Probing the Physics of Inflation with Gravitational Waves

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Thanks to all my collaborators:

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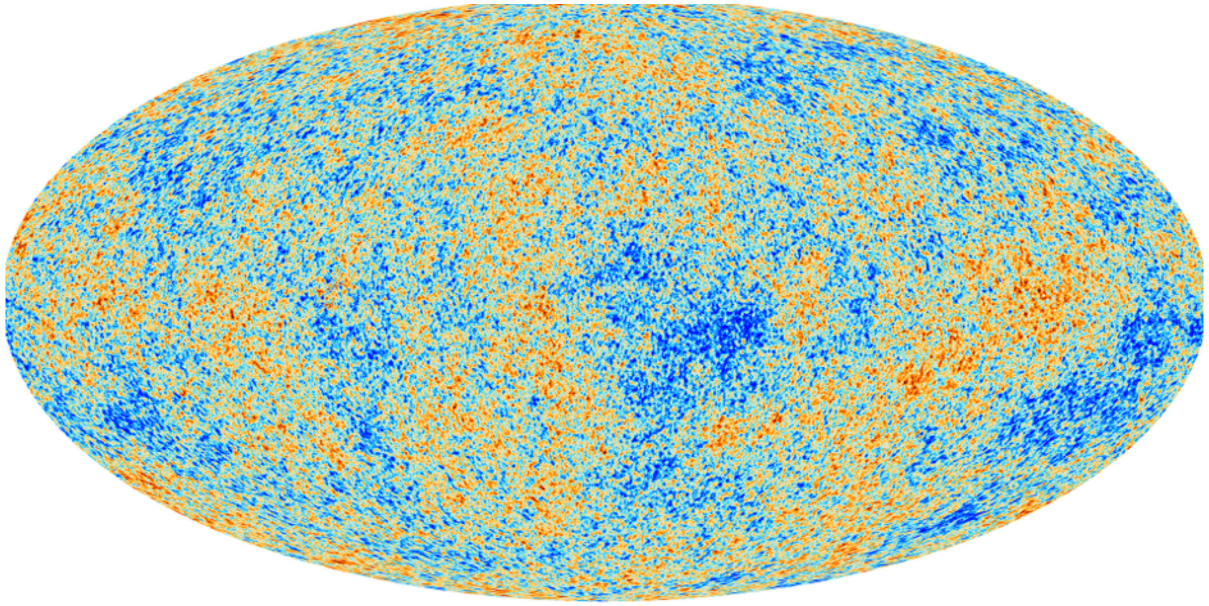
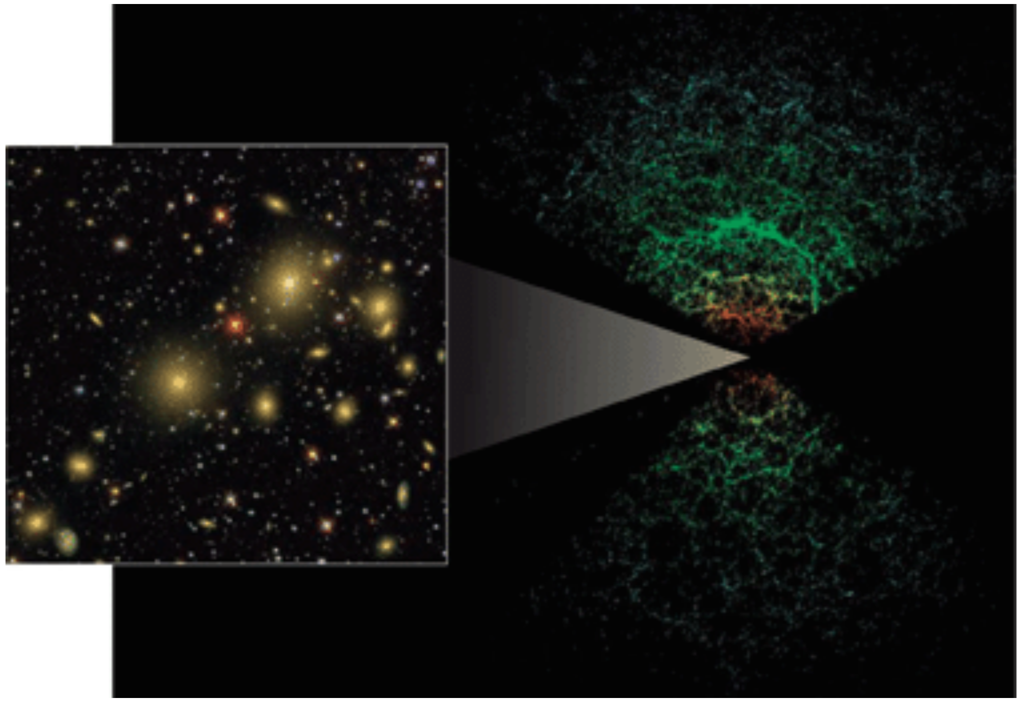
Introduction

- **Inflation** is a short period of **superluminal**, accelerated **expansion**, occurred within the first second of our universe life.
- It solves problems of big bang cosmology: `horizon, flatness, entropy problems`
- Moreover, inflation provides an **elegant mechanism** for generating the **primordial seeds** for the CMB and the LSS





