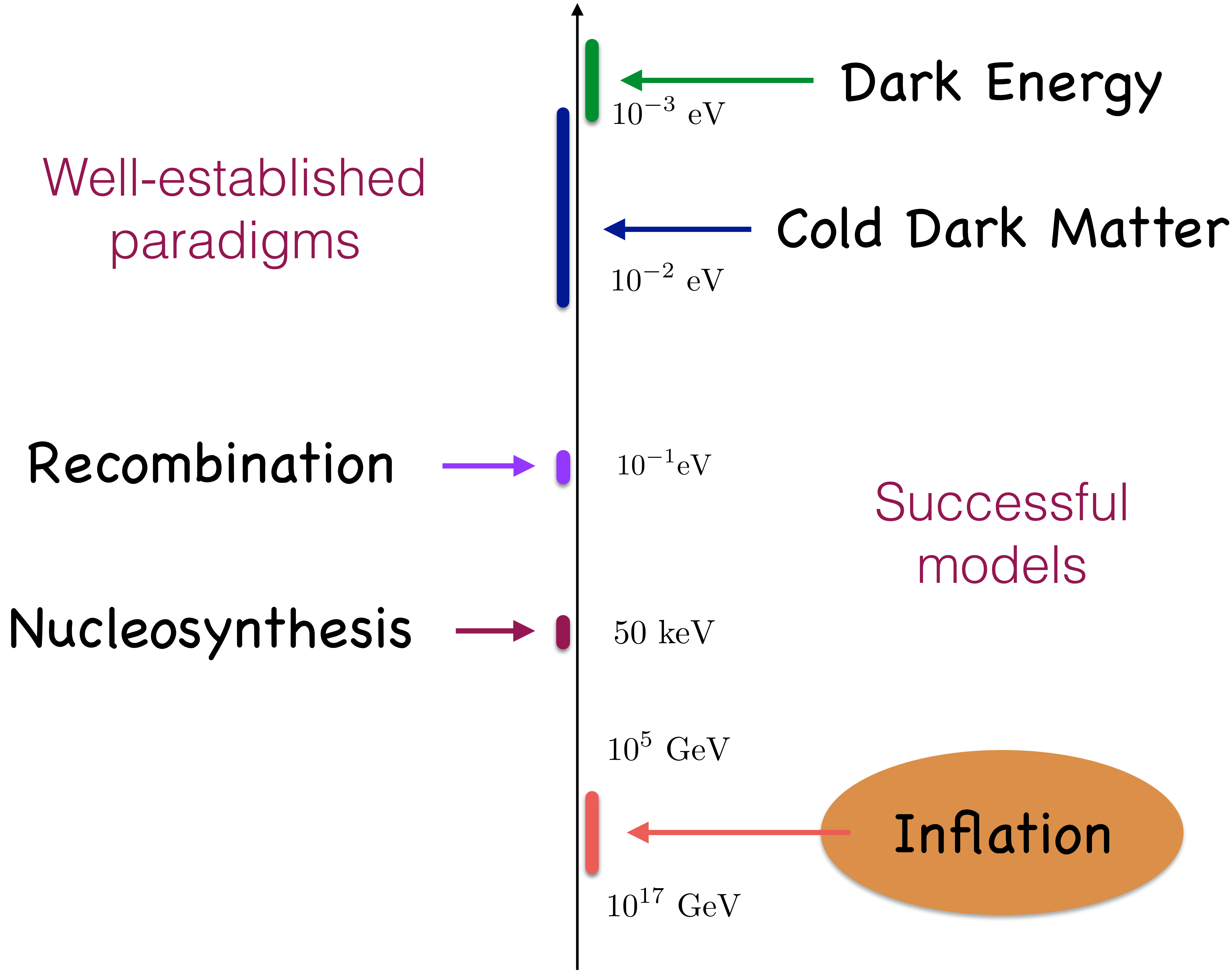
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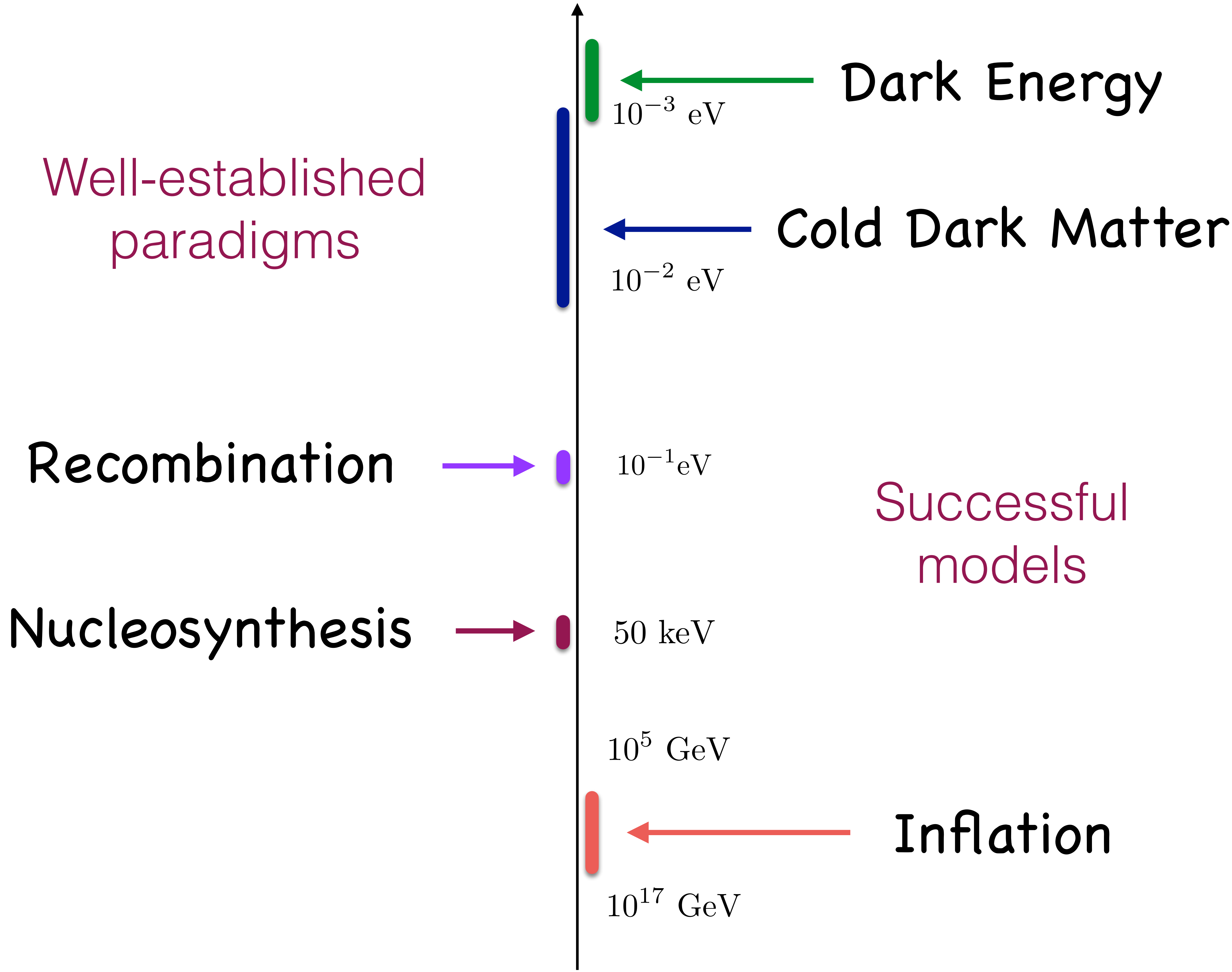

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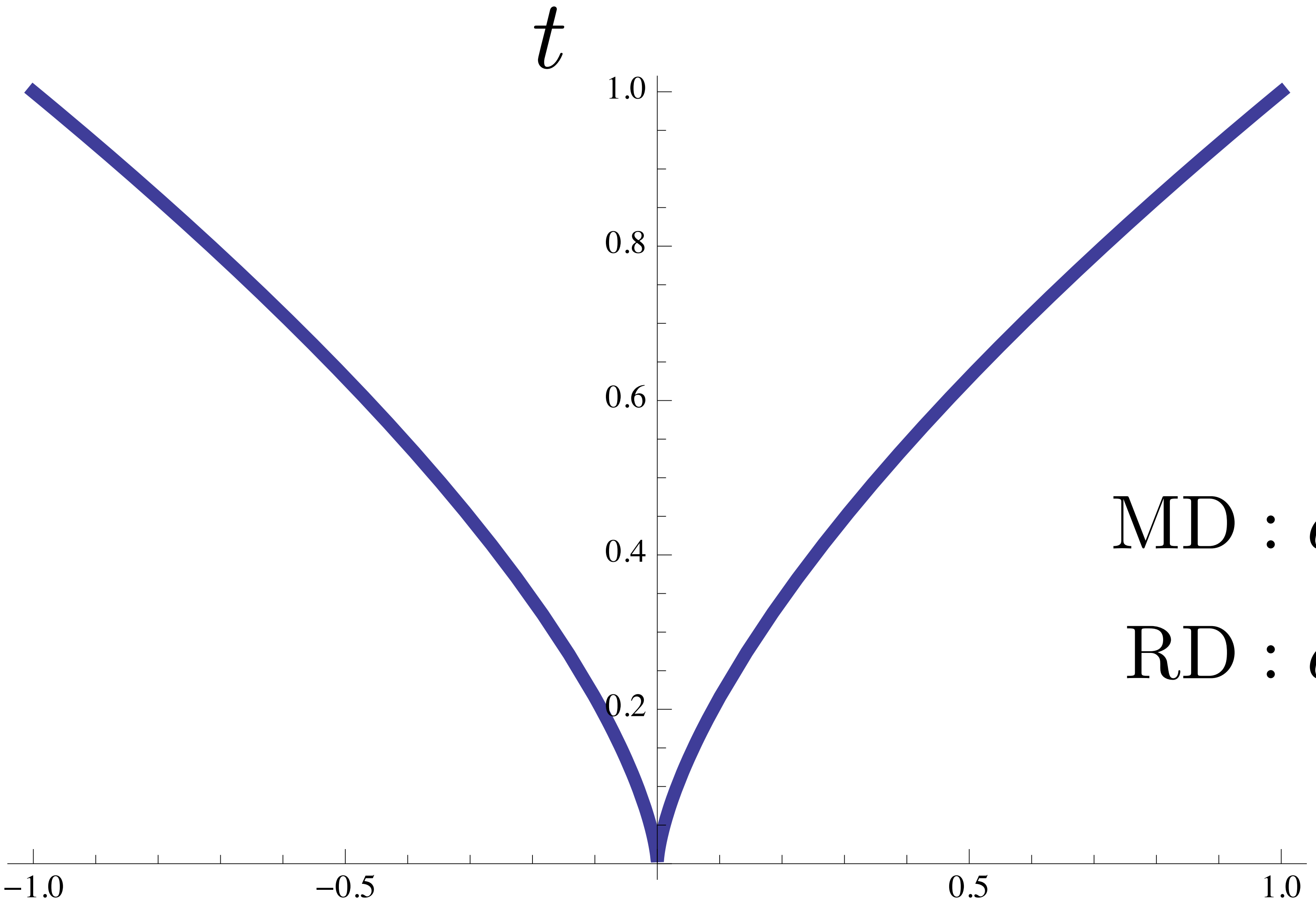
Inflation

