Deformation of the GW spectrum by density perturbations

Ryusuke Jinno (DESY)

[Domcke, Jinno, Rubira '20 (JCAP 06 (2020) 046)]

GRAVITATIONAL WAVES: A NEW PROBE TO THE UNIVERSE

- Gravitational waves:

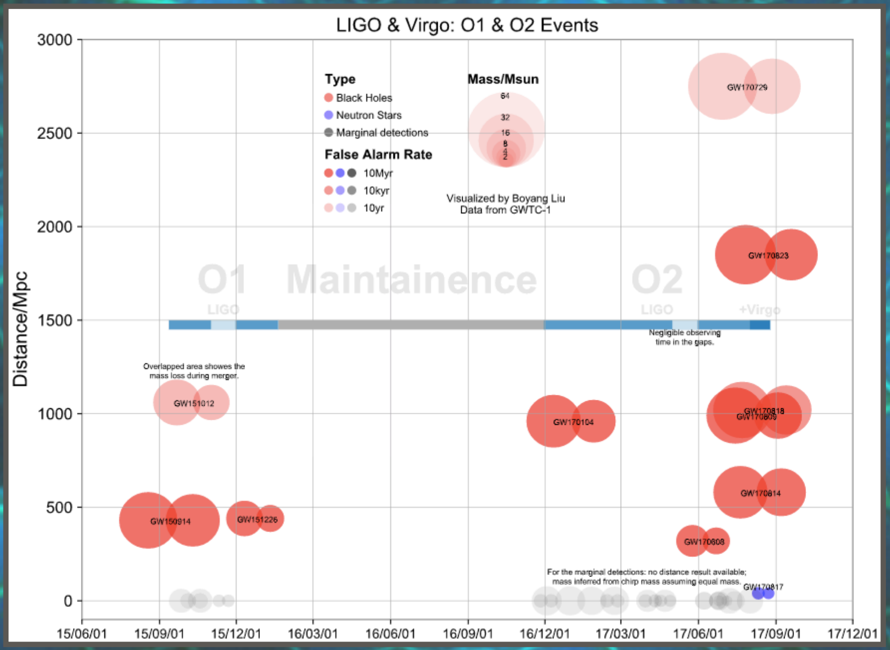
  Transverse-traceless part of the metric

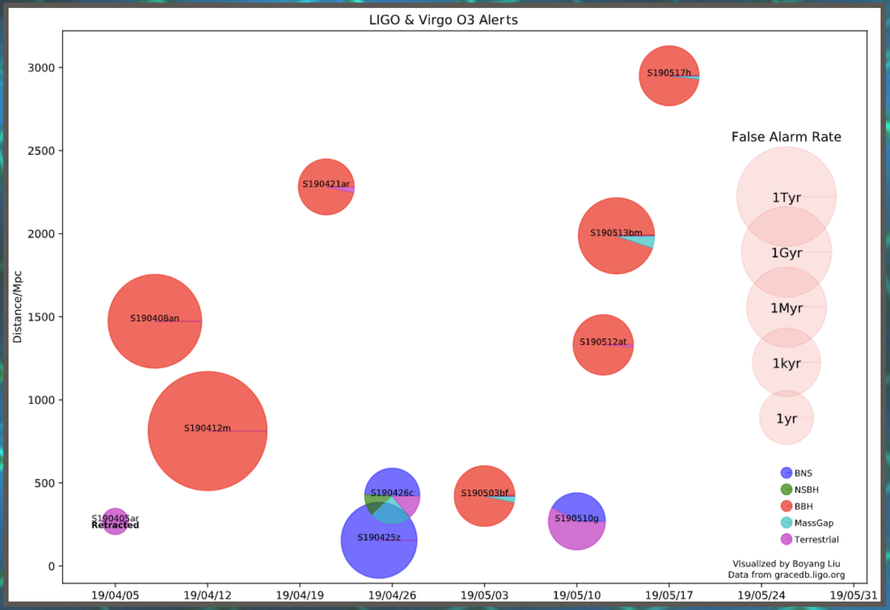
$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

  obeying the equation of motion sourced by the energy-momentum tensor of the system

$$\Box h_{ij} \sim G\Lambda_{ij,kl}T_{kl}$$

- Detections by LIGO & Virgo have been exciting us



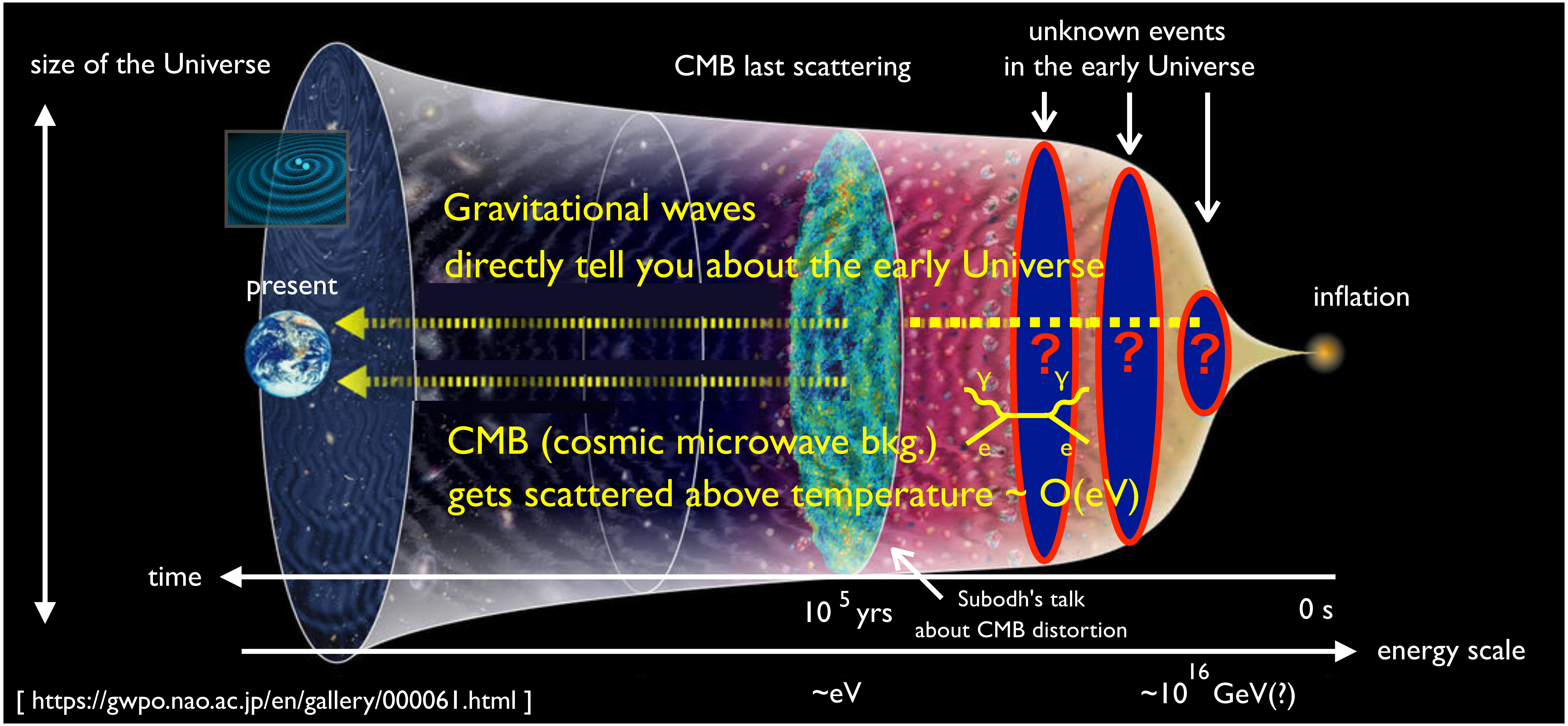


[Wikipedia "List of gravitational wave observations"]

[see also https://gracedb.ligo.org/superevents/public/O3/]

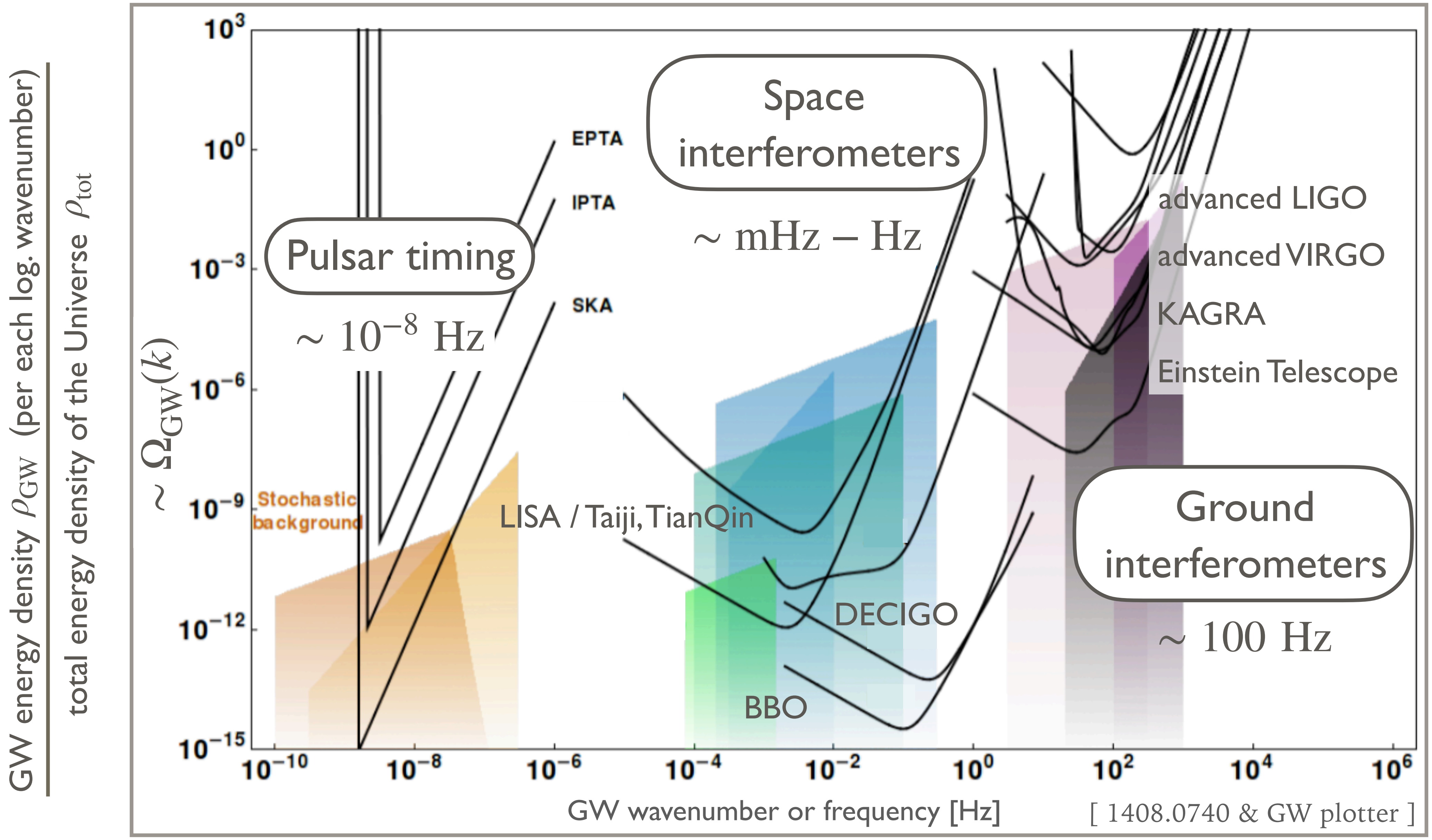
GWS FROM EARLY UNIVERSE

➤ What is special about GWs?