Theoretical Prediction of Cosmological Inflation

Stochastic background of gravitational waves

Vanilla model

$$S_h = \frac{M_{\rm Pl}^2}{4} \int dt\, d^3x\, a^3 \left[\dot{h}_{ij}^2 - \frac{1}{a^2} \left(\vec{\nabla} h_{ij} \right)^2 \right]$$

- **Tensor power spectrum** $\mathcal{P}_h = \frac{2\,k^3}{2\pi^2} \langle |h|^2 \rangle = \frac{2}{\pi^2} \frac{H^2}{M_{\rm Pl}^2}$

Tensor-to-scalar ratio: $\frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon$ → *Energy scale of inflation* → *Size of inflaton field excursion*

Tensor-to-Scalar ratio $\frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = r \leq 0.07$ [Planck, Bicep 2]

How to detect GWs from inflation?

`But what if` $r \ll 10^{-3}$ `at CMB scales?`

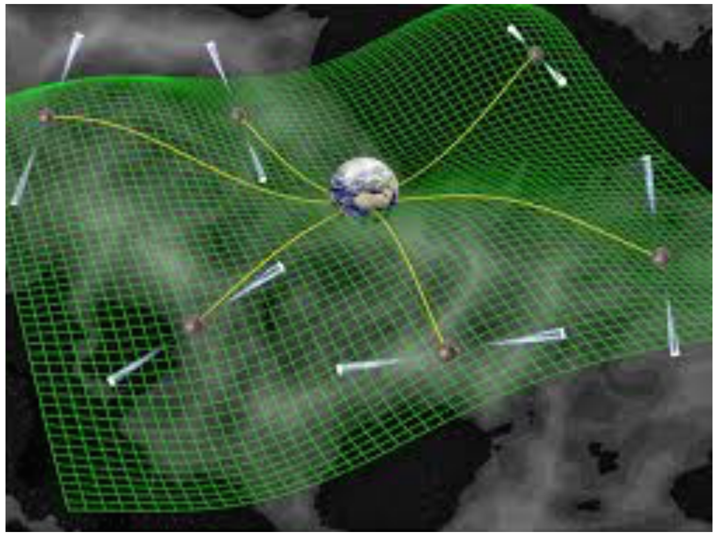
(Difficult to get large field excursions in quantum gravity embeddings of inflation)

Should we give up any attempts
to probe inflationary tensor modes?

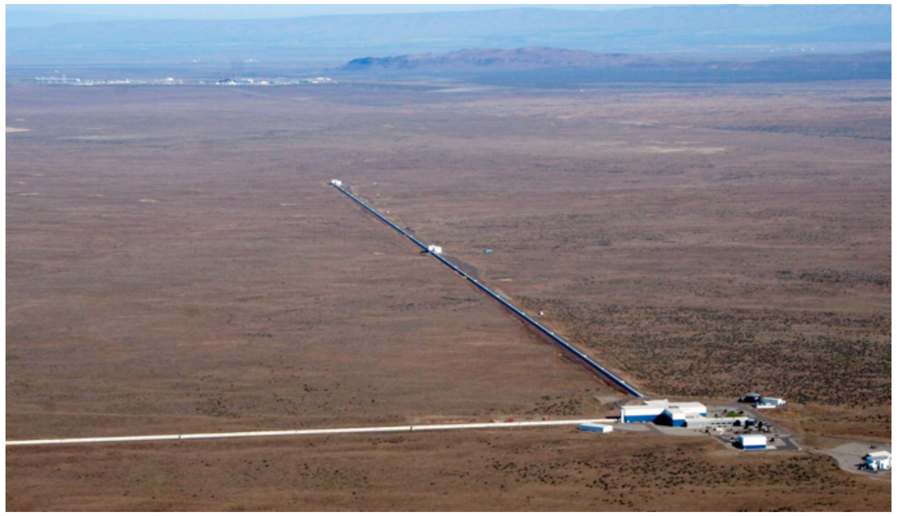
How to detect GWs from inflation?

Direct detection?

