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MATHEMATICS OF THE UNIVERSE

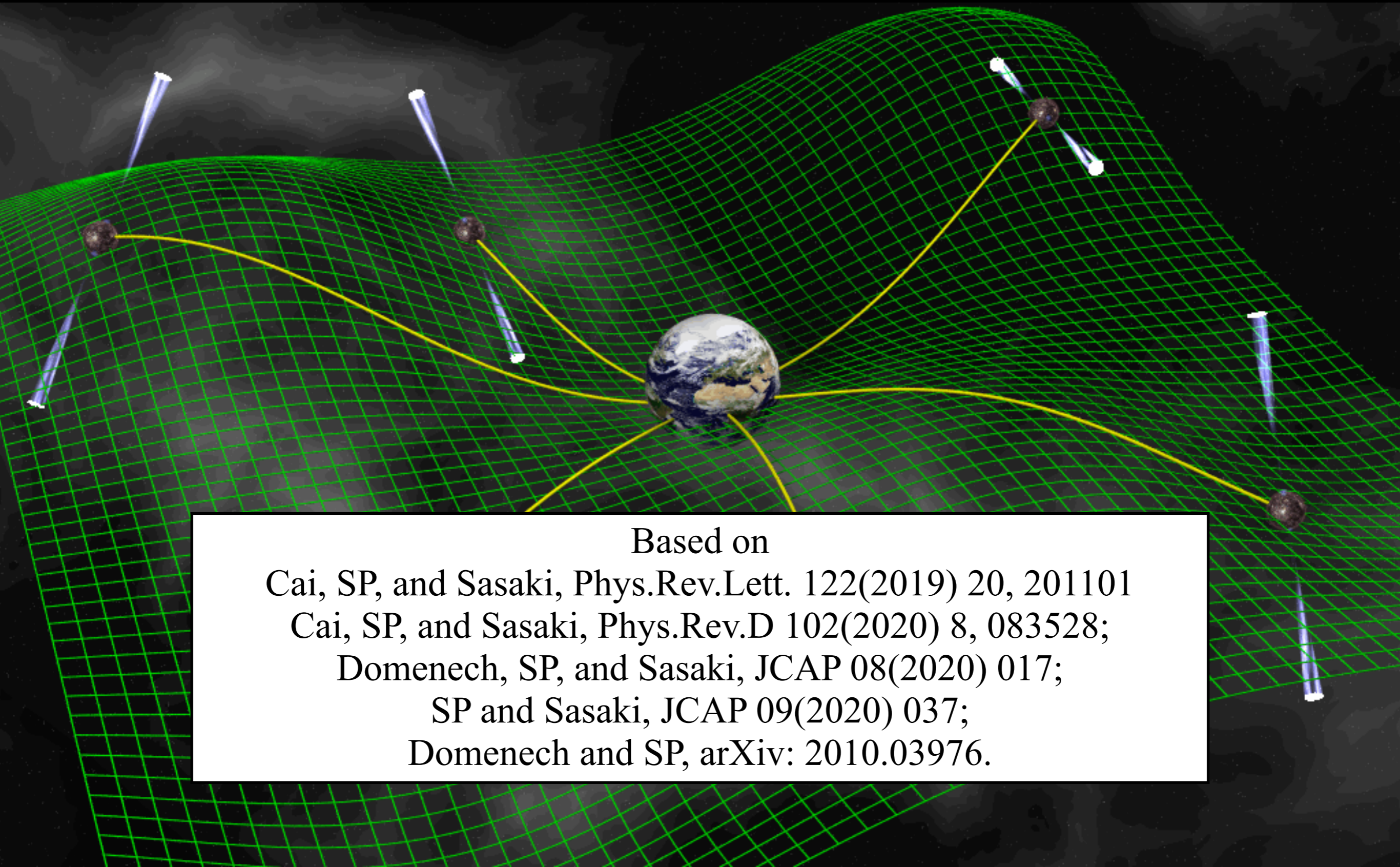


NANOGrav 12.5-yr Result and the Planet-mass PBHs

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Institute of Theoretical Physics, CAS
Kavli IPMU, the University of Tokyo

Gravitational Wave Primordial Cosmology, IAP, France
2021-5-19



Based on

Cai, SP, and Sasaki, Phys.Rev.Lett. 122(2019) 20, 201101

Cai, SP, and Sasaki, Phys.Rev.D 102(2020) 8, 083528;

Domenech, SP, and Sasaki, JCAP 08(2020) 017;

SP and Sasaki, JCAP 09(2020) 037;

Domenech and SP, arXiv: 2010.03976.

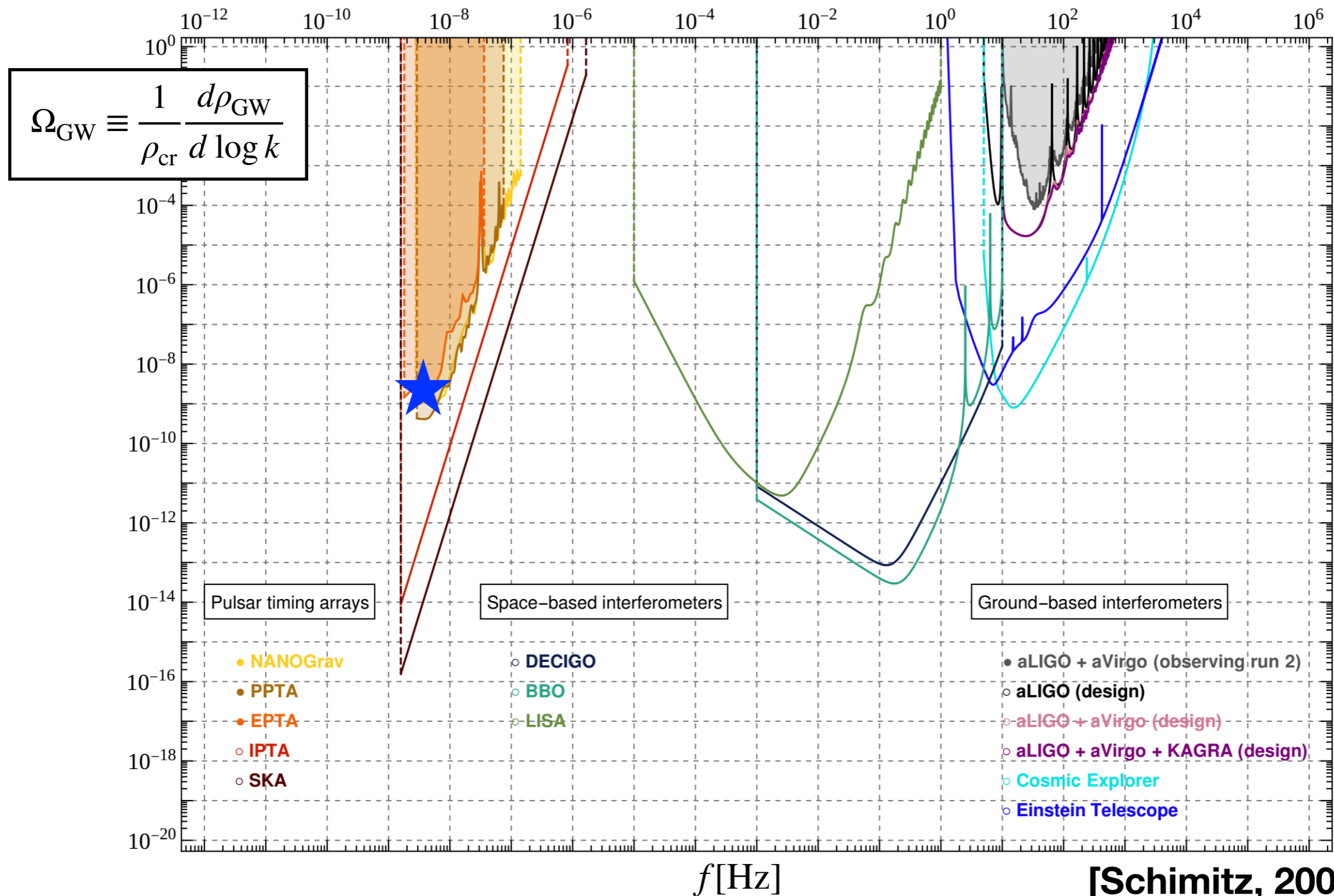
PTA discovered SGWB?

The NANOGrav 12.5-year Data Set: Search For An Isotropic Stochastic Gravitational-Wave Background

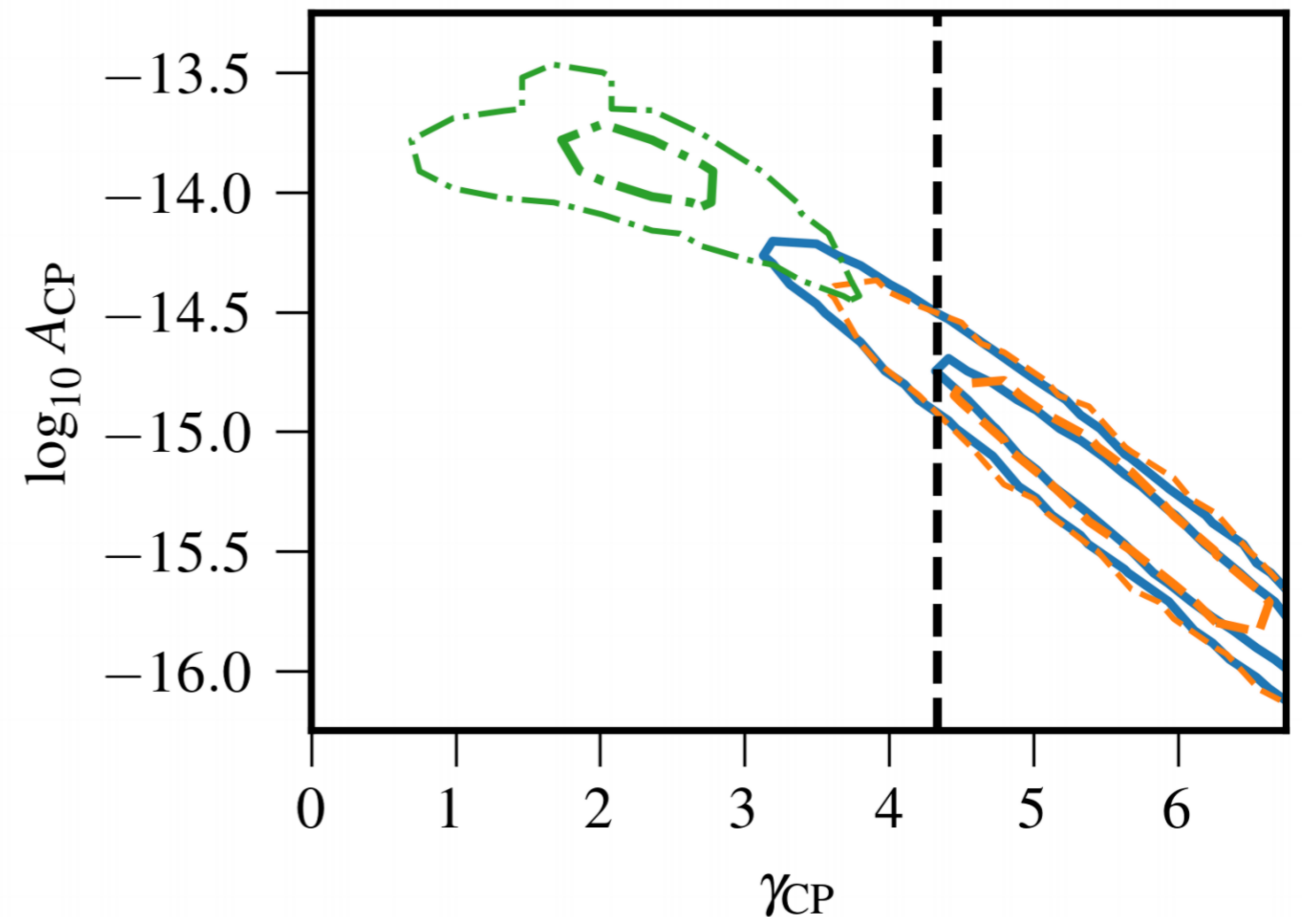
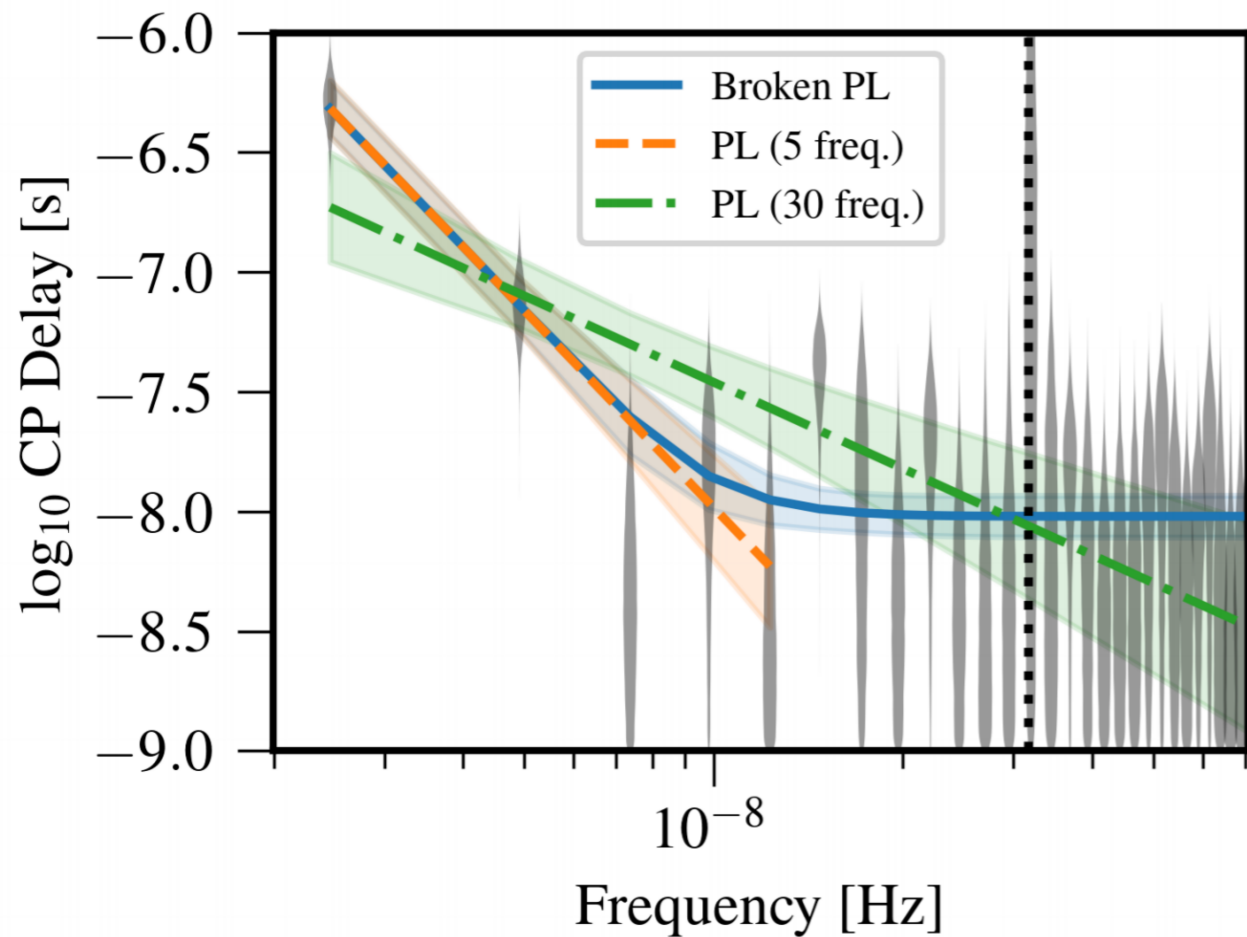
ABSTRACT

We search for an isotropic stochastic gravitational-wave background (GWB) in the 12.5-year pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars. The Bayesian posterior of the amplitude for a $f^{-2/3}$ power-law spectrum, expressed as characteristic GW strain, has median 1.92×10^{-15} and 5%–95% quantiles of 1.37 – 2.67×10^{-15} at a reference frequency of $f_{\text{yr}} = 1 \text{ yr}^{-1}$. The Bayes factor in favor of the common-spectrum process versus independent red-noise processes in each pulsar exceeds 10,000. However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with General Relativity. We find that the process has neither monopolar nor dipolar correlations, which may arise from, for example, reference clock or solar-system ephemeris systematics, respectively. The amplitude posterior has significant support above previously reported upper limits; we explain this in terms of the Bayesian priors assumed for intrinsic pulsar red noise. We examine potential implications for the supermassive black hole binary population under the hypothesis that the signal is indeed astrophysical in nature.

PTA discovered SGWB?



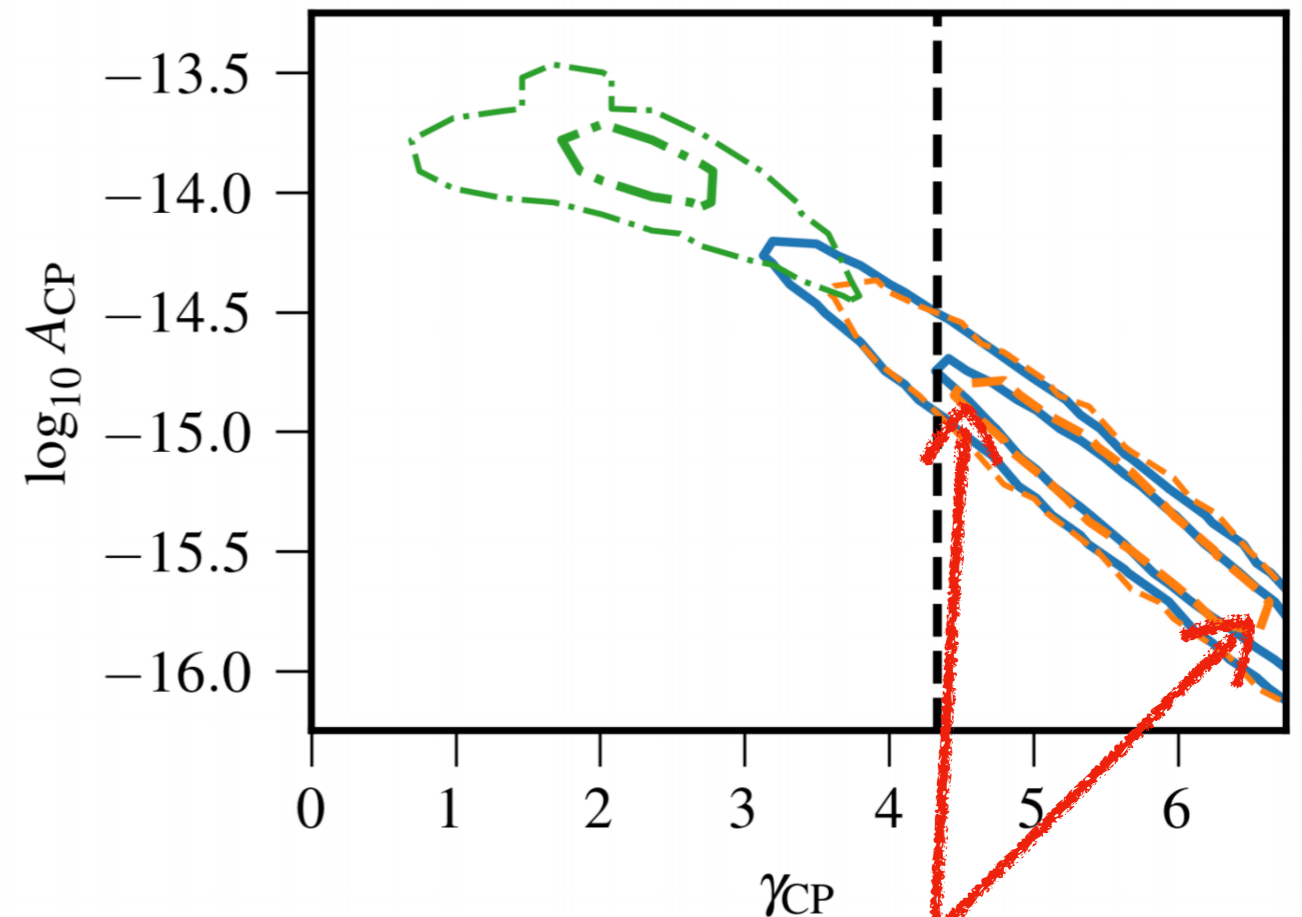
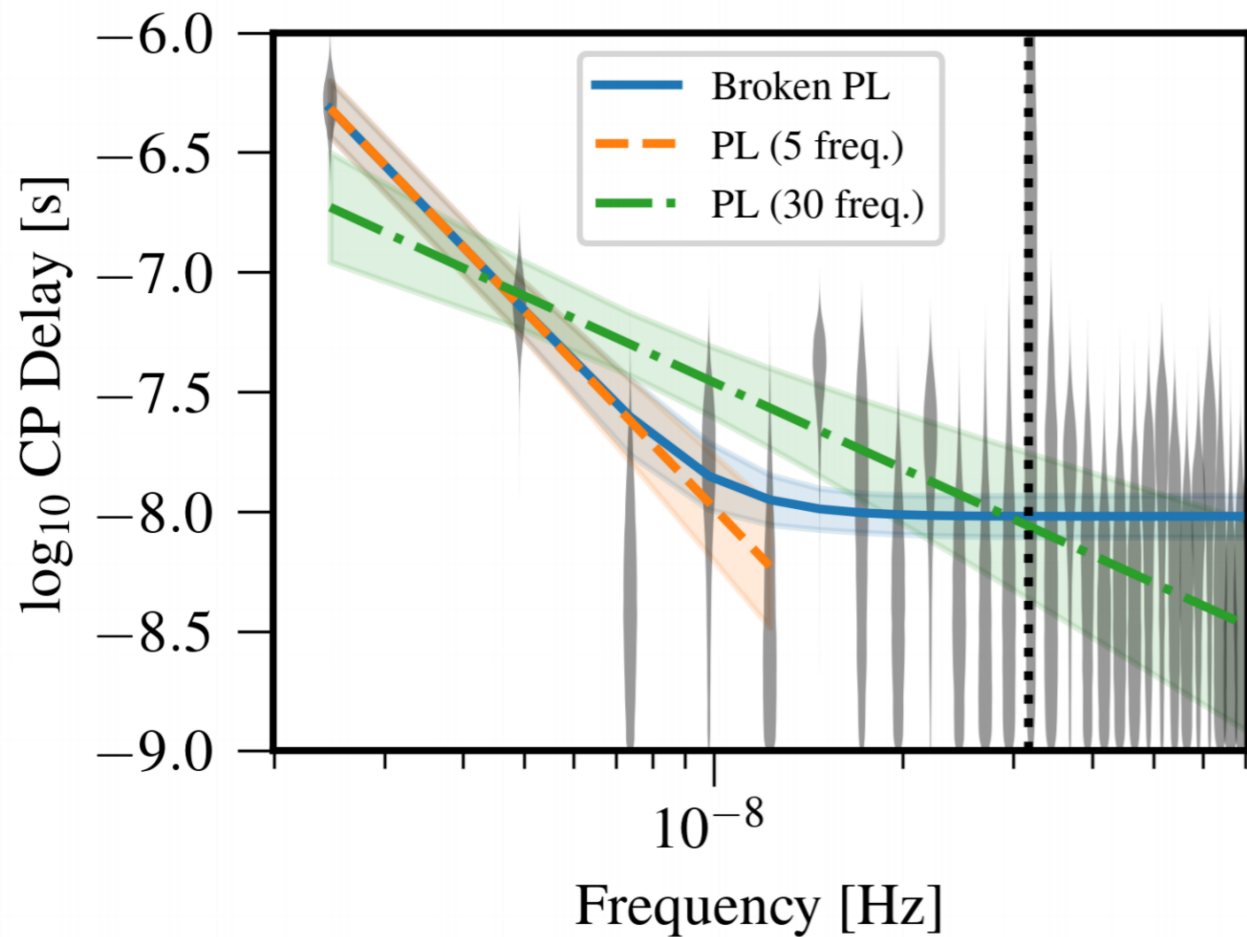
PTA discovered SGWB?



$$\Omega_{GW} = \frac{2\pi^2 f_{yr}^2}{3H_0^2} A_{SGWB}^2 \left(\frac{f}{f_{yr}} \right)^{5-\gamma}$$

$$f_{yr} = 3.17 \times 10^{-8} \text{ Hz.}$$

PTA discovered SGWB?

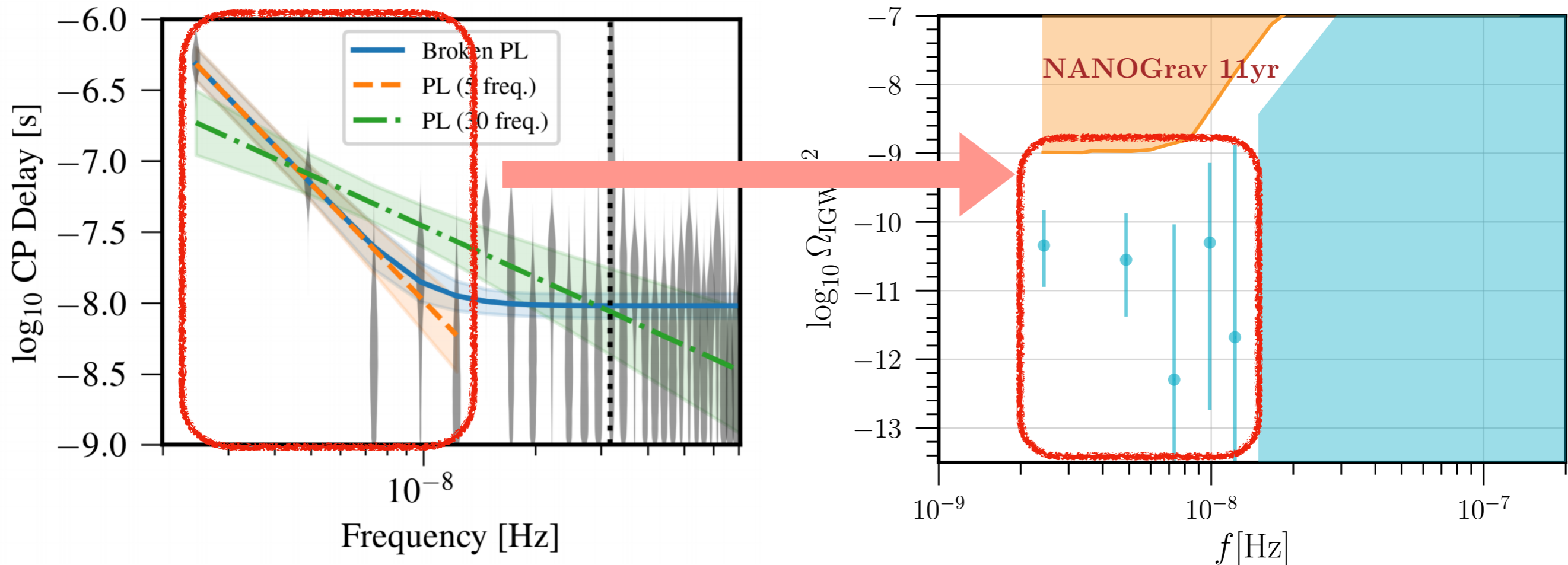


$$\Omega_{GW} = \frac{2\pi^2 f_{yr}^2}{3H_0^2} A_{SGWB}^2 \left(\frac{f}{f_{yr}} \right)^{5-\gamma}$$

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$\Omega_{GW} \propto f^{-3/2 \sim 1/2}$

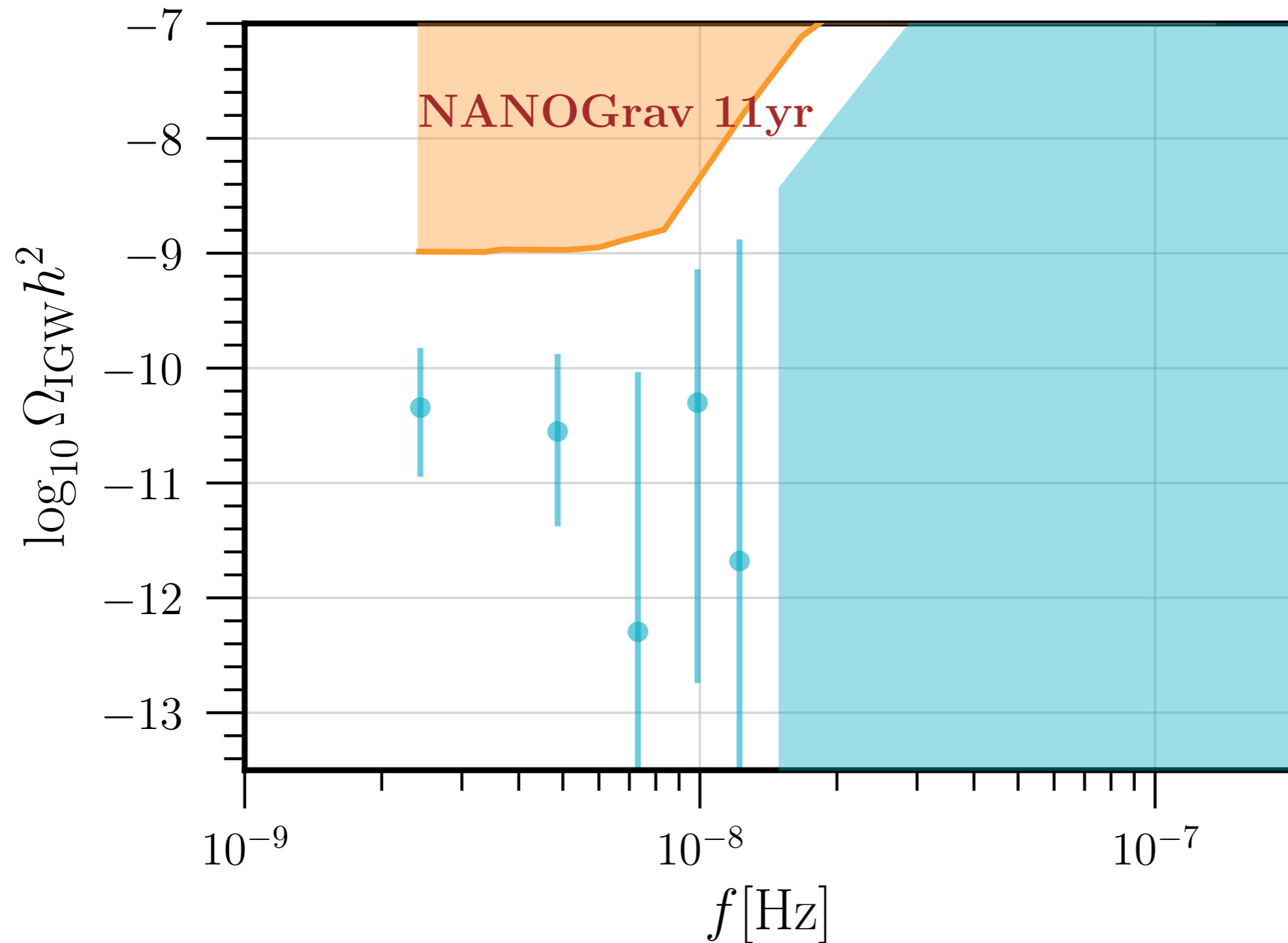
PTA discovered SGWB?



