Probing Small Scale Inflationary Fluctuations via GWs

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*Based mainly on

C. Ünal, E. D. Kovetz, S. P. Patil Phys. Rev. D 103, 063519 [arXiv: 2008.11184]

C. Ünal Phys. Rev. D 99, 041301(R) [arXiv:1811.09151]

J. Garcia-Bellido, M. Peloso, C. Ünal JCAP 1709 (2017) 013 [arXiv:1707.02441]

**Learn a lot from working with De Luca, Franciolini, Garcia-Bellido, Kehagias, Kovetz, Patil, Peloso, Riotto

Gravitational-Wave Primordial Cosmology Meeting, Paris, May 2021

Outline

- Inflationary observables and scales
- Induced GWs
- Primordial NG, topology of diagrams and imprints on GW spectrum
- Results
- Next generation experiments + PBH Origin of Stellar and Super Massive BHs
- Conclusions

Brief Summary for Inflation at CMB

- The dimensionless power spectra for scalar and tensor sectors

$$\left\langle \zeta_{\vec{k}}\, \zeta_{\vec{k}'} \right\rangle \equiv \frac{2\pi^2}{k^3}\, P_\zeta(\vec{k})\, \delta^{(3)}(\vec{k}+\vec{k}')\ ,$$
$$\left\langle h_\lambda(\vec{k})\, h_{\lambda'}(\vec{k}') \right\rangle = \frac{2\pi^2}{k^3}\, P_\lambda(k)\delta_{\lambda\,\lambda'}\, \delta^{(3)}(\vec{k}+\vec{k}') \qquad (1)$$

- The power spectrum is conventionally parametrized as

$$P_\zeta(k) = \mathcal{A}_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*)} \quad , \qquad P_{\rm gw} = \frac{2H^2}{\pi^2 M_p^2}\left(\frac{k}{k_*}\right)^{n_T} \qquad (2)$$

- The parameters in Planck '18 (for the pivot scale $k_* = 0.05\mathrm{Mpc}^{-1}$)
  - $\mathcal{A}_s = (2.1 \pm 0.03)\cdot 10^{-9}$ $\quad$ (*Planck TT, TE, EE + lowE + lensing*) , 68% CL
  - $n_s = 0.9649 \pm 0.0042$ $\quad$ (*Planck TT, TE, EE + lowE + lensing*), 68% CL
  - $\alpha_s = -0.0045 \pm 0.0067$ $\quad$ (*Planck TT, TE, EE + lowE + lensing*) , 68% CL

Inflationary models

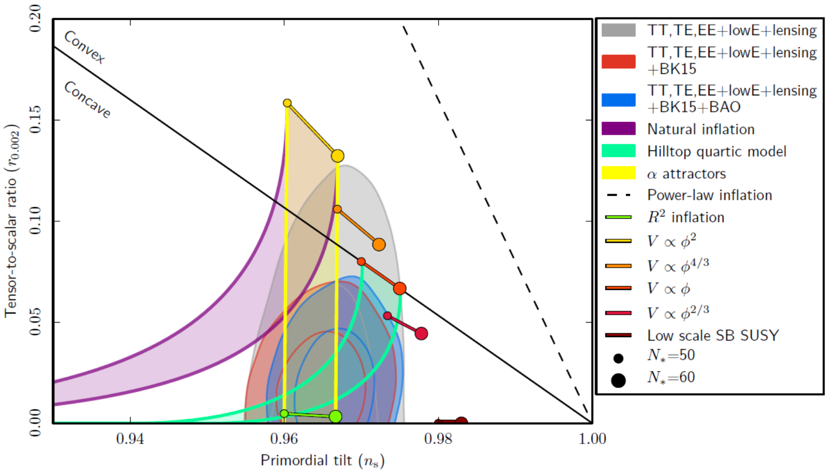


Figure: Predictions of selected inflationary models (taken from Planck '18)

How to probe smaller scales?

Inflation is expected to last roughly 60 e-folds depending on post-inflation physics.

- CMB and LSS probe the wavenumbers in the range $10^{-4} \lesssim k/\mathrm{Mpc}^{-1} \lesssim 0.1$.
- $\mu-$ and $y-$ distortions extend this range up to $\sim 10^5\,\mathrm{Mpc}^{-1}$.
- This corresponds only 18 efolds of inflation.

The rest $\sim$ 40 e-folds is unexplored apart from the bounds and potential signatures associated with primordial black holes (PBHs), and the GW signatures!