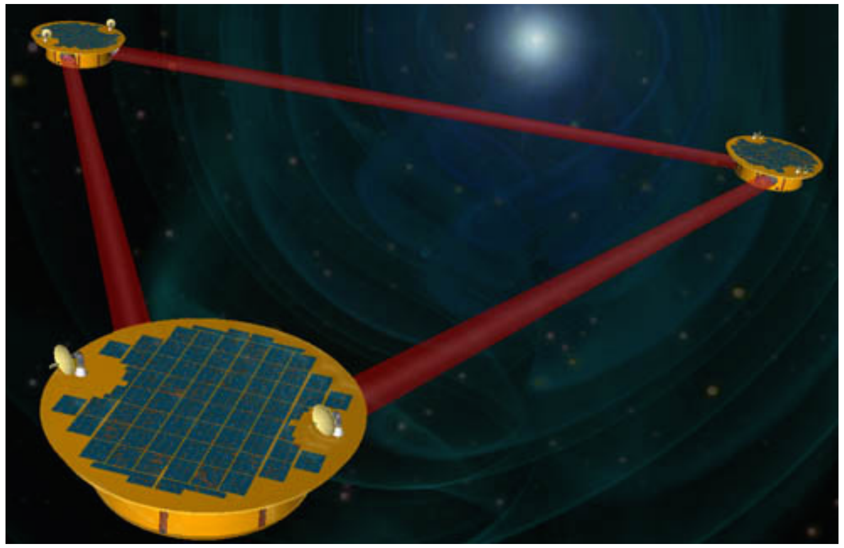
PTA



Ground-Based Intf



Space-Based Intf



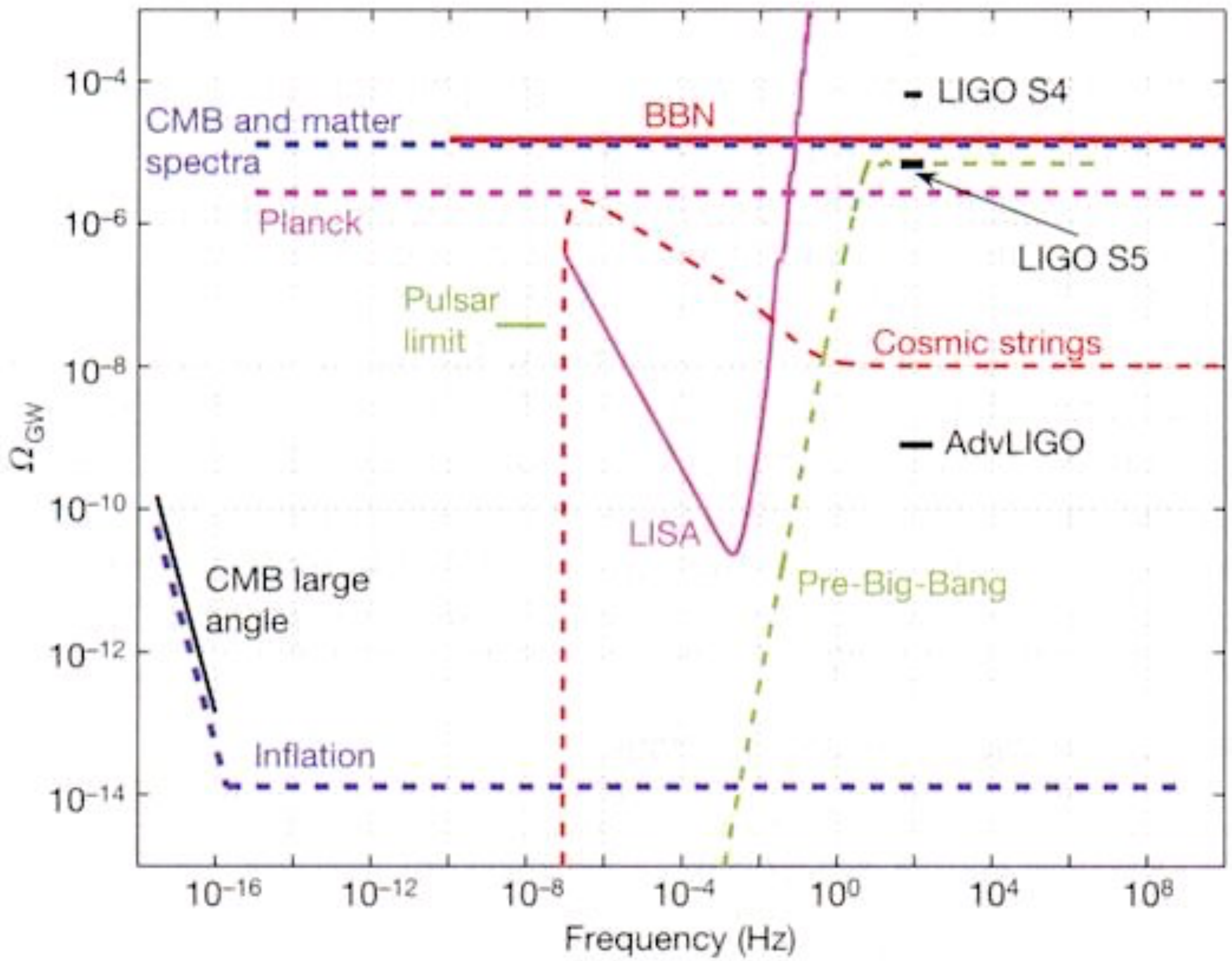
How to detect GWs from inflation?

Direct detection?