- Sets the ``correct” initial conditions
- Solves the curvature problem
- Solves the Horizon problem
- As a gift -> provides a mechanism for generating the primordial perturbations

Horizon problem

$$ds^2 = -dt^2 + a^2(t)dx^2$$



$$\mathrm{MD} : a \propto t^{2/3}$$

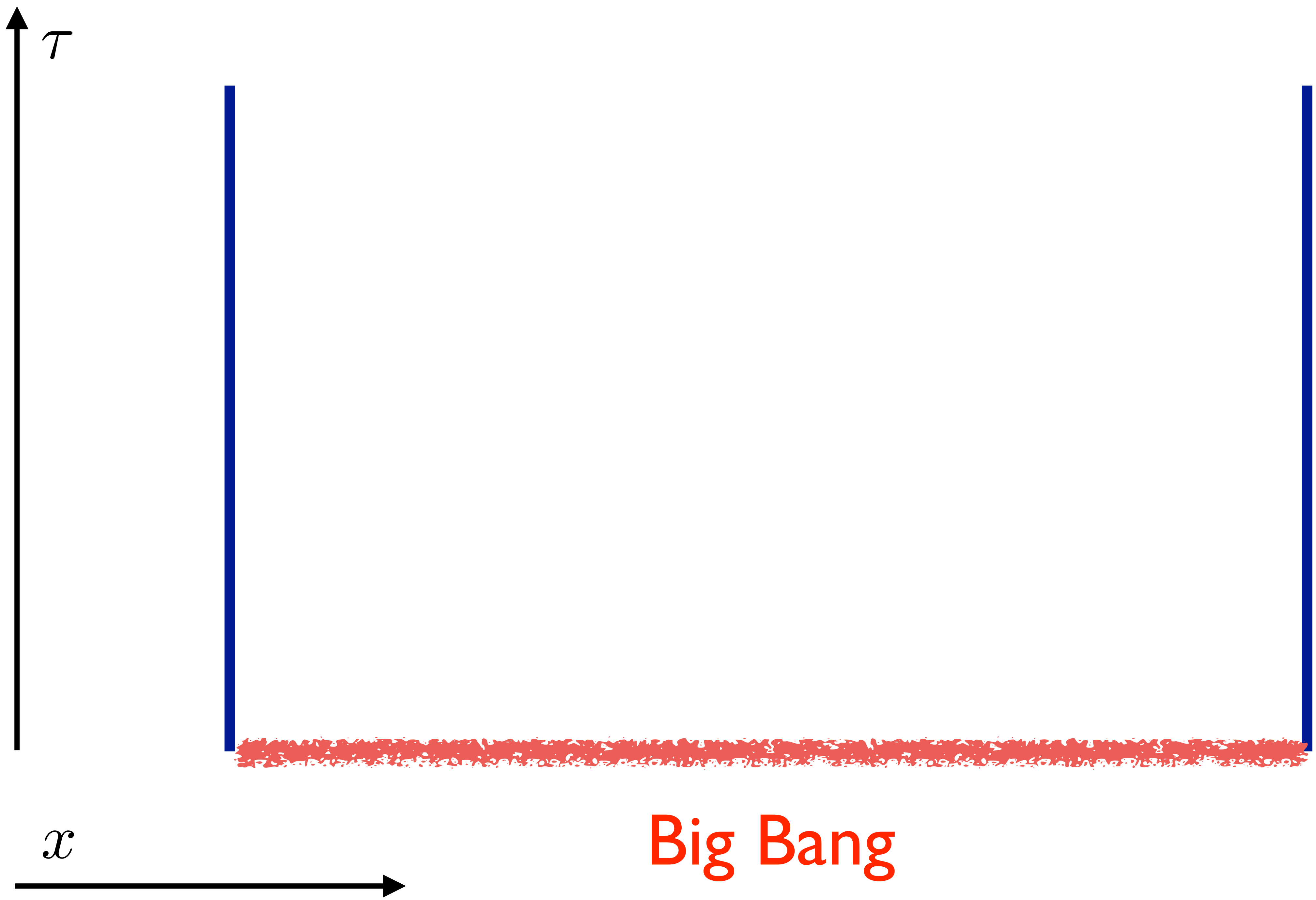
$$\mathrm{RD} : a \propto t^{1/2}$$

Big Bang

Horizon problem

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$$

$$d\tau = \frac{dt}{a(t)}$$



$$\mathrm{MD}: a \propto \tau^2$$

$$\mathrm{RD}: a \propto \tau$$

Horizon problem

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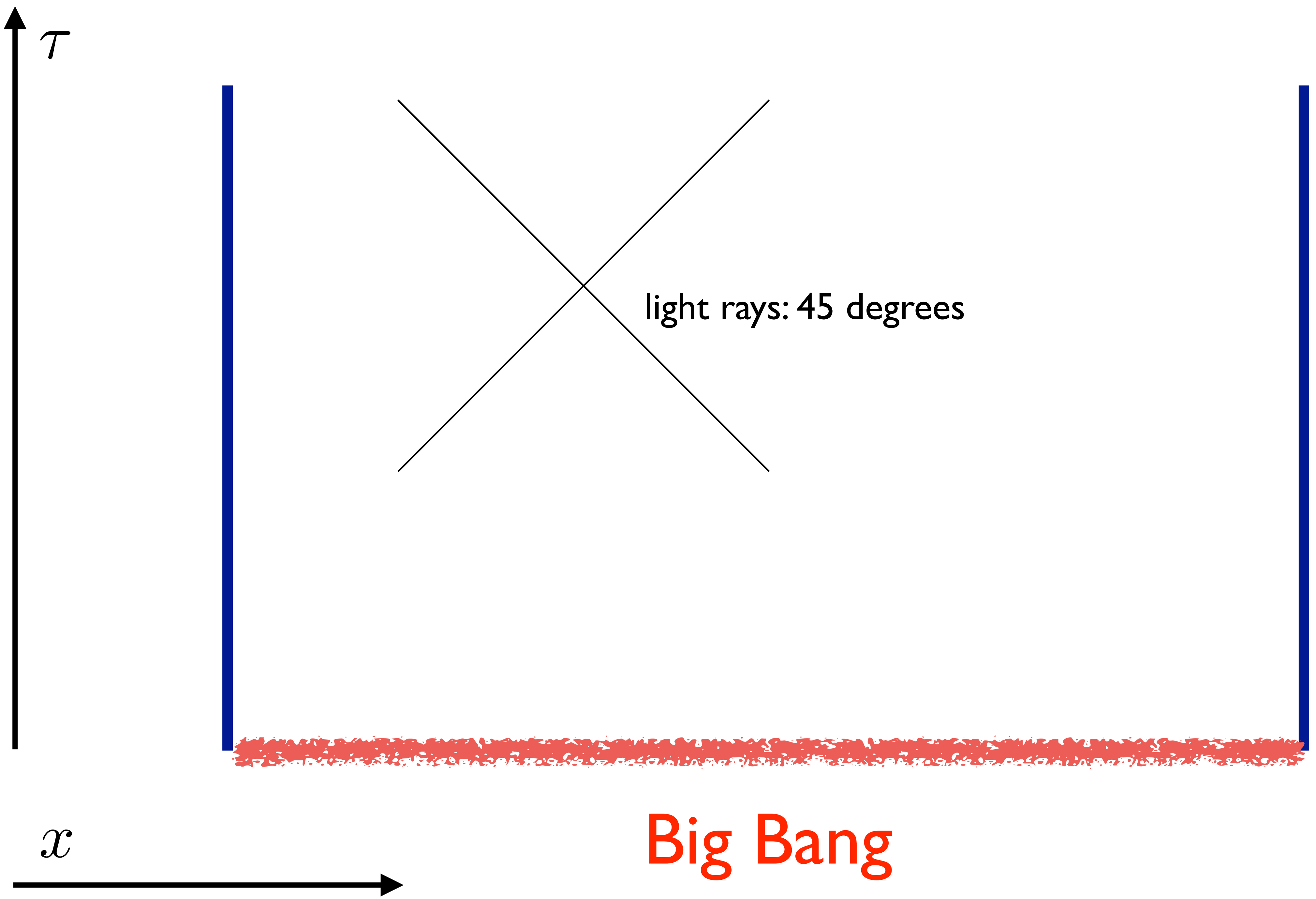