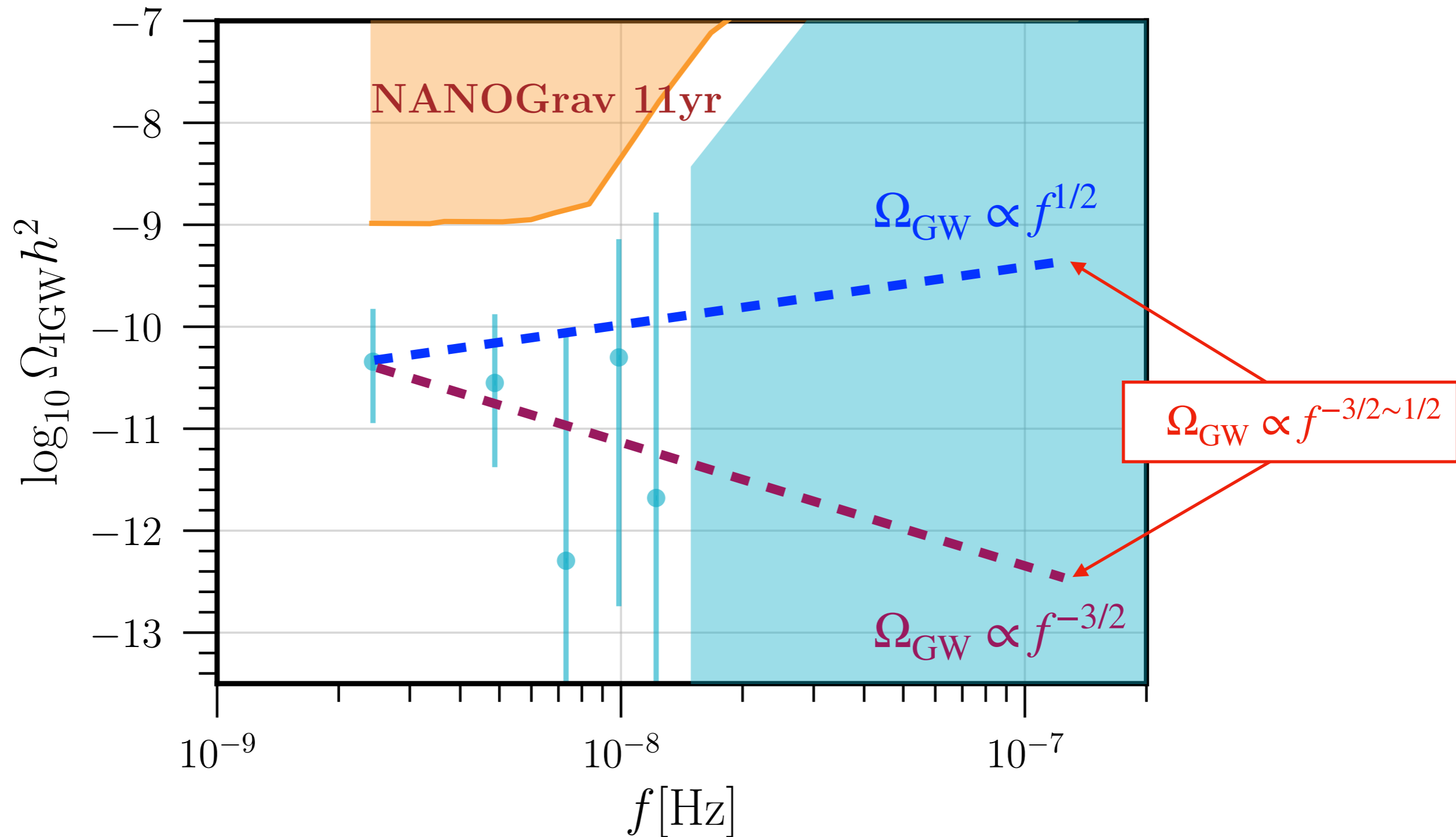
$$\Omega_{\text{GW}} = \frac{2\pi^2 f_{\text{yr}}^2}{3H_0^2} A_{\text{SGWB}}^2 \left(\frac{f}{f_{\text{yr}}} \right)^{5-\gamma}$$

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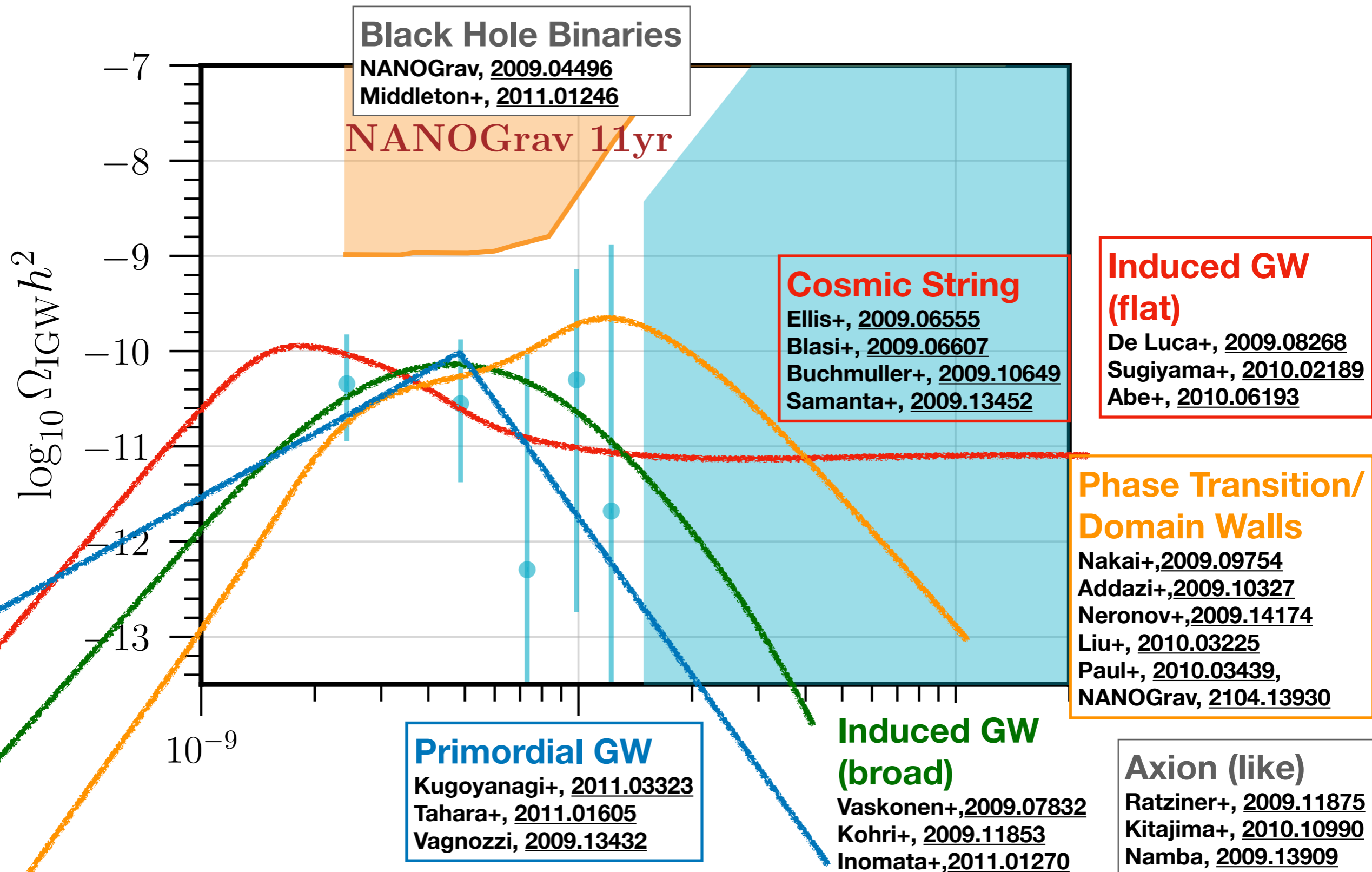
PTA discovered SGWB?



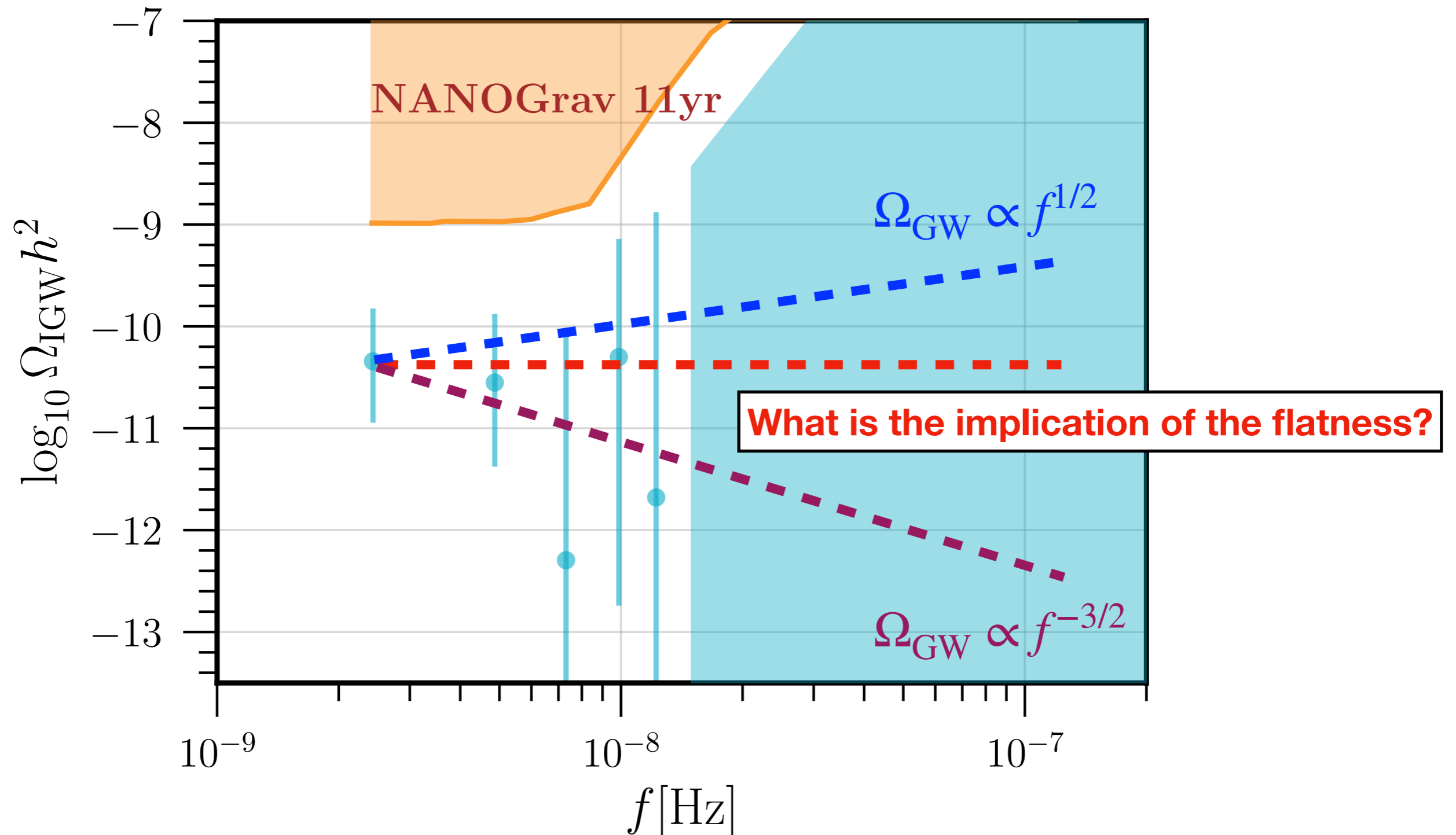
PTA discovered SGWB?



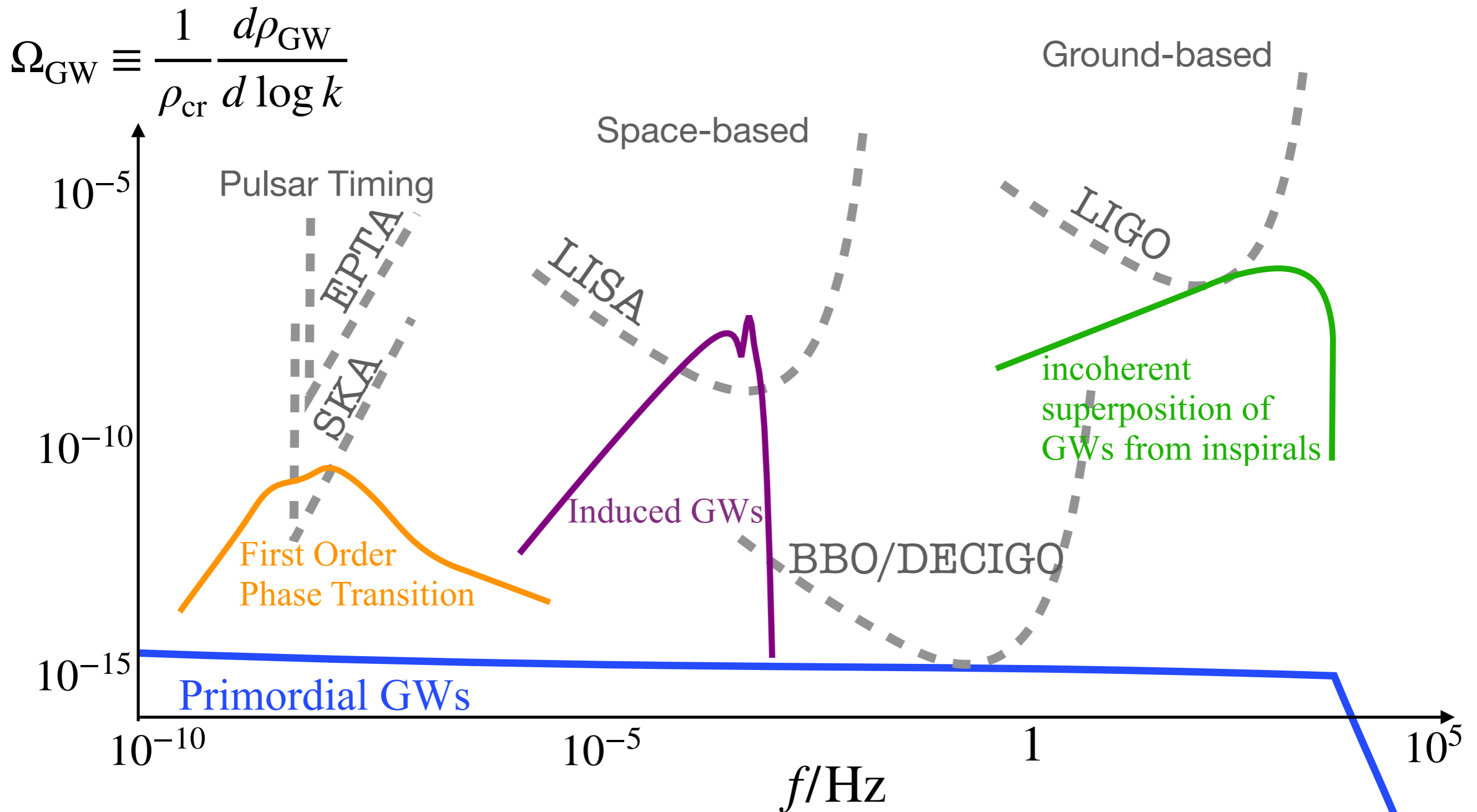
PTA discovered SGWB?



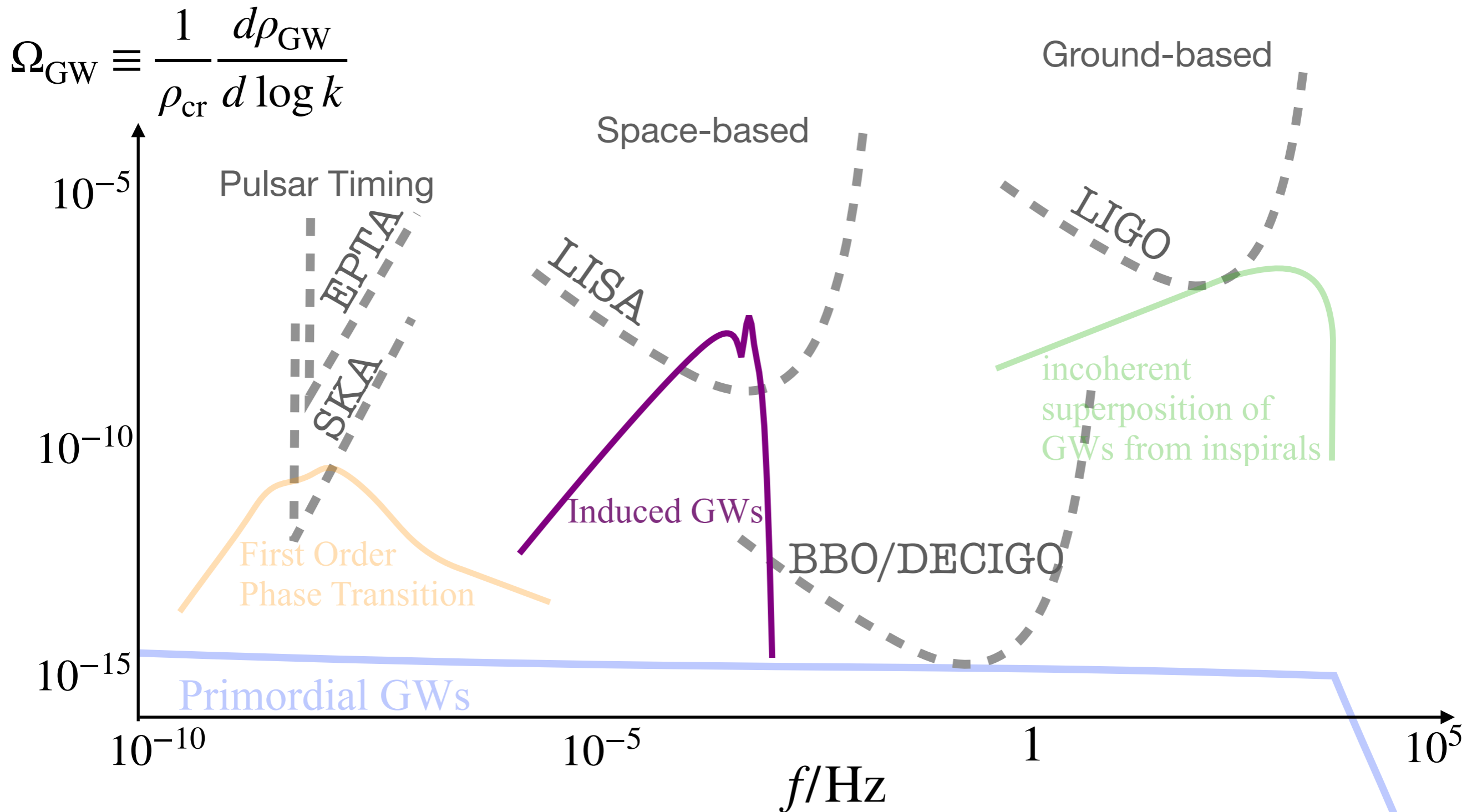
Spectral Shape of SGWB

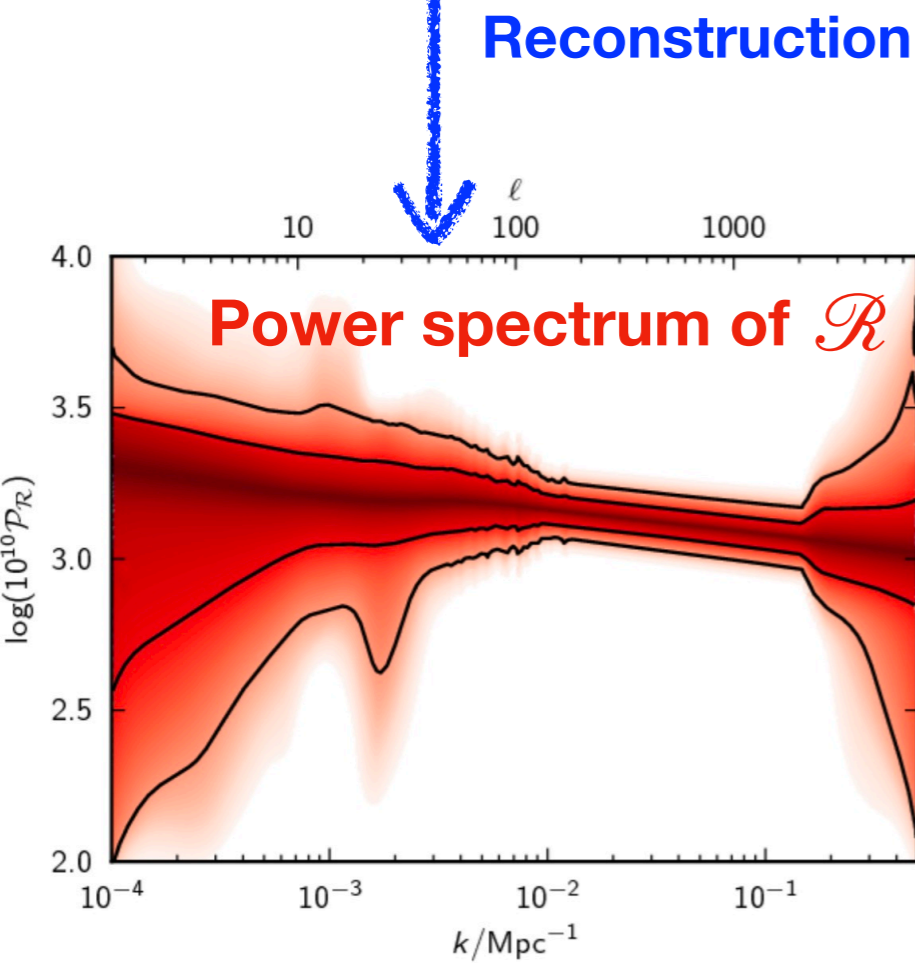
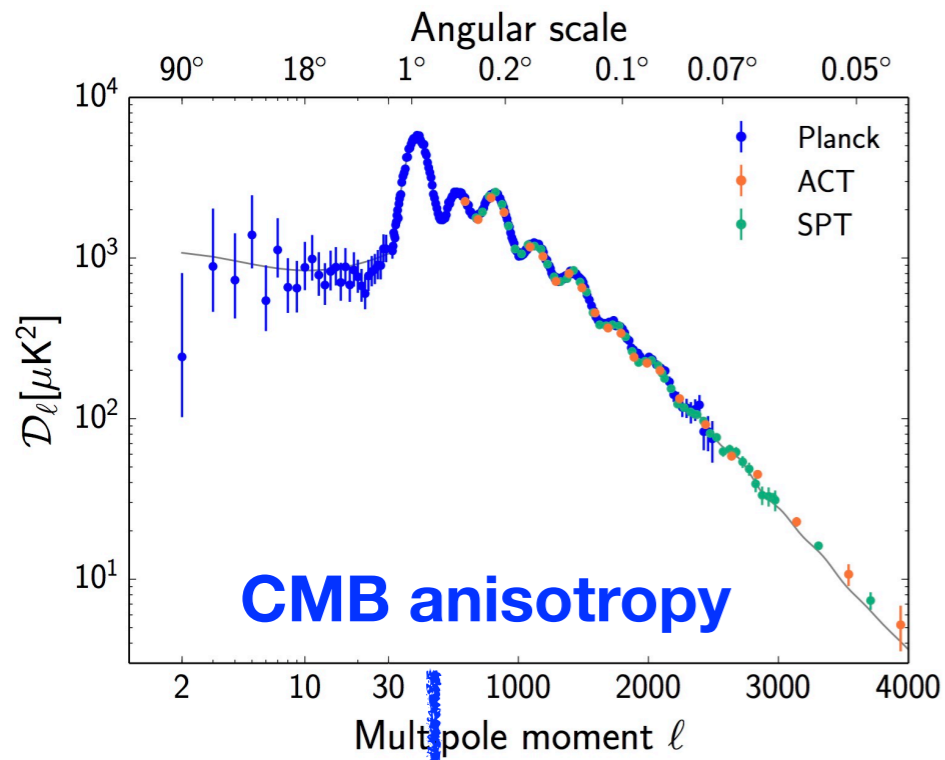


Spectral Shape of SGWB



Spectral Shape of SGWB

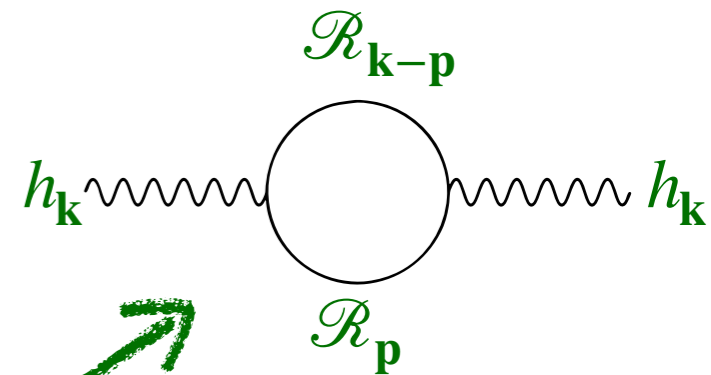




Lack of constraints on small scales

nonlinear perturbation

induced GWs

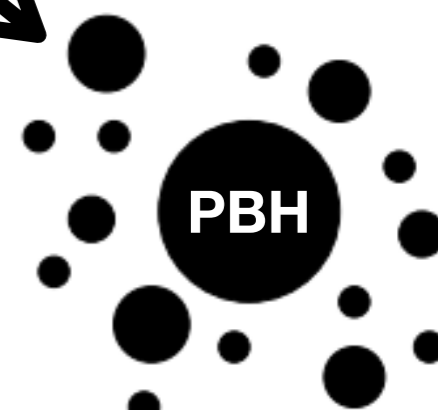


**LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO**

cross-check

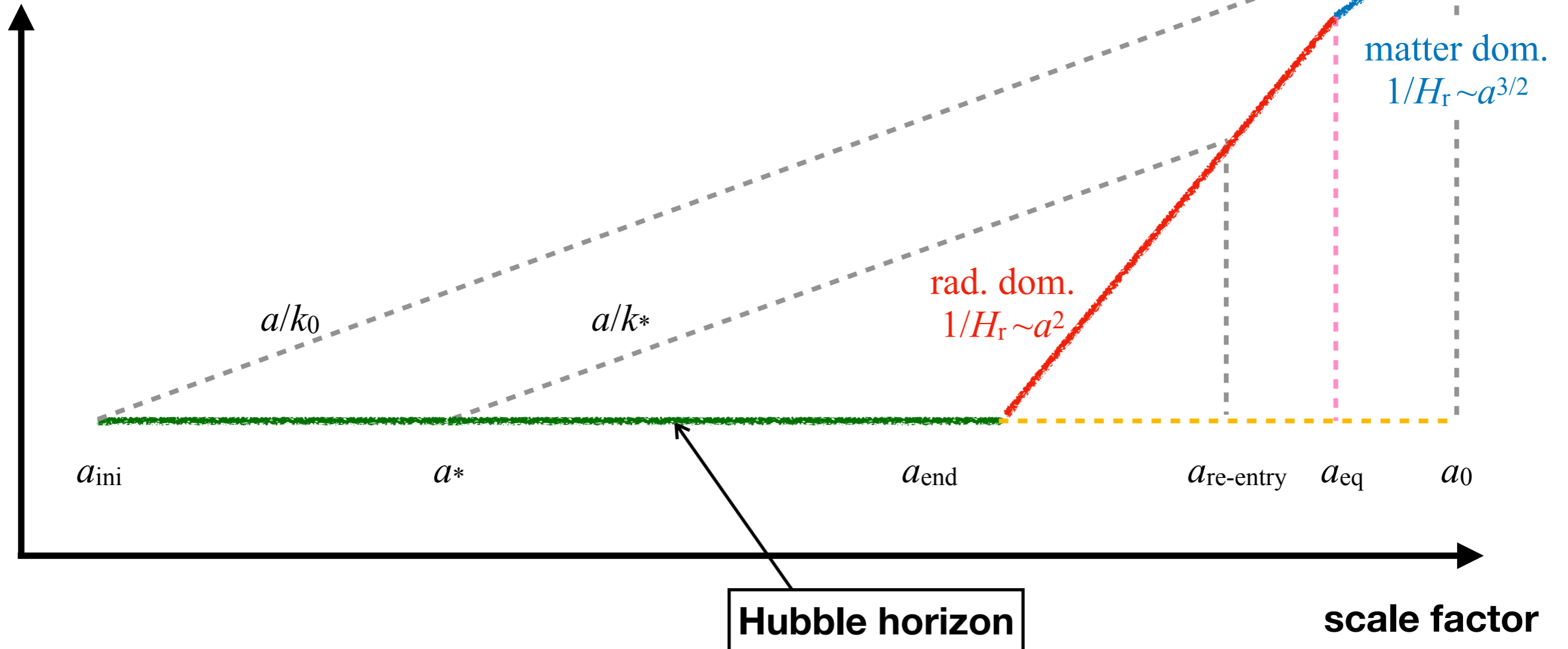
**EG γ -ray
femtolensing
microlensing
LIGO
CMB μ -distortion**