Problem

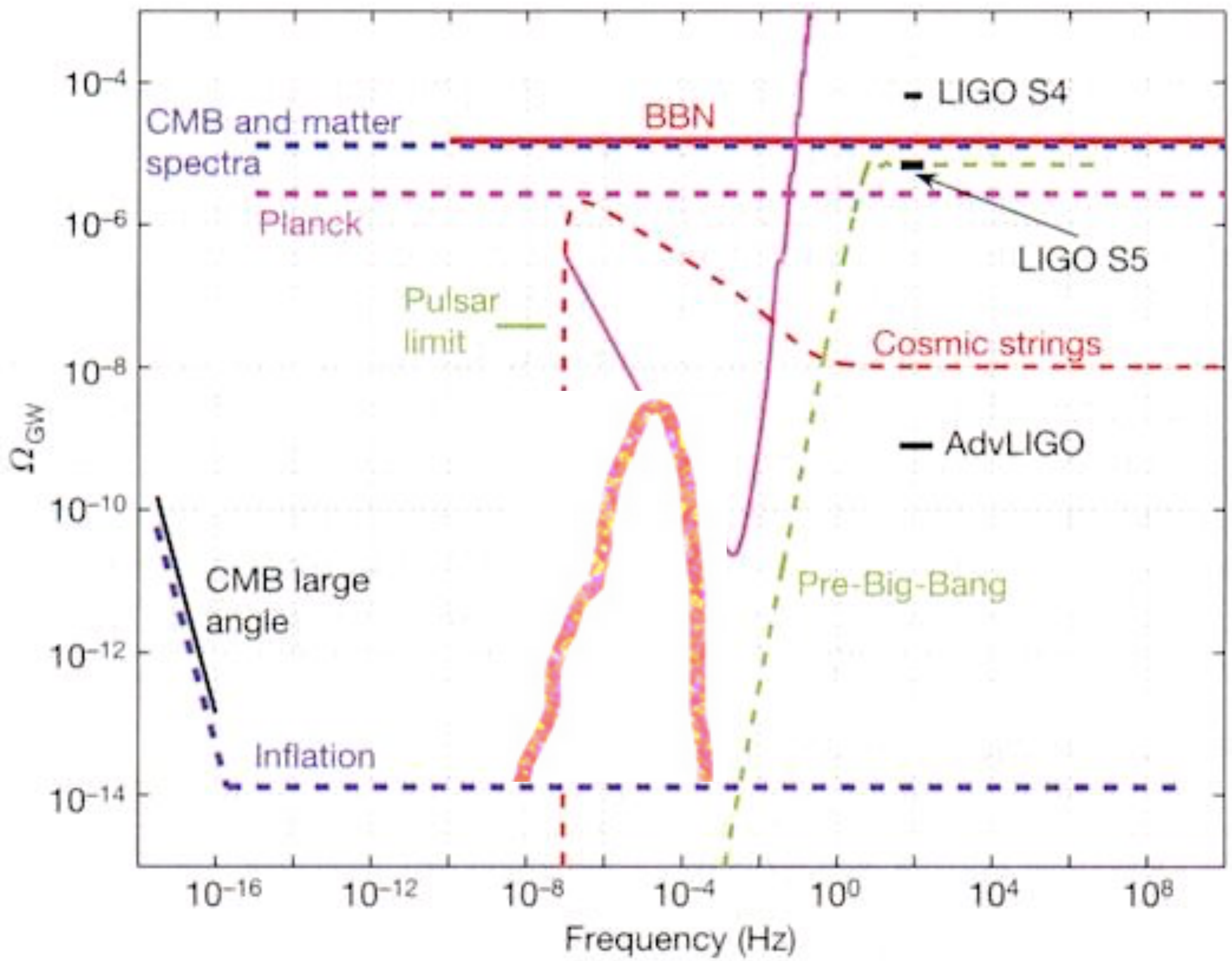
Amplitude of GWs
from vanilla models of inflation
is too small to be detected

[Thrane et al.]

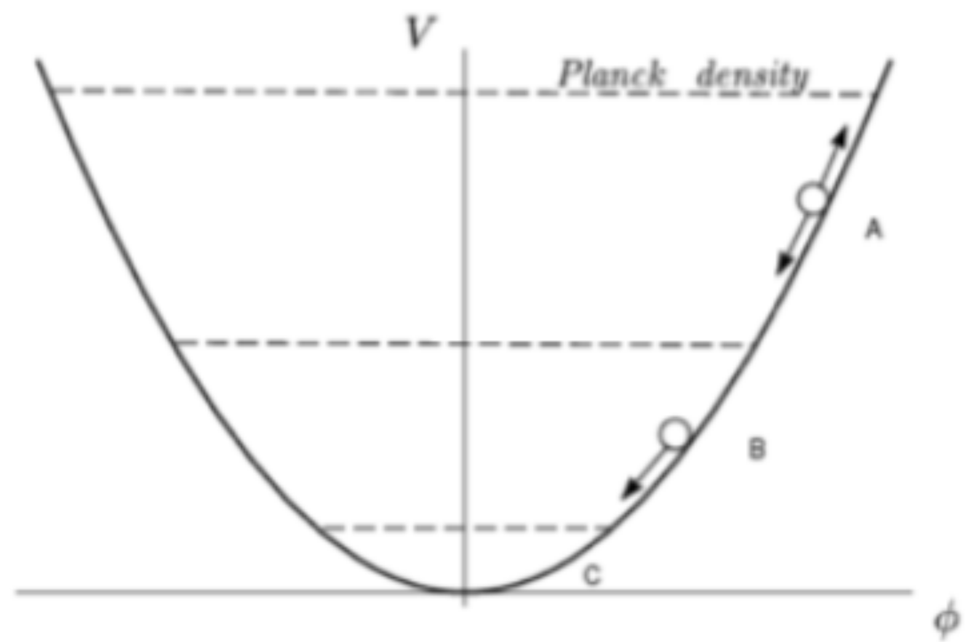


How to detect GWs from inflation?