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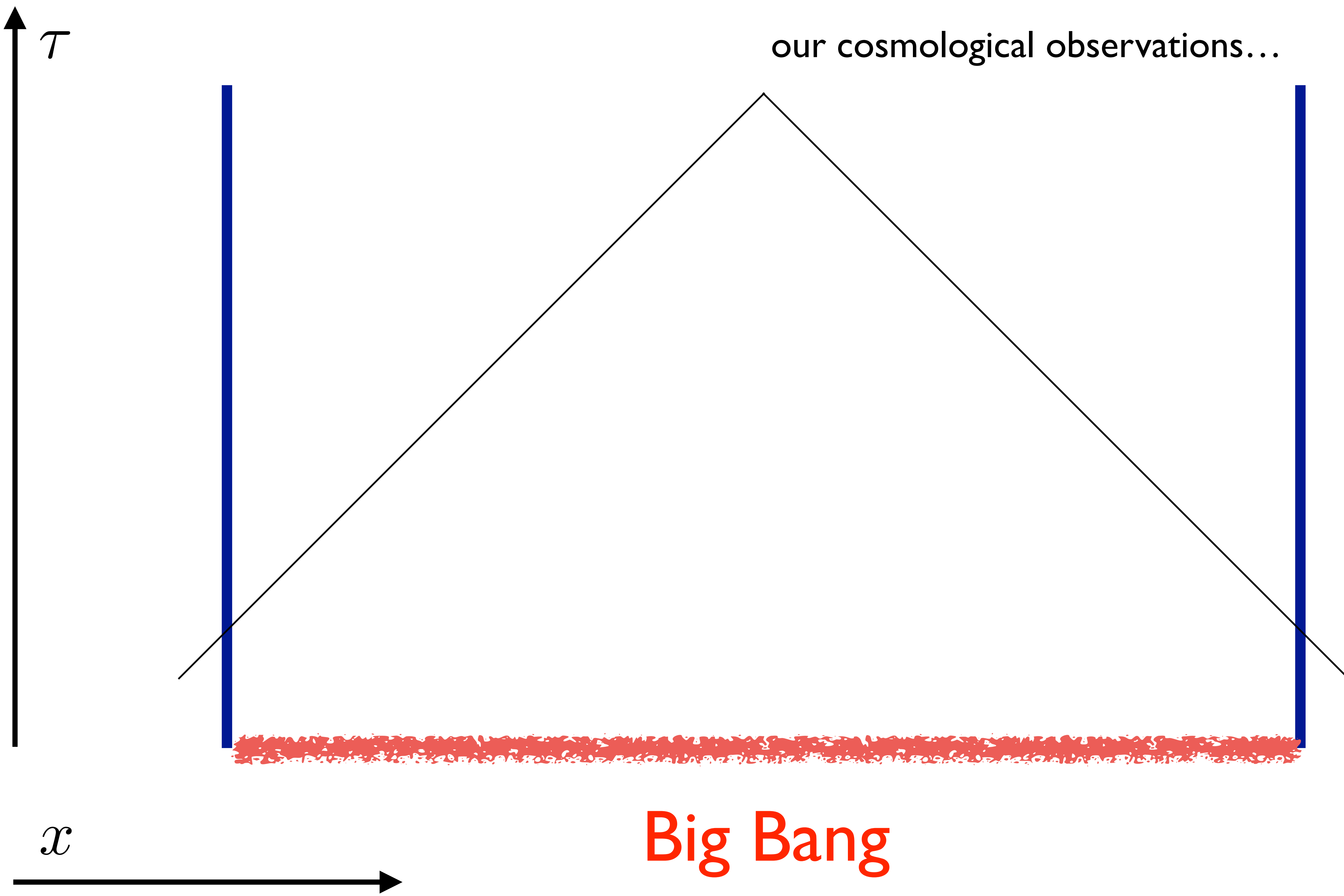

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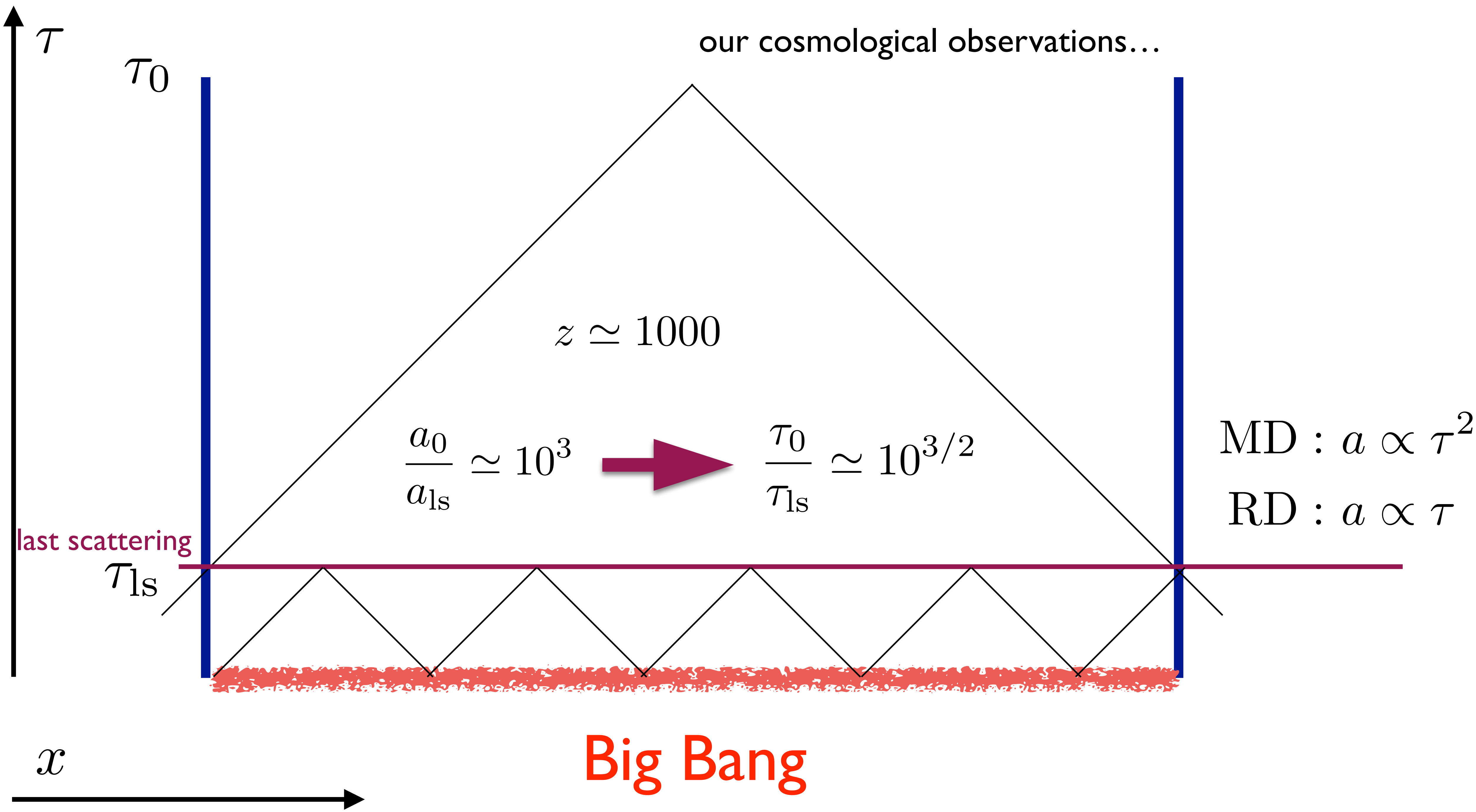