Tasinato's talk



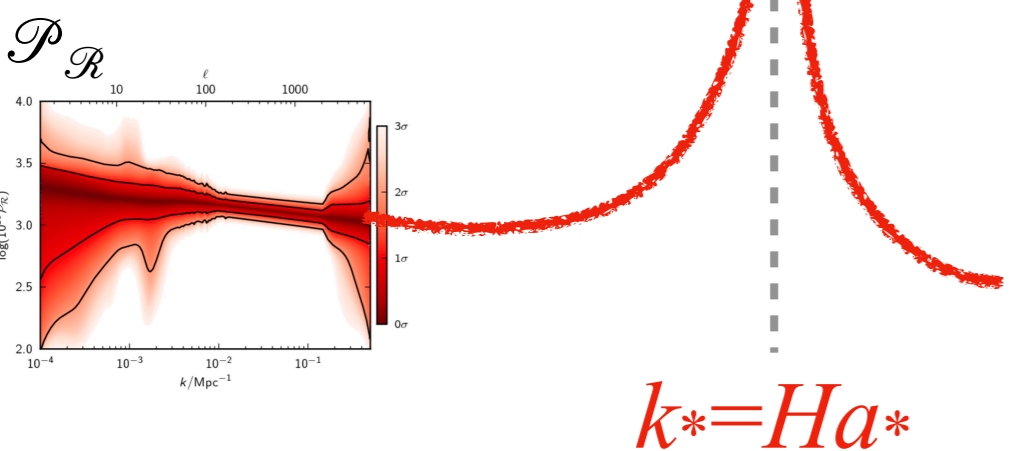
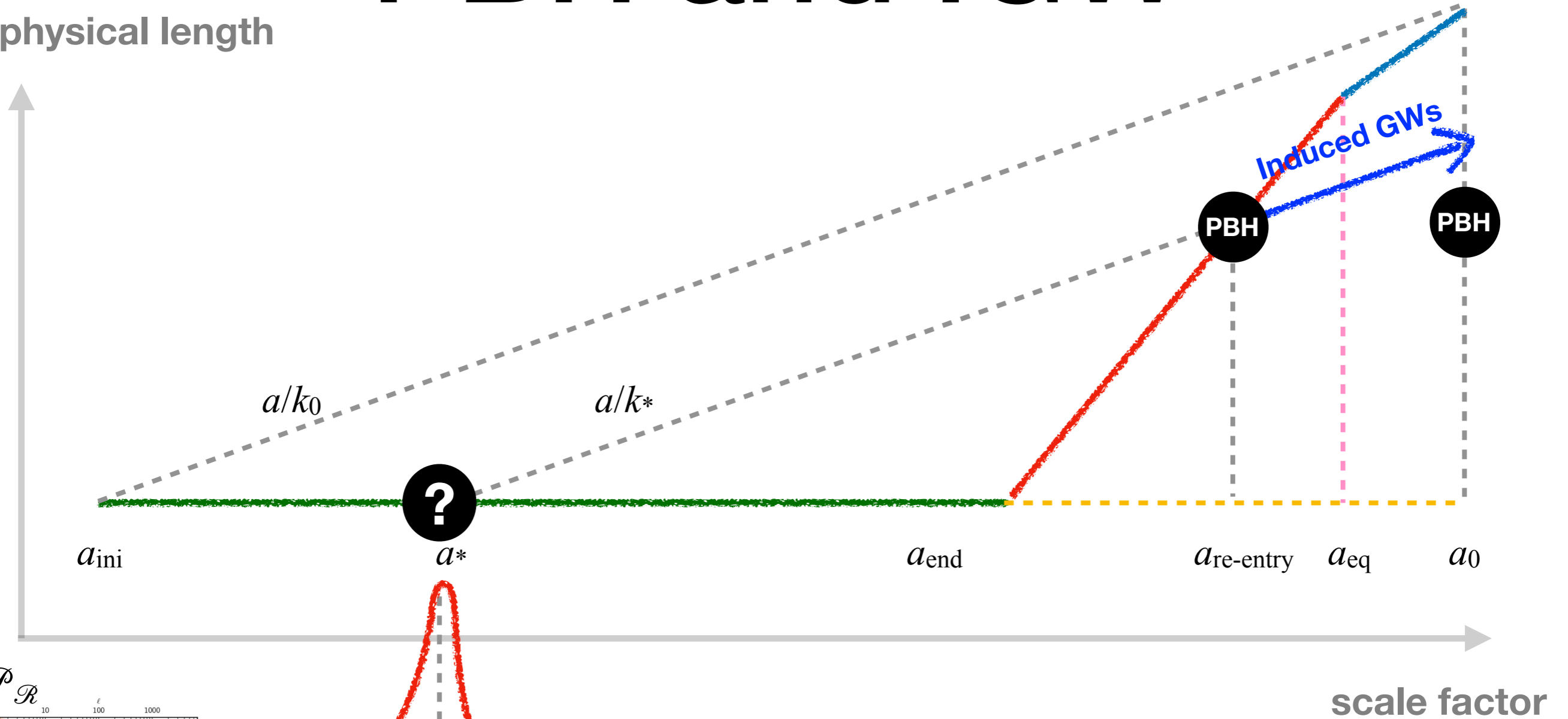
PBH and IGW

physical length



PBH and IGW

physical length



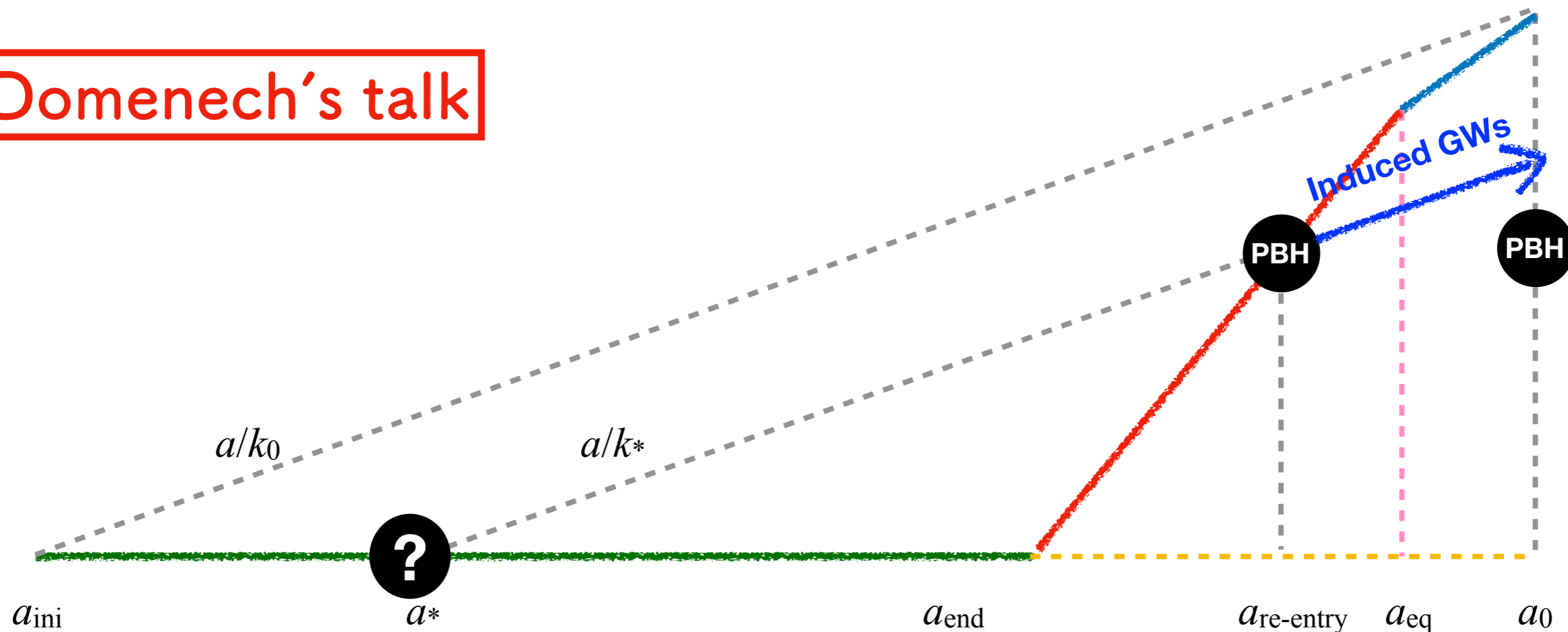
$$k^* = Ha^*$$

$$h_{\mathbf{k}} \sim \int d\eta \times (\text{Green function}) \int d^3p \times (\text{Transfer function}) \times \Phi_{\mathbf{p}} \Phi_{\mathbf{k}-\mathbf{p}}.$$

$$\Omega_{\text{GW}} \sim \Omega_r \langle hh \rangle \sim \Omega_r \langle \Phi \Phi \Phi \Phi \rangle \sim \Omega_r \mathcal{P}_{\Phi}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2$$

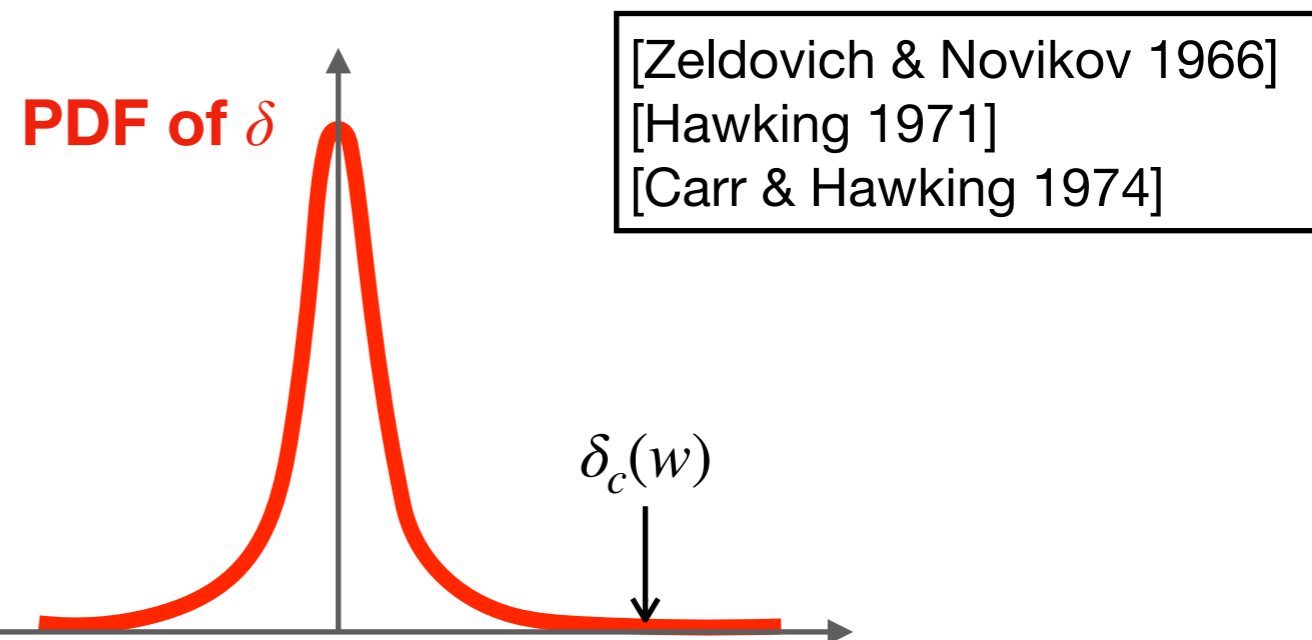
[Matarrese+, 1997]
 [Ananda+, 2007]
 [Baumann+, 2007]

Domenech's talk



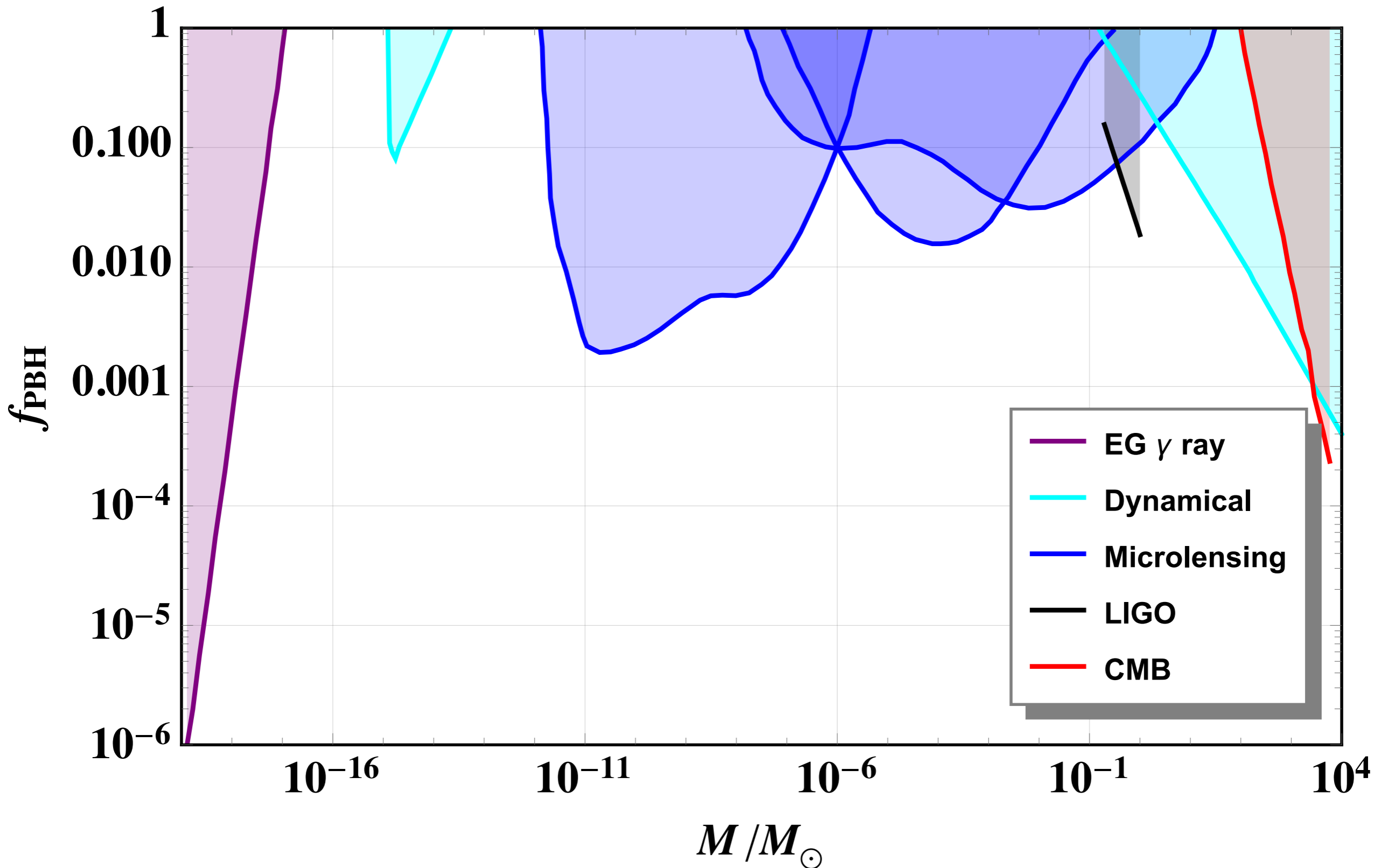
$$\beta \sim \text{erfc}\left(\frac{\mathcal{R}_c}{2\sqrt{\mathcal{P}_{\mathcal{R}}}}\right)$$

$$f_{\text{PBH}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_{\odot}}\right)^{-1/2}$$

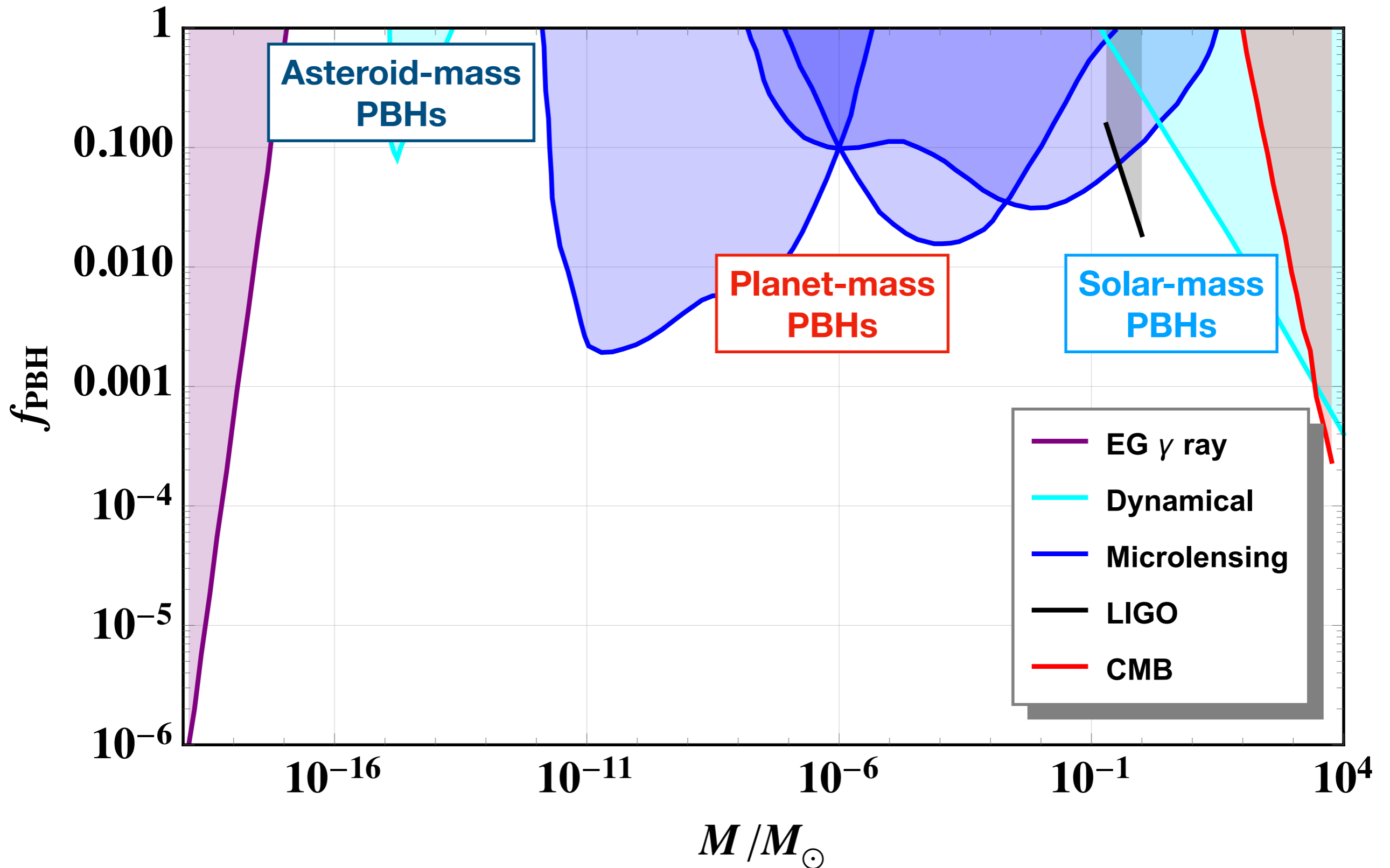


[Zeldovich & Novikov 1966]
 [Hawking 1971]
 [Carr & Hawking 1974]