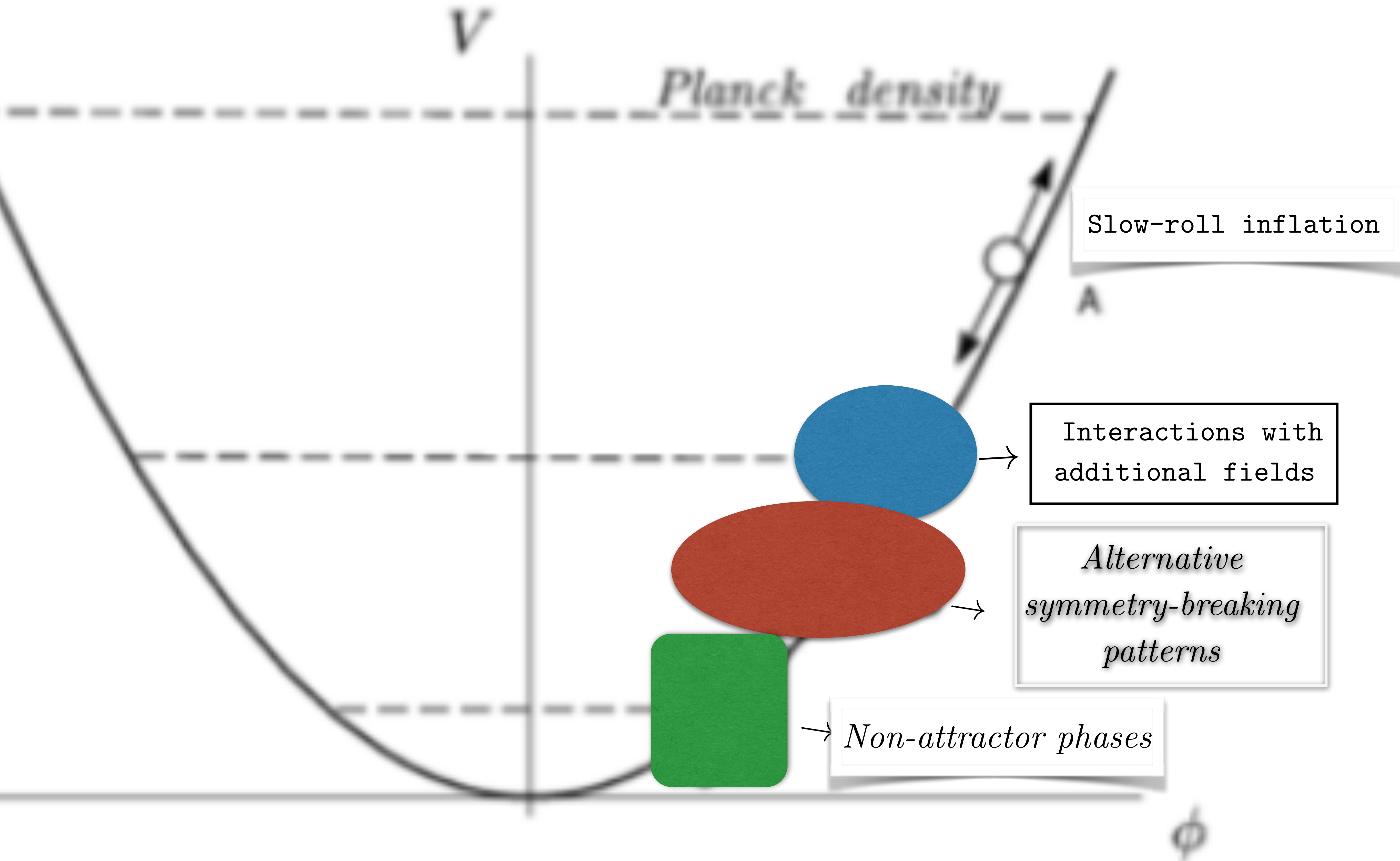
Direct detection?