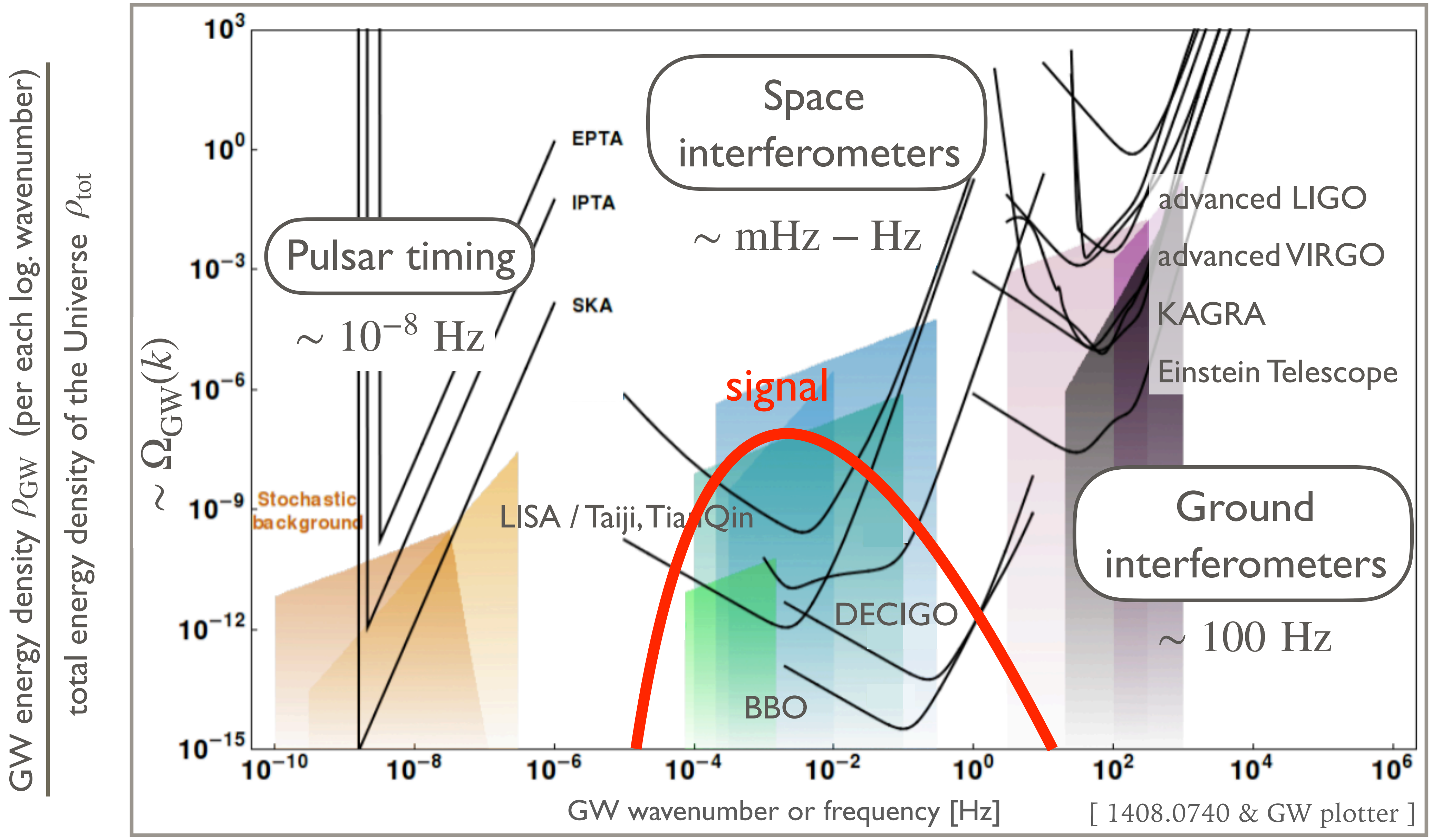
PRESENT & FUTURE OBSERVATIONS

➤ Summary of ongoing & future experiments



PRESENT & FUTURE OBSERVATIONS

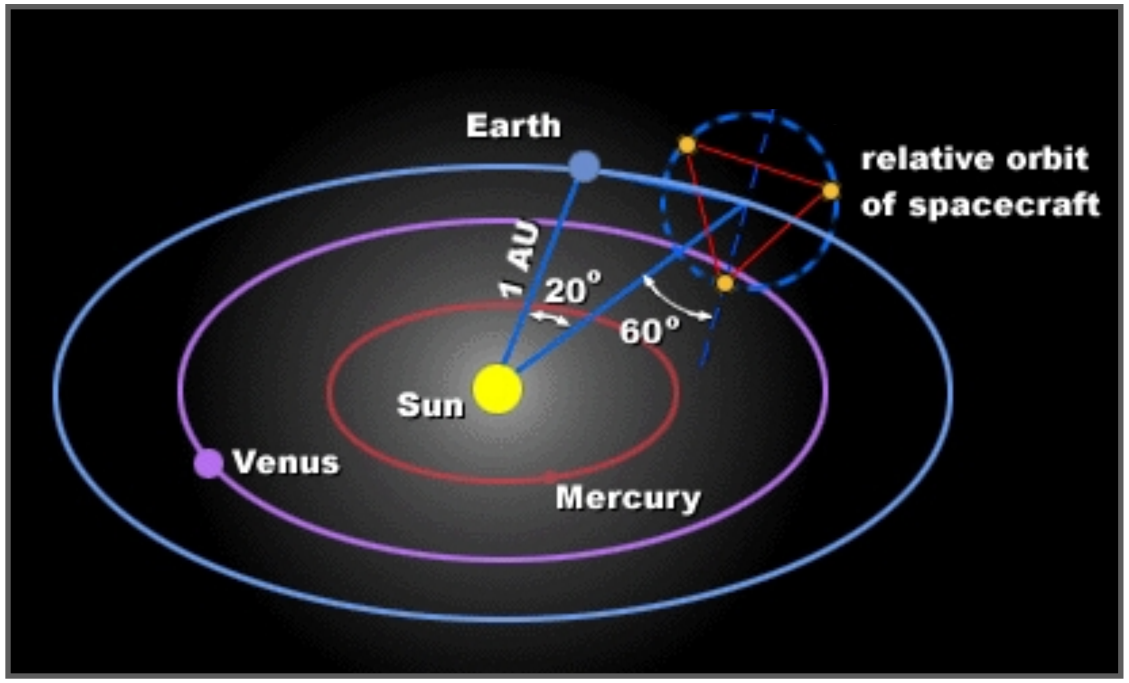
➤ Summary of ongoing & future experiments

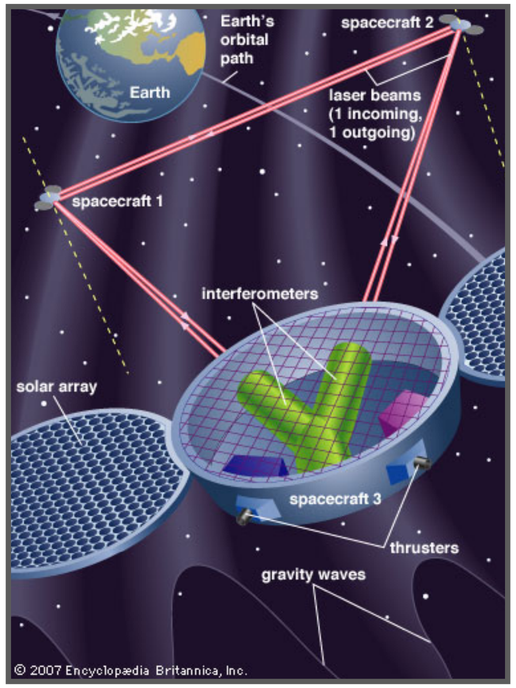


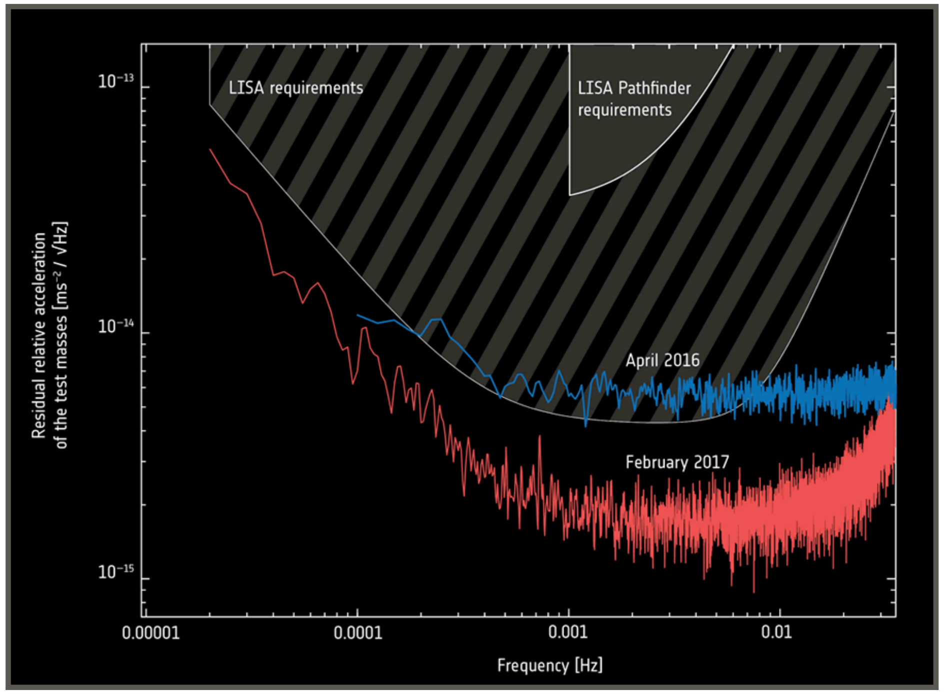
PRESENT & FUTURE OBSERVATIONS

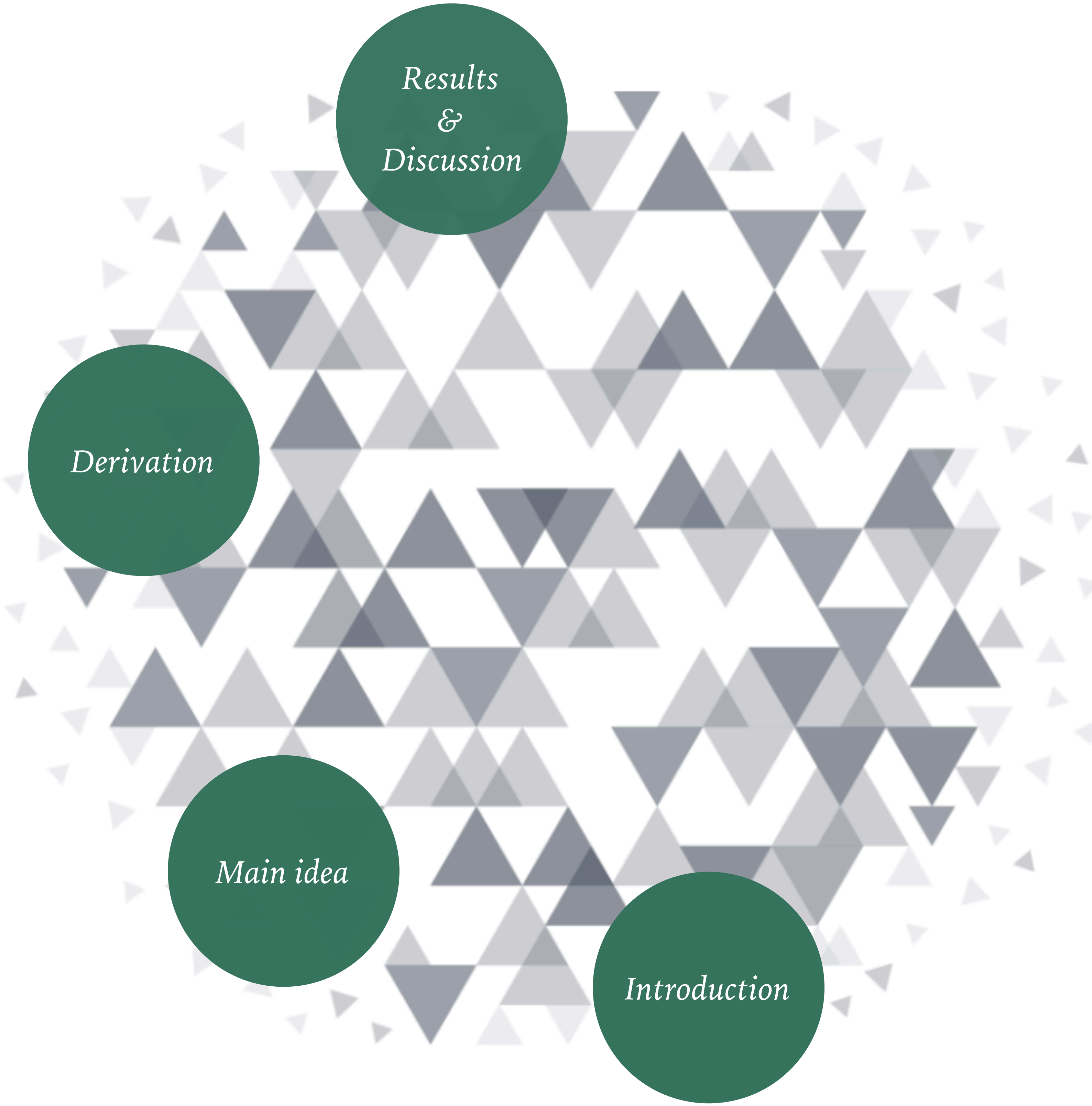
LISA (Laser Interferometer Space Antenna)