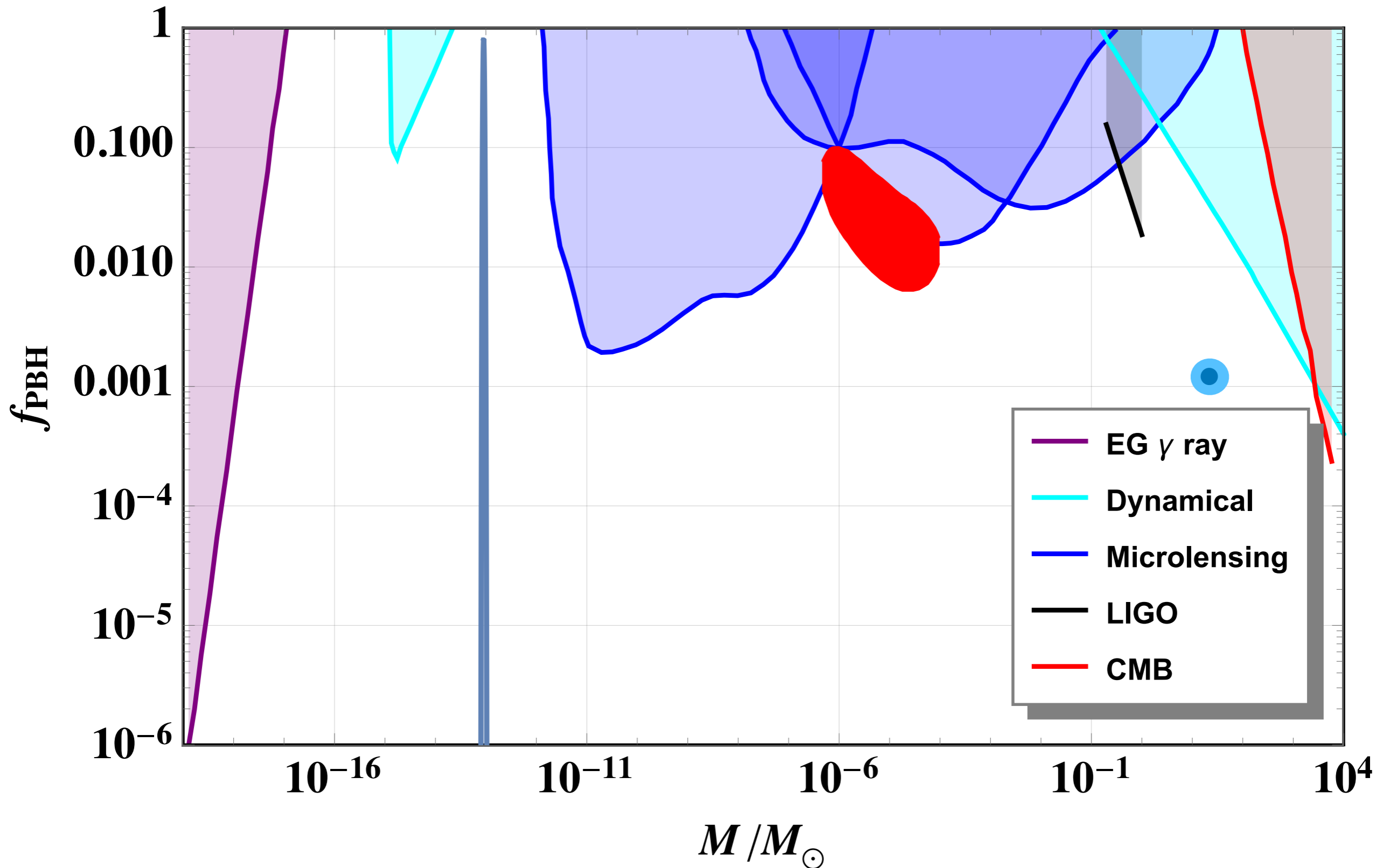
Constraints on f_{PBH}



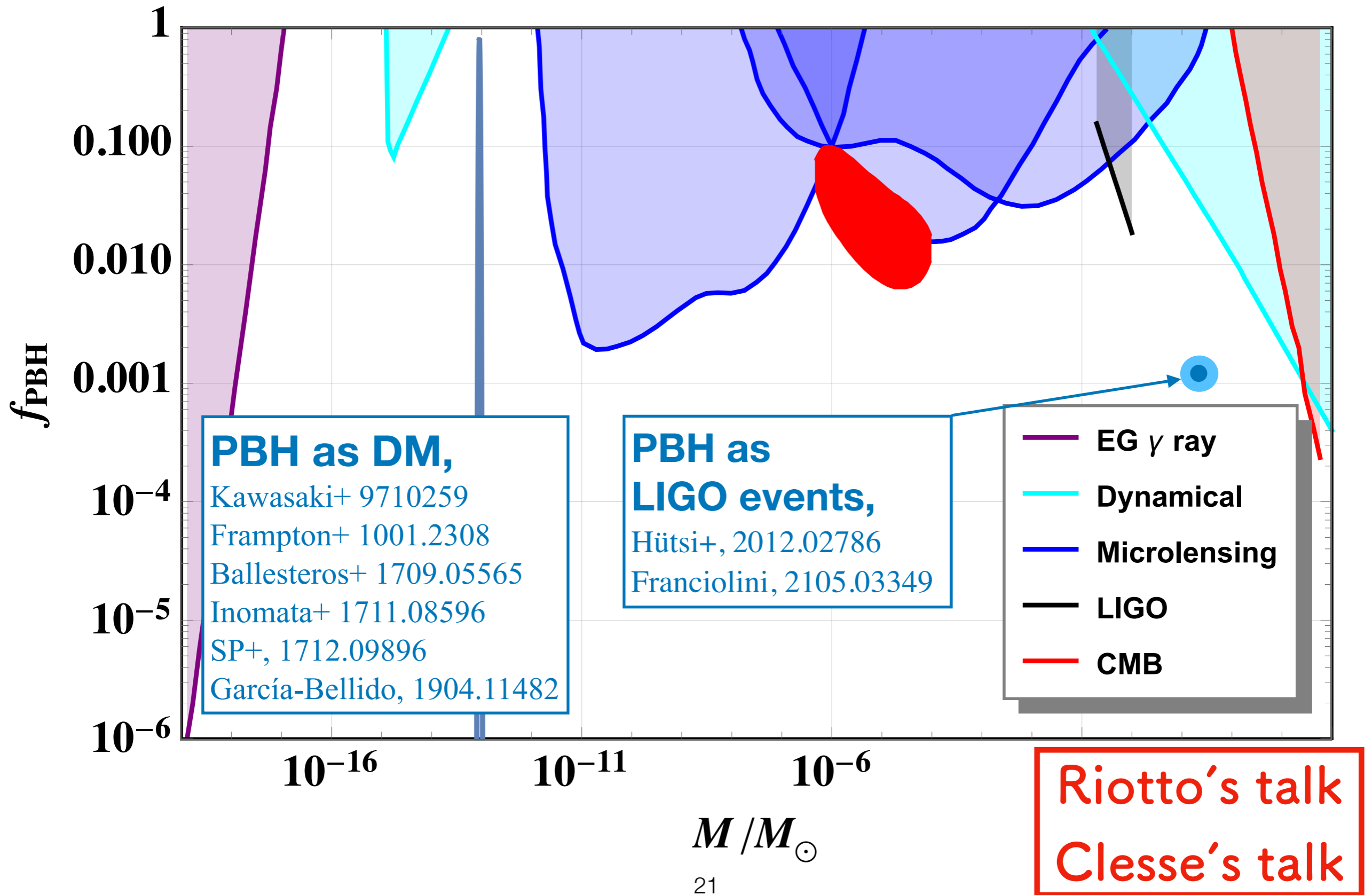
Constraints on f_{PBH}



Constraints on f_{PBH}



Constraints on f_{PBH}

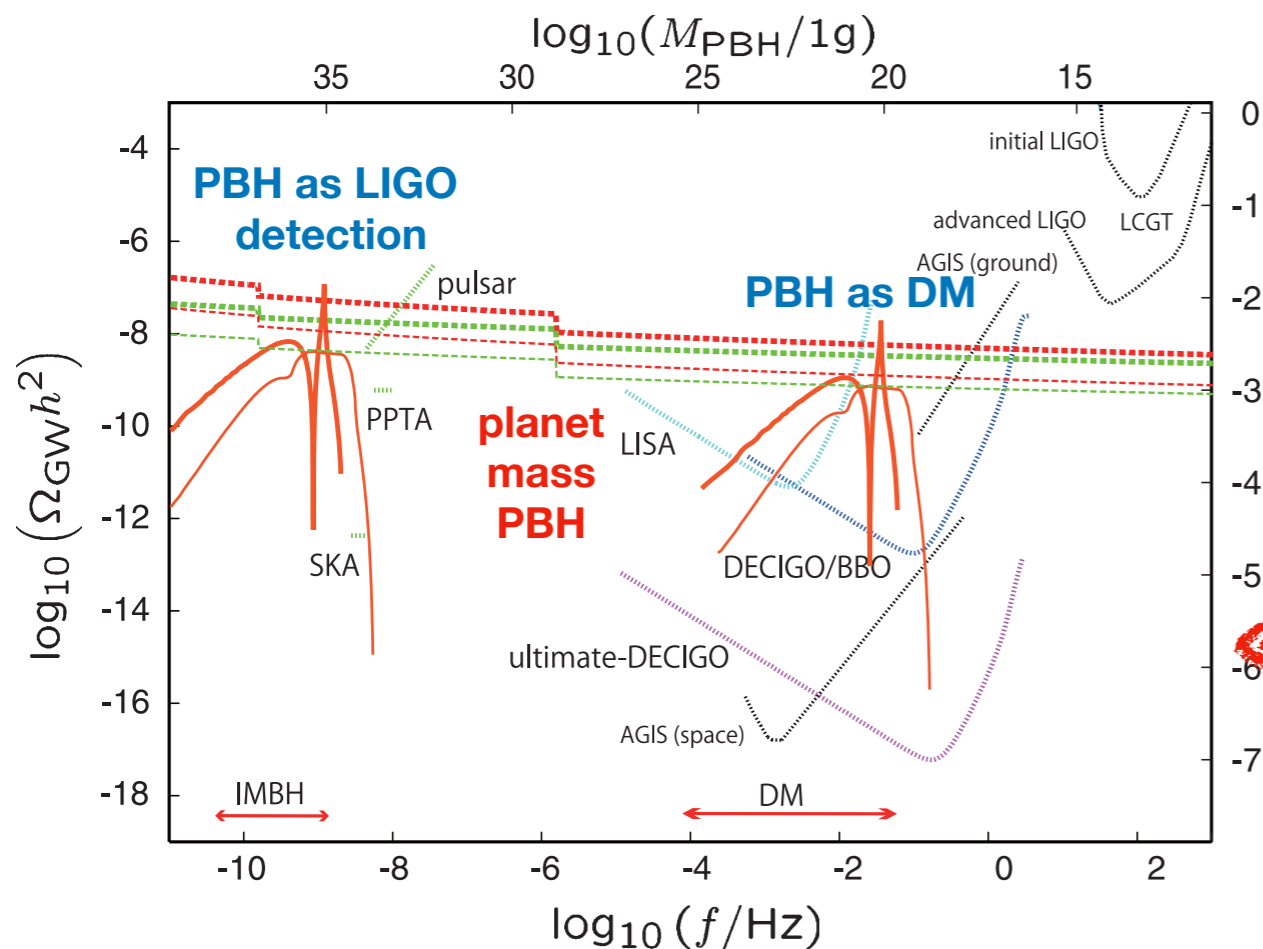


Induced GWs

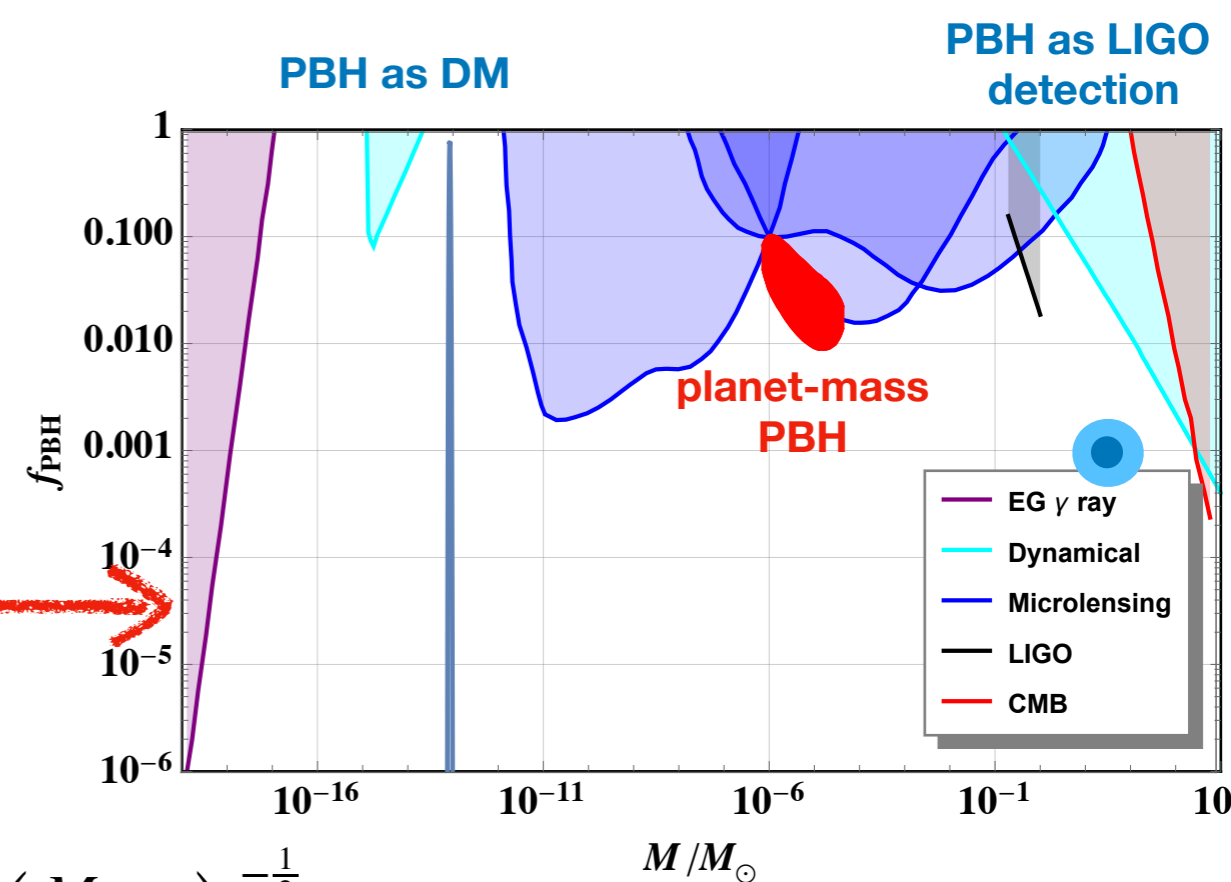
$$\Omega_{\text{GW}} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathcal{P}_{\Phi}^2 \sim \mathcal{P}_{\mathcal{R}}^2$$

PBH abundance

$$f_{\text{PBH}} \sim 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_{\odot}} \right)^{-1/2}, \quad \beta \sim \text{erfc} \left(\frac{\mathcal{R}_c}{2\sqrt{\mathcal{P}_{\mathcal{R}}}} \right)$$



$$f_{\text{IGW}} \sim 3\text{Hz} \left(\frac{M_{\text{PBH}}}{10^{16}\text{g}} \right)^{-1/2}$$



Induced GWs

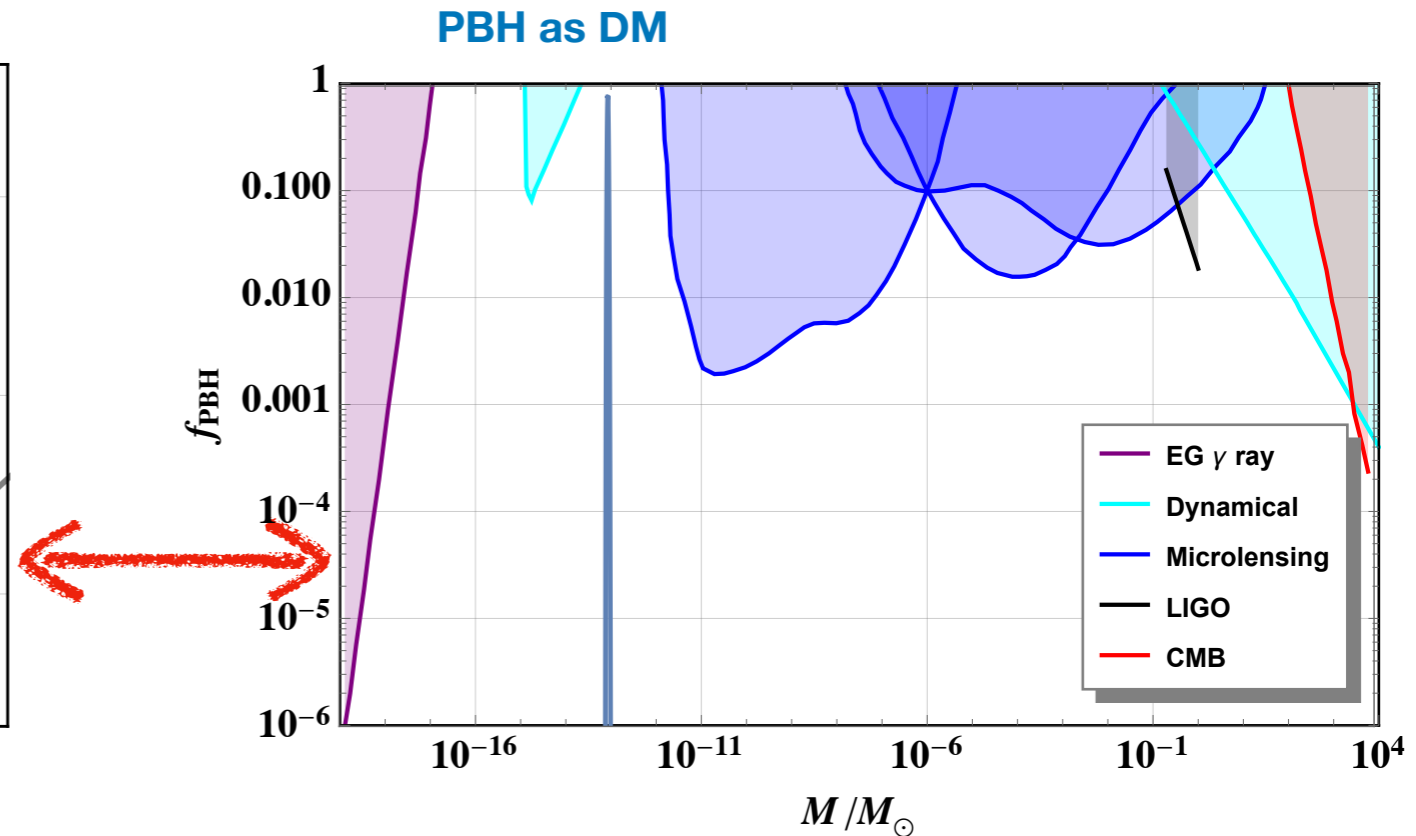
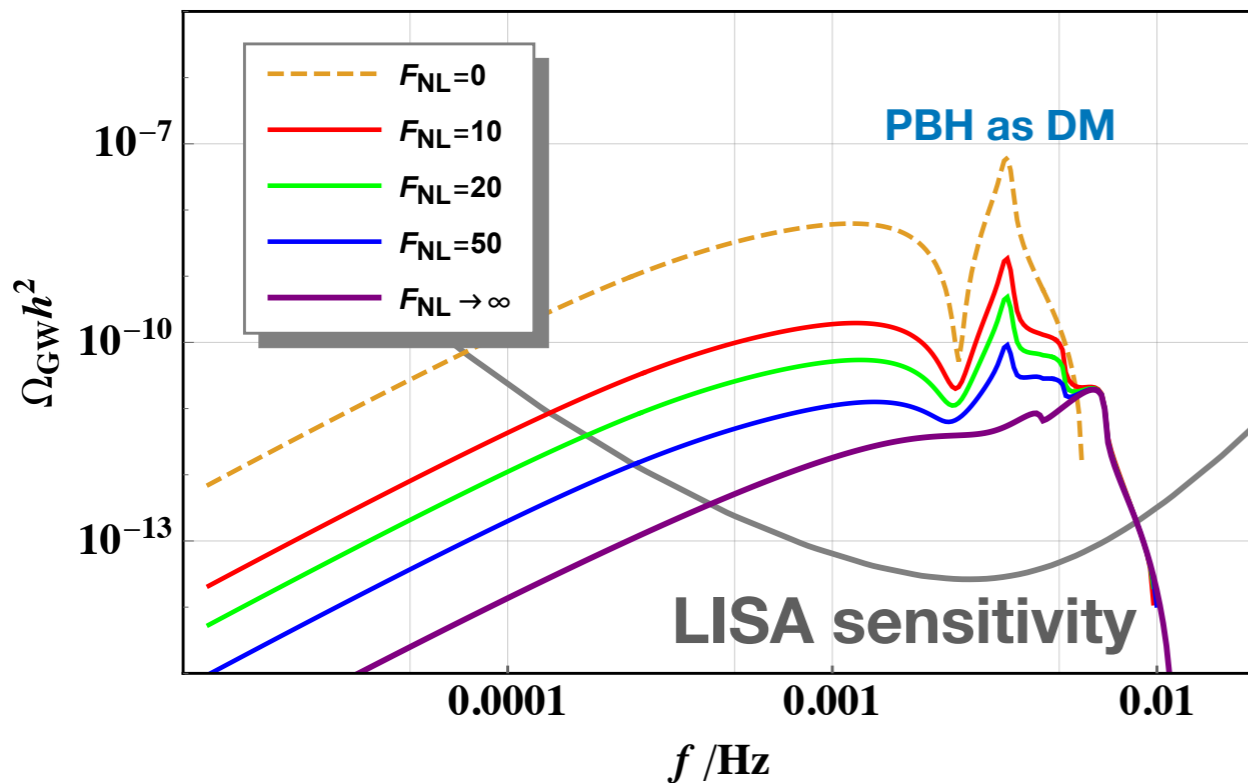
$$\Omega_{\text{GW}} \sim \mathcal{P}_{\mathcal{R},G}^2 + F_{\text{NL}}^2 \mathcal{P}_{\mathcal{R},G}^3 + F_{\text{NL}}^4 \mathcal{P}_{\mathcal{R},G}^4$$

PBH abundance

$$\mathcal{R}_{g\pm}(\mathcal{R}_c) = \frac{1}{2F_{\text{NL}}} \left(-1 \pm \sqrt{1 + 4F_{\text{NL}} (F_{\text{NL}} \mathcal{P}_{\mathcal{R},G} + \mathcal{R}_c)} \right).$$

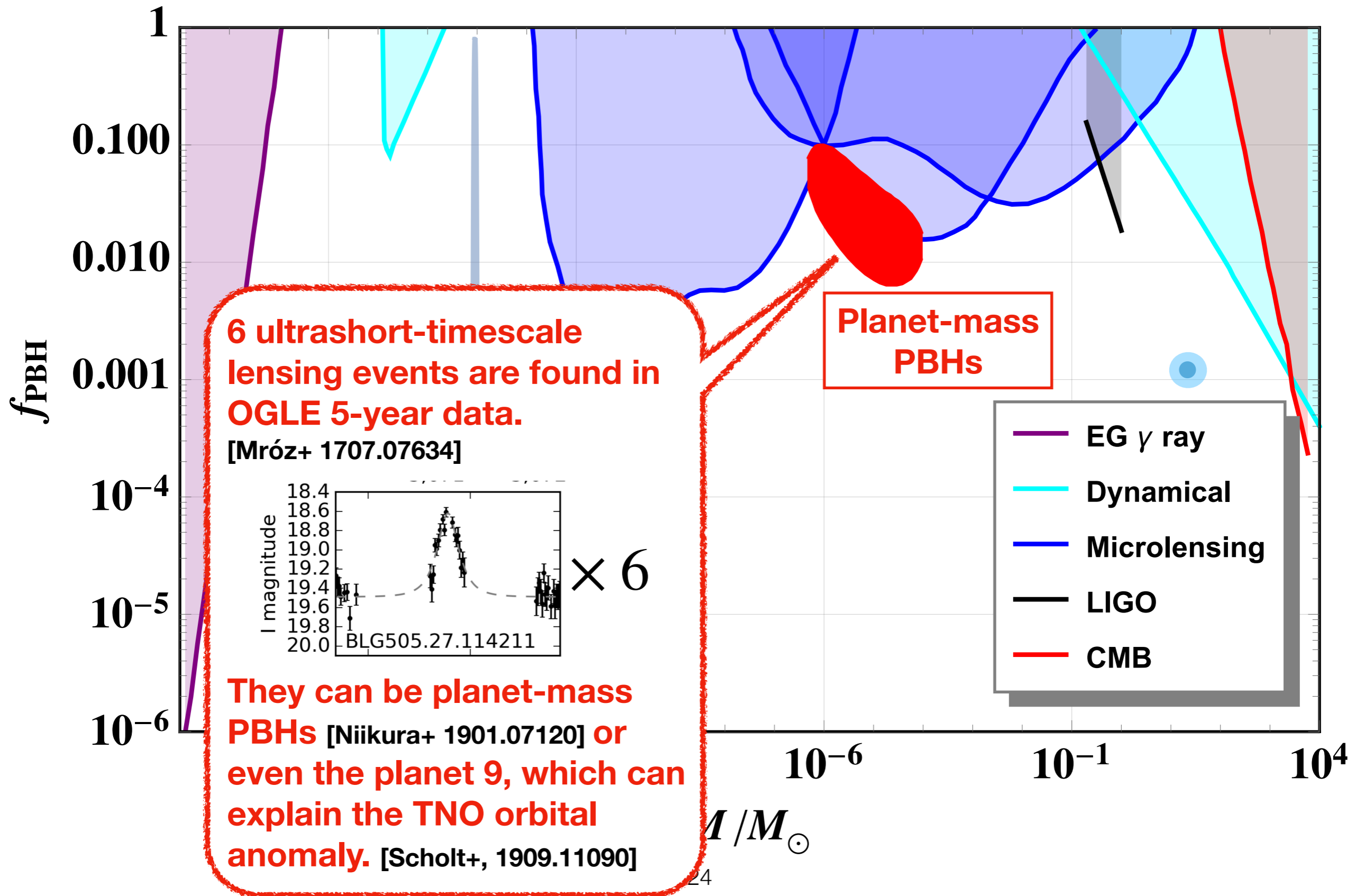
$$\beta = \frac{1}{2} \text{erfc} \left(\frac{\mathcal{R}_{g+}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}} \right) + \frac{1}{2} \text{erfc} \left(-\frac{\mathcal{R}_{g-}(\mathcal{R}_c)}{\sqrt{2\mathcal{P}_{\mathcal{R},G}}} \right); \quad f_{\text{NL}} > 0.$$