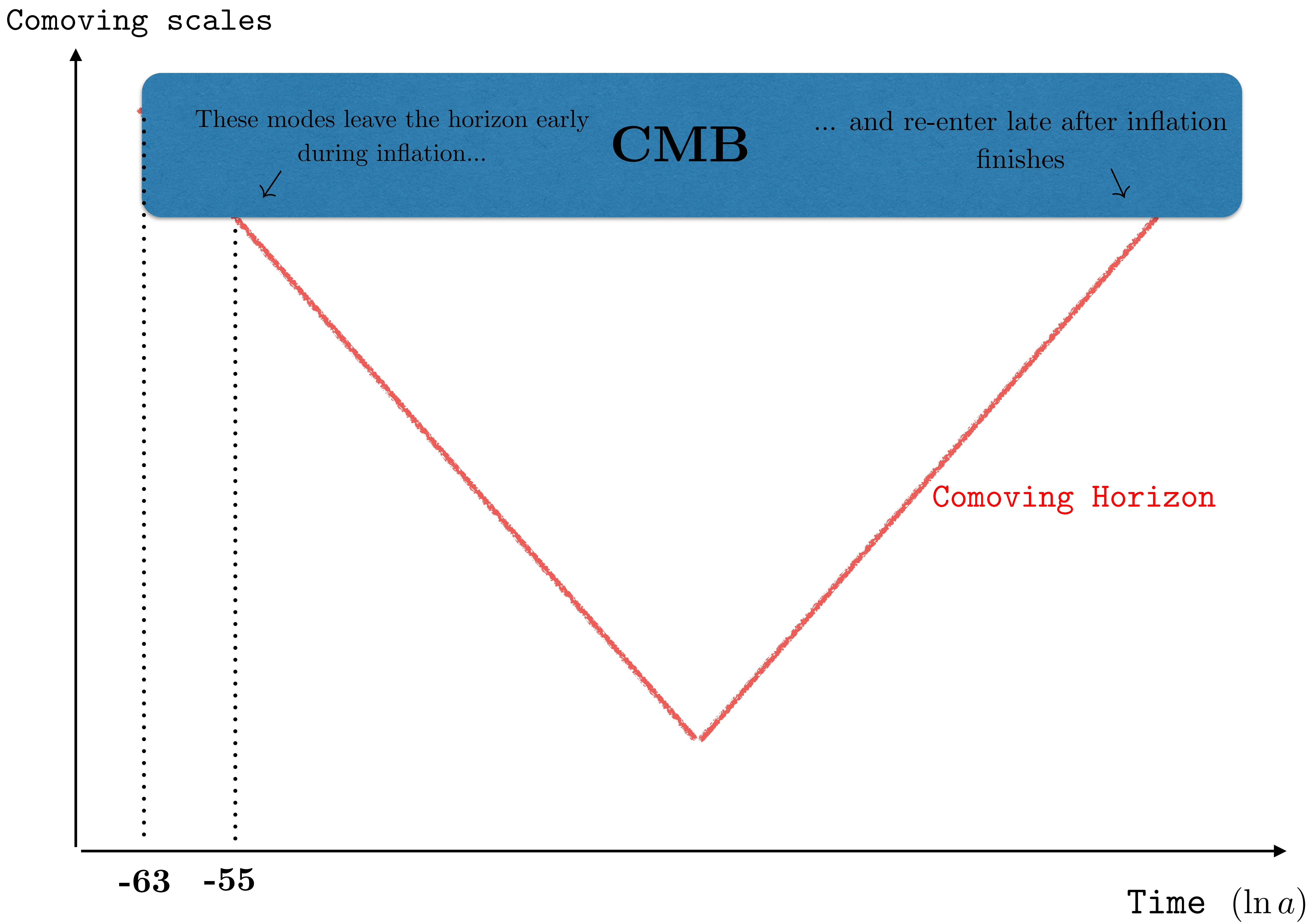
Instead of vanilla model...



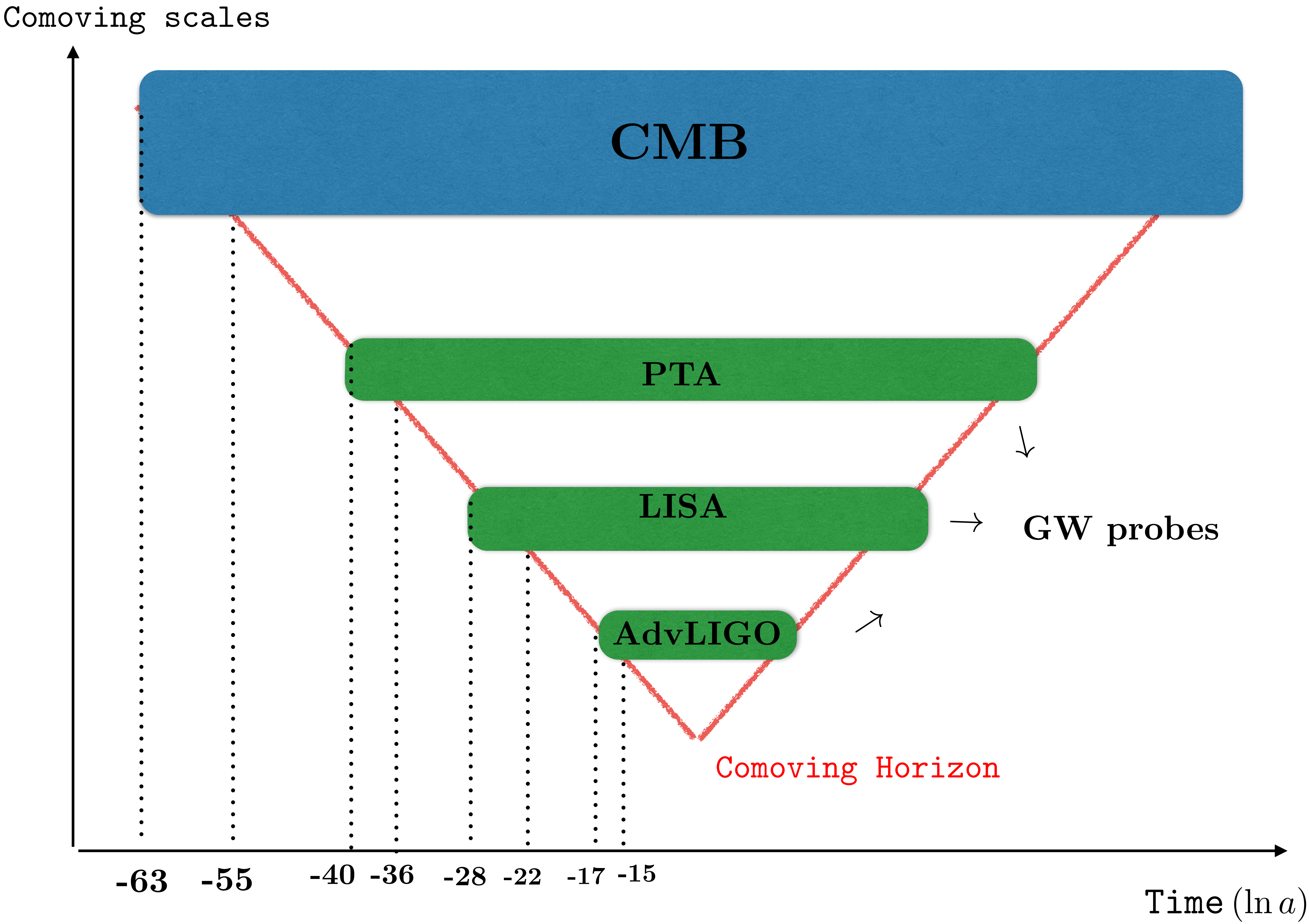
...we could have a richer dynamics after an initial phase of slow-roll...