- Space interferometer project led by ESA & NASA
- Selected as third-large class mission(L3) in 2017. Operation from 2034.
- 3 spacecrafts orbitting around the Sun. Distance btwn spacecrafts $= 2.5 \times 10^6$ km.
- Tested necessary technologies with LISA pathfinder since 2015.





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Introduction

MAIN IDEA

- What I discuss will be in parallel to Ema & Gianmassimo's talk

"***GWs propagate in an inhomogeneous Universe***"

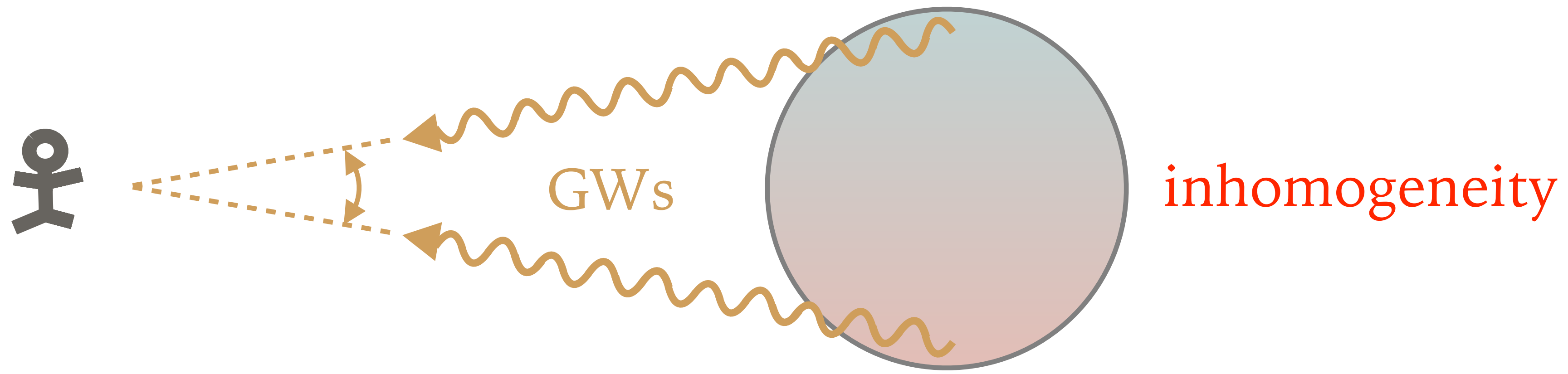
... from another aspect. Consistency will be discussed at the end.

[Laguna, Larson, Spergel, Yunes '10] [Alba, Maldacena '15]
[Bartolo, Bertacca, Matarrese, Peloso, Riccardone, Riotto, Tasinato '19]
[Adshead, Afshordi, Dimastrogiovanni, Fasiello, Lim, Tasinato '20] [Malhotra, Dimastrogiovanni, Fasiello, Shiraishi '20] ...

- Our question:

"***What is the effect of GW propagation on GW <u>isotropic</u> spectrum?***"

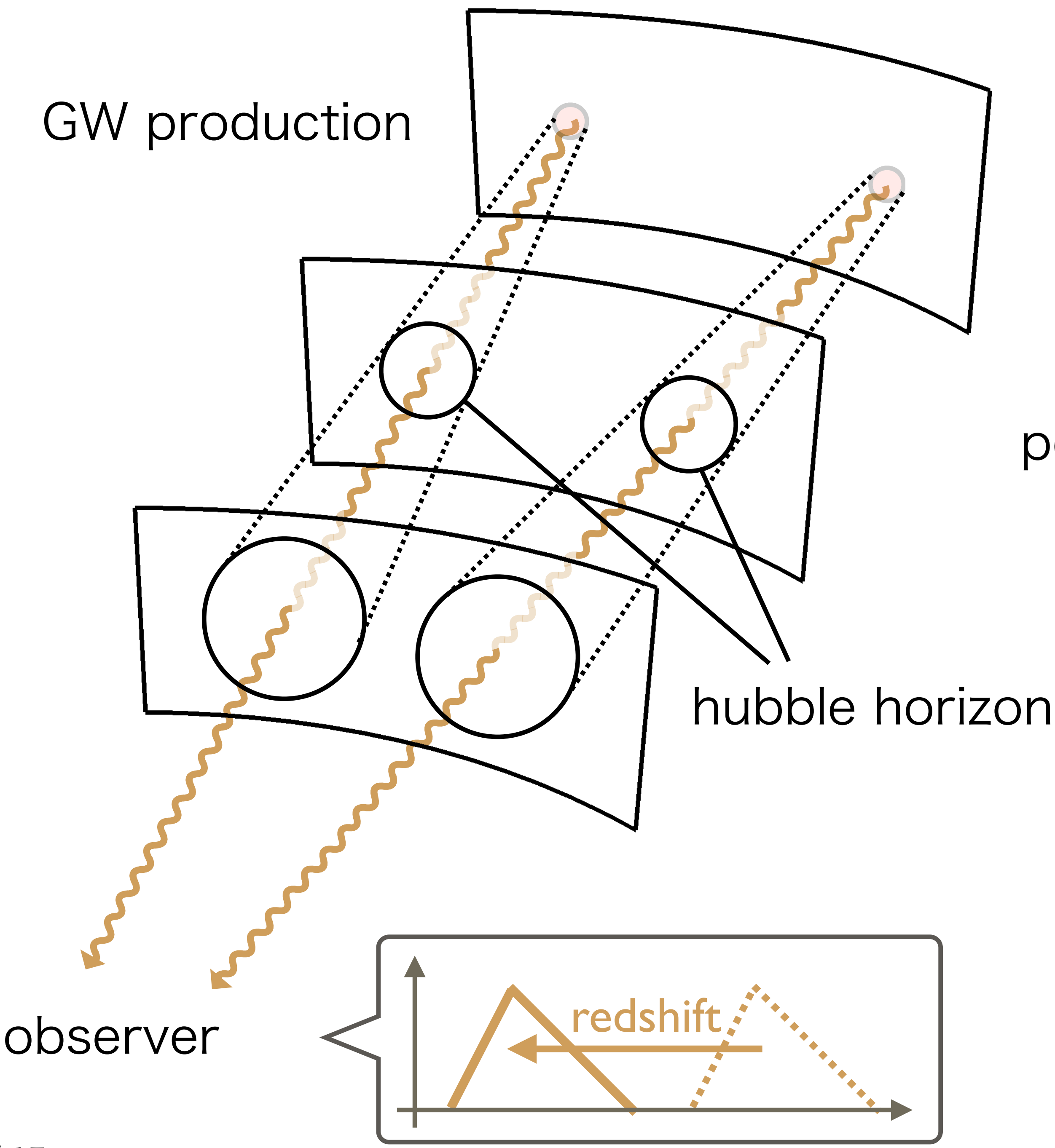
Why do we care about isotropic spectrum?



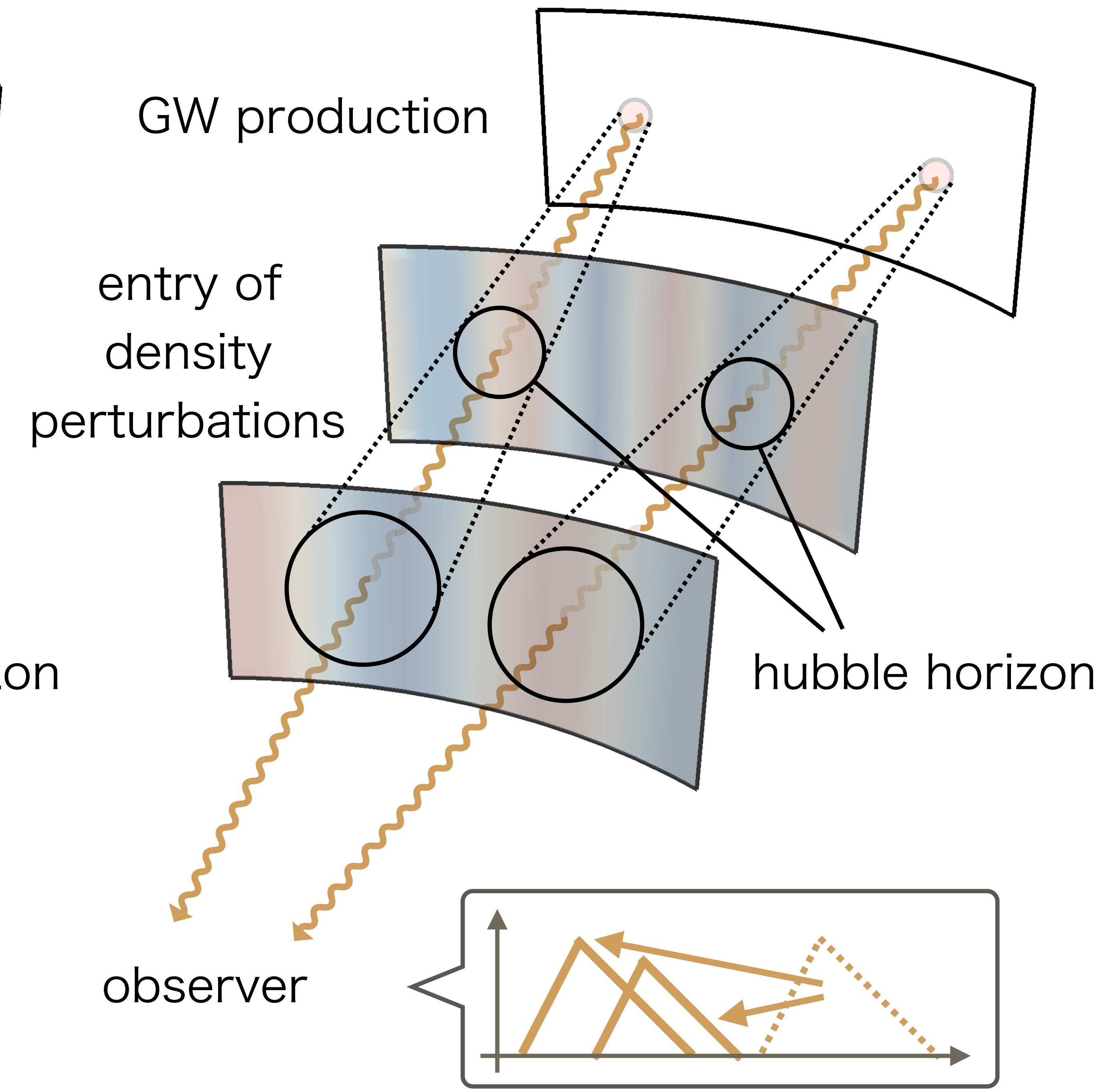
... because (typical angular scale for inhomogeneity) $\lll$ (detector resolution)