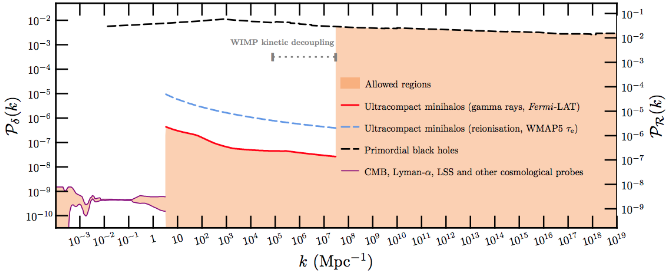


Figure: Density/curvature perturbations, taken from arXiv:1110.2484

Basic Assumptions and Observational Signatures

- Assume amplification in primordial density fluctuations at scales much smaller than CMB modes (not assume a specific mechanism)
  - Inevitable (induced) GWs from enhanced primordial density perturbations via (nonlinear coupling) $\zeta + \zeta \to h$

    Acquaviva+'02 ; Mollerach, Harari, Matarrese '03, Ananda, Clarkson, Wands '06 ; Baumann+'07

$$h''_{\lambda,\,\mathrm{k}}(\eta) + 2\mathcal{H}\, h'_{\lambda,\,\mathrm{k}}(\eta) + \mathrm{k}^2 h_{\lambda,\,\mathrm{k}}(\eta) = 2\mathcal{S}_{\lambda,\,\mathrm{k}}(\eta)\ , \tag{3}$$

$$\mathcal{S}_{\lambda,\,\mathrm{k}}(\eta) \quad \propto \quad \int d^3\mathrm{p}\ \partial\,\zeta_{\mathrm{p}}\ \partial\,\zeta_{\mathrm{k-p}} \tag{4}$$

$$\Omega_{GW} \propto P_{h_{ind}} \propto \left(\int d\tau\, G \cdot \mathcal{S}\right)^2 \propto \langle \zeta\,\zeta\,\zeta\,\zeta \rangle \tag{5}$$

  - Primordial Black Holes (may or may not be part of DM, but our conclusions are **independent** from that)

- Could we measure these observables so that we can learn more about primordial/high energy universe?
  Possible!

NonGaussianity

- When curvature fluctuations are amplified, they usually come together with non-trivial amount of NG
  - Slowing down the inflaton leads to quantum diffusion
    Pattison+ '17 ; Franciolini+ '17 ; Biagetti+ '18 ; Ezquiaga, Garcia-Bellido '18...
  - Particle production is inherently NG via $2 \rightarrow 1$ and $3 \rightarrow 1$ processes
    Barnaby, Peloso '10 ; Anber, Sorbo '12 ; Bugaev, Klimai '13 ; Garcia-Bellido, Peloso, Unal '16...

- Let's allow some NG

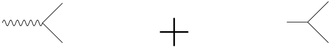
$$\zeta_{\rm k} = \zeta_{\rm k}^G + f_{NL} \int \frac{d^3 p}{(2\pi)^{3/2}} \zeta_{\rm p}^G \, \zeta_{\rm k-p}^G \,, \qquad \Rightarrow \qquad P_\zeta(k) = P_\zeta^G(k) + P_\zeta^{NG}(k) \quad (6)$$

$$\begin{aligned} P_\zeta^G(k) &= \mathcal{A} \cdot \exp\left[-\frac{\ln^2(k/k_*)}{2\sigma^2} \right] \\ P_\zeta^{NG}(k) &= 2 f_{\rm NL}^2 \int \frac{dp}{p} \frac{d\Omega}{4\pi} \frac{k^3}{|{\rm k}-{\rm p}|^3} P_\zeta^G(p) \, P_\zeta^G(|{\rm k}-{\rm p}|) \end{aligned} \quad (7)$$

- Effects of NG :Scalar modes peak at a larger frequency, more contraction due to more legs, wider signal due to convolution

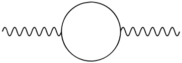
Induced GWs with Local NG Garcia-Bellido, Peloso, Unal '17 ; Unal '19

$$\begin{aligned}\Omega_{GW} \propto P_{h_{ind}} &\propto \left(\int d\tau\, G \cdot \mathcal{S}\right)^2 \propto \int d^3p \int d^3q \,\langle\, \underbrace{\zeta_{\mathrm{p}} \;\; \zeta_{\mathrm{k-p}} \;\; \zeta_{\mathrm{q}} \;\; \zeta_{\mathrm{k'-q}}}_{\left(\zeta_G + f_{NL}\zeta_G^2\right)^4} \,\rangle \\ &= \Omega_{GW}^{G} + \Omega_{GW}^{NG} \qquad (8)\end{aligned}$$