Cai, SP, and Sasaki, 1810.11000
 Ünal, 1811.09151
 Ünal+, 2008.11184

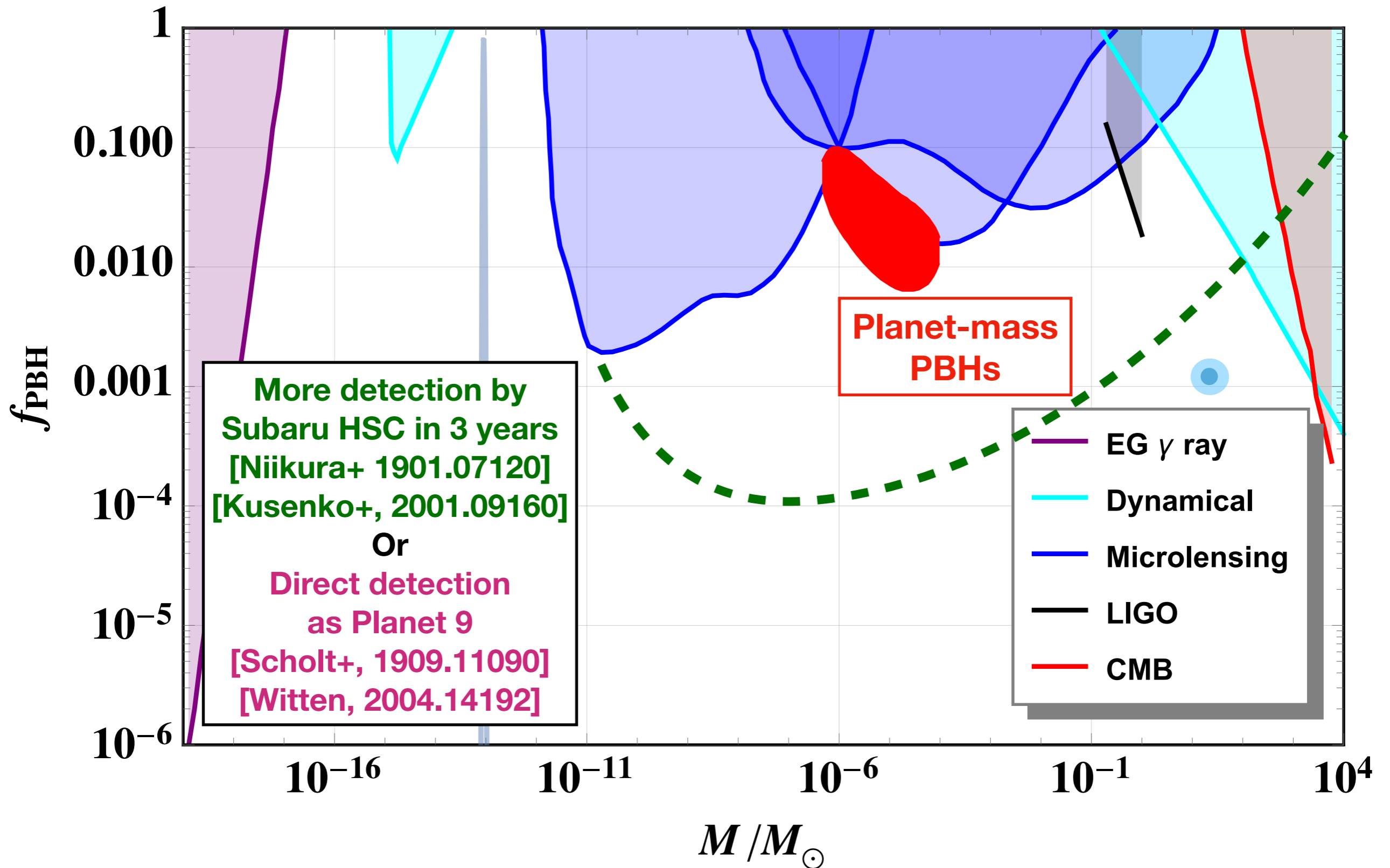


Ünal's talk

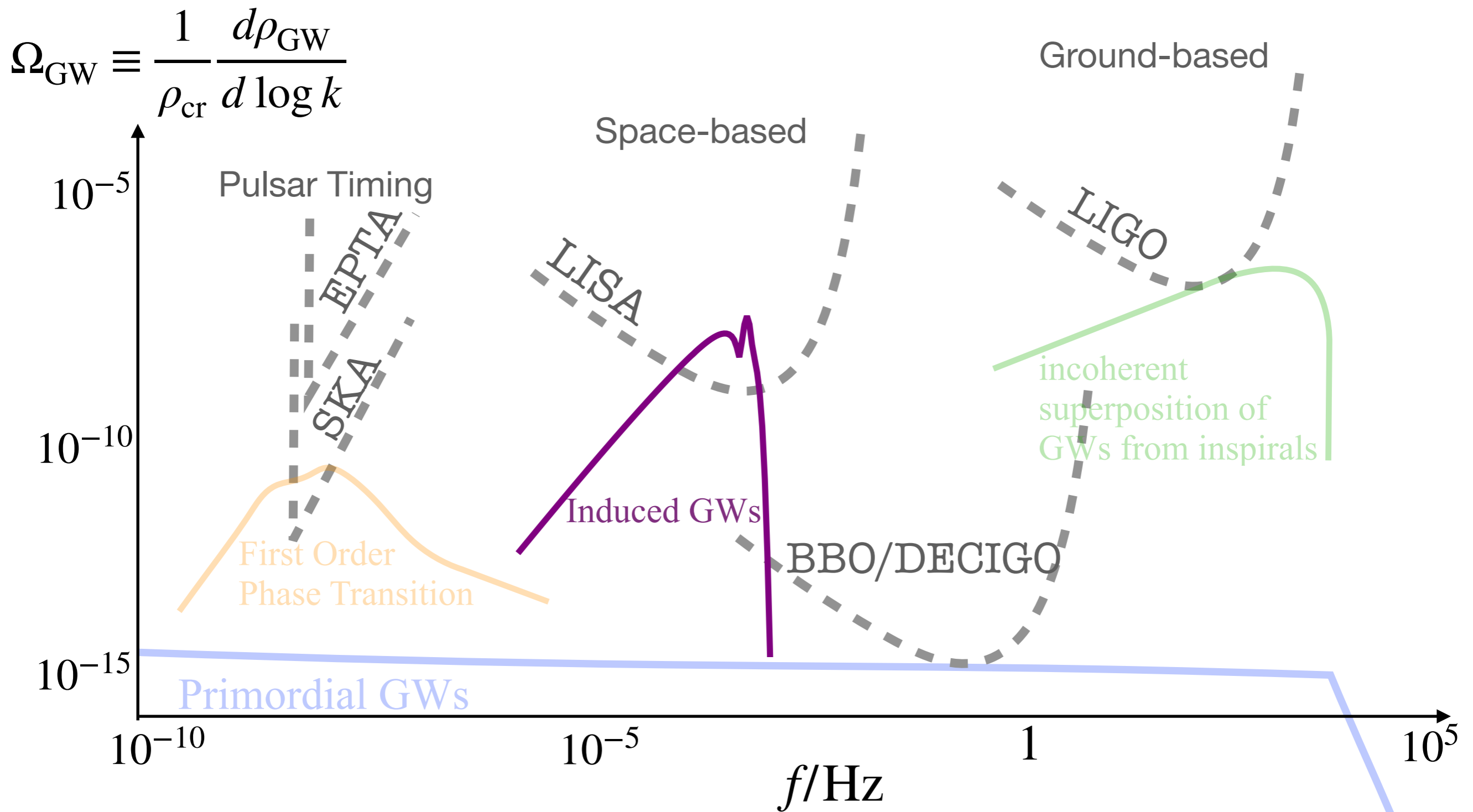
Constraints on f_{PBH}



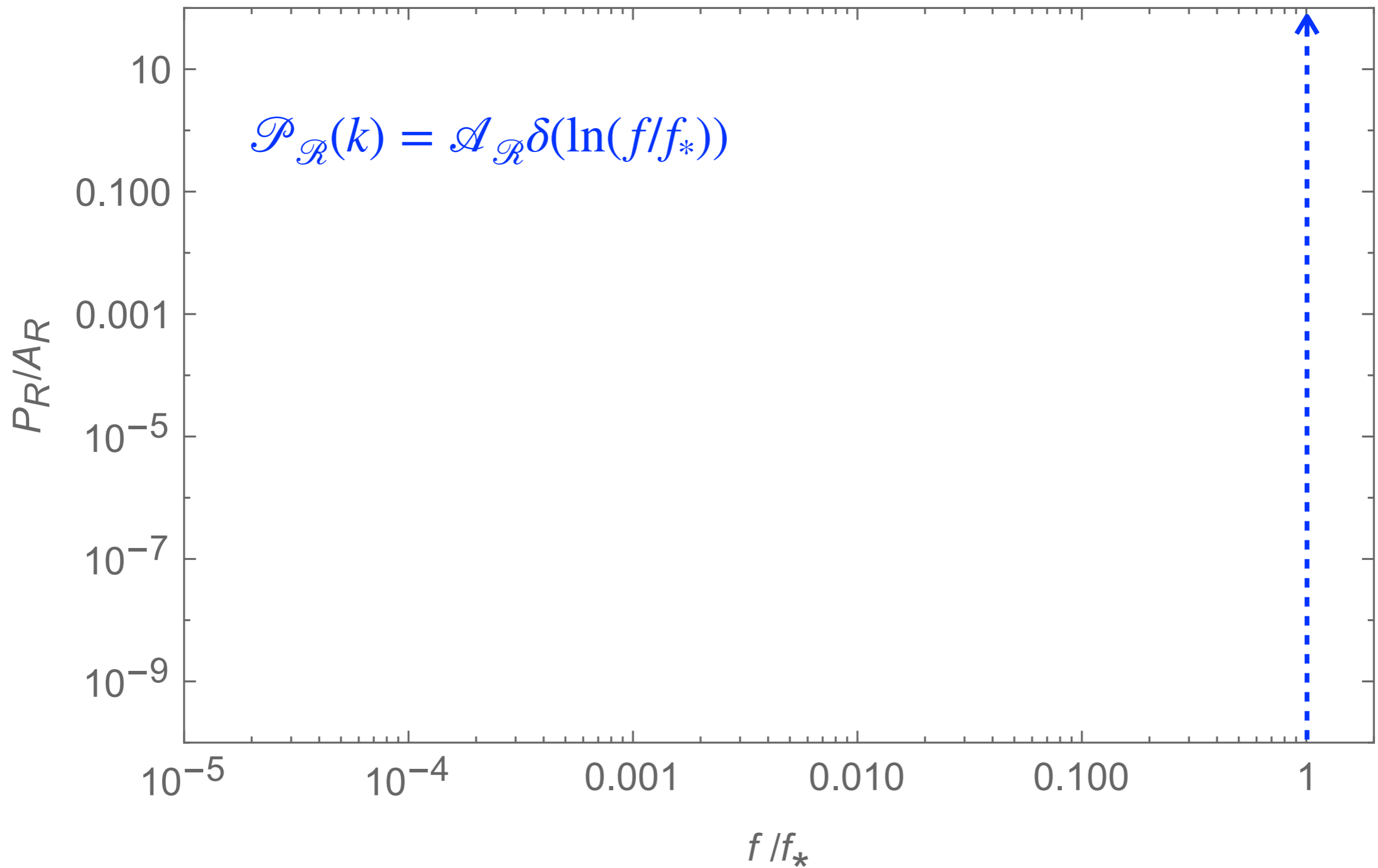
Constraints on f_{PBH}



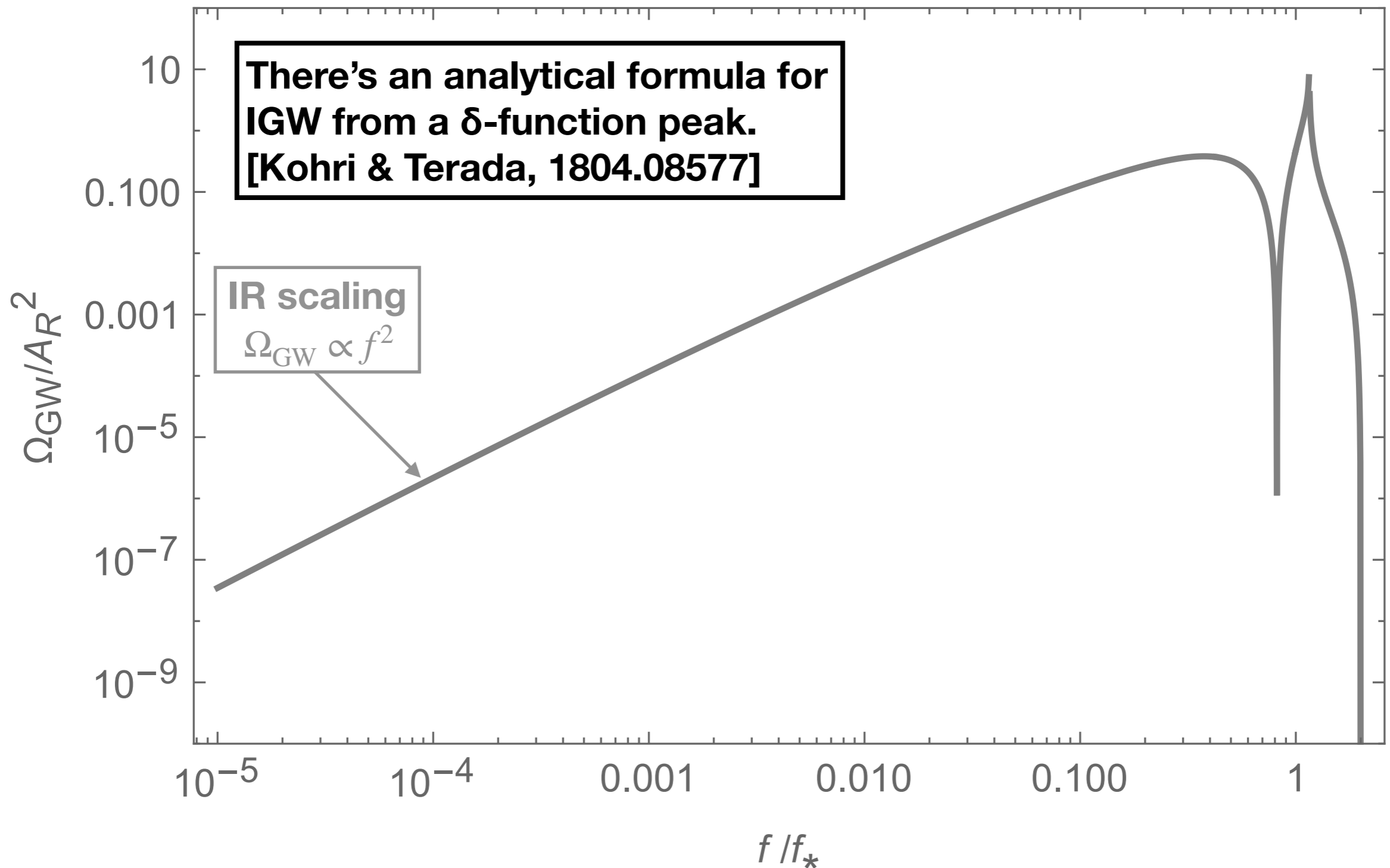
Shape of Ω_{IGW}



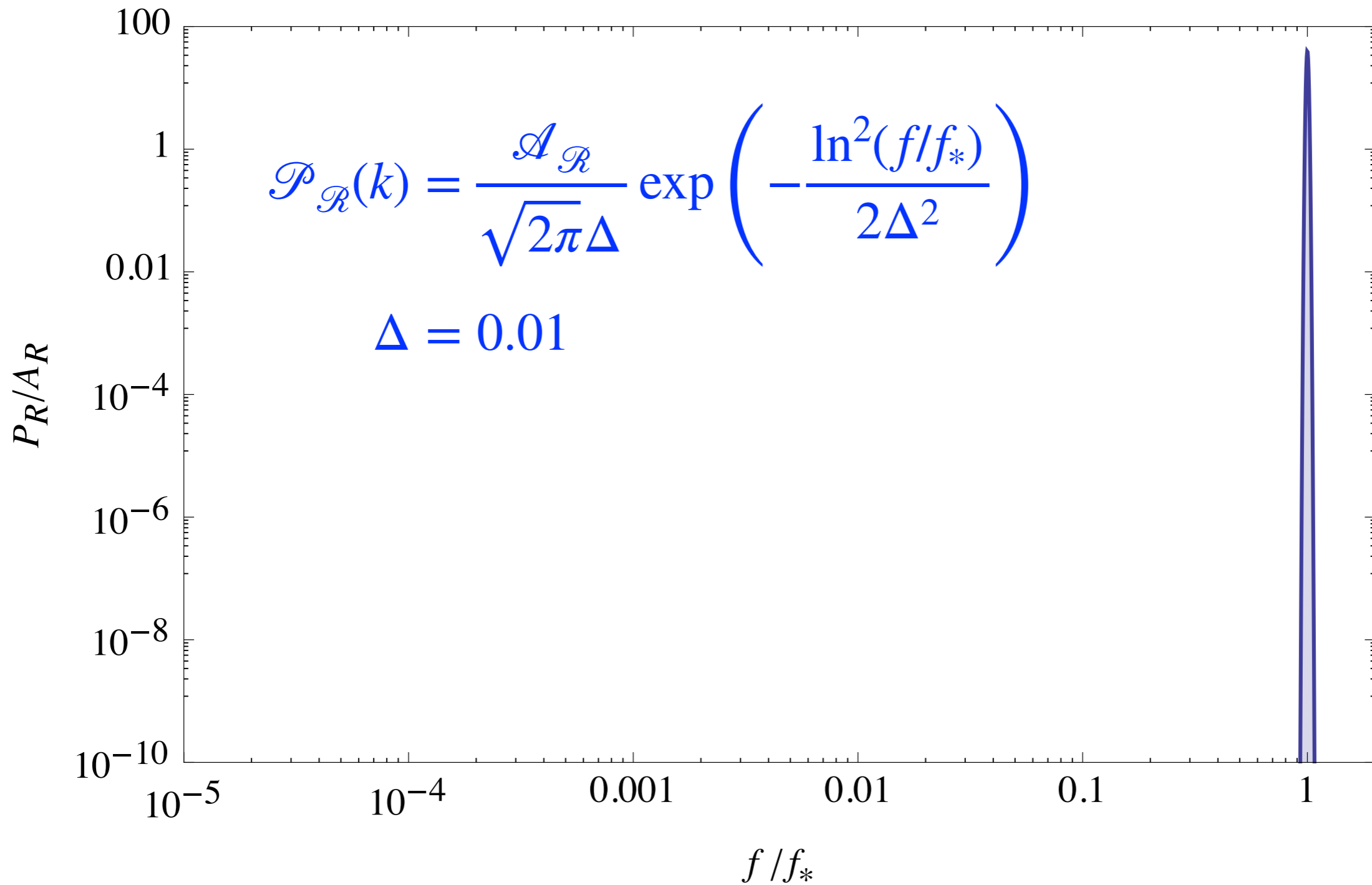
Shape of Ω_{IGW}



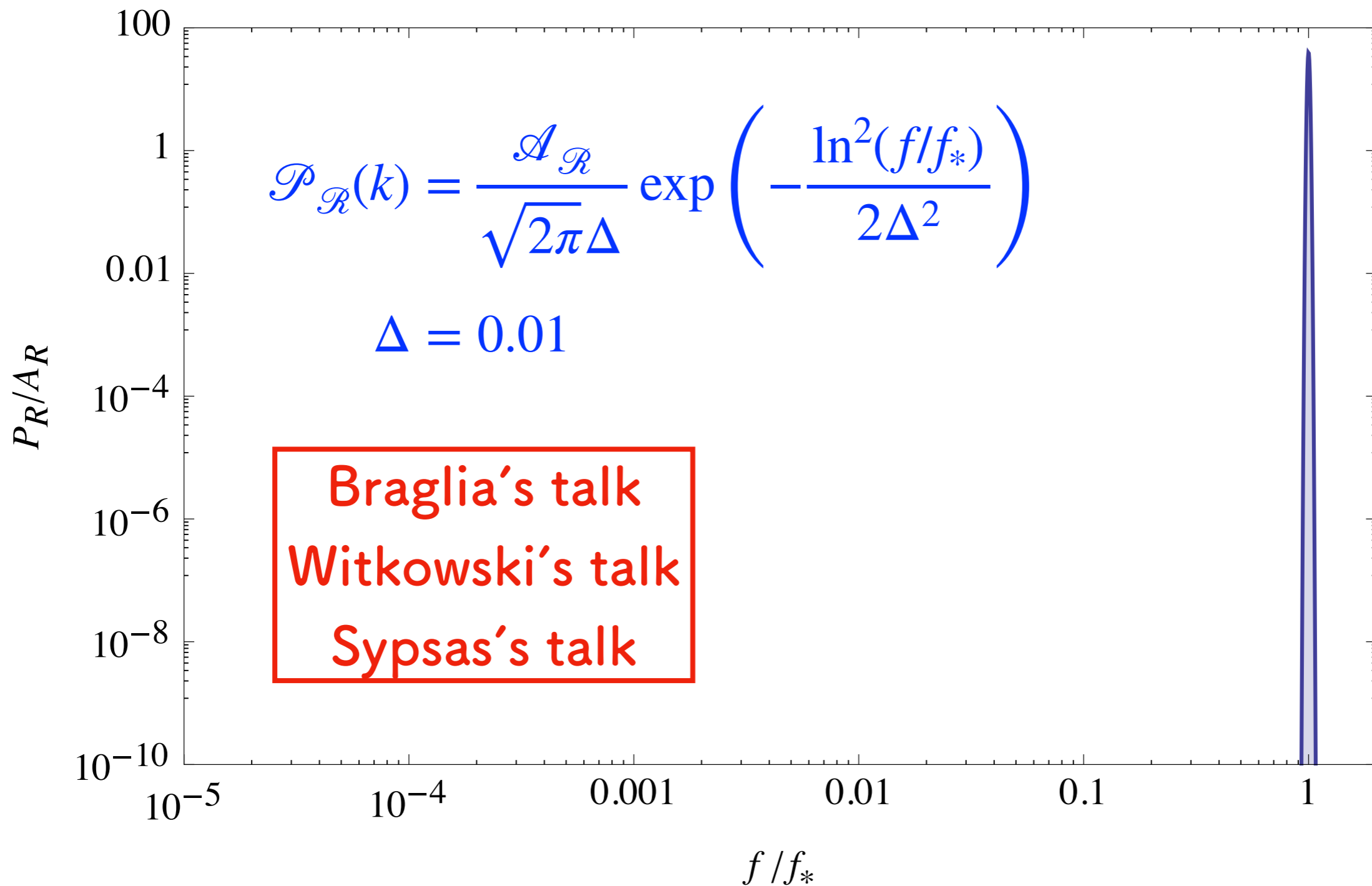
Shape of Ω_{IGW}



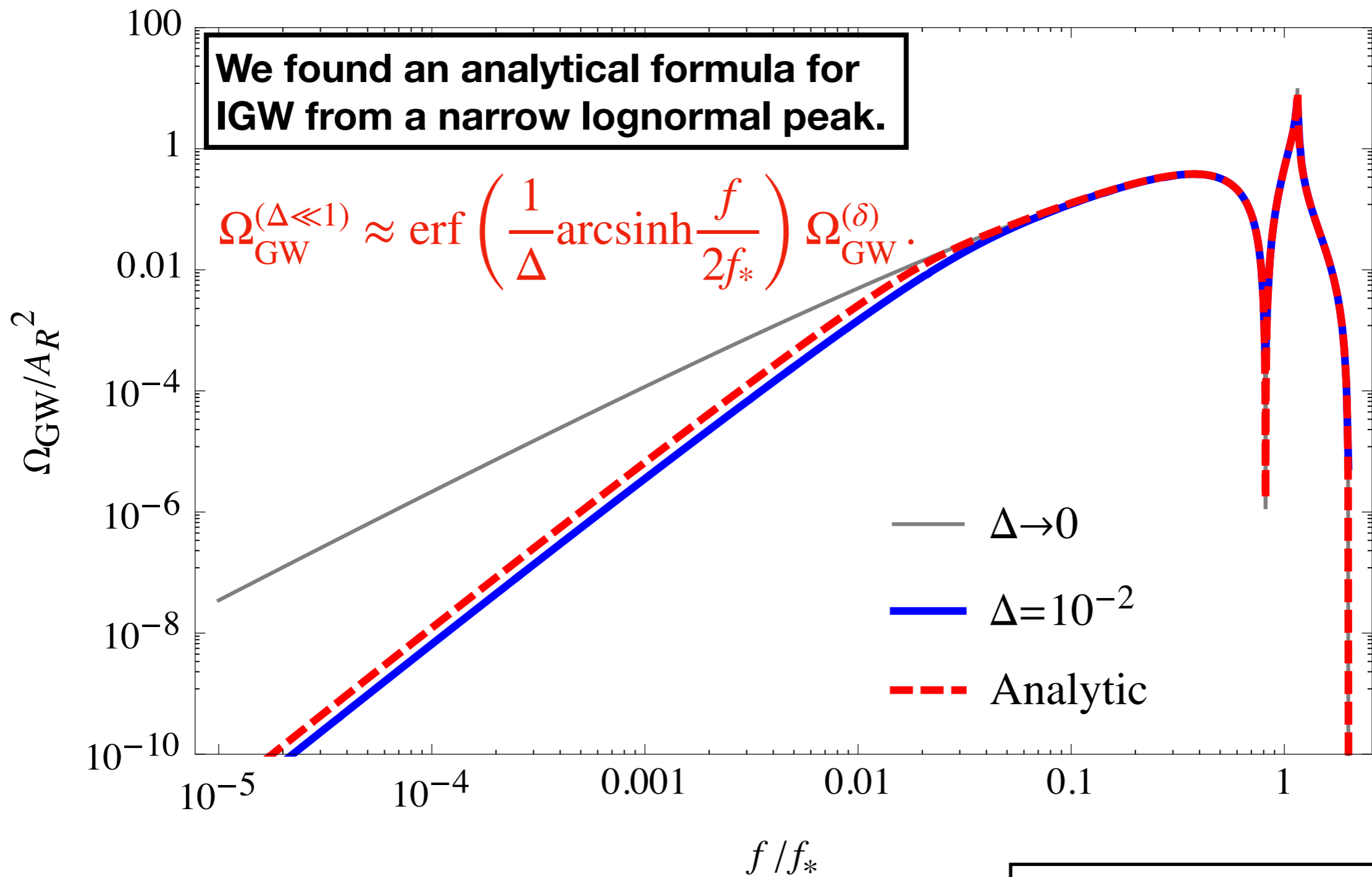
Shape of Ω_{IGW}



Shape of Ω_{IGW}

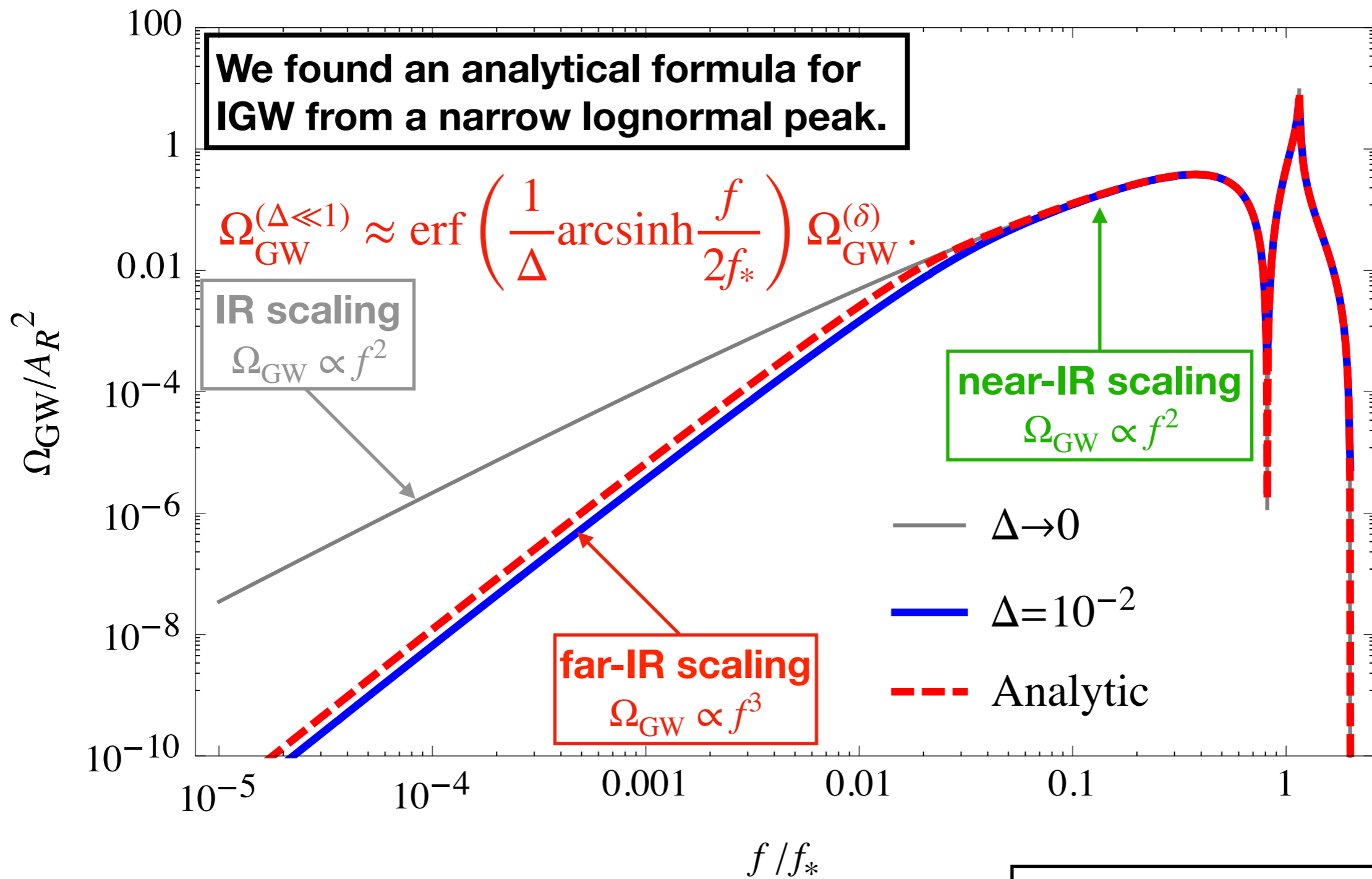


Shape of Ω_{IGW}



SP and Sasaki, 2005.12306

Shape of Ω_{IGW}

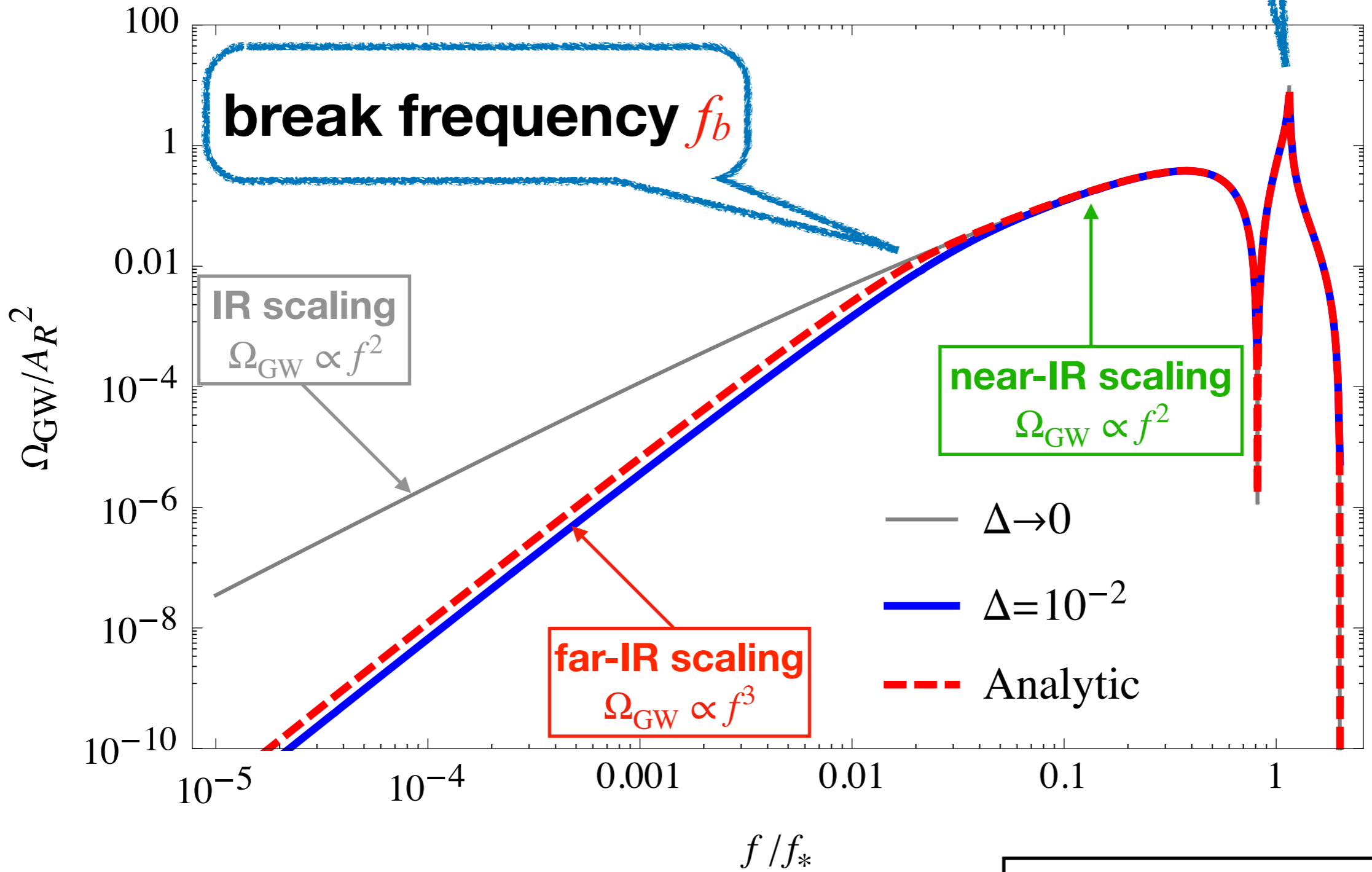


SP and Sasaki, 2005.12306

$$\Delta \approx \frac{f_b}{\sqrt{3}f_p}$$

peak frequency f_p

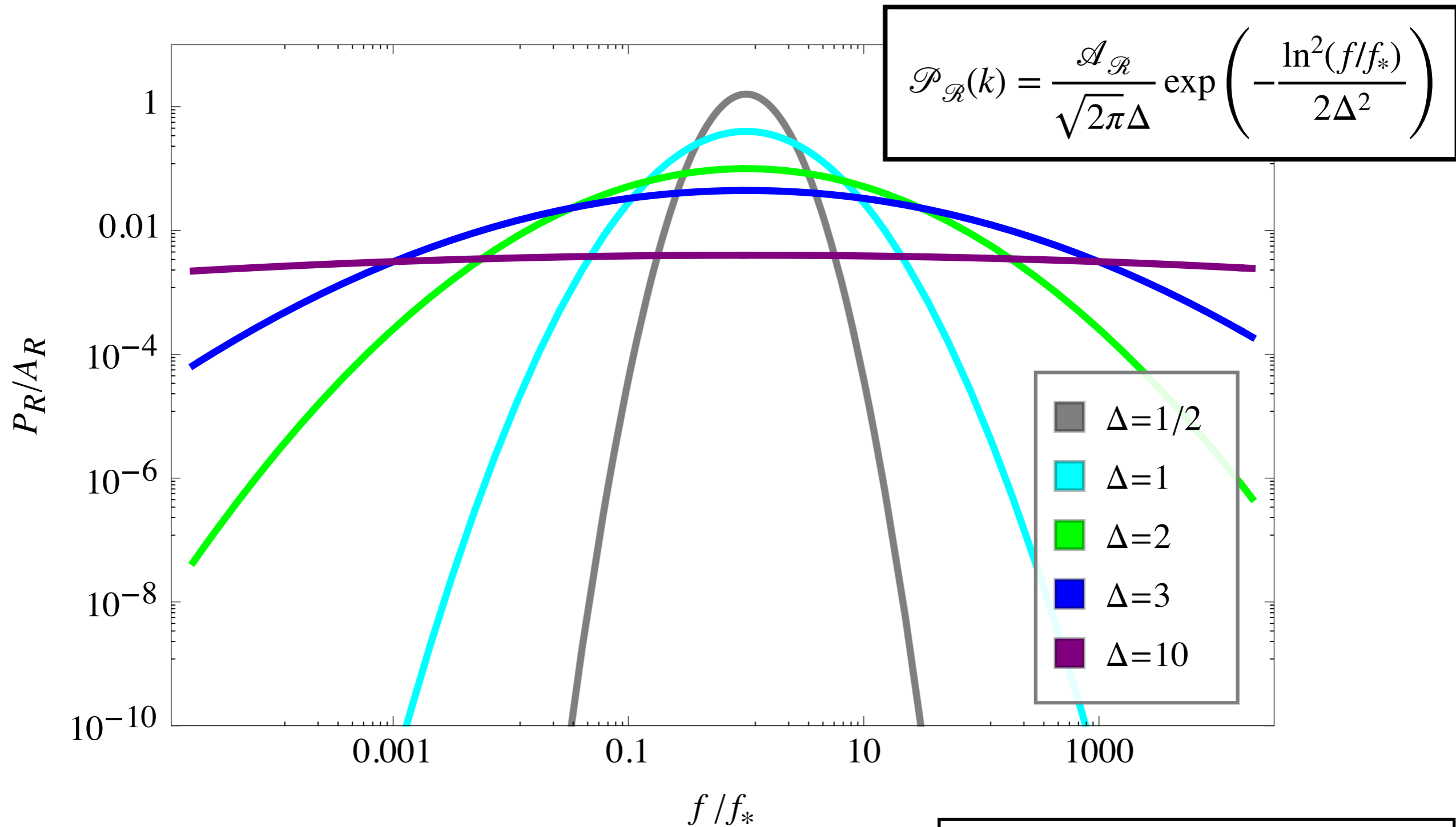
break frequency f_b



- $\Delta \rightarrow 0$
- $\Delta = 10^{-2}$
- - - Analytic

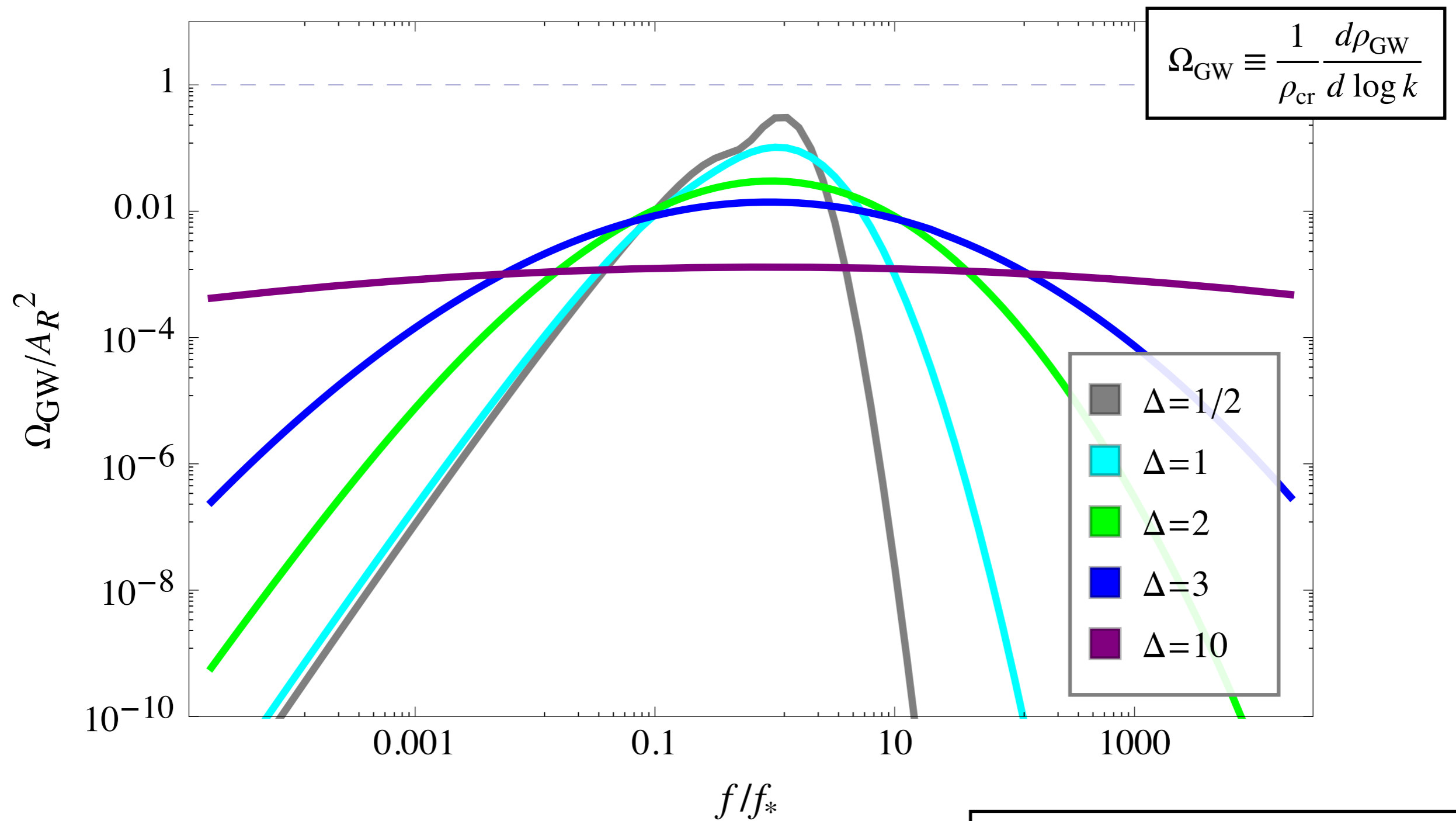
SP and Sasaki, 2005.12306

Shape of Ω_{IGW}



SP and Sasaki, 2005.12306

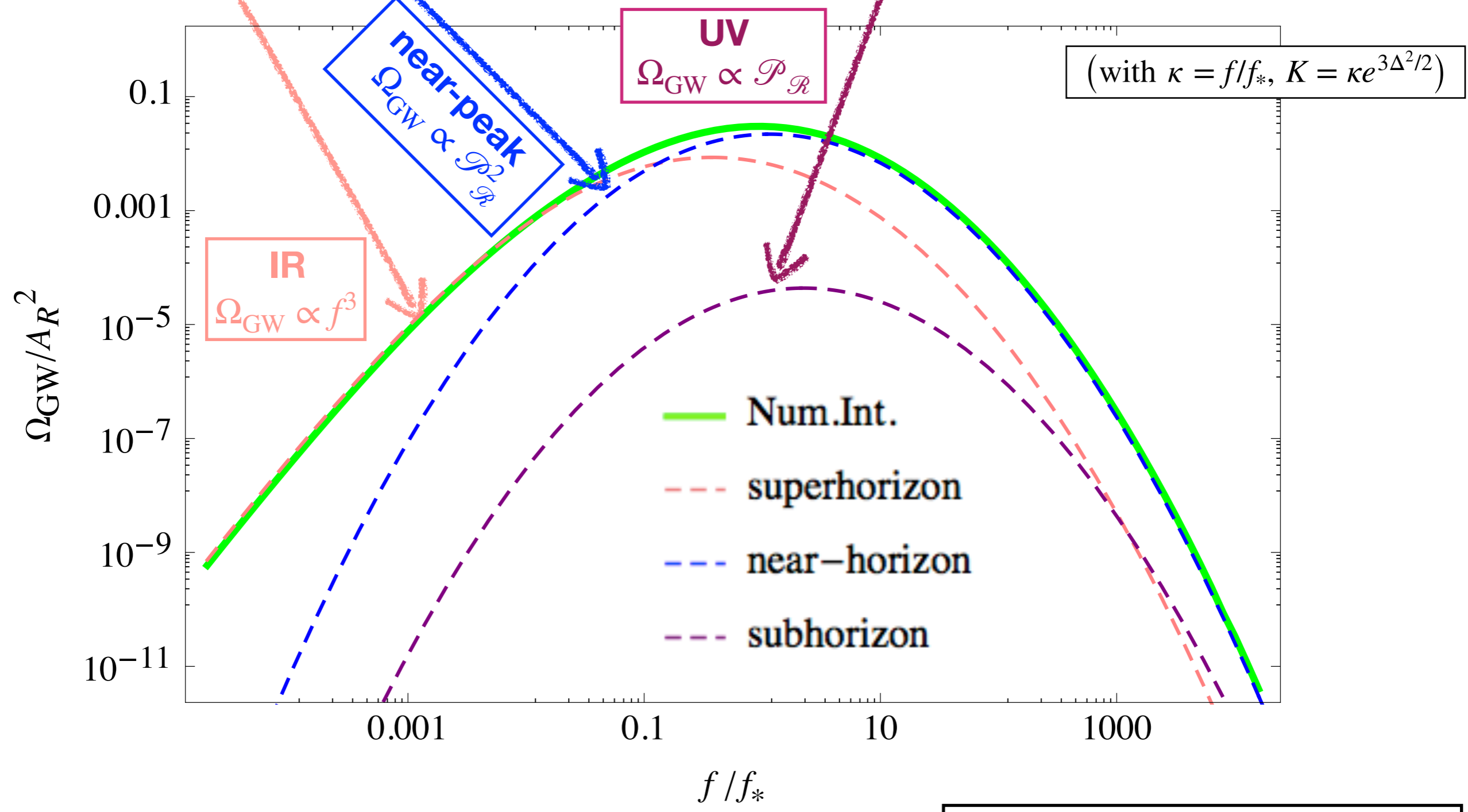
Shape of Ω_{IGW}



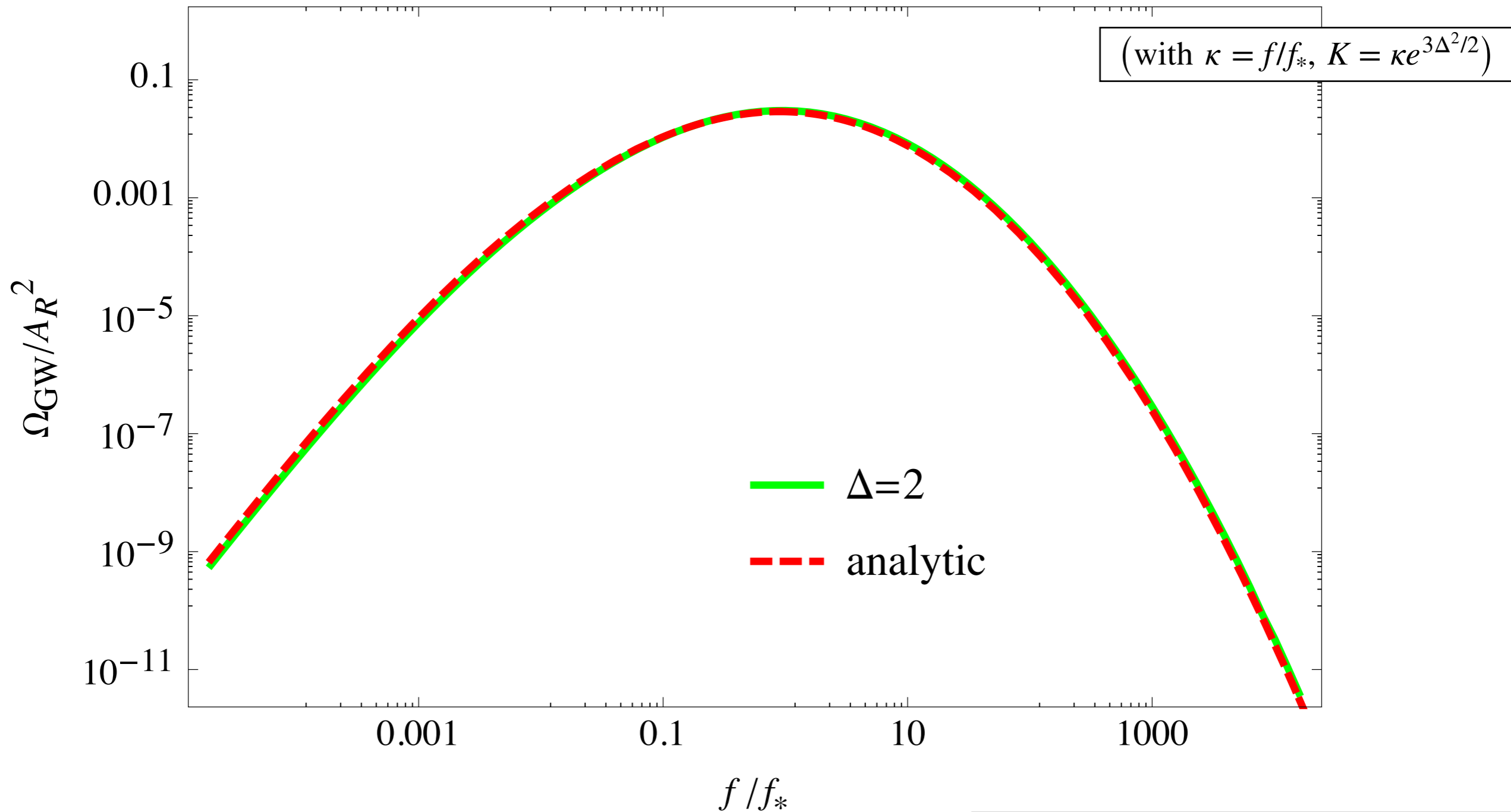
SP and Sasaki, 2005.12306

$$\frac{\Omega_{\text{GW}}}{\mathcal{A}_R^2} \approx \frac{4}{5\sqrt{\pi}} \kappa^3 \frac{e^{\frac{9\Delta^2}{4}}}{\Delta} \left[\left(\ln^2 K + \frac{\Delta^2}{2} \right) \text{erfc} \left(\frac{\ln K + \frac{1}{2} \ln \frac{3}{2}}{\Delta} \right) - \frac{\Delta}{\sqrt{\pi}} \exp \left(-\frac{\left(\ln K + \frac{1}{2} \ln \frac{3}{2} \right)^2}{\Delta^2} \right) \left(\ln K - \frac{1}{2} \ln \frac{3}{2} \right) \right]$$

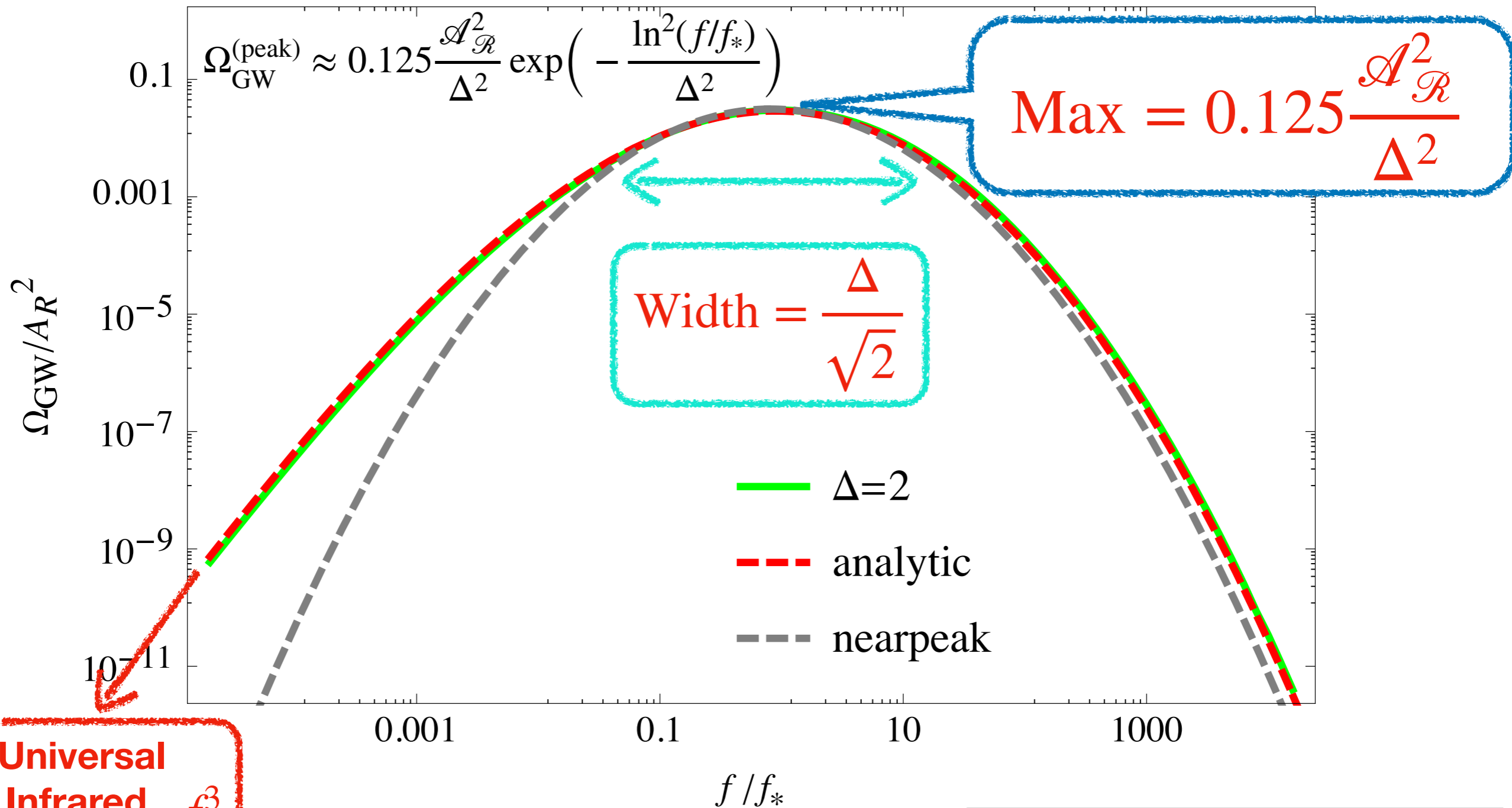