Horizon problem

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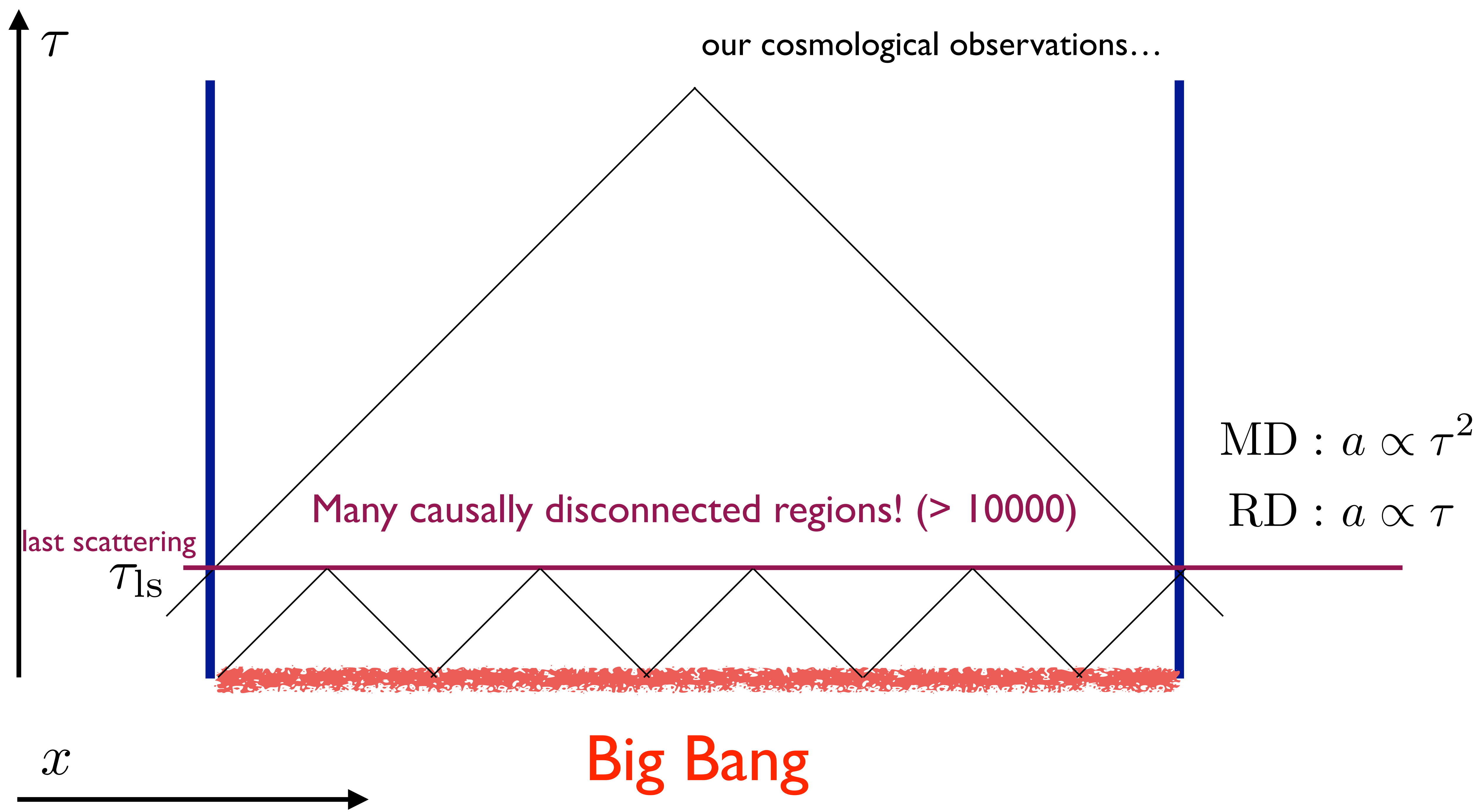

Horizon problem

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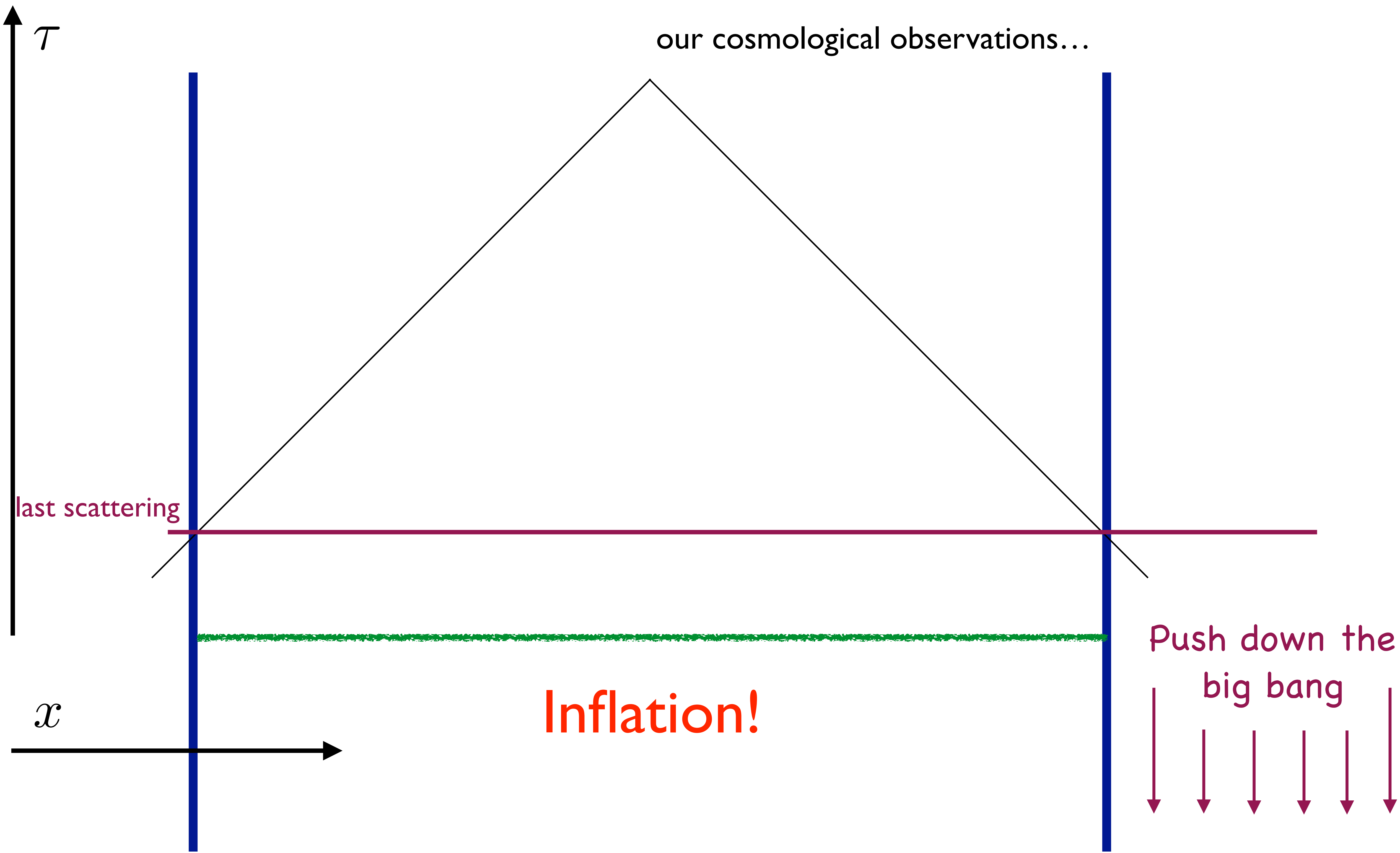


Horizon problem

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$$

We want a primordial epoch as long as possible in τ time

$a(t) \propto t^\alpha$

$$\tau = \int_0^t \frac{d\tilde{t}}{a(\tilde{t})} = \left.\frac{\tilde{t}^{1-\alpha}}{1-\alpha}\right|_0^t$$

If $\ddot{a} > 0,$ τ is even divergent in 0!

Primordial accelerated expansion = Inflation

Fourier space - picture

Every wavelength $\ell \propto a$. How does this length compare with H^{-1} ?