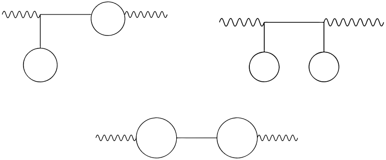


Contractions

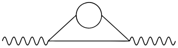
- $\mathcal{O}(f_{NL}^0)$



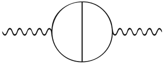
- Contractions vanishing due to zero momentum propagator or symmetry



- $\mathcal{O}(f_{NL}^2)$



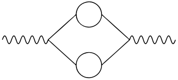
hybrid



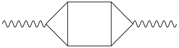
walnut

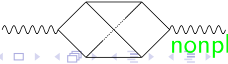
- $\mathcal{O}(f_{NL}^4)$

reducible



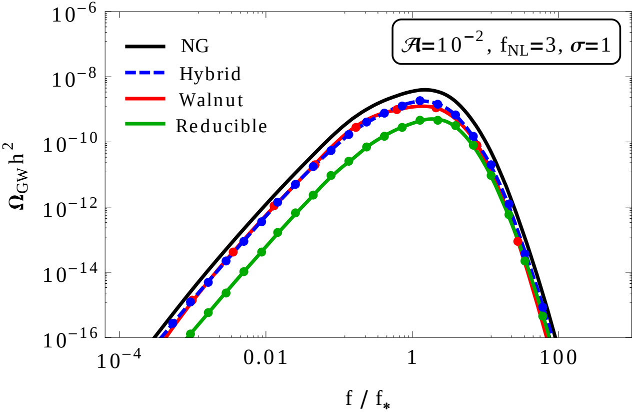
planar





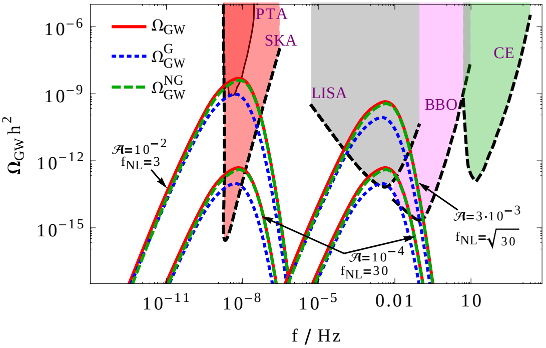
nonplanar

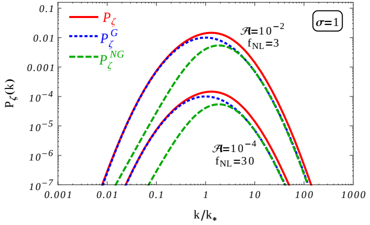
Results for small/mild NG [1] [2] Unal '19



[1] Large f_{NL} limit studied by Nakama, Kamionkowski, Silk '16 ; Garcia-Bellido, Peloso, Unal '17
[2] Also recent works with similar results : Atal, Domenech '21 ; Adshead, Lozanov, Weiner '21

Ω_{GW}^{NG} peaks at larger freq + wider + larger amplitude



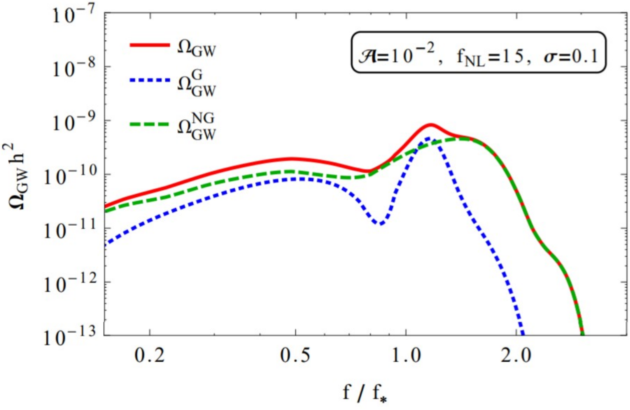


$\rho_{PBH} \simeq \rho_{DM}$: (i)$10 - 100 M_\odot \leftrightarrow f_{PTA-SKA}$ and (ii)$10^{-14} - 10^{-11} M_\odot \leftrightarrow f_{LISA}$)