$$+ \frac{0.0659}{\Delta^2} \kappa^2 e^{\Delta^2} \exp \left(-\frac{\left(\ln \kappa + \Delta^2 - \frac{1}{2} \ln \frac{4}{3} \right)^2}{\Delta^2} \right) + \frac{1}{3} \sqrt{\frac{2}{\pi}} \kappa^{-4} \frac{e^{8\Delta^2}}{\Delta} \exp \left(-\frac{\ln^2 \kappa}{2\Delta^2} \right) \text{erfc} \left(\frac{4\Delta^2 - \ln(\kappa/4)}{\sqrt{2}\Delta} \right)$$



$$\frac{\Omega_{\text{GW}}}{\mathcal{A}_R^2} \approx \frac{4}{5\sqrt{\pi}} \kappa^3 \frac{e^{\frac{9\Delta^2}{4}}}{\Delta} \left[\left(\ln^2 K + \frac{\Delta^2}{2} \right) \text{erfc} \left(\frac{\ln K + \frac{1}{2} \ln \frac{3}{2}}{\Delta} \right) - \frac{\Delta}{\sqrt{\pi}} \exp \left(-\frac{\left(\ln K + \frac{1}{2} \ln \frac{3}{2} \right)^2}{\Delta^2} \right) \left(\ln K - \frac{1}{2} \ln \frac{3}{2} \right) \right] \\ + \frac{0.0659}{\Delta^2} \kappa^2 e^{\Delta^2} \exp \left(-\frac{\left(\ln \kappa + \Delta^2 - \frac{1}{2} \ln \frac{4}{3} \right)^2}{\Delta^2} \right) + \frac{1}{3} \sqrt{\frac{2}{\pi}} \kappa^{-4} \frac{e^{8\Delta^2}}{\Delta} \exp \left(-\frac{\ln^2 \kappa}{2\Delta^2} \right) \text{erfc} \left(\frac{4\Delta^2 - \ln(\kappa/4)}{\sqrt{2}\Delta} \right)$$



Shape of Ω_{IGW}

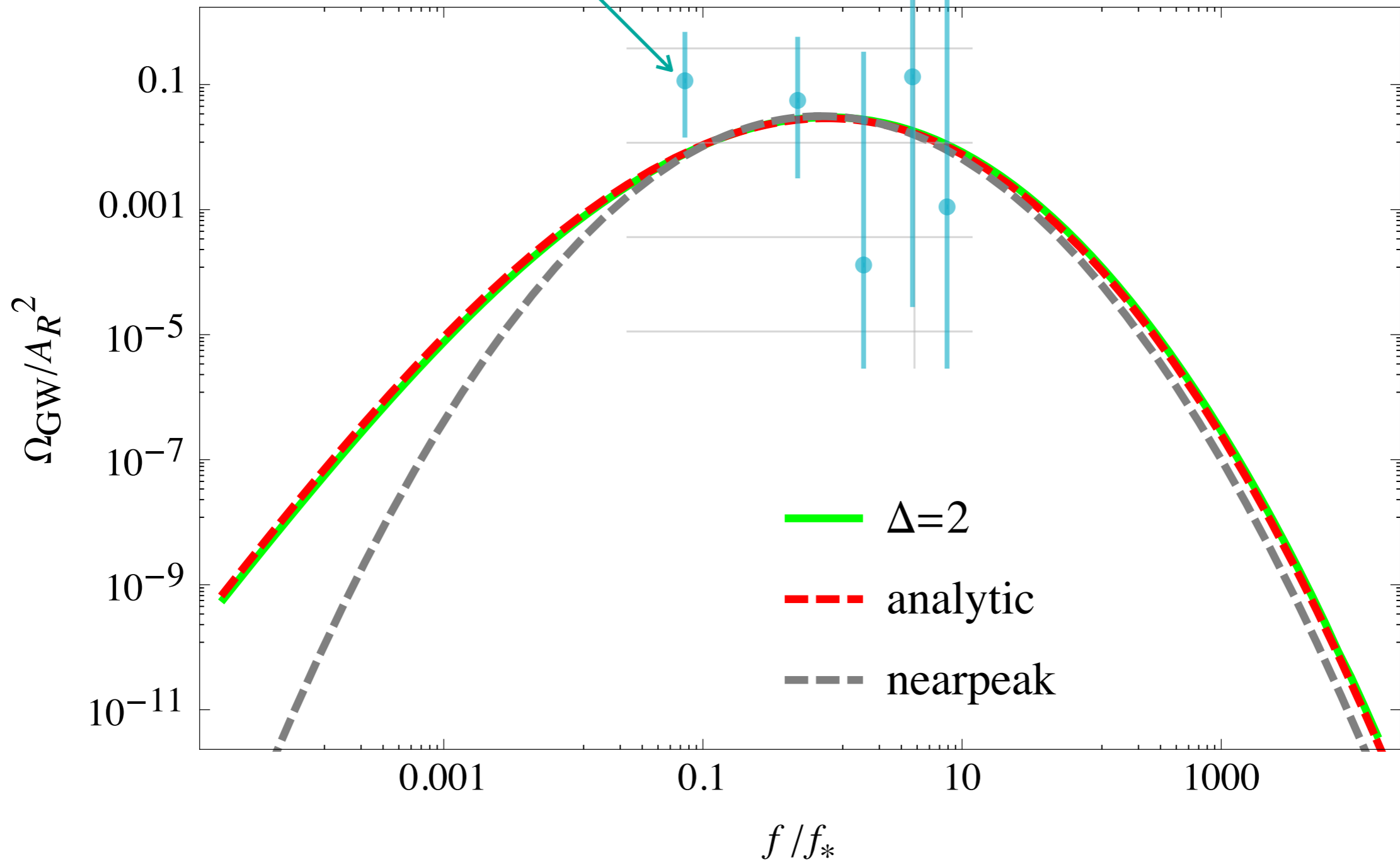


**Universal
 Infrared
 Scaling:** f^3

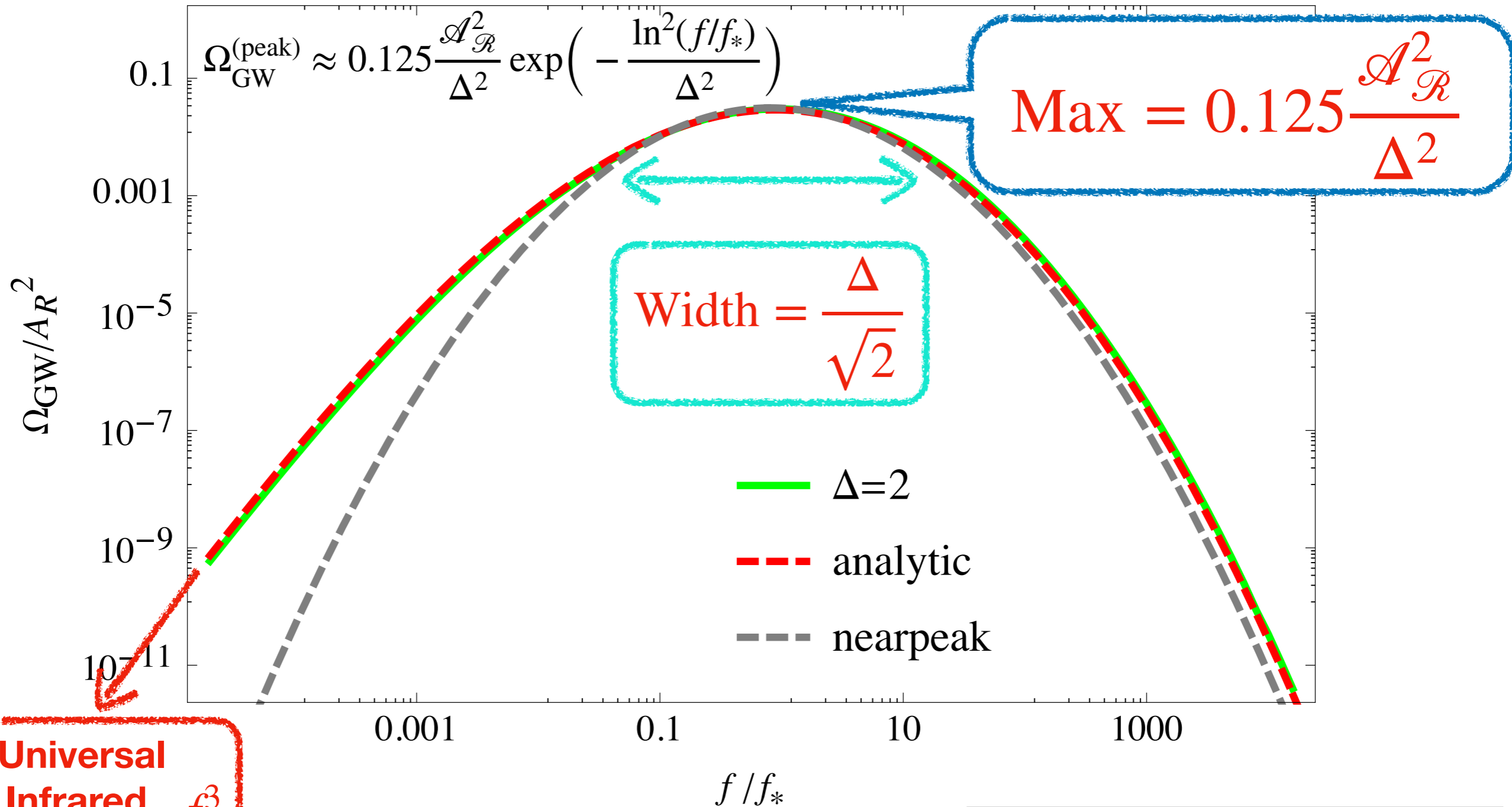
SP and Sasaki, 2005.12306

NANOGrav Results

The flat near-peak spectrum can be used to interpret NANOGrav result.
Kohri+, [2009.11853](#)
Inomata+, [2011.01270](#)



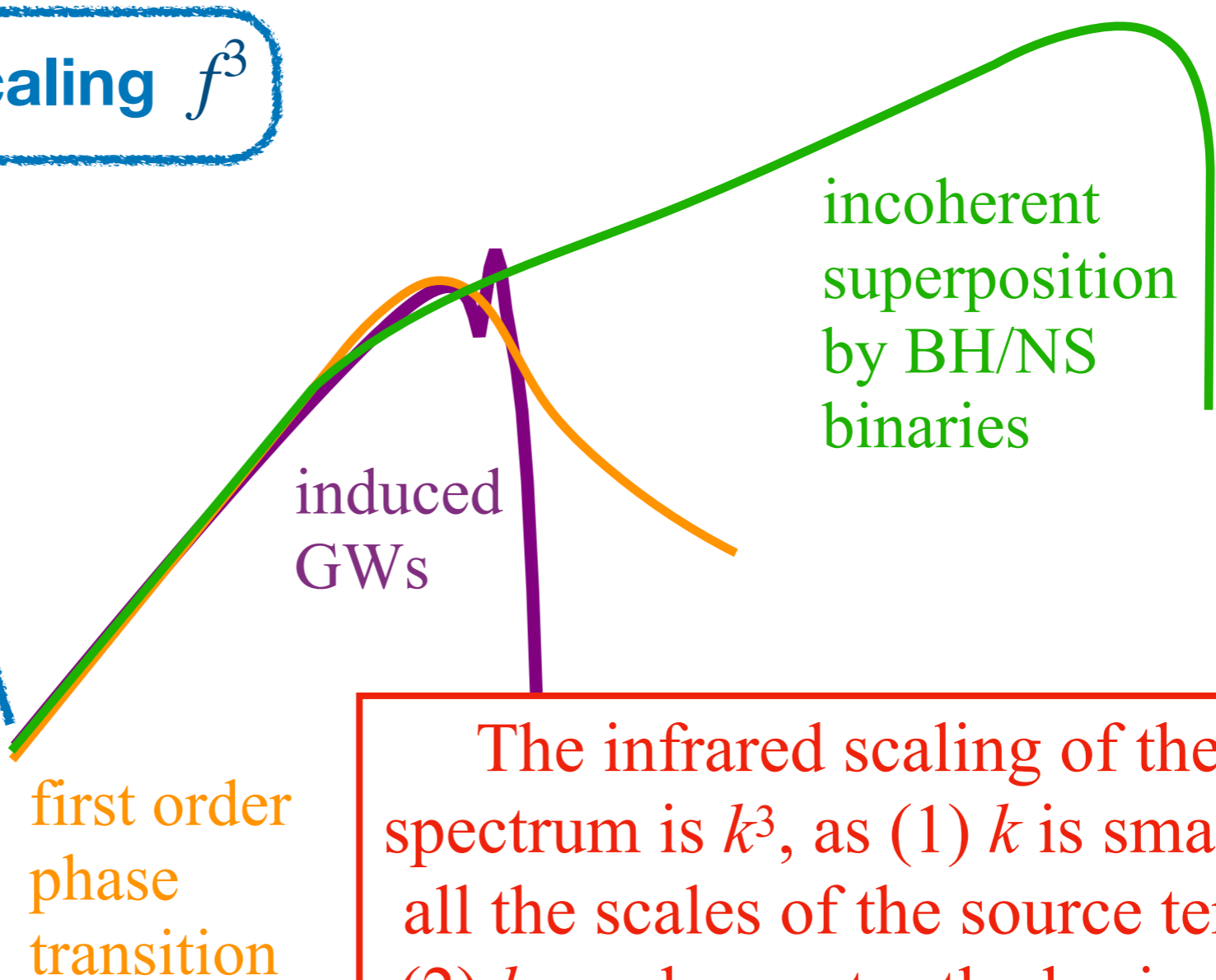
Infrared scaling of Ω_{IGW}



**Universal
 Infrared
 Scaling:** f^3

Infrared scaling of Ω_{IGW}

Universal infrared scaling f^3



first order
phase
transition

induced
GWs

incoherent
superposition
by BH/NS
binaries

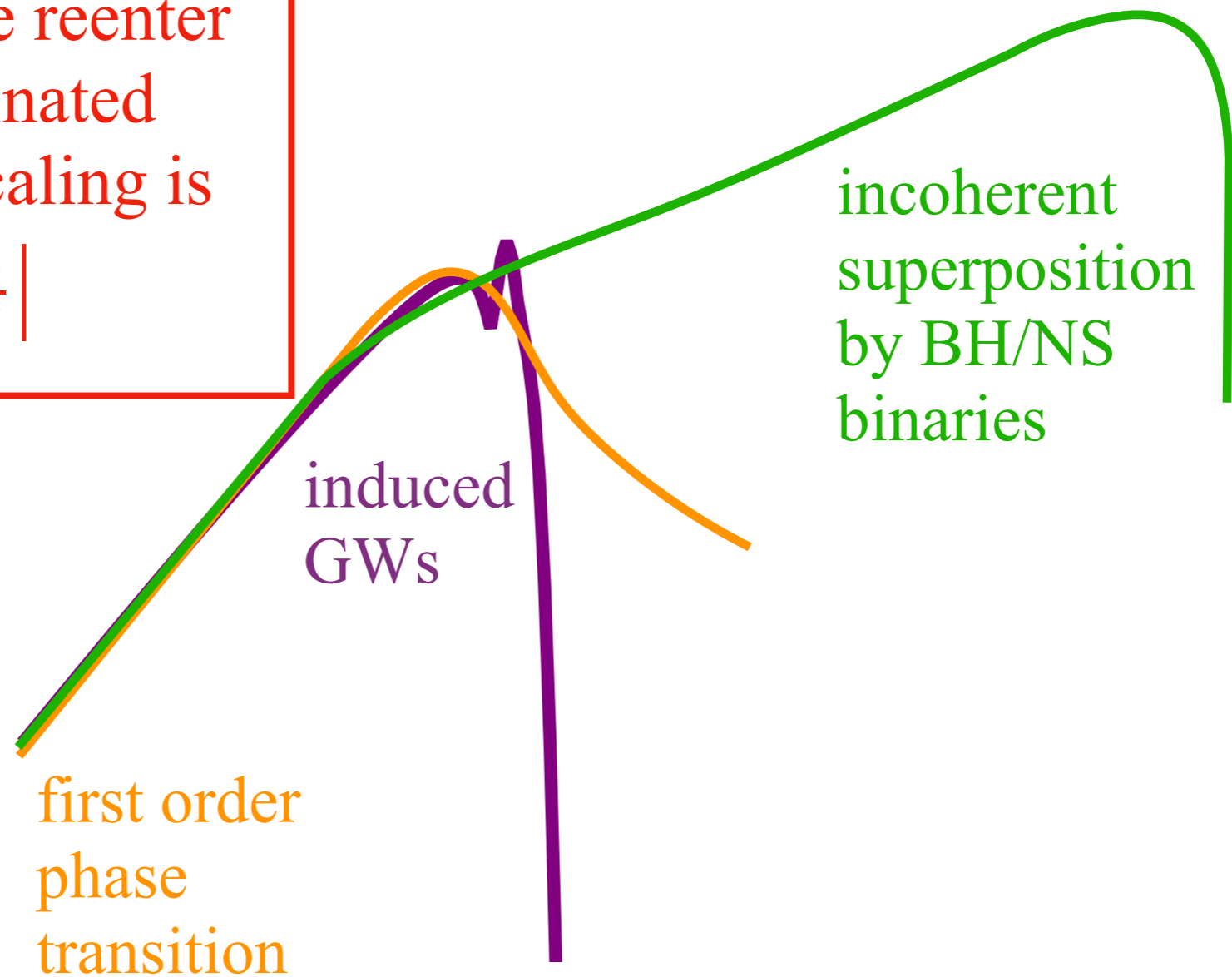
The infrared scaling of the GW spectrum is k^3 , as (1) k is smaller than all the scales of the source term; and (2) k -mode reenter the horizon in the radiation dominated universe.