V
Planck density
Slow-roll inflation
A
Interactions with additional fields
Alternative symmetry-breaking patterns
Non-attractor phases
φ



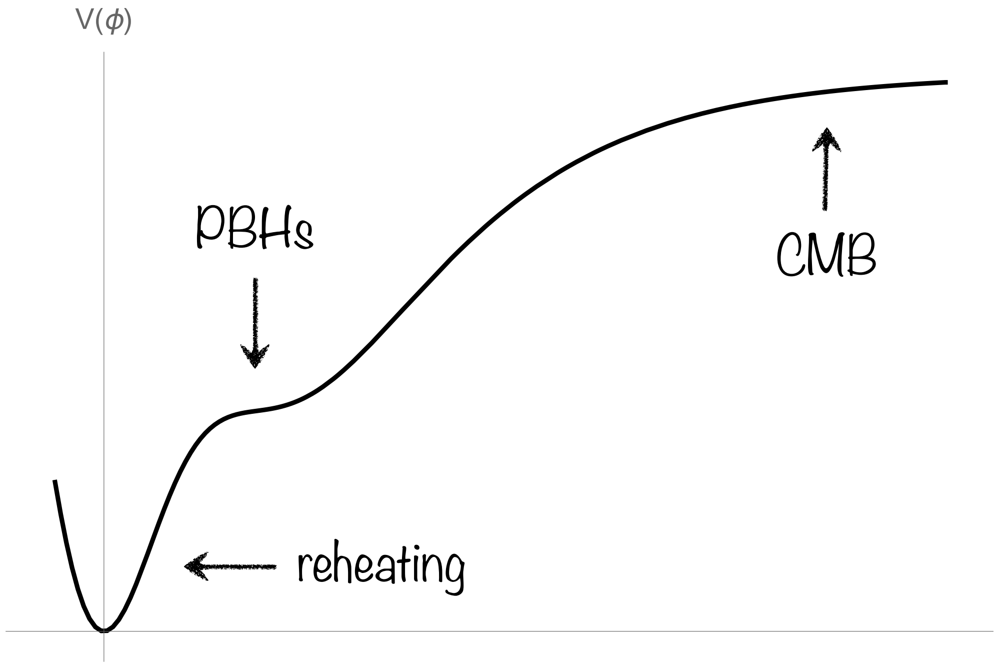
Comoving scales
These modes leave the horizon early
during inflation...
CMB
... and re-enter late after inflation
finishes
Comoving Horizon
-63
-55
Time $(\ln a)$