$$\frac{d}{dt}(aH) < 0 \Longleftrightarrow \ddot{a} < 0$$

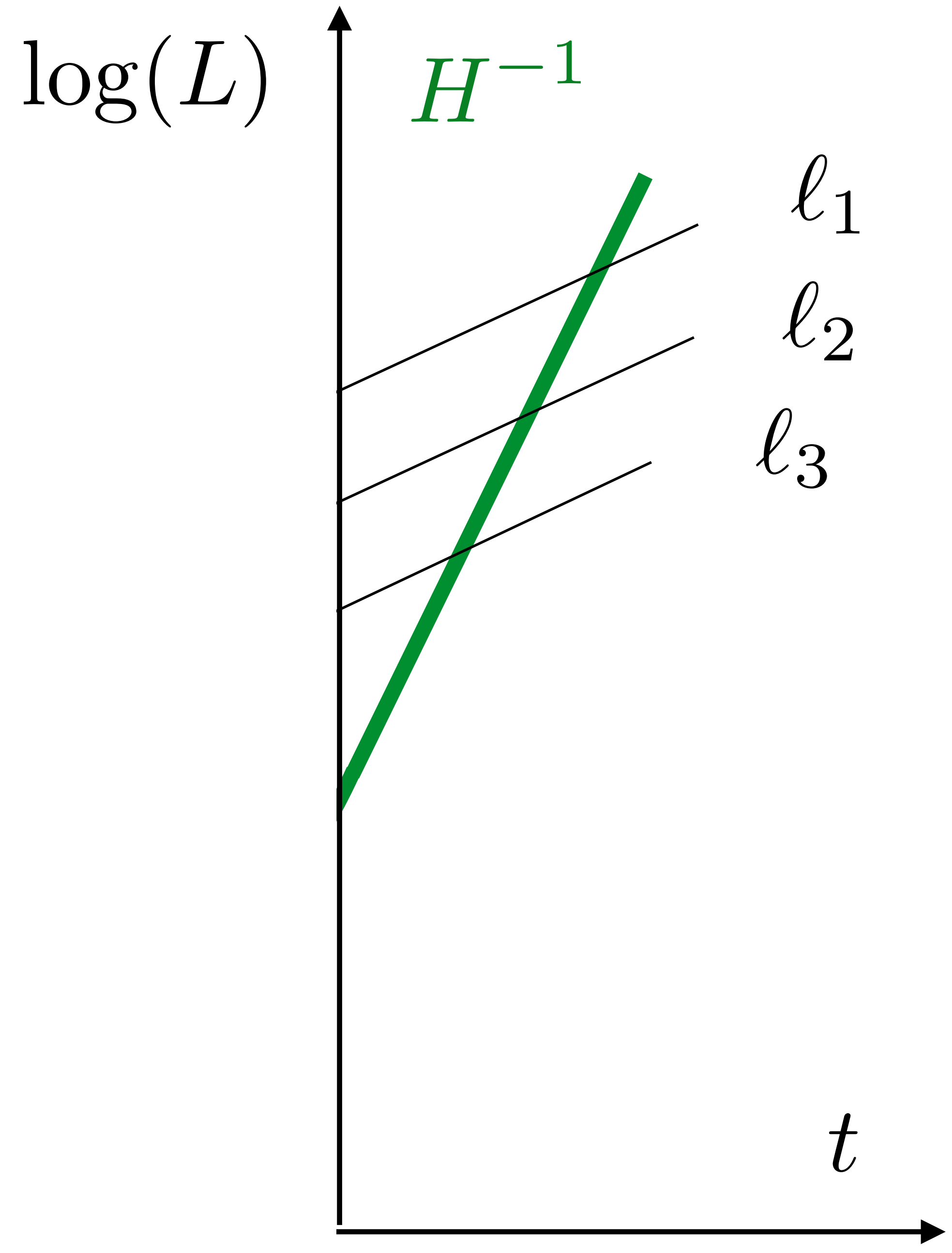
ℓ grows slower

$$\frac{d}{dt}(aH) > 0 \Longleftrightarrow \ddot{a} > 0$$

ℓ grows faster

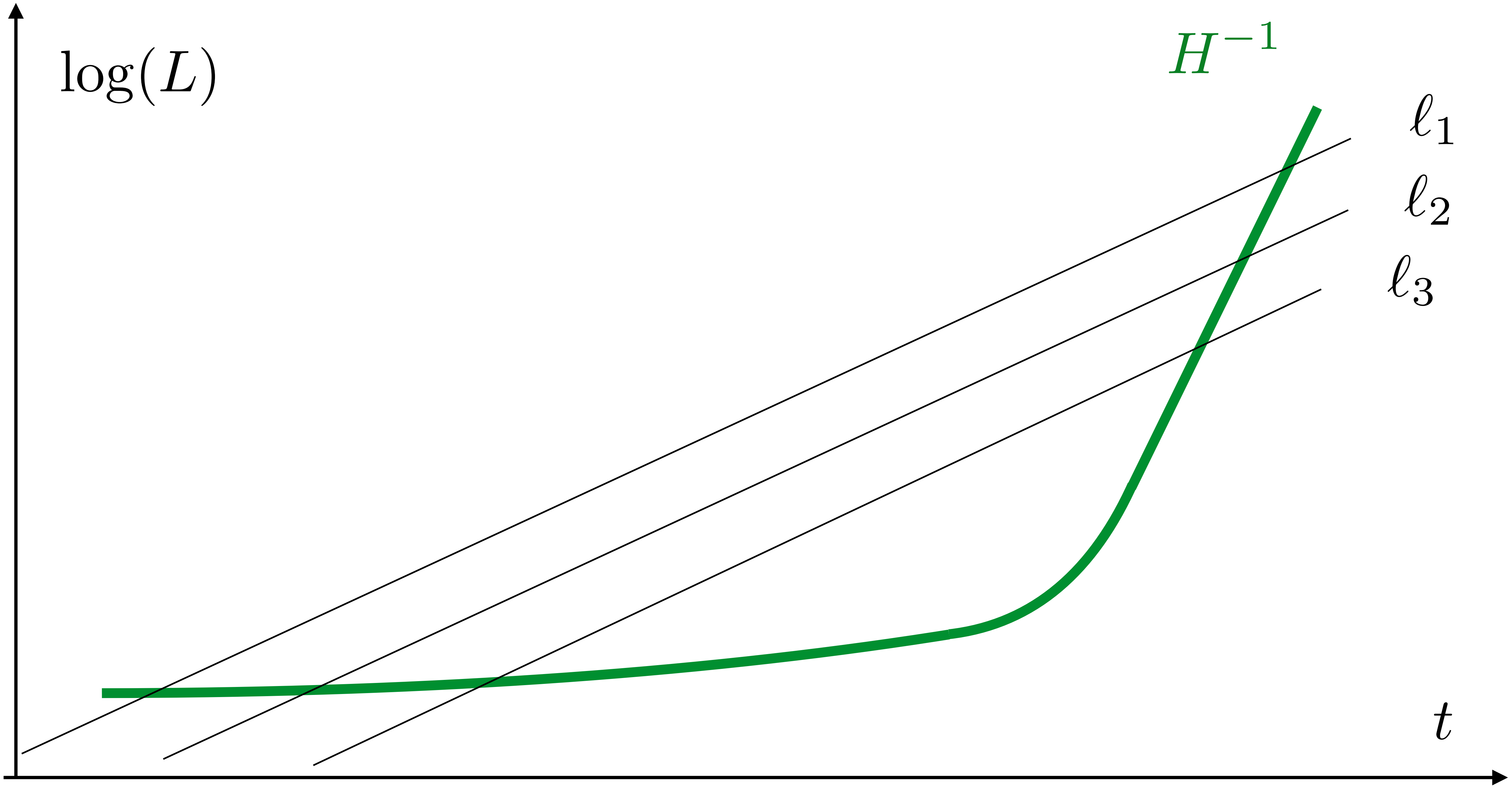
Fourier space - picture

Every wavelength $\ell \propto a$. How does this length compare with H^{-1} ?



Fourier space - picture

Every wavelength $\ell \propto a$. How does this length compare with H^{-1} ?



Fourier space - picture

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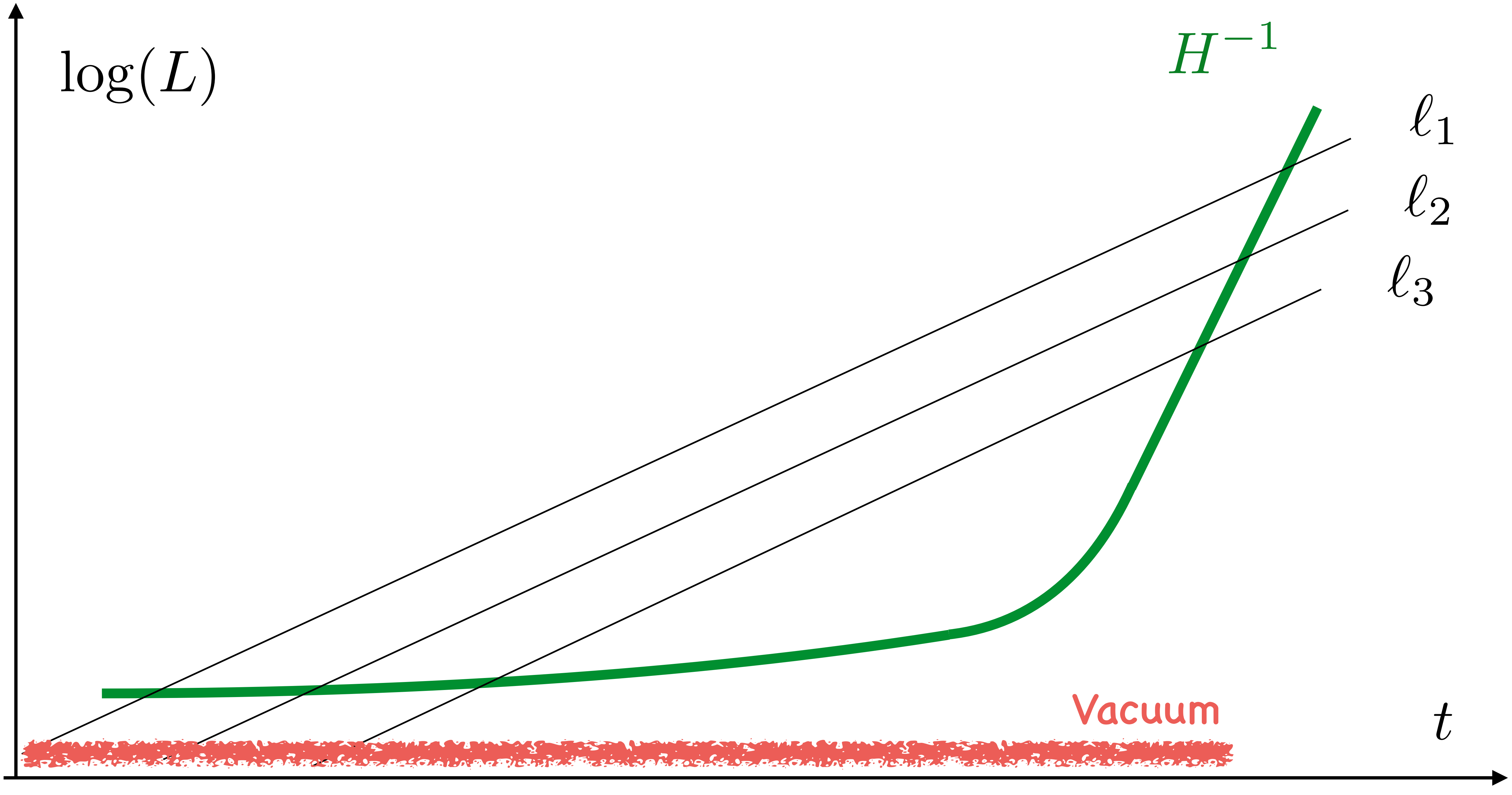