SKETCH OF THE MAIN RESULT

FRW case

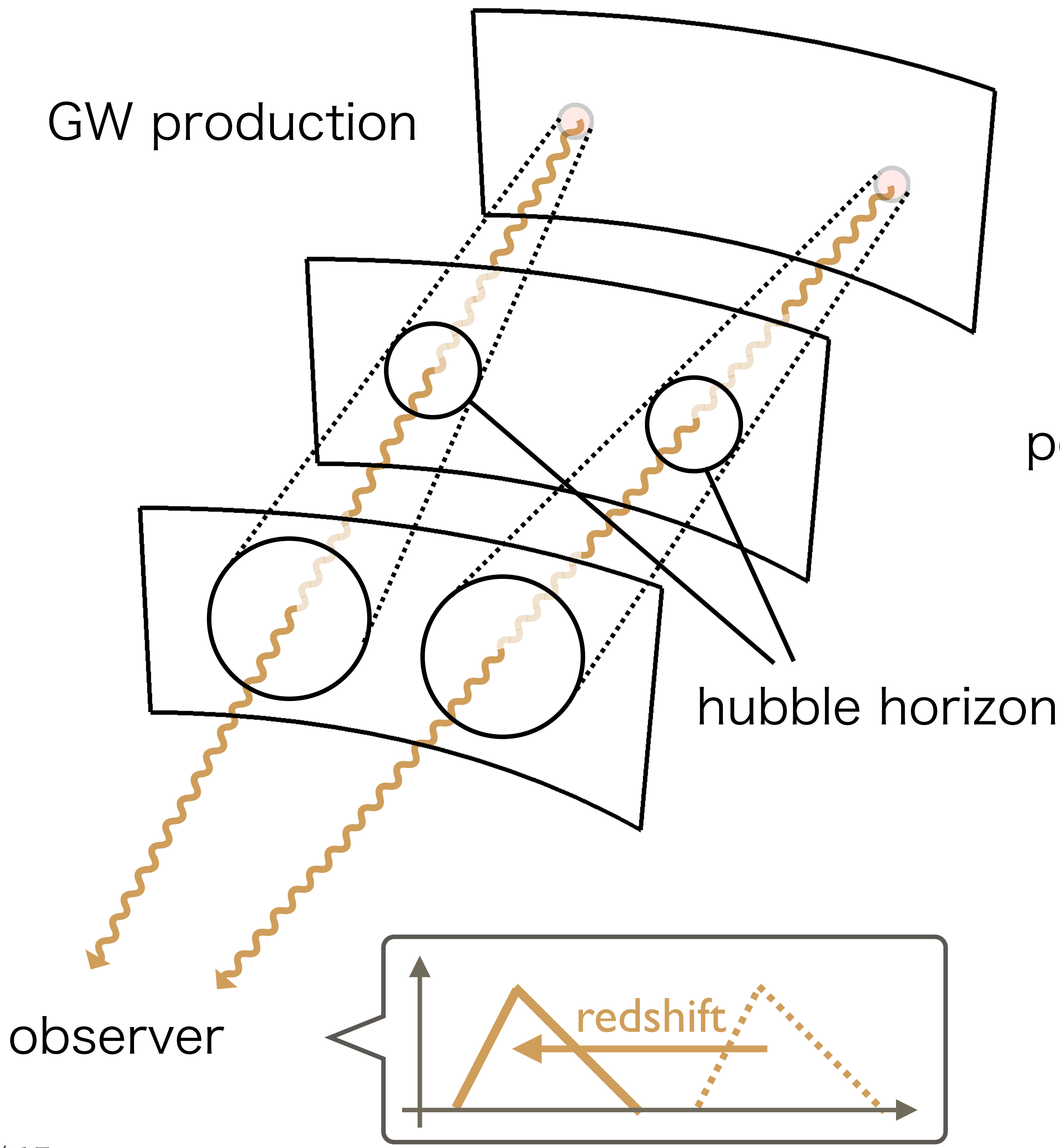


Inhomogeneous case

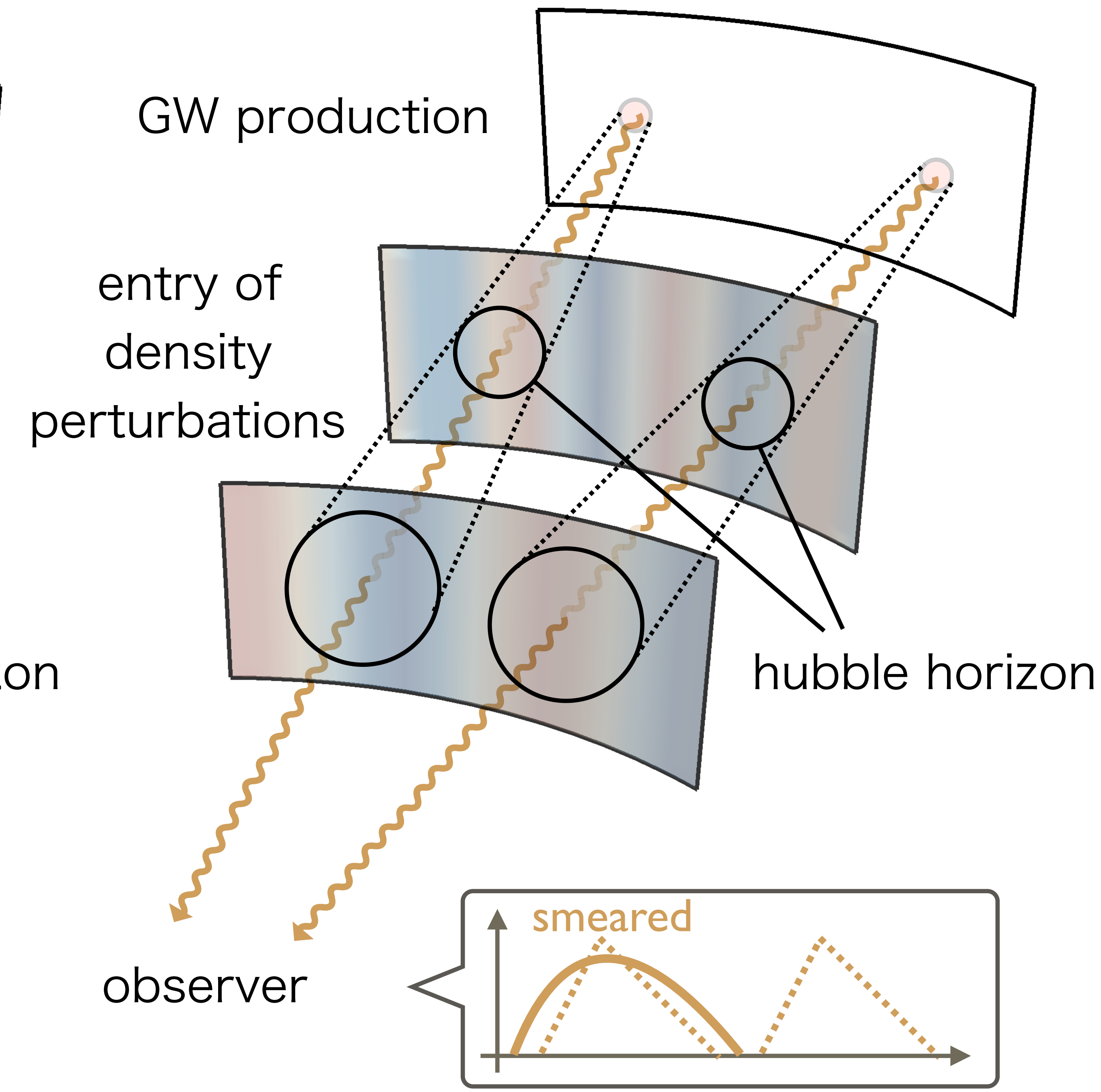


SKETCH OF THE MAIN RESULT

FRW case

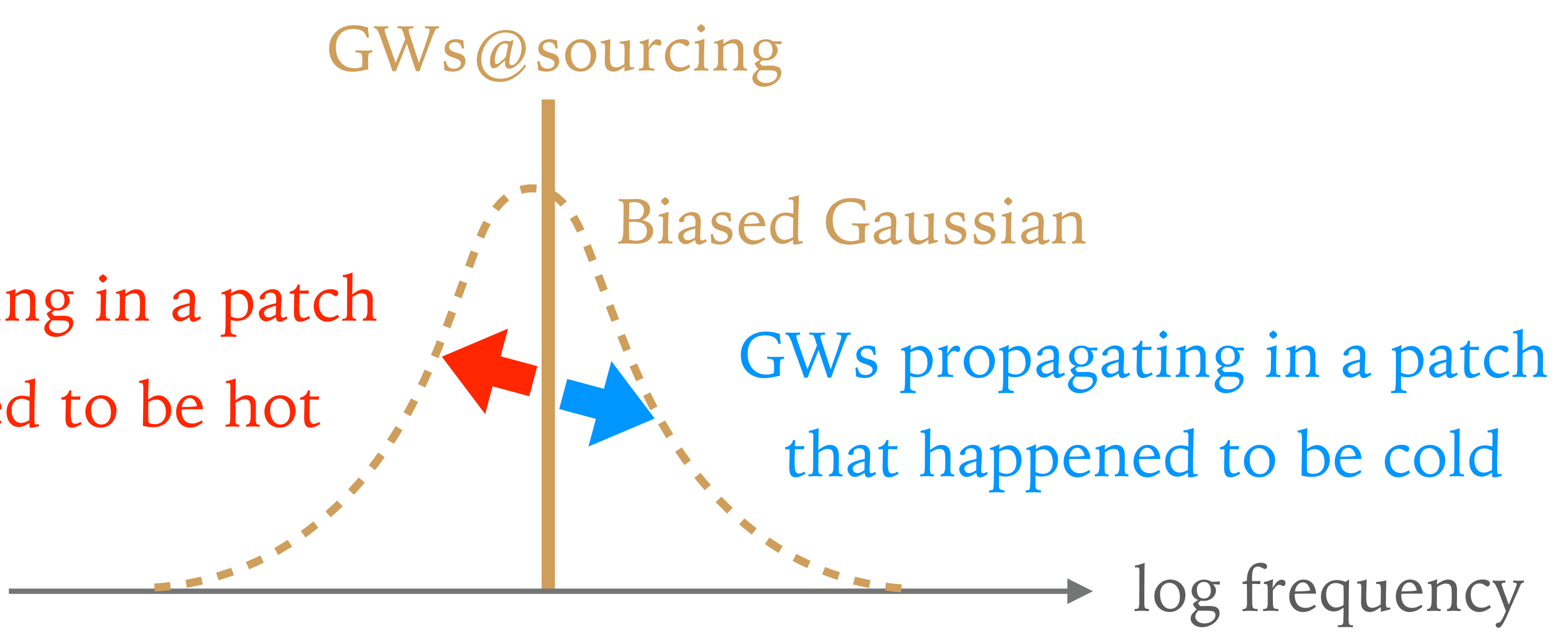


Inhomogeneous case



SKETCH OF THE MAIN RESULT (CONT'D)

➤ Each frequency bin experiences random walk



- For a more generic GW spectrum@sourcing, we convolute the biased Gaussian

$$\underbrace{\Delta_h^{2,(o)}(\ln f)}_{\text{observer}} \simeq \int d\ln f' \; \underbrace{\Delta_h^{2,(s)}(\ln f')}_{\text{source}} \; \underbrace{K(f, f')}_{\text{biased Gaussian}}$$