(i)Bird et al '16 ; Clesse, Garcia-Bellido '16 ; Sasaki et al '16

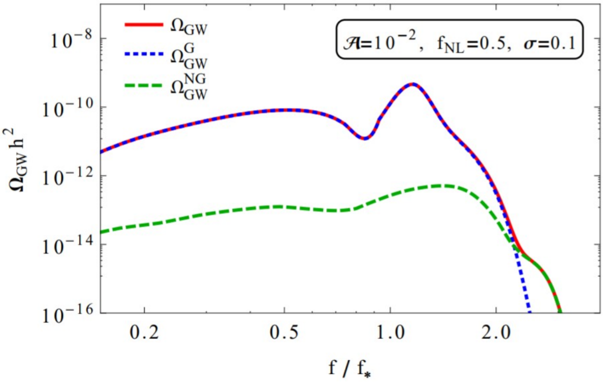
(ii) Garcia-Bellido, Peloso, Unal '17 ; Bartolo et al '19 ; Cai, Pi, Sasaki '19

Signature 1: A not-very-well-resolved double peak.



Signatures for Narrow Spectra - II ($\sigma << 1$) Unal '19

Signature 2: A bump in UV tail even if GWs from NG fluctuations are completely subdominant.

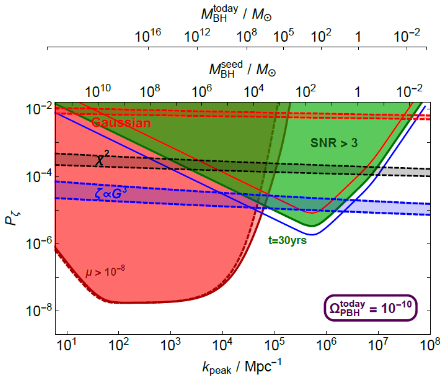


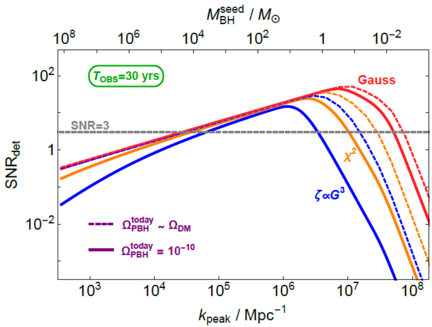
With PTA-SKA and LISA, probing $f_{NL} \sim \mathcal{O}(0.1 - 10)$ is possible
(could be better probe than next generation CMB+LSS !)

Sensitivity of Next Generation Experiments Unal, Kovetz, Patil '21

- CMB Spectral Distortions
  $< \mu_{PIXIE} > \simeq 10^{-8} \simeq 2.3\,\Delta N\, P_\zeta \;\rightarrow\; P_\zeta \sim 10^{-8}$
- Stochastic Gravitational Wave Background
  $\Omega_{GW} \simeq (\mathrm{sym\,fact}) \cdot \Omega_{rad} \cdot P_\zeta^2 \;\rightarrow\; P_\zeta \sim \left(\Omega_{GW} \cdot 10^4\right)^{1/2}$
  $P_\zeta(k_{PTA-SKA}) \sim 10^{-5.5}$, $P_\zeta(k_{LISA}) \sim 10^{-4.5}$, $P_\zeta(k_{ET/CE}) \sim 10^{-4}$

See also Byrnes, Cole, Patil '18 ; Inomata, Nakama '18 ; Cai, Pi, Wang, Yang '19





Summary

- Enhanced perturbations (usually contain NG) lead inevitably induced GWs

  $+$ may also produce PBHs in large abundance

- GW spectrum can probe small scale inflationary perturbations $f_{NL} \sim (0.1 - 1)$ with PTA-SKA and LISA

  ! which is is even better than next generation CMB+LSS

- If no signal from (distortions + SKA GWs), then we obtain the strongest constraints on primordial fluctuations at small scales.

  $+$ rule out "*robustly*" PBH ($M > 0.1 M_{\odot}$) and the intriguing possibility that PBH are SMBH seeds

  independent of inflationary fluctuation statistics, merger history, accretion rates and clustering properties

- Similar conclusions will be valid for LISA scales/frequencies, namely ballpark of $10^{-12} M_{\odot}$ PBHs will be probed conclusively (detected or ruled out).

Imprints of Primordial Non-Gaussianity on Gravitational Wave Spectrum
Supplemental Material

In this supplementary section, we give explicit expressions for the Hybrid, Walnut and Reducible diagrams.