Fourier space - picture

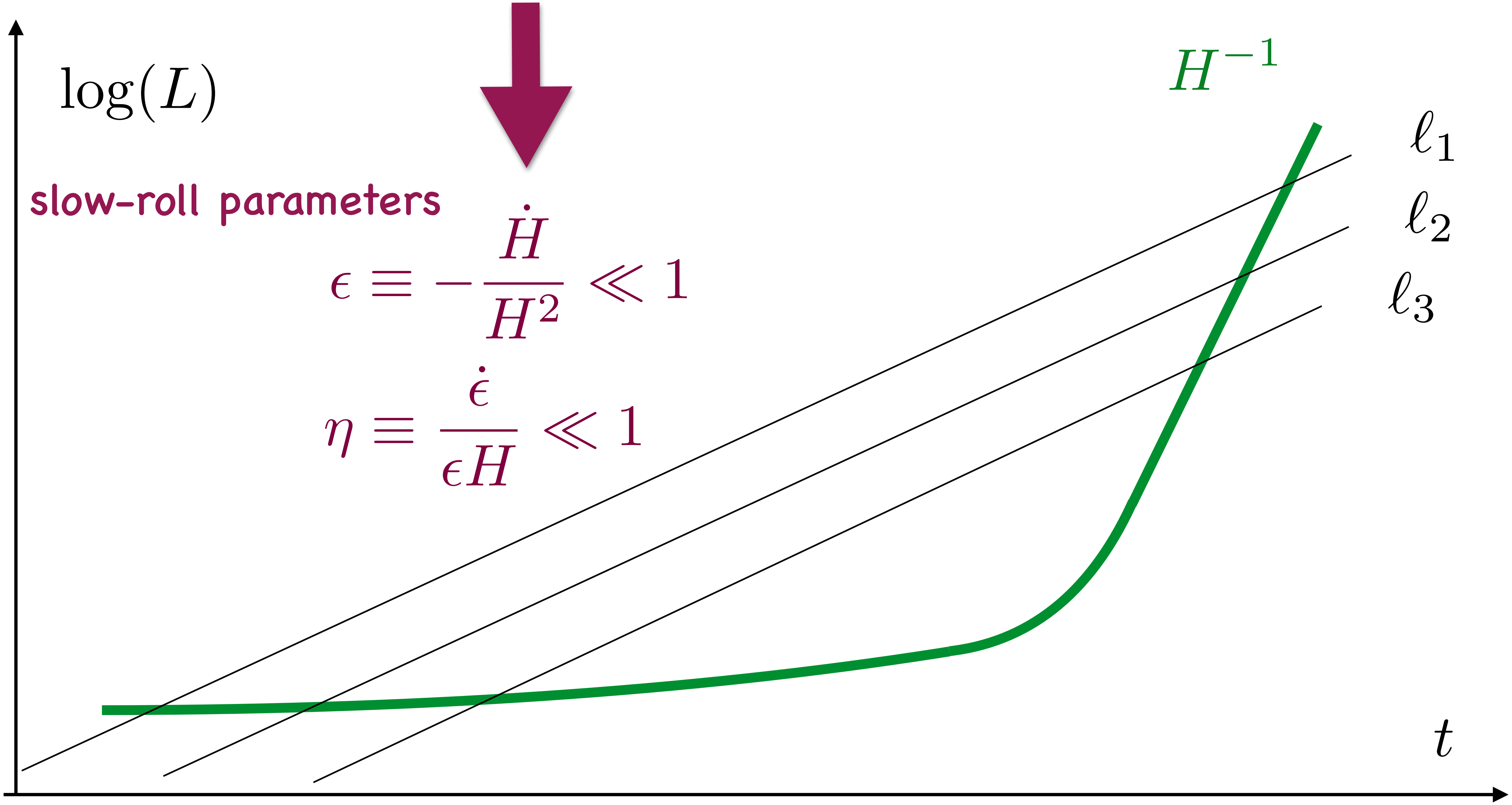
Null energy condition guarantees that $\dot{H} \leq 0$.

During inflation $H \simeq \text{const.}$

slow-roll parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$

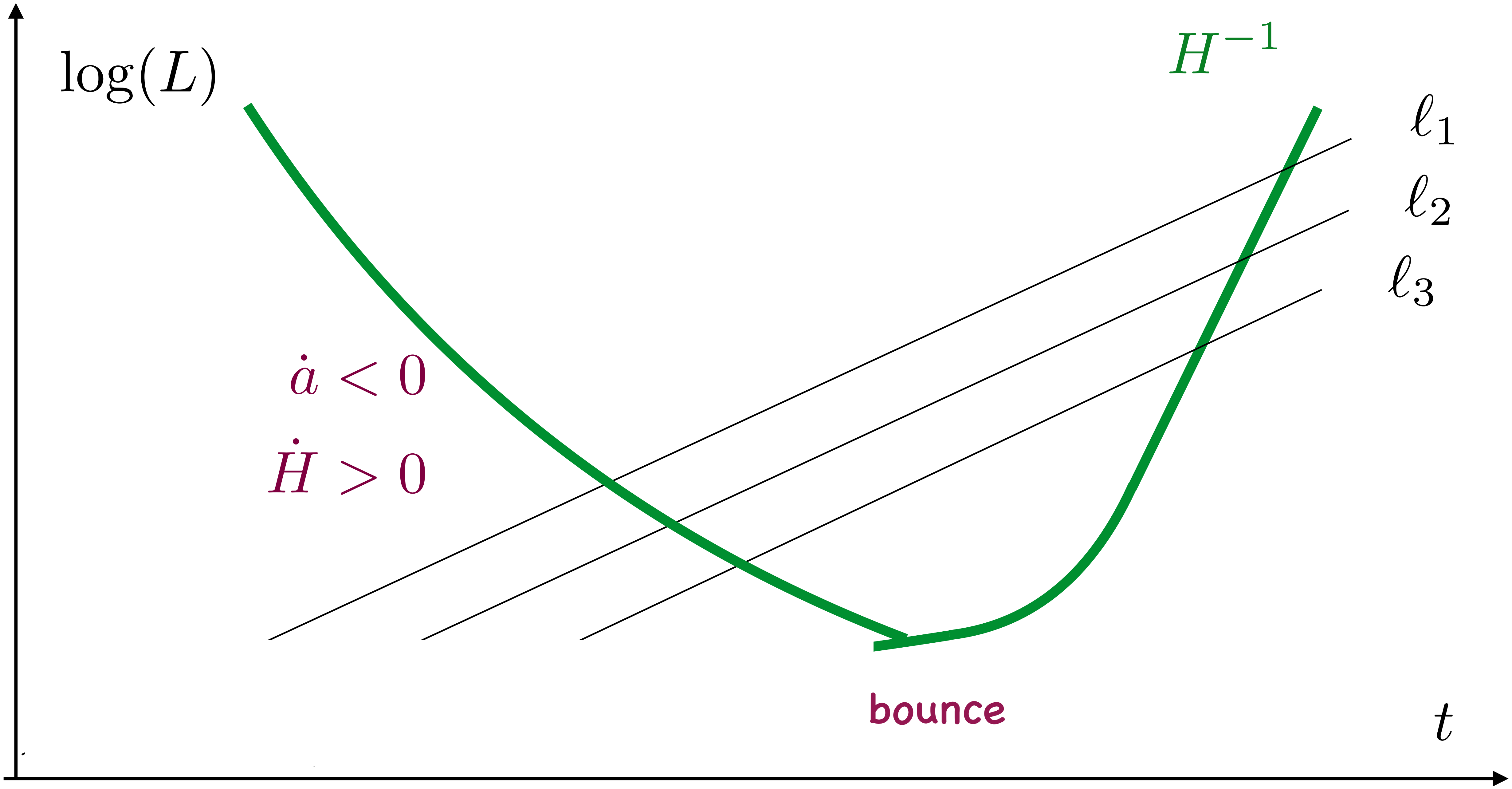
$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ll 1$$



Aside: bouncing alternatives to Inflation

Null energy condition guarantees that $\dot{H} \leq 0$.

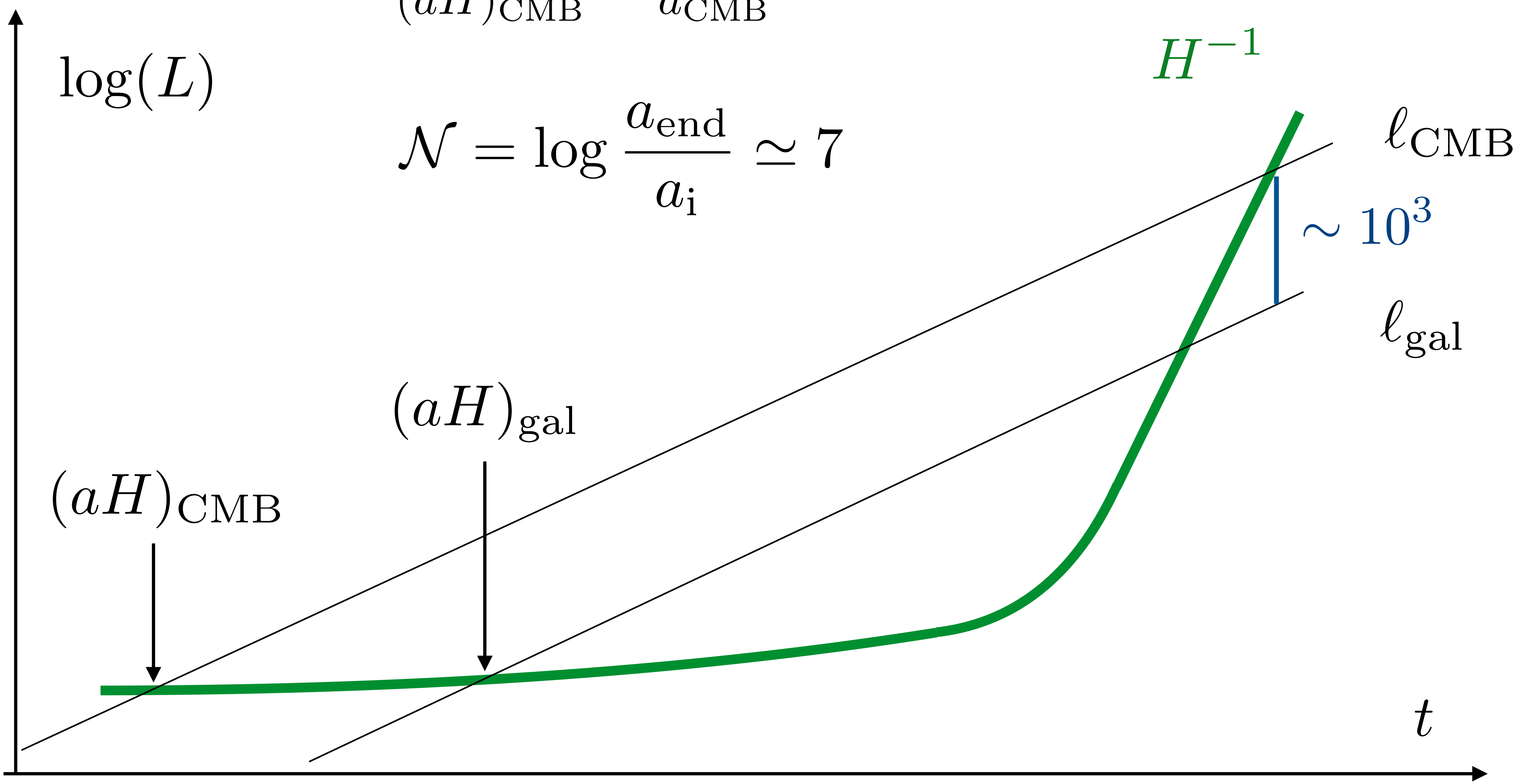
However: Pre-big bang, cyclic cosmologies, LQC etc.