➤ This picture takes into account at least

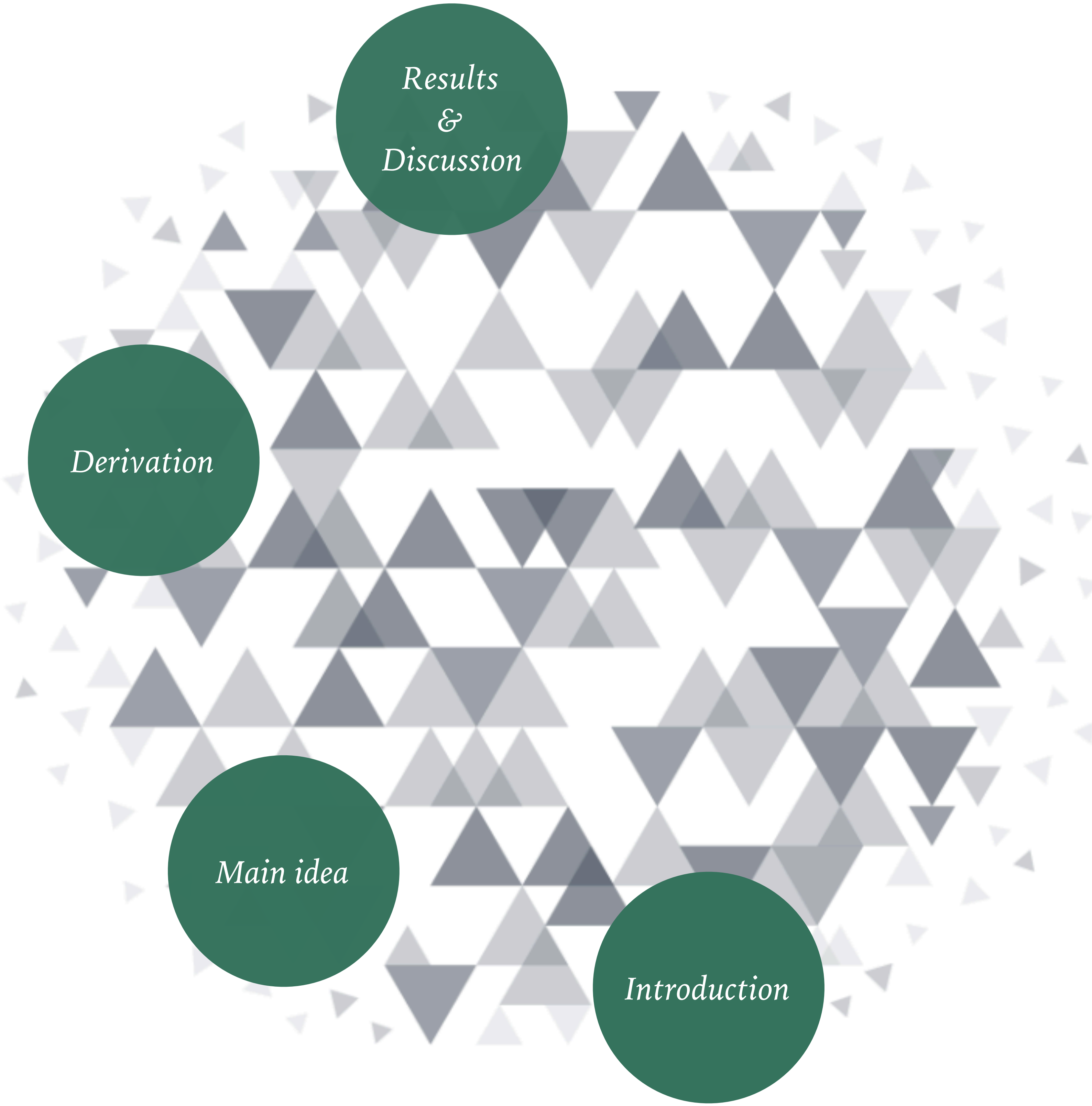
<u>part of</u> the relevant contributions (→ later)

DEFINITION OF THE SETUP

- Assume any GW source in very early Universe
  - First-order phase transitions e.g. [Witten '84] [Hogan ' 86] [Kosowski, Turner, Watkins '92] [Kamionkowski, Kosowski, Turner '93] [Huber, Konstandin '08] [Caprini et al. '16] [Hindmarsh, Huber, Rummukainen, Weir '13] [Ellis, Lewicki, No '18] ...
  - Gauge field production e.g. [Cook, Sorbo '11] [Sorbo '11] [Malek-Nejad, Sheikh-Jabbari '11] [Anber, Sorbo '12] [Namba, Peloso, Shiraishi, Sorbo, Unal '15] [Domcke, Pieroni, Binétury '16] ...
  - Preheating don't miss Dani's talk
  - Topological defects e.g. [Zeldovich, Kobzarev, Okun '74] [Kibble '76] [Vilenkin '81] [Gleiser, Roberts '98] [Battye, Shellard '93 & '96] [Figueroa, Hindmarsh, Urrestilla '13] [Ramberg, Visinelli '19] [Chang, Cui '20] ...
  - ...
- Curvature perturbations (from inflation) enter the horizon later
  - We assume single clock: each horizon patch is different only by time shift.

    GW production occurs in the same way among all the patches, up to time shift.
  - Preferably enhanced: Talks by Matteo, Spyros, Lukas (w/ Sébastien, Jacopo), Gianmassimo, Caner, Guillem, Shi, Antonio, Sébastien
- We assume hierarchy in scales

  (detector resolution) >>> (length of curvature pert'n) >>> (GW wavelength)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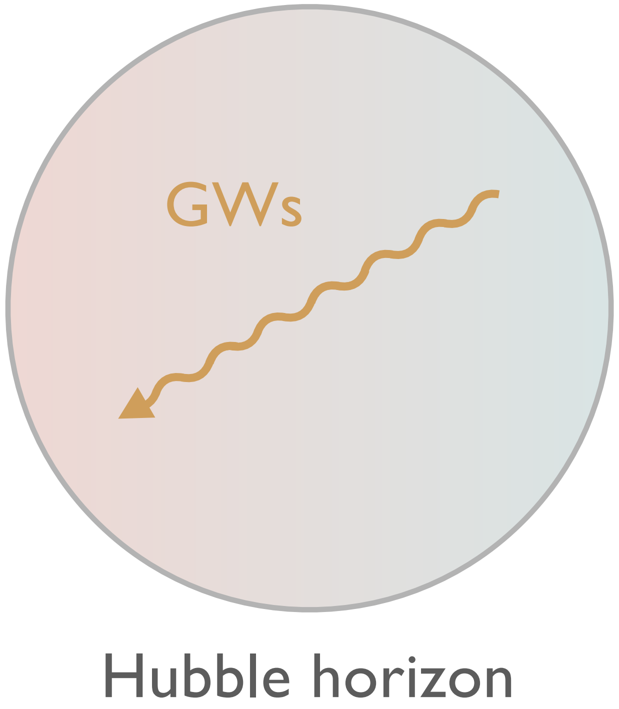
QUICK "DERIVATION"

➤ Hierarchy in scales justifies using geometric optics [Laguna, Larson, Spergel, Yunes '10]

- Infinitely short-wave GW rays propagating inside density perturbations



➤ Just like CMB, we have several effects on GW rays

Sachs-Wolfe / Integrated Sachs-Wolfe / Doppler / Lensing

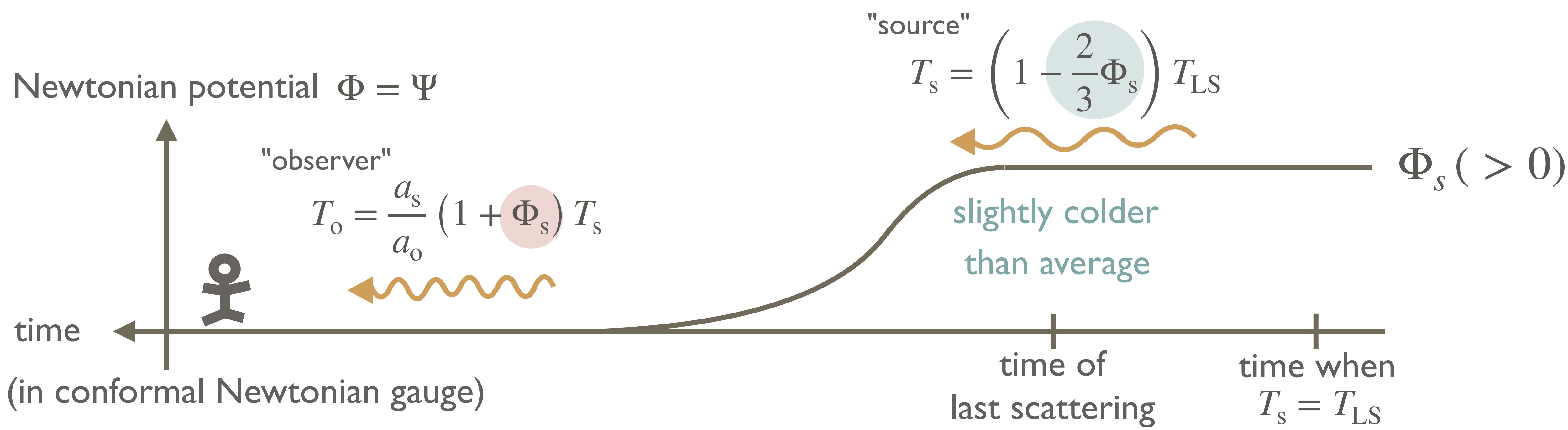
QUICK "DERIVATION"

- Sachs-Wolfe effect (in CMB context) [Sachs & Wolfe '67] [Hu & White '97]

$$\frac{\Delta T}{T} = \Phi_{\rm s} - \frac{2}{3}\Phi_{\rm s}$$

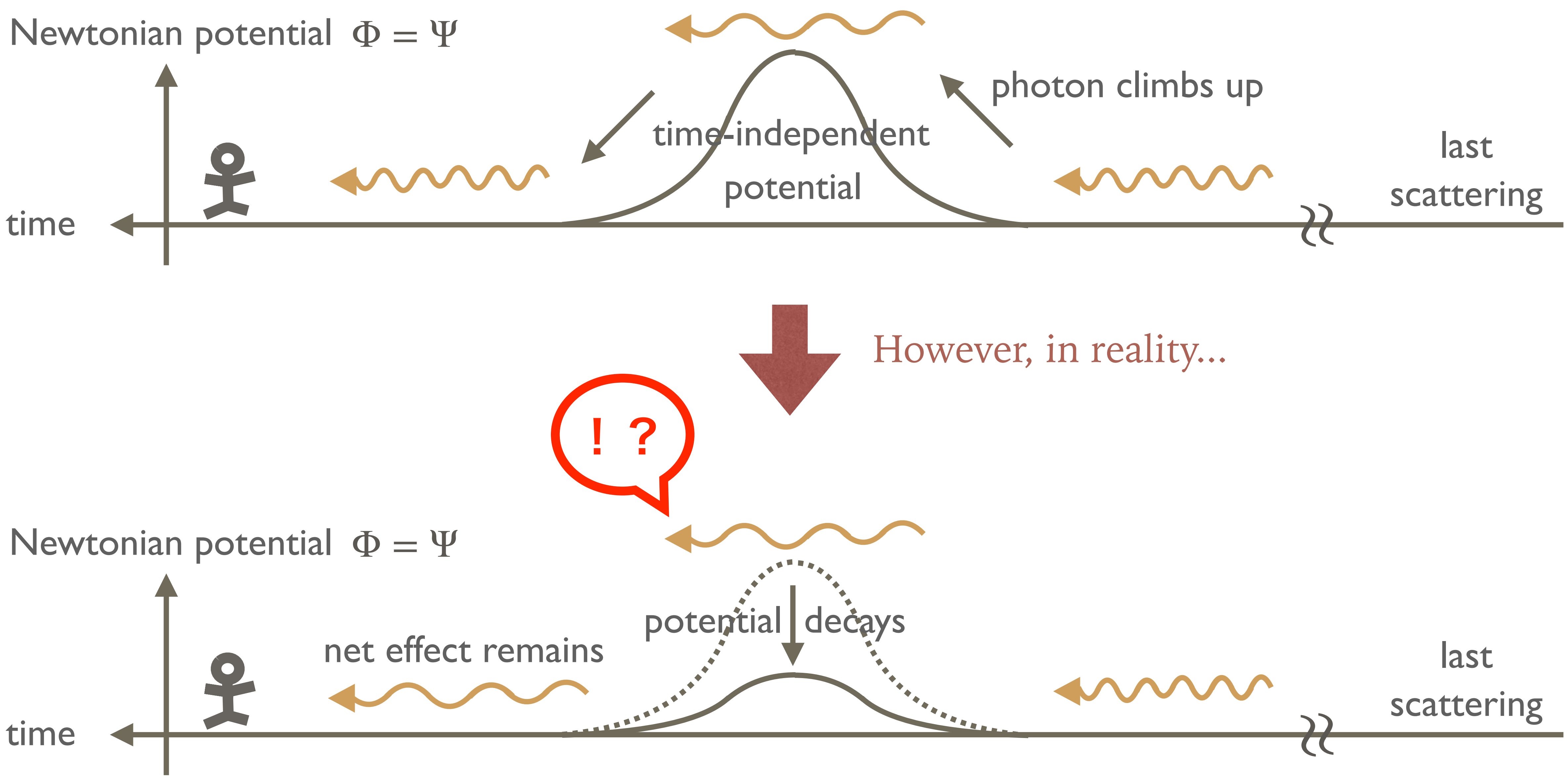
where $ds^2 = -a^2(1+2\Phi)\,d\tau^2 + a^2\delta_{ij}(1-2\Psi)\,dx^i dx^j$ (conformal Newtonian gauge)