Cai, SP, and Sasaki, 1909.13728
Domènech, 1912.05583
Domènech, SP, and Sasaki, 2005.12314

Infrared scaling of Ω_{IGW}

When the infrared mode reenters the horizon in a w -dominated universe, the infrared scaling is

$$\Omega_{\text{GW}} \propto f^{3-2 \left| \frac{1-3w}{1+3w} \right|}$$

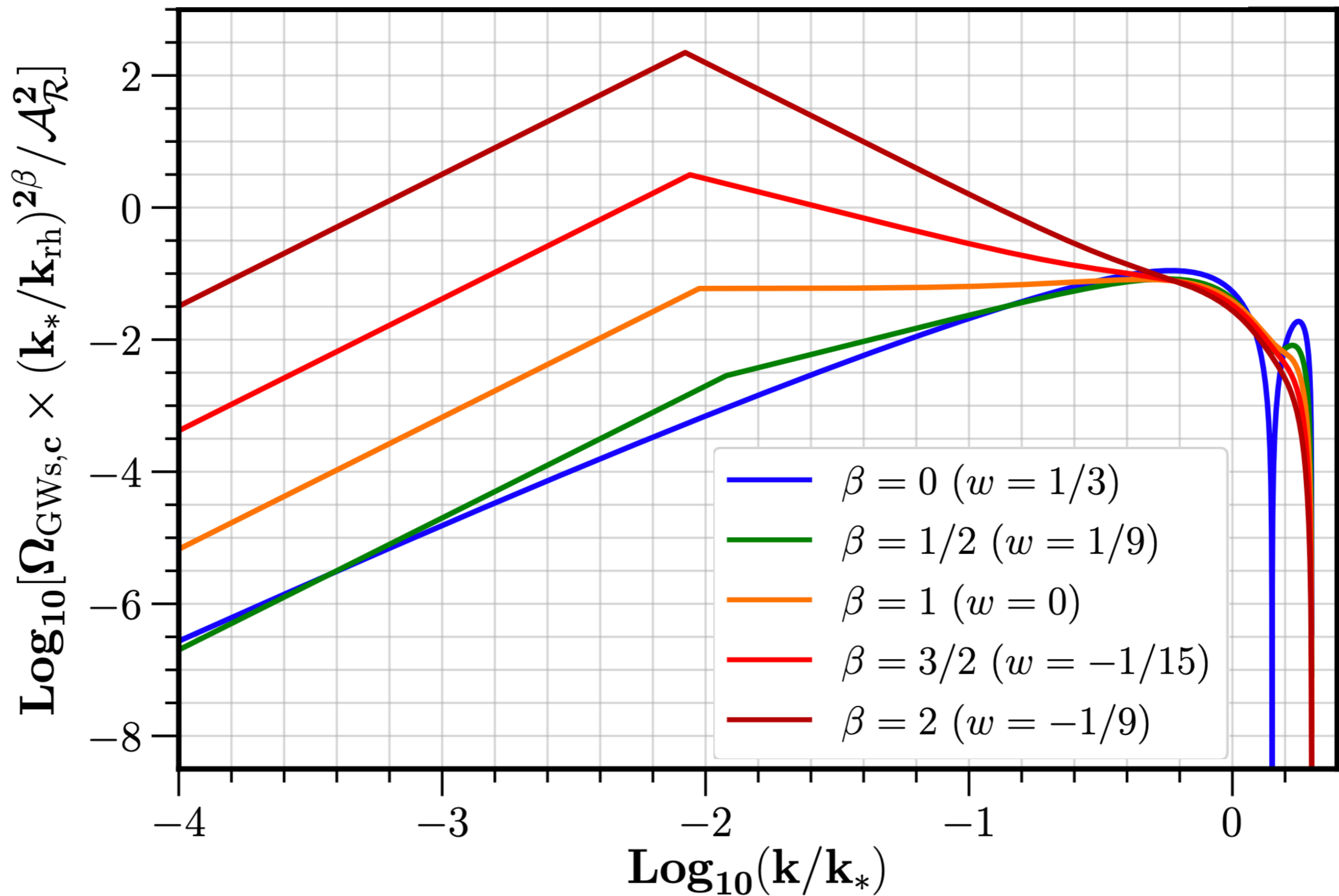


Cai, SP, and Sasaki, 1909.13728

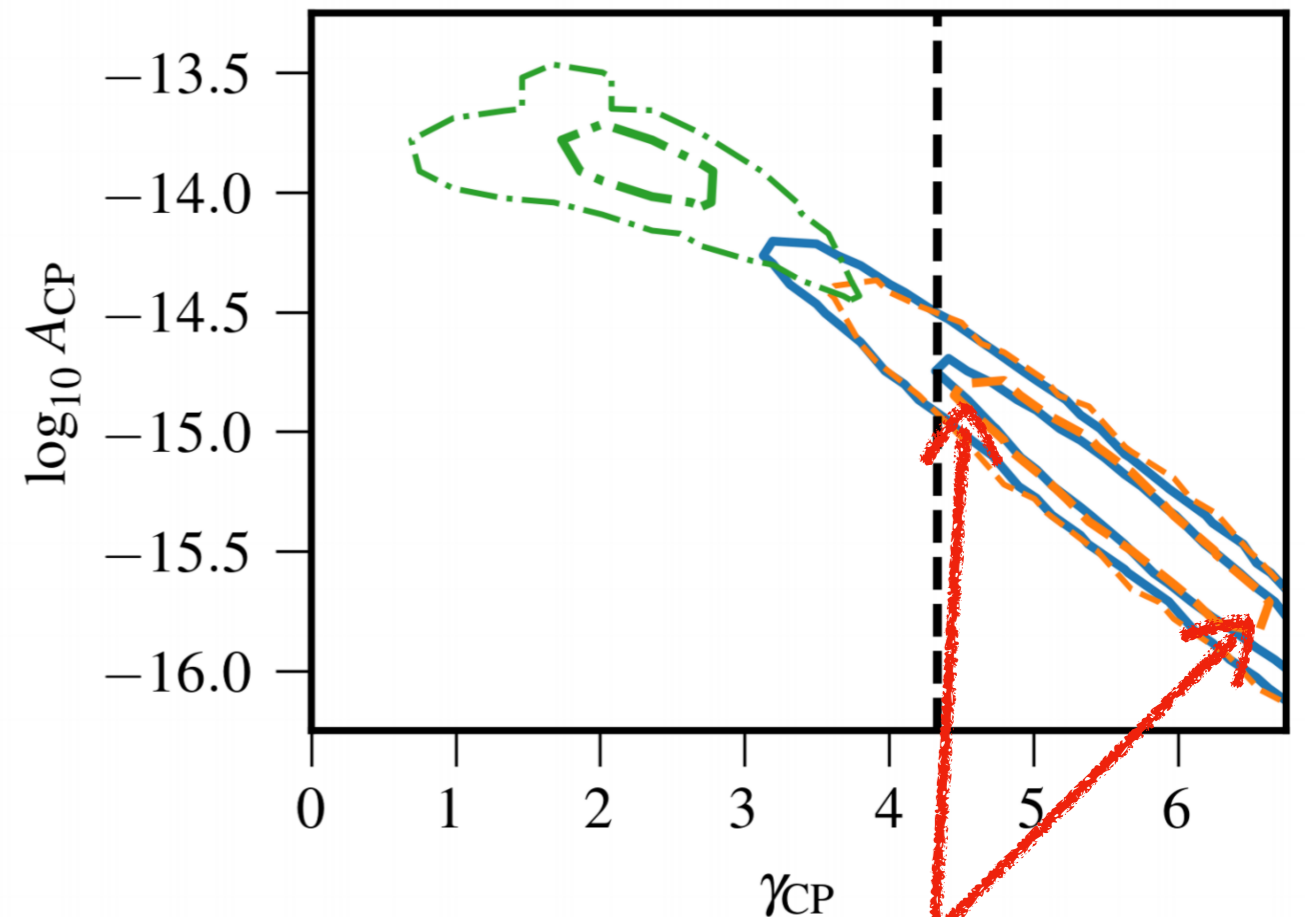
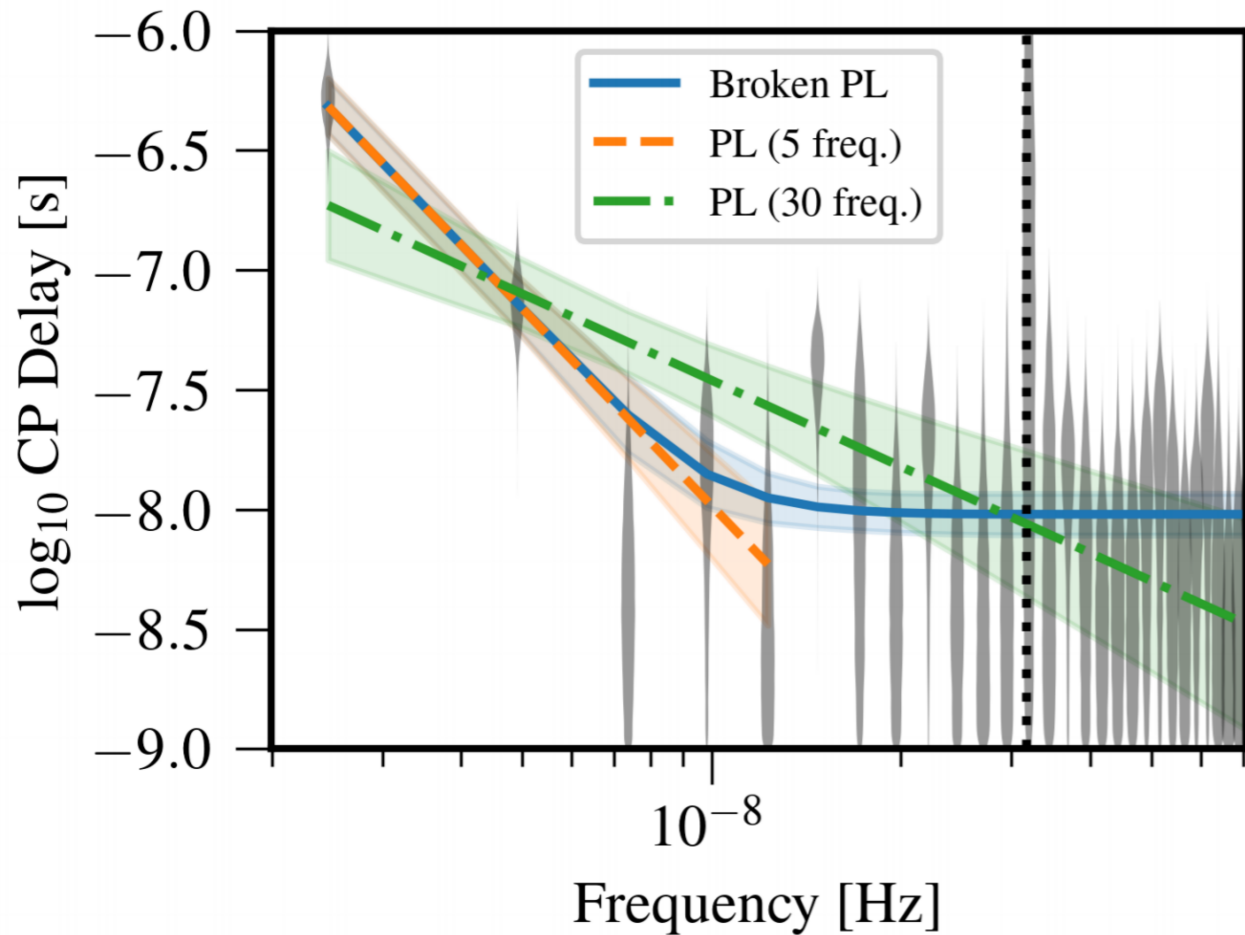
Domènech, 1912.05583

Domènech, SP, and Sasaki, 2005.12314

Infrared scaling of Ω_{IGW}



Back to NANOGrav

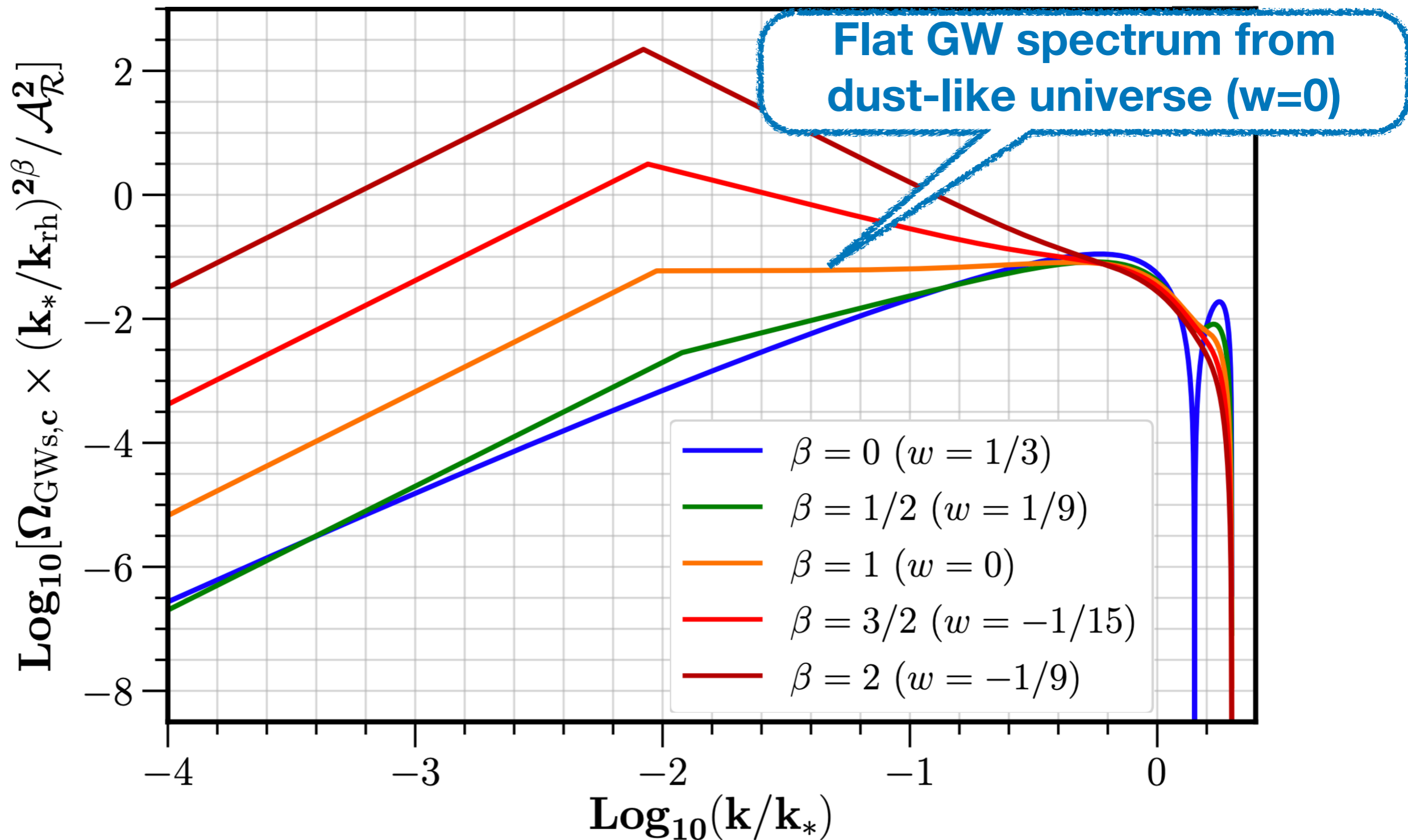


$$\Omega_{GW} = \frac{2\pi^2 f_{yr}^2}{3H_0^2} A_{SGWB}^2 \left(\frac{f}{f_{yr}} \right)^{5-\gamma}$$

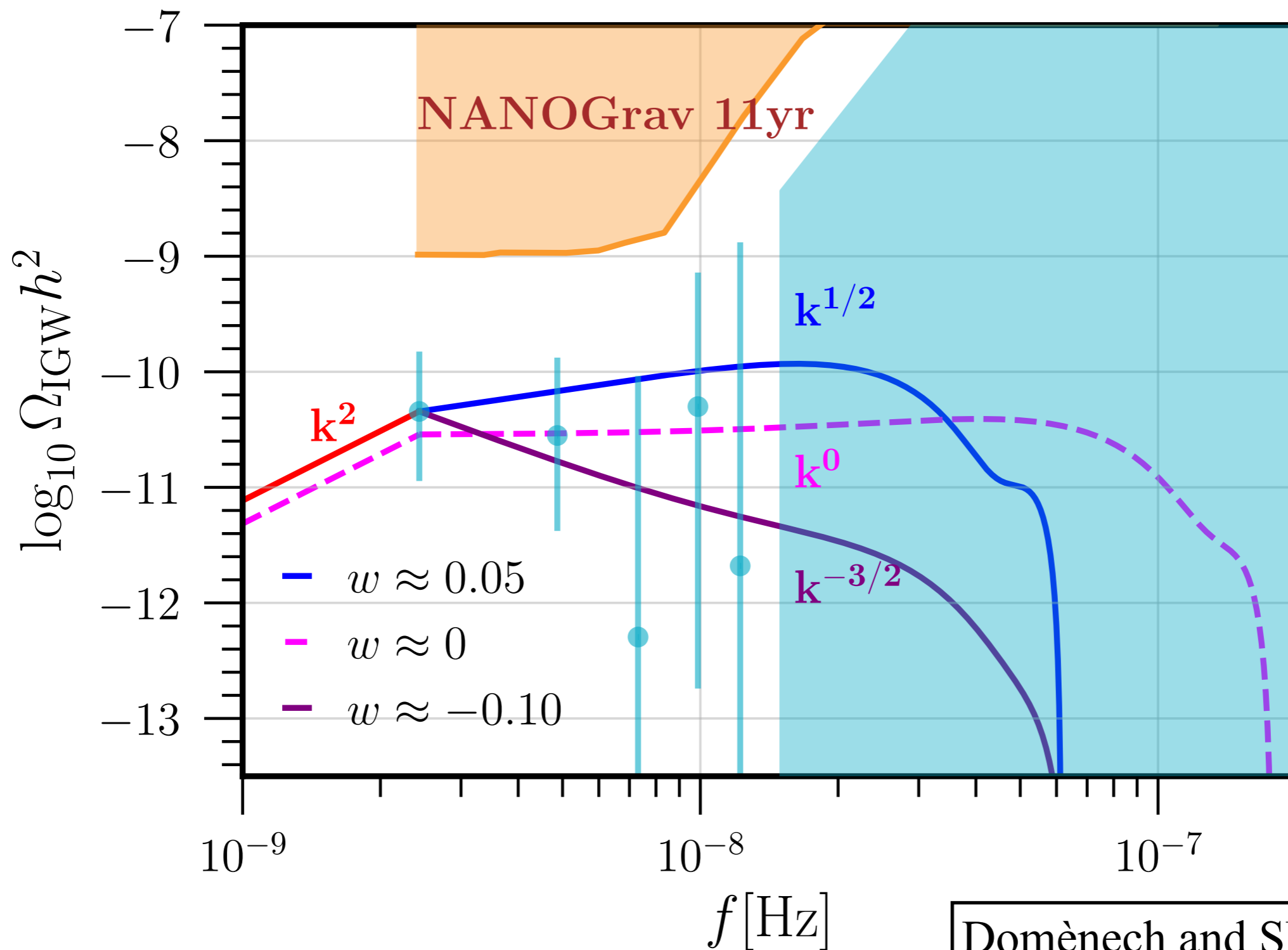
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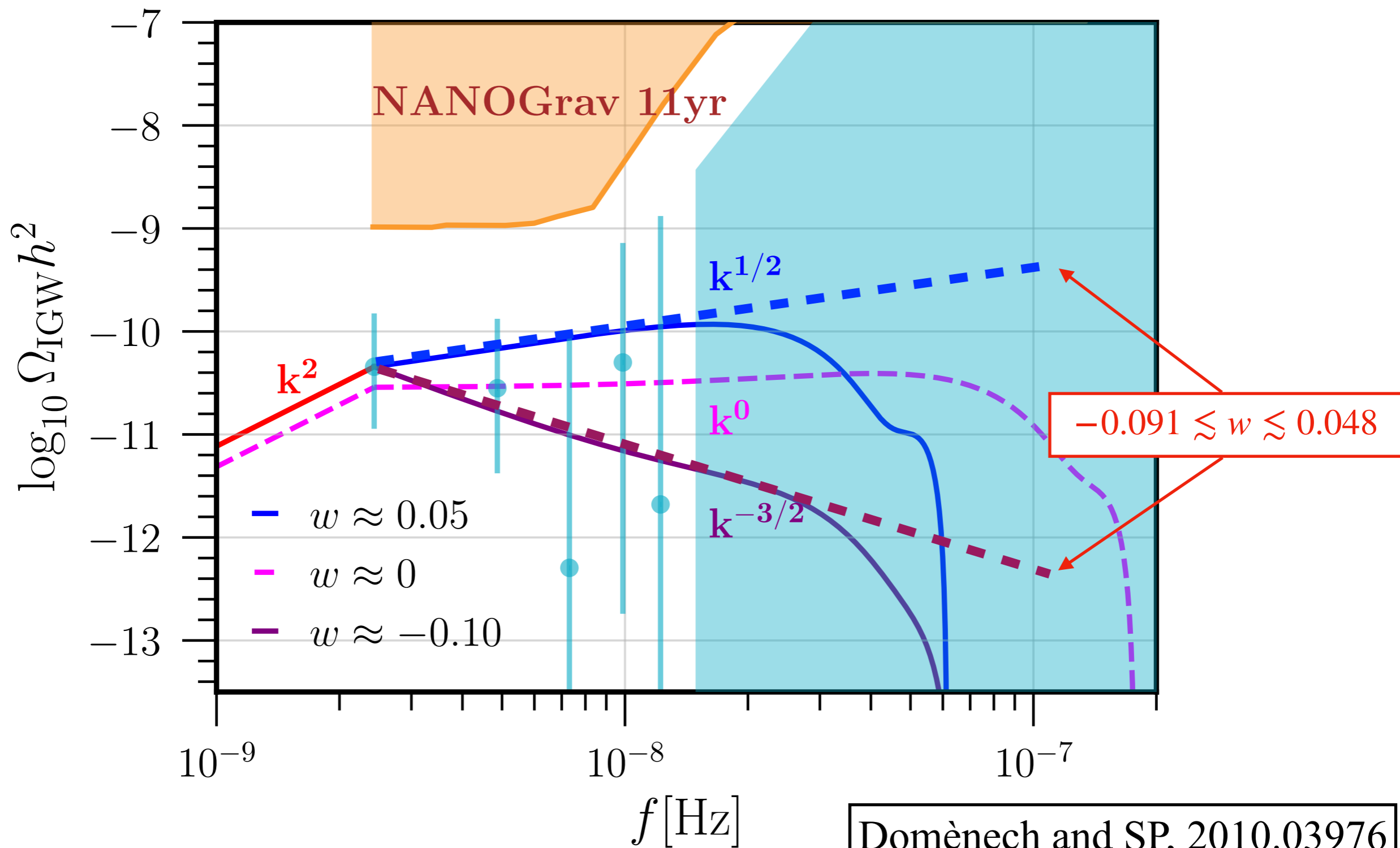
Infrared scaling of Ω_{IGW}