How much of inflation?

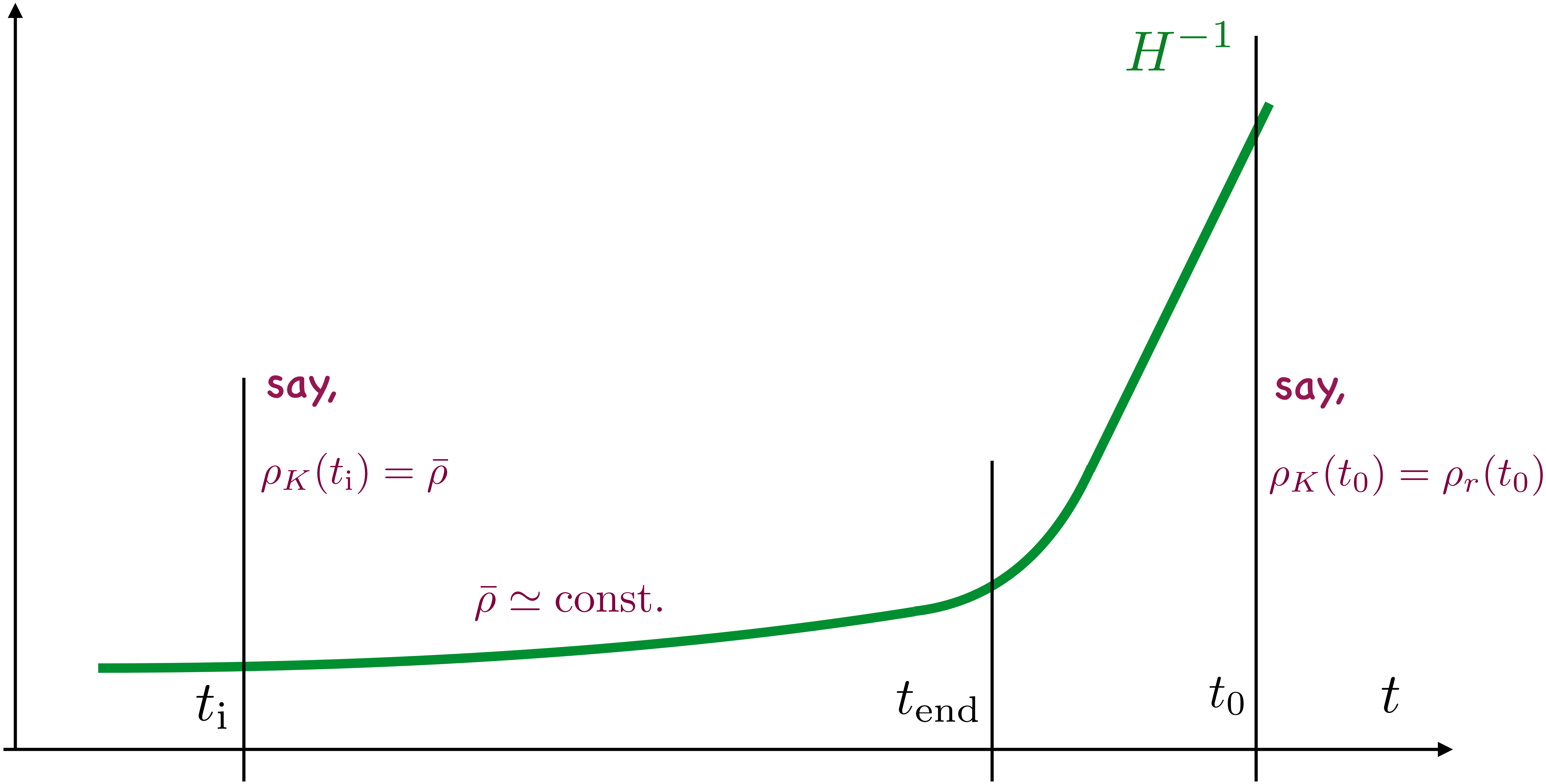
$$\frac{(aH)_{\rm gal}}{(aH)_{\rm CMB}} \simeq \frac{a_{\rm gal}}{a_{\rm CMB}} > 10^3$$

$$\mathcal{N} = \log \frac{a_{\rm end}}{a_{\rm i}} \simeq 7$$



How much of inflation?

Solving the spatial curvature problem is a little more difficult

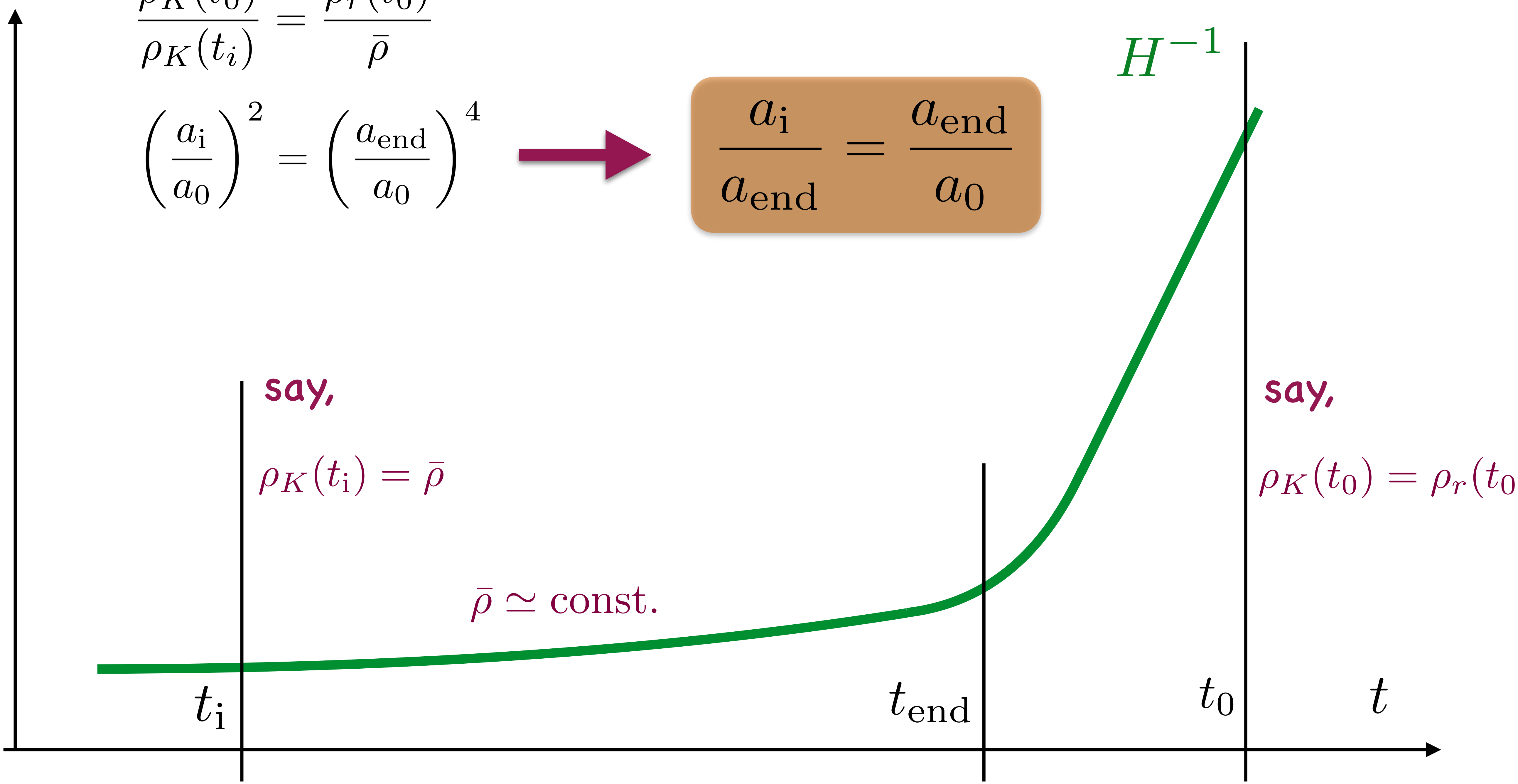


How much of inflation?