$\Phi = \Psi$ (absense of anisotropic stress)



QUICK "DERIVATION"

➤ Integrated Sachs-Wolfe effect (in CMB context) [Rees & Sciama '68]



QUICK "DERIVATION"

- We average the frequency and amplitude random walk over the curvature perturbations

$$\Delta_h^{2,(\mathrm{o})}(\ln f) \simeq \left\langle \left(1 + 2\underline{\Delta \ln A}\right) \Delta_h^{2,(\mathrm{s})}(\ln f - \underline{\Delta \ln f}) \right\rangle_{\text{scalar ens. ave.}}$$

...using linear-order results from geometric optics [Laguna, Larson, Spergel, Yunes '10]

$$\text{amplitude} \quad \Delta \ln A = \boxed{-\Psi_\mathrm{s} - \frac{1}{2}\Phi_\mathrm{s}} + \text{lensing (neglected)}$$

$$\text{frequency} \quad \Delta \ln f = \boxed{\Phi_\mathrm{s} - \frac{1}{2}\Phi_\mathrm{s}} \boxed{+ \int_{\lambda_\mathrm{s}}^{\lambda_\mathrm{o}} d\lambda\, \partial_\tau(\Phi + \Psi)}$$

SW ISW

- Then the scalar average is strictly calculable

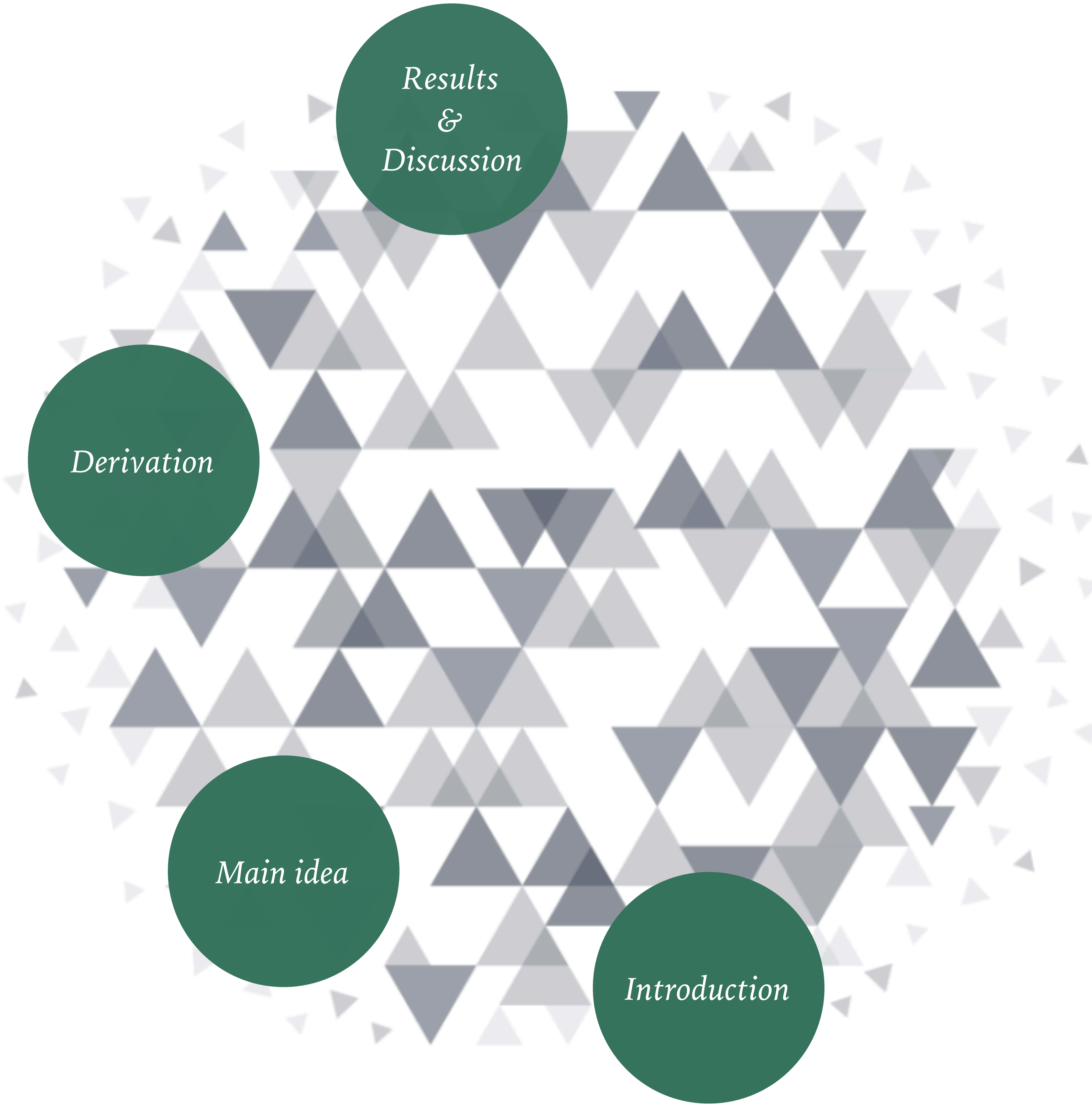
$$\Delta_h^{2,(\mathrm{o})}(\ln f) \simeq \int d\ln f'\, \Delta_h^{2,(\mathrm{s})}(\ln f')\, K(f, f')$$

bias $b \simeq -0.52$

$$K(f, f') = \frac{1}{\sqrt{2\pi\sigma^2}}[1 + b(\ln f - \ln f')]e^{-\frac{(\ln f - \ln f')^2}{2\sigma^2}}$$

linearly biased Gaussian

variance $\sigma^2 \sim \int d\ln k\, \Delta_{\mathcal{R}}^2$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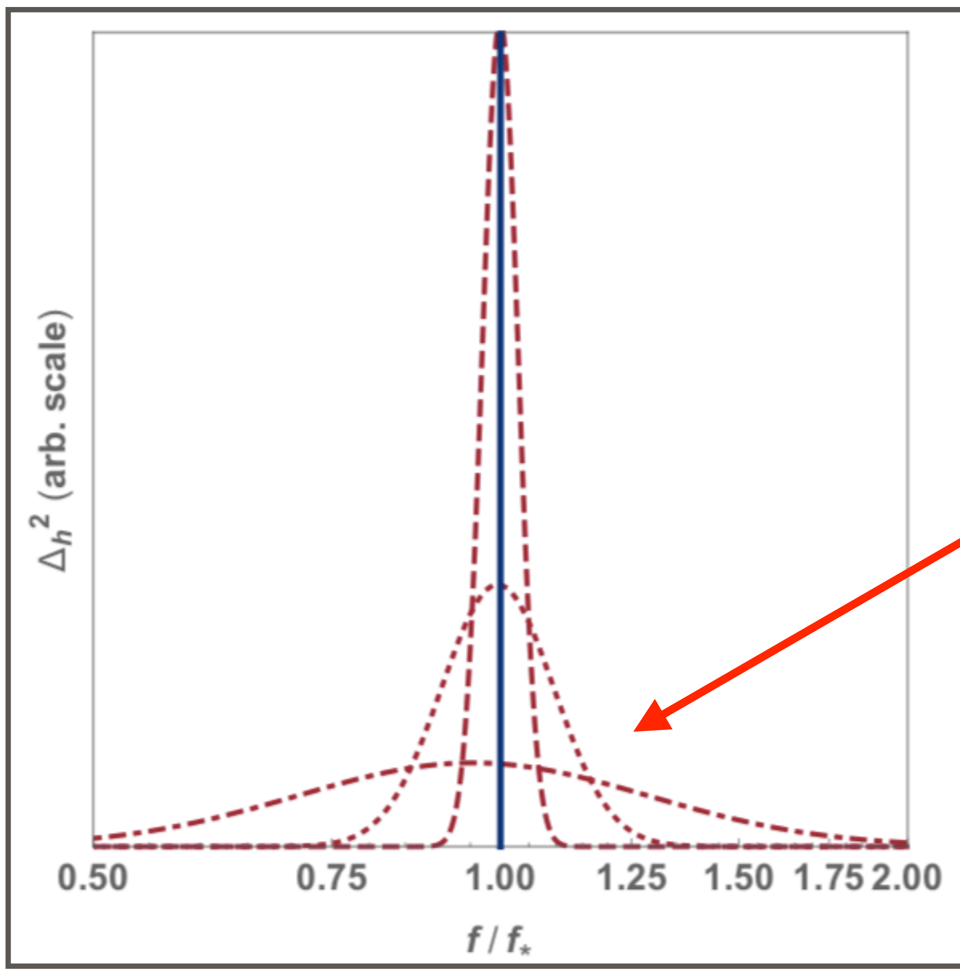
RESULTS

- For a spiky GW spectrum at the sourcing time

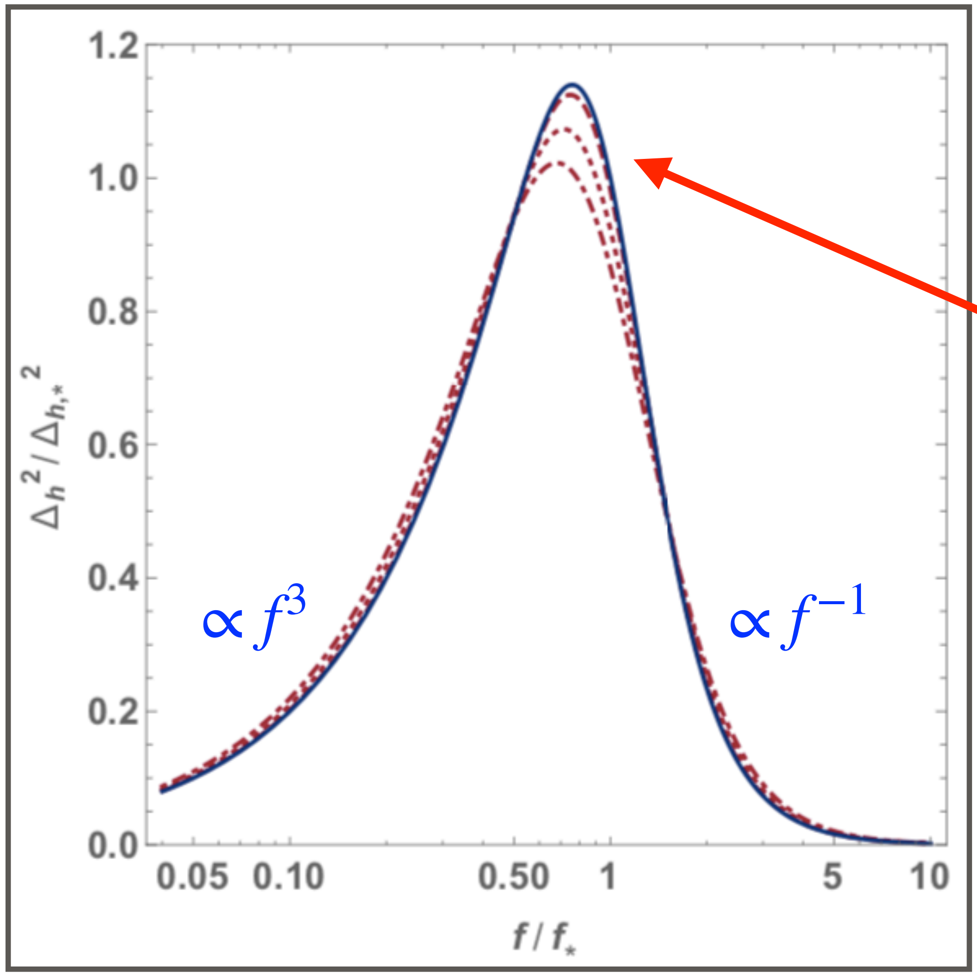


$$\int d\ln k\, \Delta_{\mathcal{R}}^2 = 10^{-3}, 10^{-2}, 10^{-1}$$

blue = original (= source) / red = deformed (= observed)