For the numerical evolution, time variables, η_c, η', η'', are rescaled with external momentum, k, such that $k\eta_c = x_c$, $k\eta' = x'$, $k\eta'' = x''$, and internal momentum variables are rescaled with peak scale, k_*, such that $\tilde{p} = p/k_*$, $\tilde{q} = q/k_*$. Finally, these dimensionless expressions are evaluated numerically when the corresponding mode is deep inside the horizon, namely $k\eta_c = x_c \gg 1$.

Hybrid Diagram

$$\Omega_{\rm GW}^{\rm Hyb}(k,\eta_0)h^2 \simeq \Omega_{\gamma,0}\, h^2 \, \frac{8\cdot \mathrm{S}\cdot f_{\rm NL}^2}{12\cdot 81\cdot 4\pi^2}\, k^3\, \overline{\mathcal{I}_{\rm hyb}}\,, \tag{1}$$

where S indicates the possible contractions and it is 8, the overbar is the period average, and

$$\mathcal{I}_{\rm hyb} = \int^{\eta_c} d\eta' \int^{\eta_c} d\eta''\eta'\,\eta'' \sin(k\eta-k\eta')\,\sin(k\eta-k\eta'')\,\times$$
$$\int d^3p\, p^4 \sin^4(\theta)\; U(\hat{\mathbf{p}})^2\, F_T(p\,\eta', |\mathbf{k}-\mathbf{p}|\eta')\, F_T(q\,\eta'', |\mathbf{k}-\mathbf{p}|\;\eta'')\;\; \frac{\mathcal{C}(p)\,\mathcal{P}_\zeta(|\mathbf{k}-\mathbf{p}|)}{|\mathbf{k}-\mathbf{p}|^3} \tag{2}$$

where η_c is the conformal time for the evaluation during radiation dominated era and

$$\mathcal{C}(p) \equiv \int \frac{d^3q}{q^3}\, \frac{P_\zeta(q)\; P_\zeta(|\mathbf{p}-\mathbf{q}|)}{|\mathbf{p}-\mathbf{q}|^3} \tag{3}$$

Walnut Diagram

$$\Omega_{\rm GW}^{\rm Wal}(k,\eta_0)h^2 \simeq \Omega_{\gamma,0}\, h^2 \, \frac{8\cdot S\cdot f_{\rm NL}^2}{12\cdot 81\cdot 4\pi^2}\, k^3\, \overline{\mathcal{I}_{\rm wal}}\,, \tag{4}$$

where $S = 32$ and

$$\mathcal{I}_{\rm wal} = \int^{\eta_c} d\eta' \int^{\eta_c} d\eta''\,\eta'\,\eta'' \sin(k\eta-k\eta')\,\sin(k\eta-k\eta'')\,\times$$
$$\int d^3p\; d^3q\; p^2\; q^2 \sin^2(\theta_p) \sin^2(\theta_q)\; U(\hat{\mathbf{p}})\, U(\hat{\mathbf{q}}) F_T(p\,\eta', |\mathbf{k}-\mathbf{p}|\eta') F_T(q\,\eta'', |\mathbf{k}+\mathbf{q}|\;\eta'')\,\times$$
$$\frac{\mathcal{P}_\zeta(p)\,\mathcal{P}_\zeta(|\mathbf{k}-\mathbf{p}|)\,\mathcal{P}_\zeta(|\mathbf{k}-\mathbf{p}+\mathbf{q}|)}{p^3\;|\mathbf{k}-\mathbf{p}|^3\;|\mathbf{k}-\mathbf{p}+\mathbf{q}|^3} \tag{5}$$

Reducible Diagram

$$\Omega_{\rm GW}^{\rm Red}(k,\eta_0)h^2 \simeq \Omega_{\gamma,0}\, h^2 \, \frac{8\cdot S\cdot f_{\rm NL}^4}{12\cdot 81\cdot 16\pi^3}\, k^3\, \overline{\mathcal{I}_{\rm Red}}\,, \tag{6}$$

where $S = 8$ and

$$\mathcal{I}_{\rm Red} = \int^{\eta_c} d\eta' \int^{\eta_c} d\eta''\,\eta'\,\eta'' \sin(k\eta-k\eta')\,\sin(k\eta-k\eta'')\,\times$$
$$\int d^3p\, p^4 \sin^4(\theta)\; U(\hat{\mathbf{p}})^2\, F_T(p\,\eta', |\mathbf{k}-\mathbf{p}|\eta')\, F_T(q\,\eta'', |\mathbf{k}-\mathbf{p}|\;\eta'')\;\; \mathcal{C}(p)\cdot\mathcal{C}(|\mathbf{k}-\mathbf{p}|) \tag{7}$$

where $\mathcal{C}$ is defined in (3).