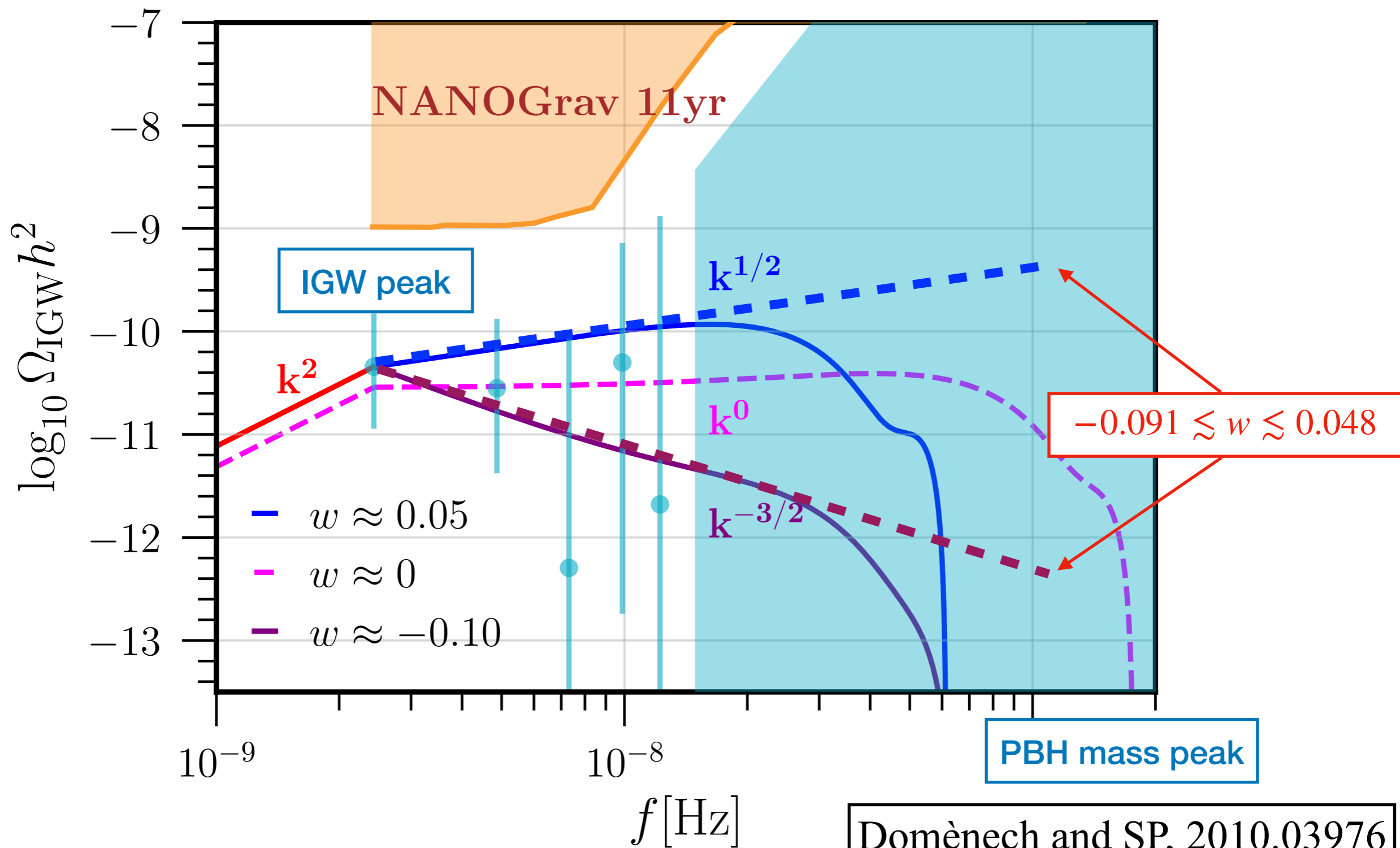
NANOGrav PBHs



NANOGrav PBHs

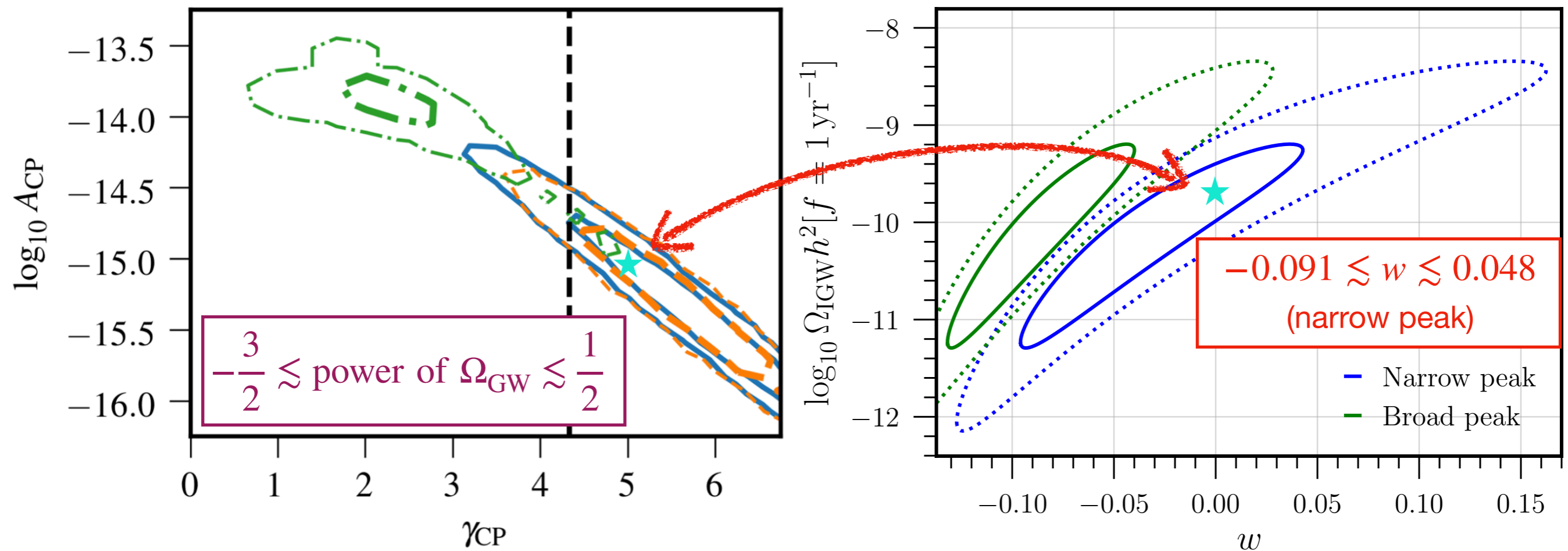


NANOGrav PBHs

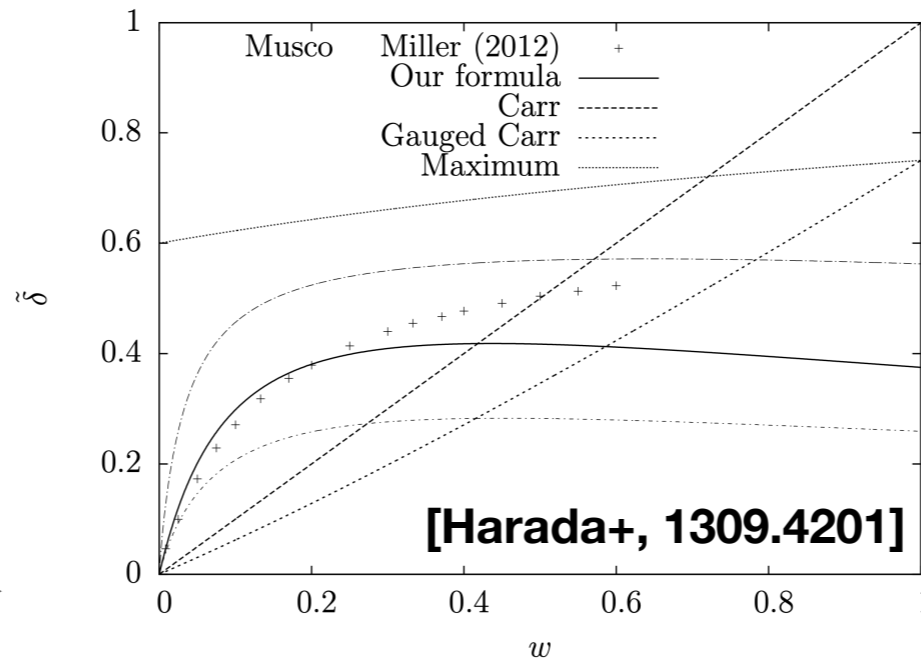
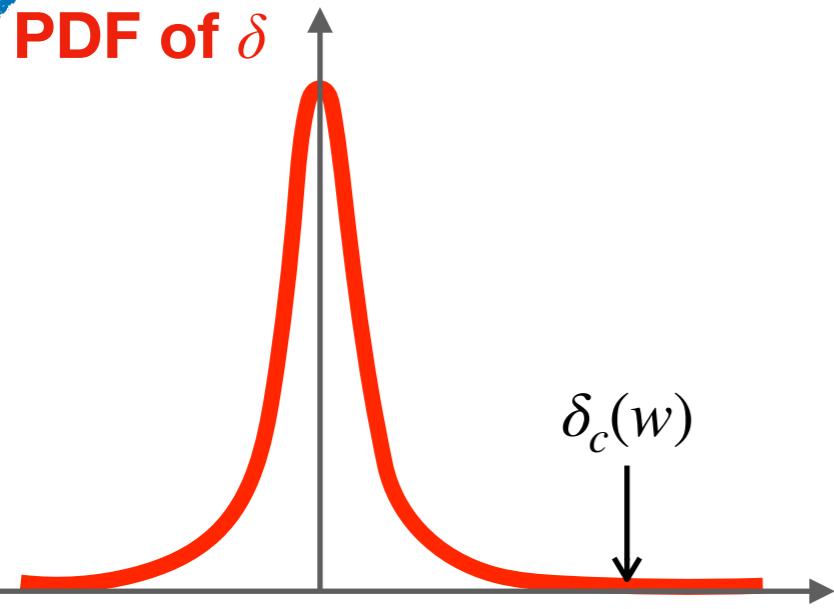


NANOGrav PBHs

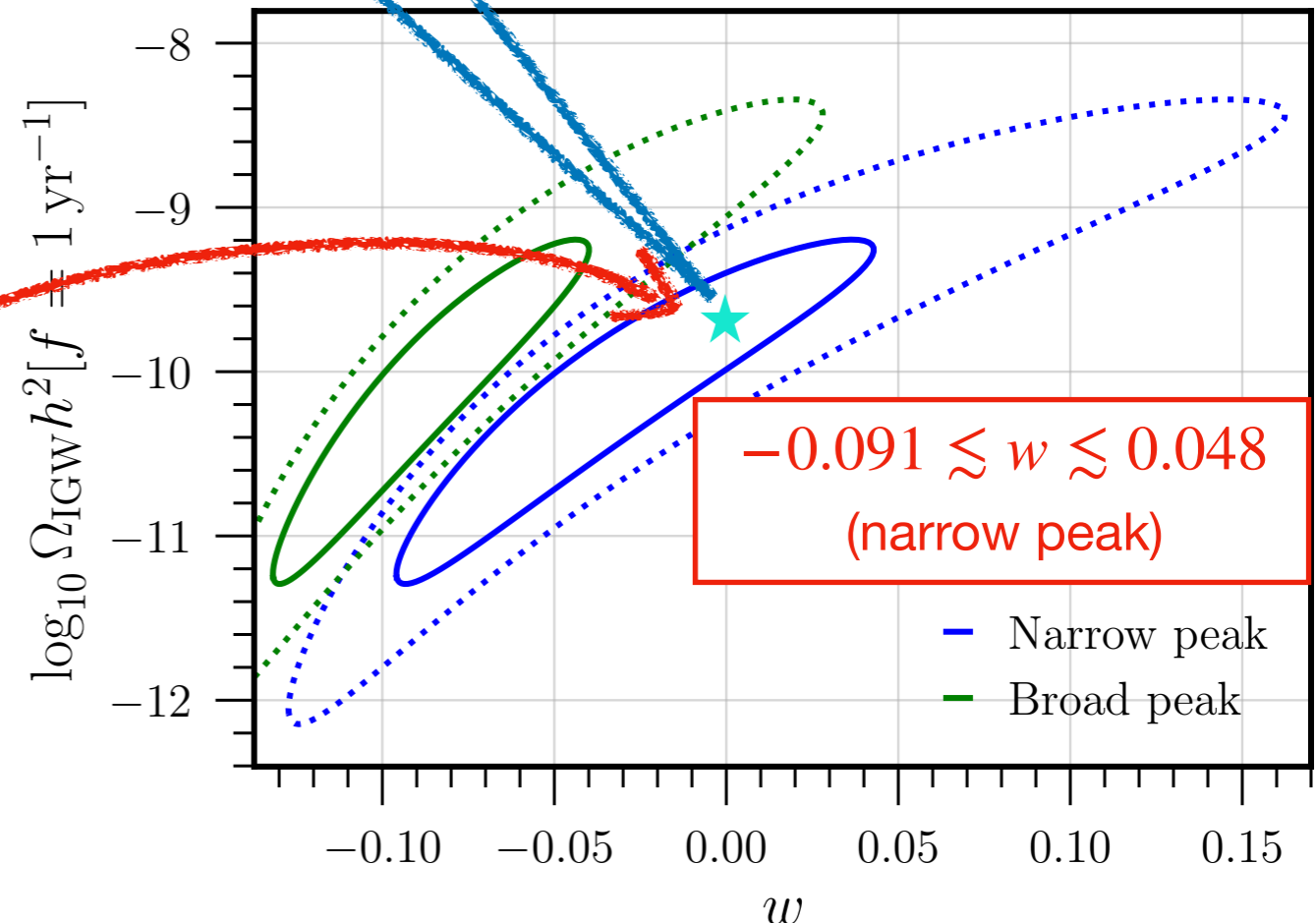
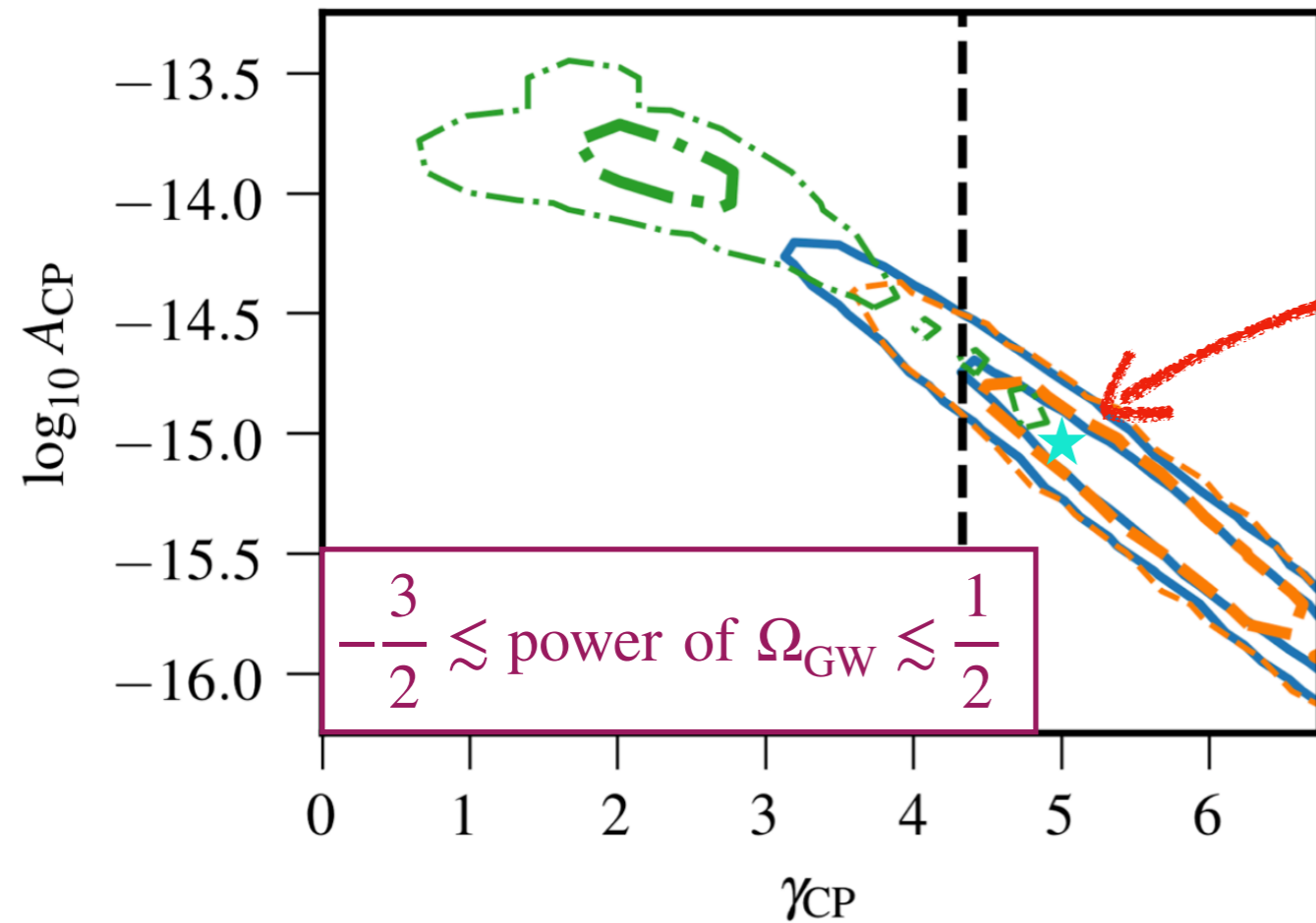
$$\Omega_{\text{GW}} = \frac{2\pi^2 f_{\text{yr}}^2}{3H_0^2} A_{\text{SGWB}}^2 \left(\frac{f}{f_{\text{yr}}} \right)^{5-\gamma_{\text{CP}}}$$



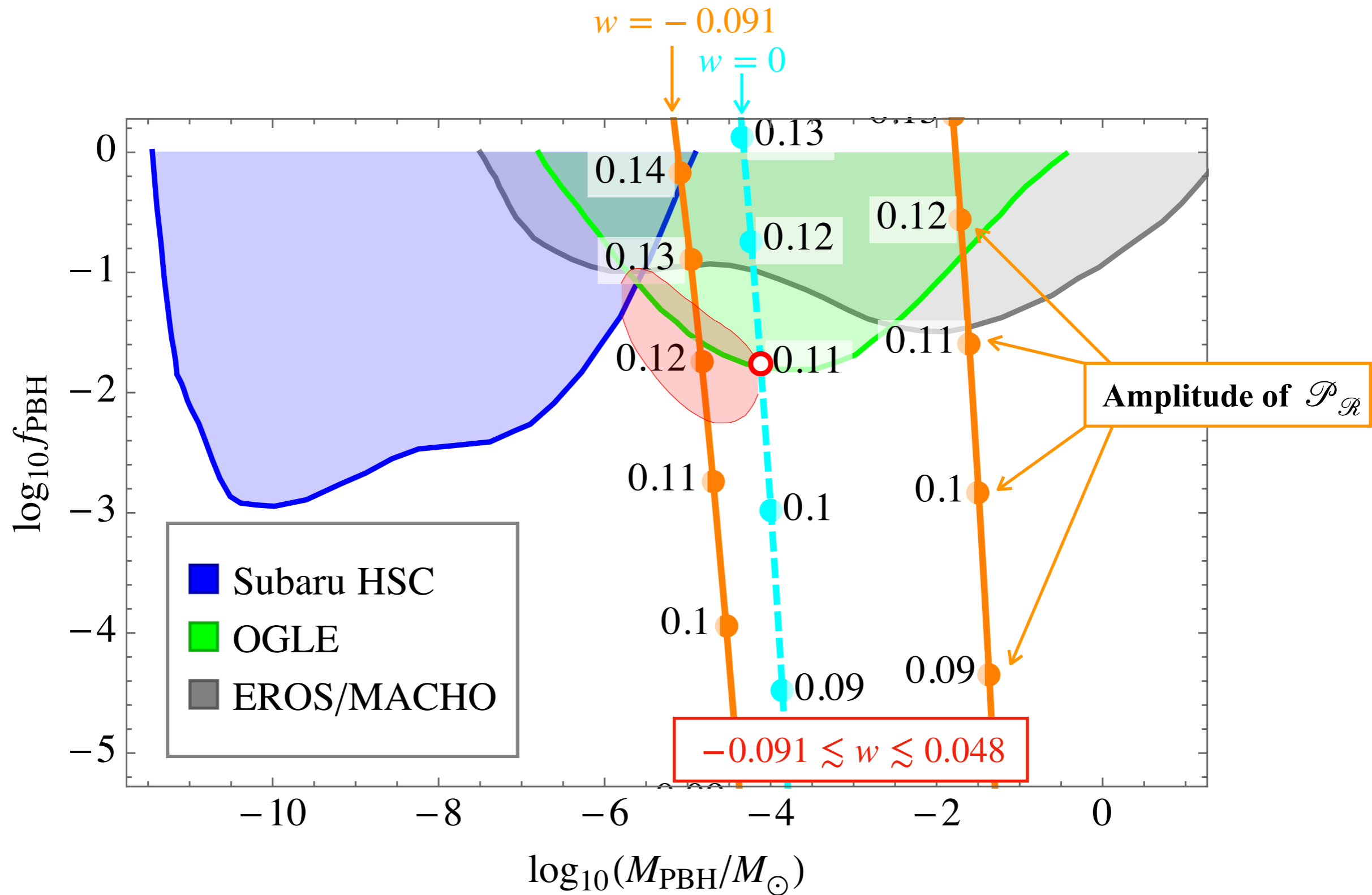
PDF of δ



Softening of EoS can decrease δ_c , which enhances the PBH abundance. We use a scalar-field dominated universe with exponential potential to avoid instability of $w < 0$.

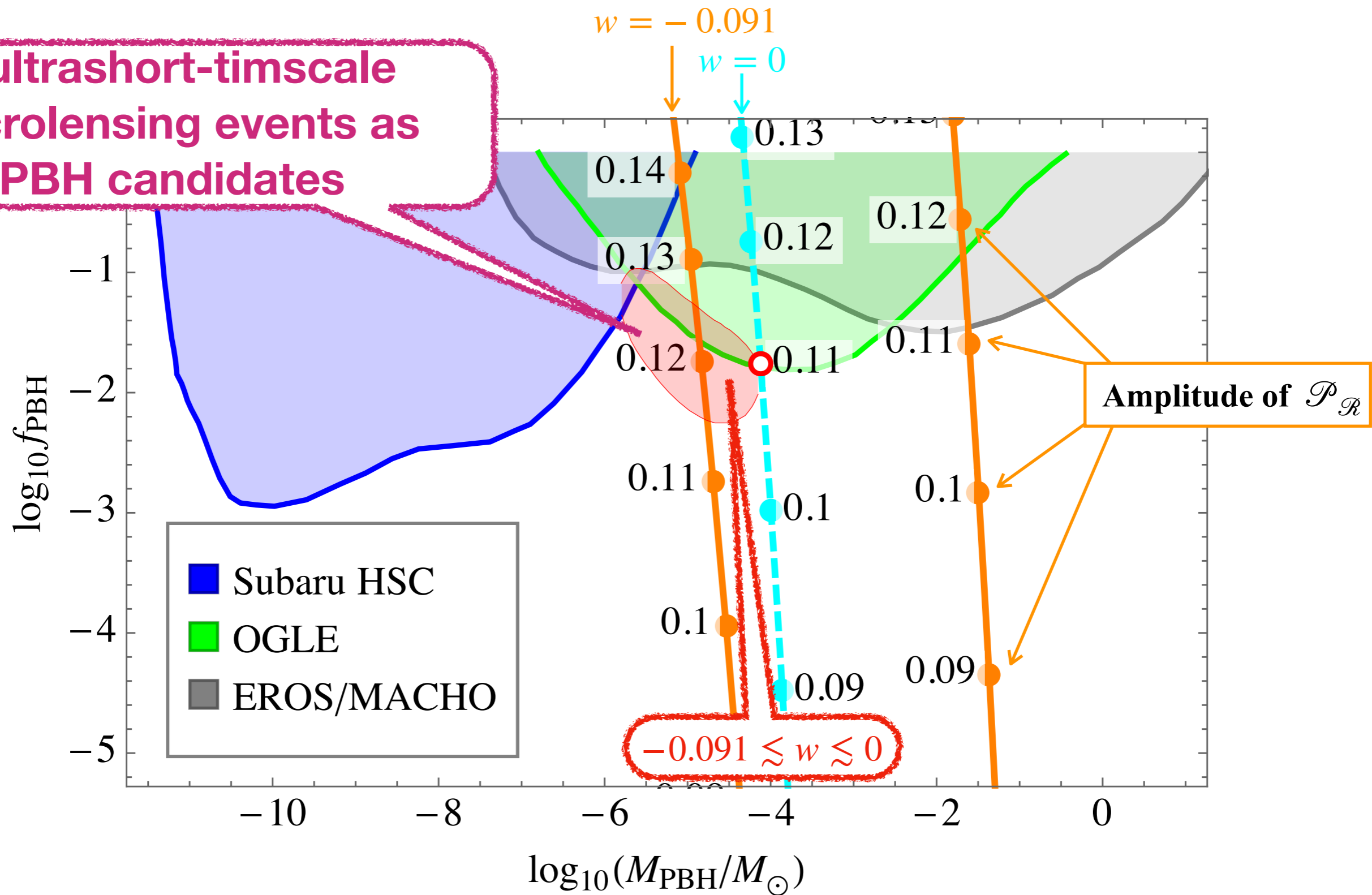


NANOGrav PBHs



NANOGrav PBHs

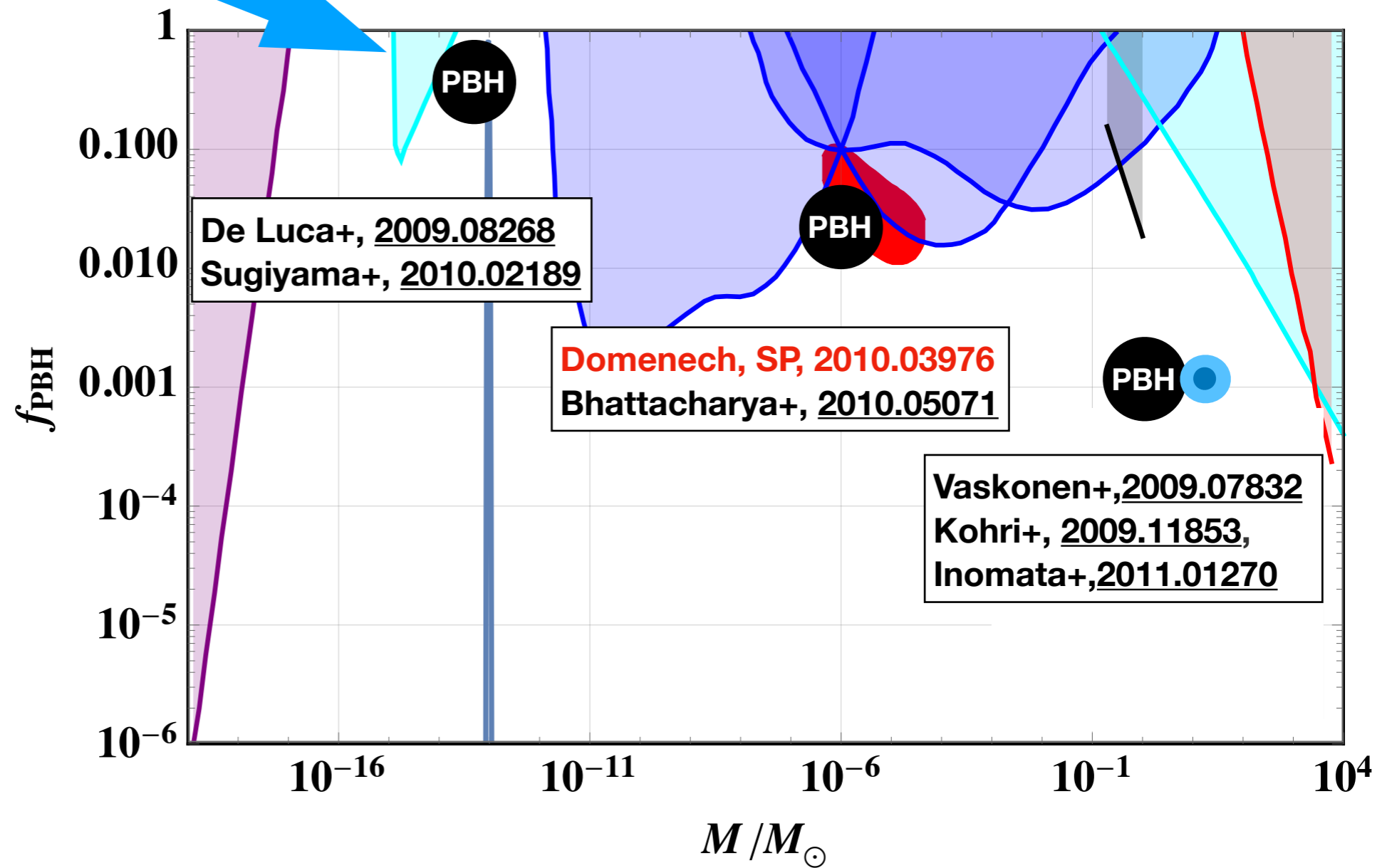
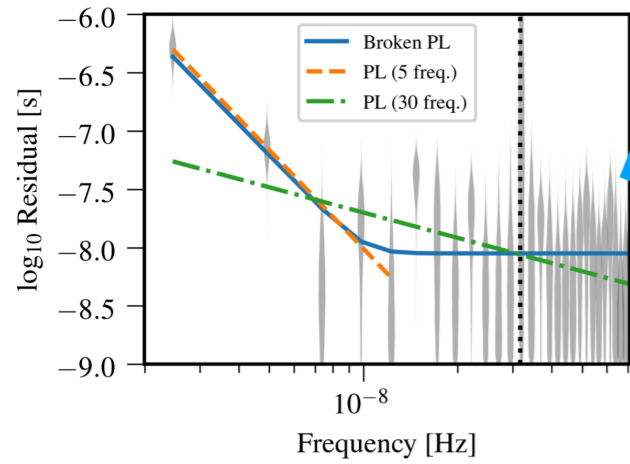
6 ultrashort-timscale
microlensing events as
PBH candidates



Amplitude of $\mathcal{P}_{\mathcal{R}}$

$-0.091 \lesssim w \lesssim 0$

Summary



Summary

- **Infrared scaling** can help us probe the thermal history of the universe and the energy scale of the source.
- **Shape of IGW**: The most crucial factor is the width of the peak of the scalar spectrum. We found analytical formulas for lognormal peaks (for RD), which is useful for signal searching in the future.
- **NANOGrav result and planet-mass PBH**: Possible detection of SGWB by NANOGrav can be connected to the recently reported planet-mass PBHs, if there is a dust-like era before ~ 150 MeV.

Thank you !