RESULTS

- For a smooth GW spectrum at the sourcing time



$\int d\ln k\, \Delta_{\mathscr{R}}^2$

$= 0.01,\ 0.05,\ 0.1$

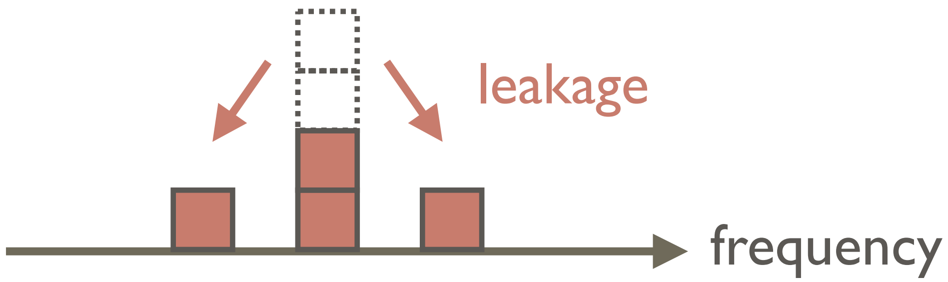
This result should be taken as order-of-magnitude estimate at best (→ discussion 2)

blue = original (= source) / red = deformed (= observed)

DISCUSSION 1

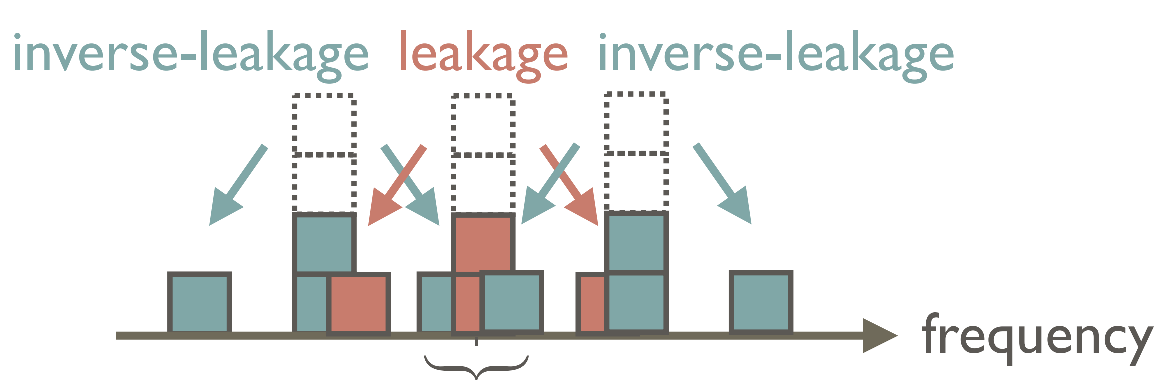
➤ For the smooth spectrum, the deformation looks smaller. Why?

- For the spiky case (= seen bin-by-bin), only leakage occurs



In this case, our prescription is safe (→ backup)

- For the smooth case, leakage & inverse-leakage almost cancel out



In this case, our calculation takes into account part of the whole effect

DISCUSSION 1 (CONT'D)

- For the smooth case, we had two nontrivial steps:

1. Taylor exp. $$\Delta_h^{2,(\mathrm{o})}(\ln f) = \left\langle e^{2\Delta \ln A} \Delta_h^{2,(\mathrm{s})}(\ln f - \Delta \ln f) \right\rangle_{\text{scalar ens. ave.}}$$
$$\stackrel{!}{\simeq} \left\langle \left(1 + 2\Delta \ln A\right) \Delta_h^{2,(\mathrm{s})}(\ln f - \Delta \ln f) \right\rangle_{\text{scalar ens. ave.}}$$

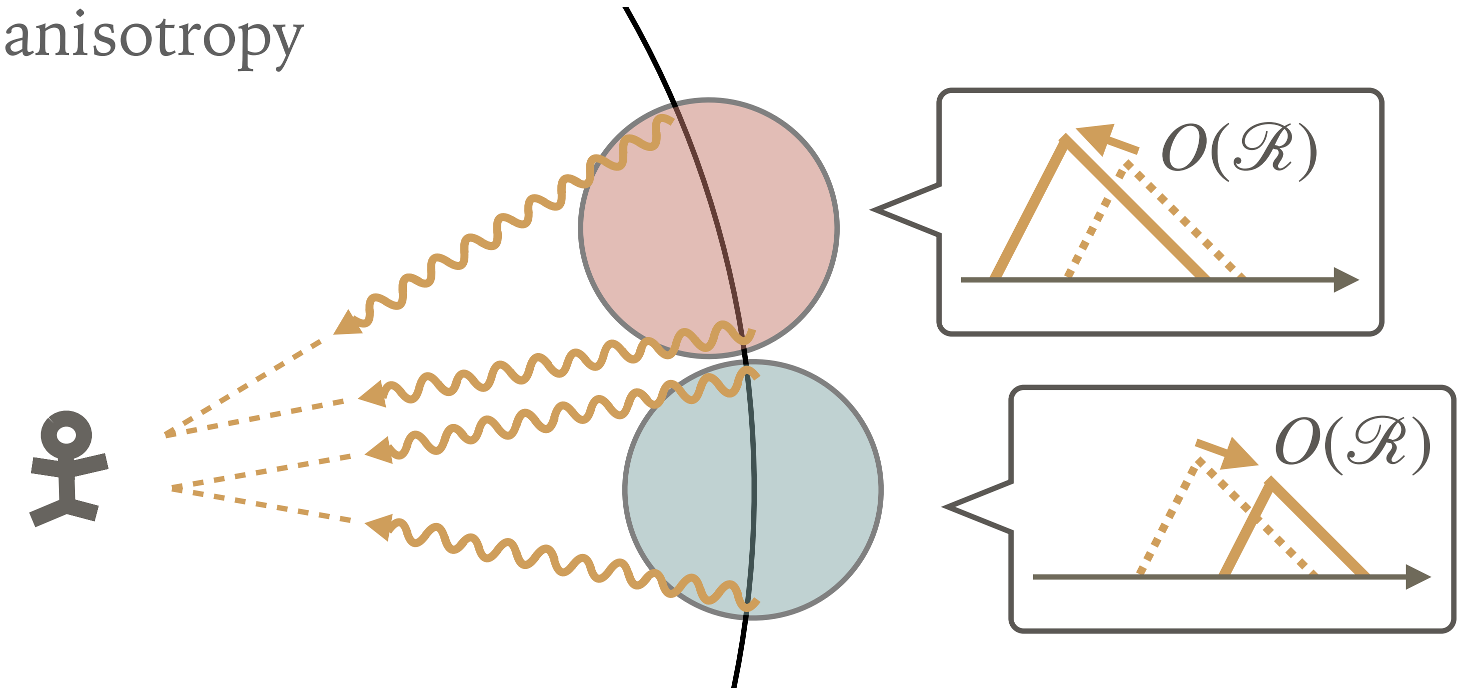
2. Linear-order result $$\Delta \ln A \stackrel{!}{=} -\Psi_{\mathrm{s}} - \frac{1}{2}\Phi_{\mathrm{s}}$$
$$\Delta \ln f \stackrel{!}{=} \Phi_{\mathrm{s}} - \frac{1}{2}\Phi_{\mathrm{s}} + \int_{\lambda_{\mathrm{s}}}^{\lambda_{\mathrm{o}}} d\lambda\, \partial_\tau(\Phi + \Psi)$$

- At each step, part of $\left\langle (\text{scalar})^2 \right\rangle_{\text{scalar ens. ave.}}$ terms are neglected

DISCUSSION 2

➤ Consistency with GW anisotropy

- GW anisotropy

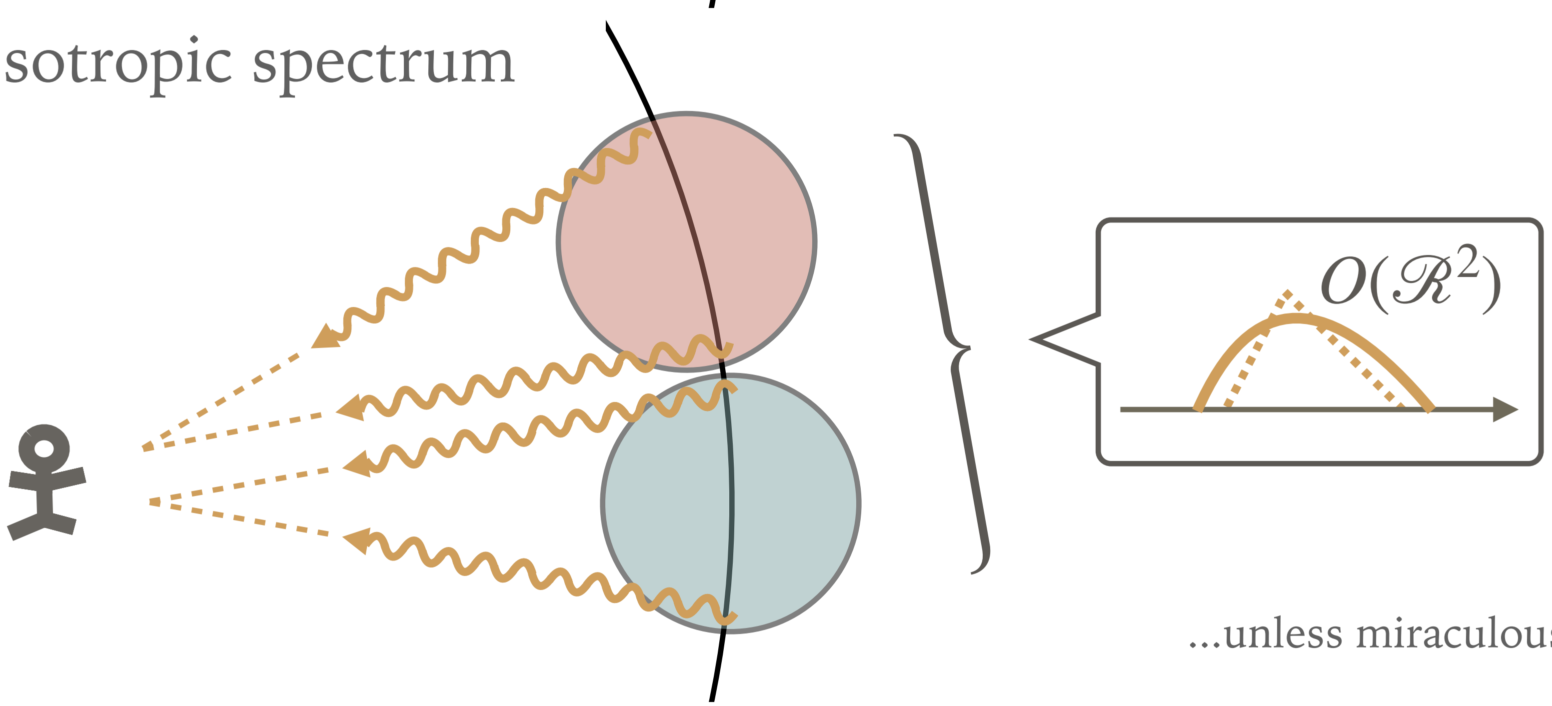


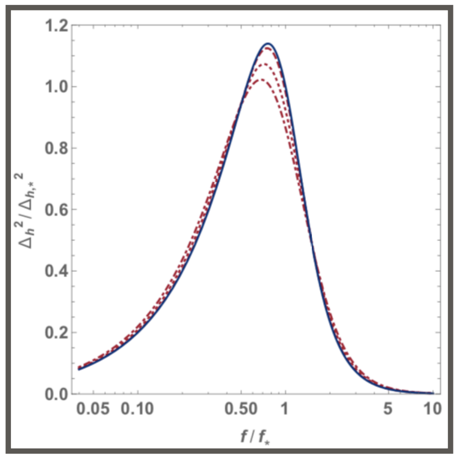
$$\delta_{\rm GW}(f,\hat{n}) \simeq \frac{\alpha}{5}\cdot\zeta_L(\hat{n}\cdot\eta_0)$$

$$\bar{\Omega}_{\rm GW}(f) \sim \left(\frac{f}{f_0}\right)^{\alpha}$$

[From Ema's talk slide]