Solving the spatial curvature problem is a little more difficult

$$\frac{\rho_K(t_0)}{\rho_K(t_i)} = \frac{\rho_r(t_0)}{\bar{\rho}}$$

$$\left(\frac{a_{\rm i}}{a_0}\right)^2 = \left(\frac{a_{\rm end}}{a_0}\right)^4 \longrightarrow \boxed{\frac{a_{\rm i}}{a_{\rm end}} = \frac{a_{\rm end}}{a_0}}$$



The scale of inflation

Spoiler: the simplest model of inflation happens at roughly
the grand-unification scale. With more fine tuning down to the EW scale

$$\left(\frac{a_{\rm end}}{a_0}\right)^4 \simeq \left(\frac{10^{-4}{\rm eV}}{10^{15}{\rm GeV}}\right)^4 \quad \Rightarrow \quad \boxed{\mathcal{N} \simeq 60}$$

Inflation

- Sets the ``correct” initial conditions
- Solves the curvature problem
- Solves the Horizon problem
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How to get inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$w = \frac{\frac{\dot{\phi}^2}{2} - V}{\frac{\dot{\phi}^2}{2} + V}$$

slow-roll:

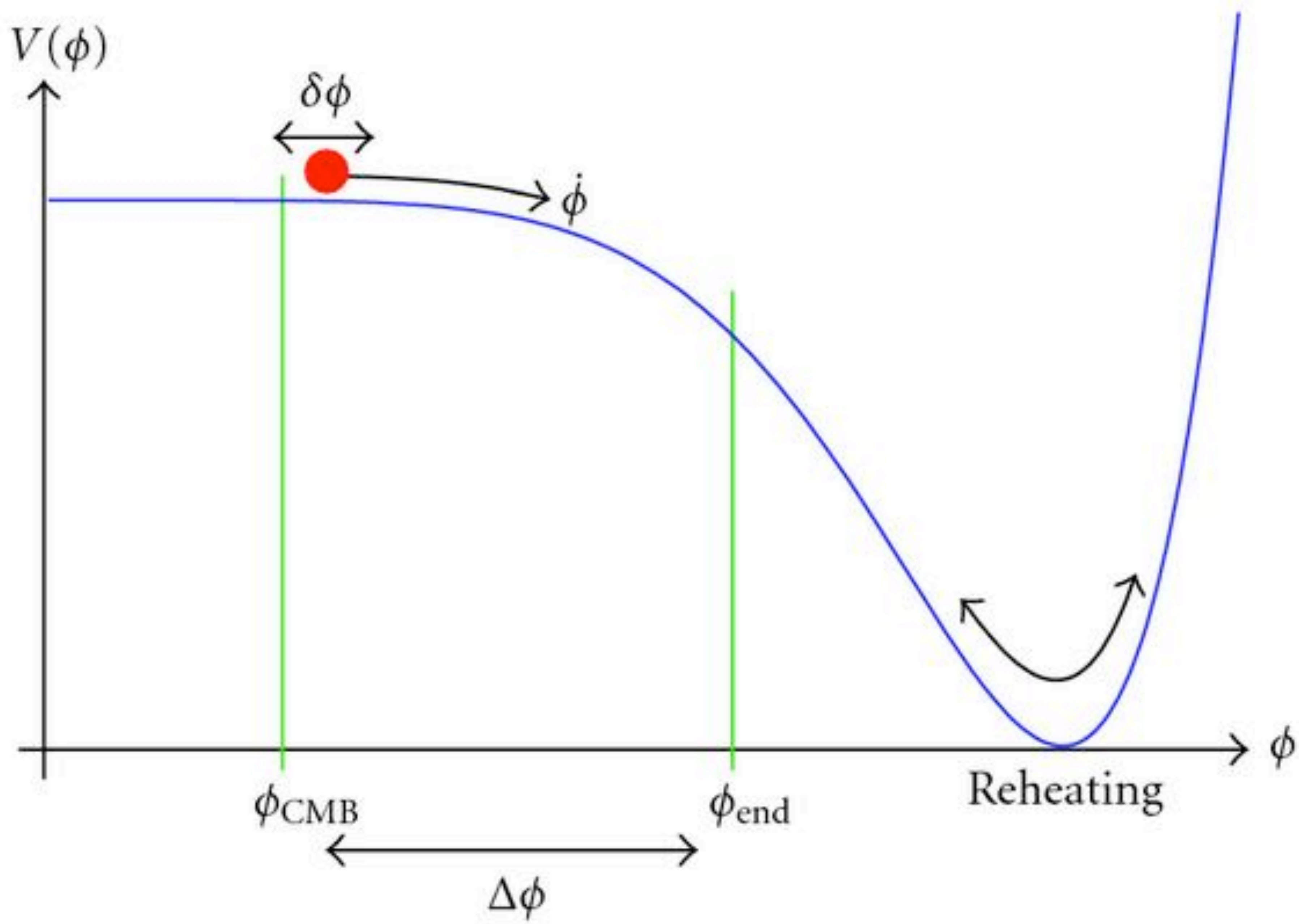
$$\epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1$$

$$\eta = M_P^2 \frac{V''}{V} \ll 1$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3M_P^2} \left(\frac{\dot{\phi}^2}{2} + V \right)$$

Typical scenario



Case study: $V(\phi) = \frac{m^2}{2}\phi^2$

$$\epsilon = \frac{M_P^2}{2\phi^2}$$

$$\eta = \frac{2M_P^2}{\phi^2}$$

$$\mathcal{N} = \log\frac{a_{\rm end}}{a_{\rm i}} = \int d\log a = \int dtH = \int \frac{d\phi}{\dot\phi}H = \int d\phi\frac{3H^2}{V'} = \int d\phi\frac{V}{M_P^2 V'} = \int d\phi\frac{\phi}{2M_P^2} = \frac{\phi_{\rm i}^2}{4M_P^2}$$

$$\phi_{\rm i} \simeq 15M_P, \qquad \phi_{\rm end} \simeq M_P$$

$$\Delta_s = \frac{H^2}{\epsilon M_P^2} = \frac{m^2\phi_{\rm i}^4}{M_P^6} \simeq 10^{-9} \qquad\qquad m \simeq 10^{-6}M_P$$

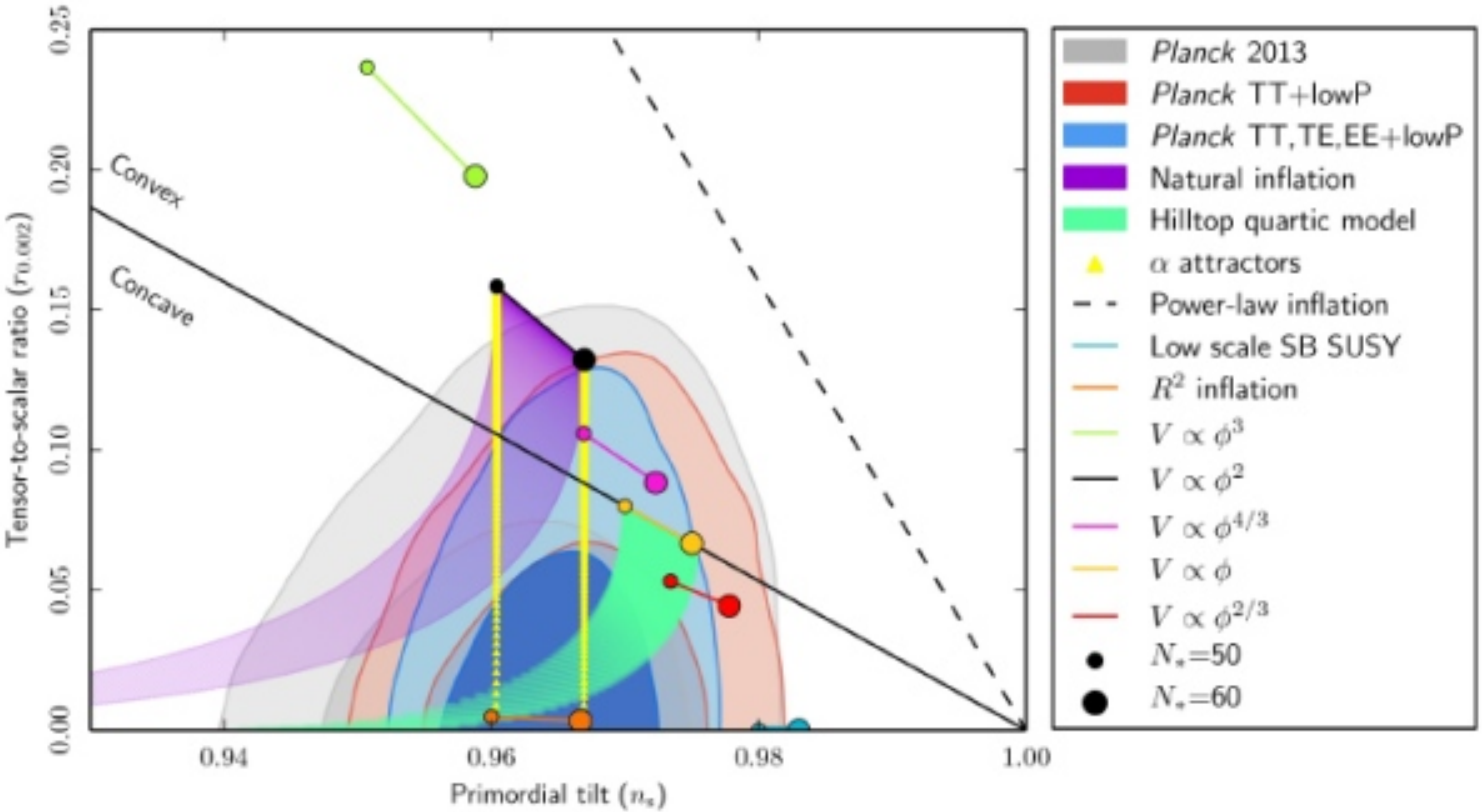


Fig. 12. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets, compared to the theoretical predictions of selected inflationary models.