- GW isotropic spectrum





...unless miraculous cancellation occurs

SUMMARY

- GWs propagate in an inhomogeneous Universe.

  The inhomogeneity deforms the original isotropic spectrum.

  - If the original GW spectrum is { sufficiently spiky → we can show this effect clearly; smooth → our result is just implicative }

  - GW anisotropy is also important

- Careful comparison btwn. theoretical and observed GW spectra may reveal the intermediate-scale curvature perturbations

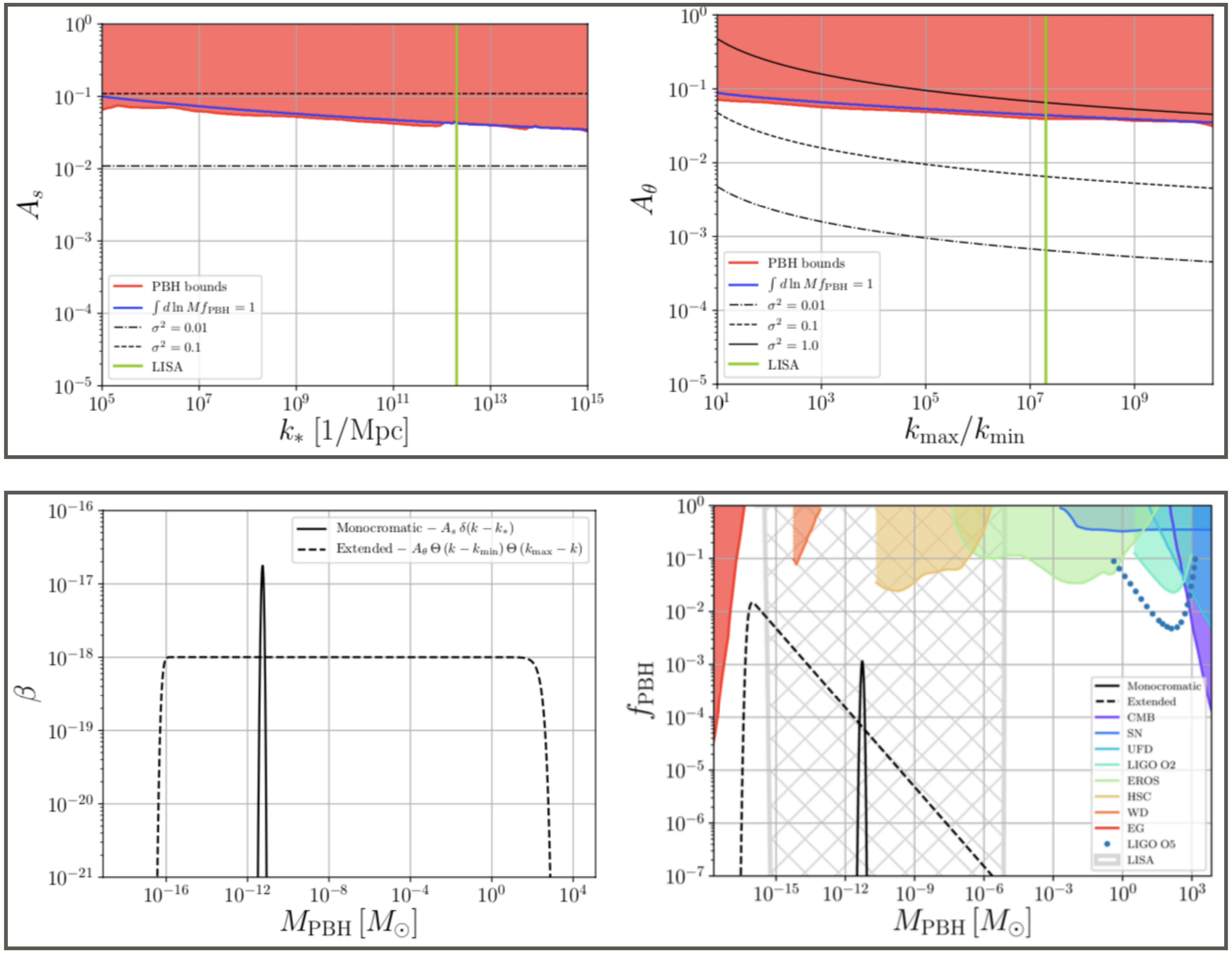
- To fully pin down the effect, we need 2nd-order pert'n theory

Backup

PBH CONSTRAINTS

$$\Delta^2_{\mathcal{R}}(k) = A_s\, k_* \,\delta(k - k_*) \qquad \Delta^2_{\mathcal{R}}(k) = A_\theta\, \Theta(k - k_{\rm min})\, \Theta(k_{\rm max} - k)$$

$$k_{\rm min} = 10^5\,{\rm Mpc}^{-1}$$



NEGLECTED TERMS

- Taylor expansion

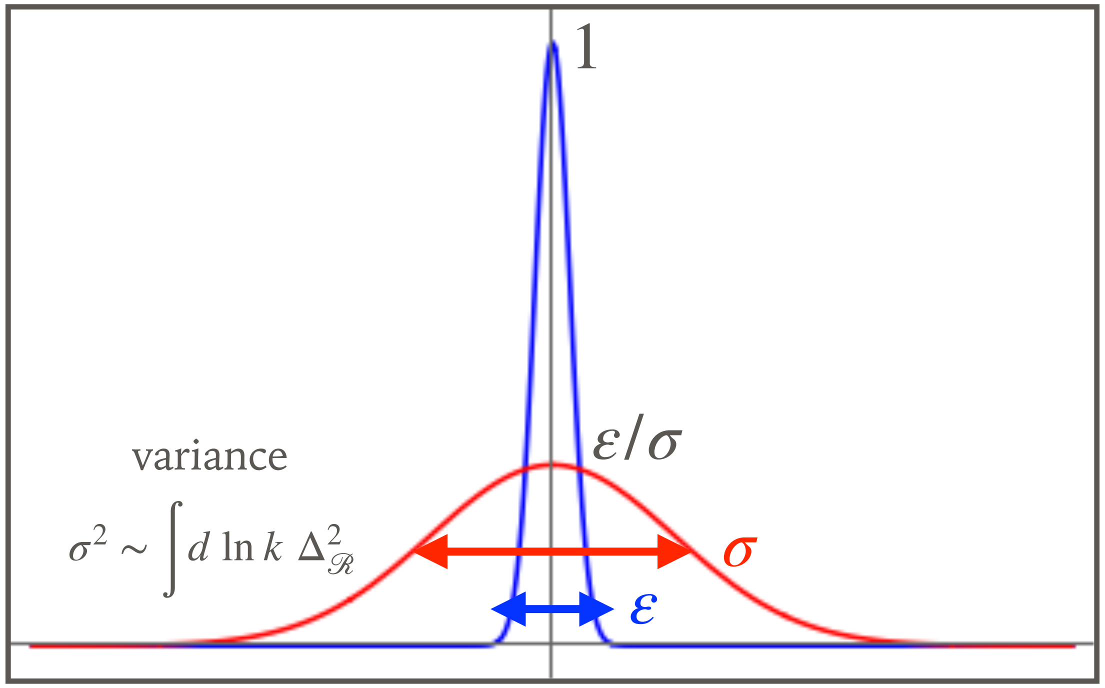
$$\begin{aligned}
\Delta_h^{2(\mathrm{o})}(\ln f) &= \left\langle e^{2\Delta \ln A} \Delta_h^{2(\mathrm{s})}\left(\ln f - \Delta \ln f\right)\right\rangle_{\mathrm{ens(s)}} \\
&= \left\langle \left(1 + 2\Delta \ln A^{(1)}\right) \Delta_h^{2(\mathrm{s})}\left(\ln f - \Delta \ln f^{(1)}\right)\right\rangle_{\mathrm{ens(s)}} && \text{: we took this} \\
&\quad + \left\langle 2\left(\Delta \ln A^{(1)}\right)^2 + 2\Delta \ln A^{(2)}\right\rangle_{\mathrm{ens(s)}} \Delta_h^{2(\mathrm{s})}(\ln f) && \text{: vertical shift} \\
&\quad + \left\langle \Delta_h^{2(\mathrm{s})}\left(\ln f - \Delta \ln f^{(2)}\right)\right\rangle_{\mathrm{ens(s)}} - \Delta_h^{2(\mathrm{s})}(\ln f) && \text{: } \mathcal{O}(\sigma^2) \text{ horizontal shift} \\
&\quad + \mathcal{O}(\sigma^3)\,.
\end{aligned}$$

SPIKY ⇄ SMOOTH

- δ-function type source spectrum $$\Delta_h^{2(\mathrm{s})}(f) = \frac{\Delta_{h,*}^2}{\sqrt{2\pi\varepsilon^2}} \exp\left[-\frac{(\ln f - \ln f_*)^2}{2\varepsilon^2}\right]$$ (δ-func. for $\varepsilon \to 0$)

- Our expression $$\Delta_h^{2(\mathrm{o})}(f) = \int d\ln f' \; \Delta_h^{2(\mathrm{s})}(f') \; K(f, f')$$ gives

  the observed spectrum $$\Delta_h^{2(\mathrm{o})}(f) = \frac{\Delta_{h,*}^2}{\sqrt{2\pi(\varepsilon^2+\sigma^2)}} \left[1 + \frac{\sigma^2}{\varepsilon^2+\sigma^2} b(\ln f - \ln f_*)\right] \exp\left[-\frac{(\ln f - \ln f_*)^2}{2(\varepsilon^2+\sigma^2)}\right]$$



DERIVATION OF THE LINEARLY BIASED GAUSSIAN

- Imagine a Gaussian variable X obeying distribution $P(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{X^2}{2\sigma^2}}$
- Suppose you would like to calculate $\langle (1+2X) f(x-X) \rangle$
- You will get $\langle (1+2X) f(x-X) \rangle = \int dX \, \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{X^2}{2\sigma^2}} (1+2X) f(x-X)$

$$= \int dx' \, \frac{1}{\sqrt{2\pi\sigma^2}} [1 + 2(x - x')] e^{-\frac{(x-x')^2}{2\sigma^2}} f(x')$$

$(x - X = x')$

- In the present case, we have infinitely many Gaussian variables $X_1, X_2, \cdots$ (corresponding to each k mode), but the calculation is essentially the same

ANALOGY TO CMB