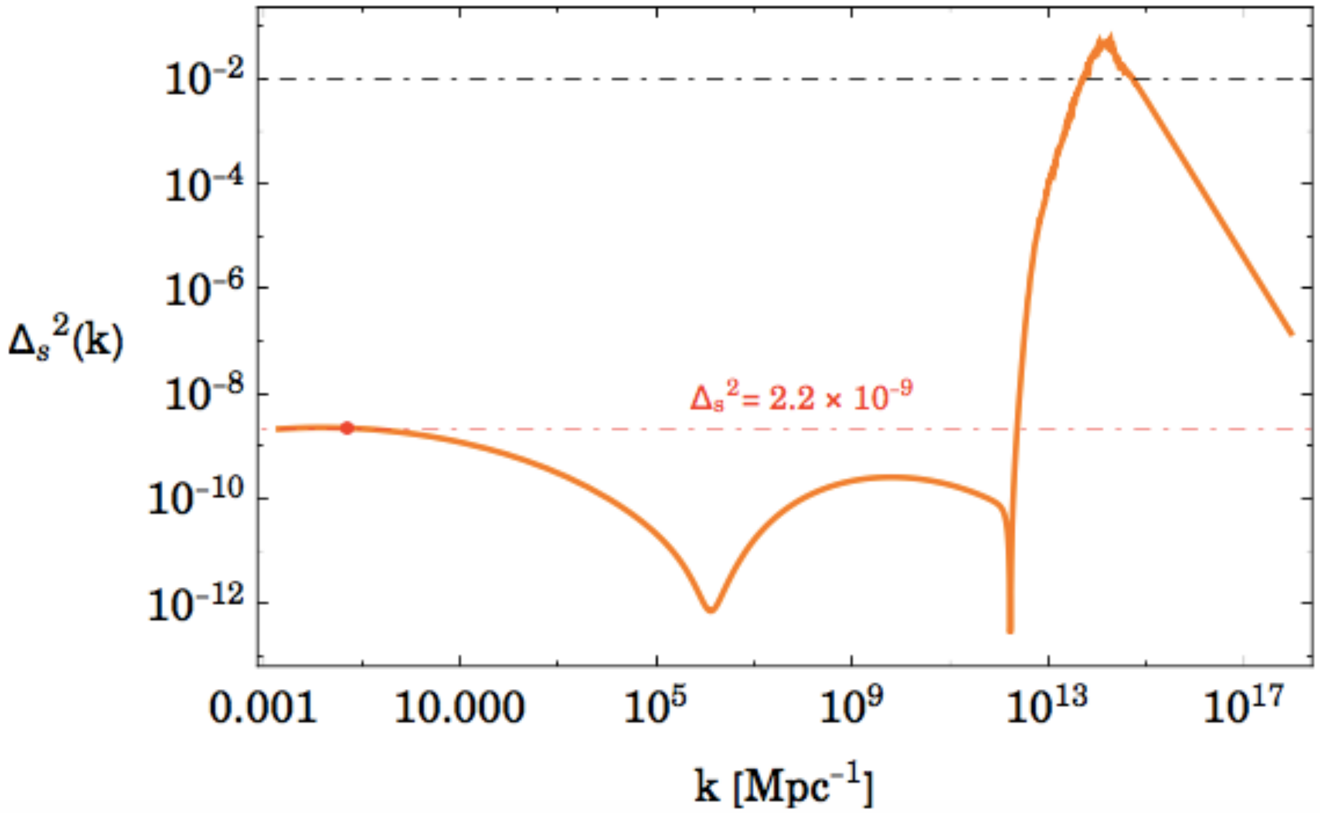


Comoving scales
CMB
PTA
LISA
GW probes
AdvLIGO
Comoving Horizon
-63
-55
-40
-36
-28
-22
-17
-15
Time $(\ln a)$

Are there additional motivations for examining models enhancing fluctuations at small scales?

Yes, formation of Primordial Black Holes





Theory

`Survey of scenarios amplifying tensor modes at small scales`

Phenomenology

`Smoking gun observables for cosmological origin of GW signal`

...paying special emphasis on distinctive features to distinguish them from astrophysical backgrounds

Mechanisms to enhance primordial GWs at small scales

We need to change the evolution eq for primordial tensor modes in inflationary background

$$\boxed{\ddot{h}_{ij}(t,\,\vec{k}) + 3H\,\dot{h}_{ij}(t,\,\vec{k}) + \frac{k^2}{a^2}\,h_{ij}(t,\,\vec{k}) \;=\; 0}$$

1. Include a source term

$$\ddot{h}_{ij}\left(\mathbf{k},t\right) + 3H\,\dot{h}_{ij}\left(\mathbf{k},t\right) + \frac{k^2}{a^2}\,h_{ij}\left(\mathbf{k},t\right) = \boxed{\frac{2}{M_{\mathrm{Pl}}^2}\Pi_{ij}^{TT}\left(\mathbf{k},t\right)}$$

Mechanisms to enhance primordial GWs at small scales

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$$\boxed{\ddot{h}_{ij}(t,\,\vec{k}) + 3H\,\dot{h}_{ij}(t,\,\vec{k}) + \frac{k^2}{a^2}\,h_{ij}(t,\,\vec{k}) \;=\; 0}$$

1. Include a source term

2. Include a ‘mass term’ (more in general, potential)
   leading to alternative symmetry-breaking patterns

$$\ddot{h}_{ij}(t,\,\vec{k}) + 3H\,\dot{h}_{ij}(t,\,\vec{k}) + \frac{k^2}{a^2}\,h_{ij}(t,\,\vec{k}) \boxed{+\, m^2 h_{ij}(t,\,\vec{k})} = \;0$$

$\Downarrow$

can induce blue spectrum

Mechanisms to enhance primordial GWs at small scales

We need to change the evolution eq for primordial tensor modes in inflationary background

$$\boxed{\ddot{h}_{ij}(t,\,\vec{k}) + 3H\,\dot{h}_{ij}(t,\,\vec{k}) + \frac{k^2}{a^2}\,h_{ij}(t,\,\vec{k}) \;=\; 0}$$

1. Include a source term
2. Include a 'mass term' (more in general, potential)
3. Modify kinetic terms leading to non-attractor eras

$$\ddot{h}_{ij}(t,\,\vec{k}) + 3H\,\boxed{f(t)}\,\dot{h}_{ij}(t,\,\vec{k}) + \boxed{c_T^2(t)}\,\frac{k^2}{a^2}\,h_{ij}(t,\,\vec{k}) \;=\; 0$$

First possibility: source terms

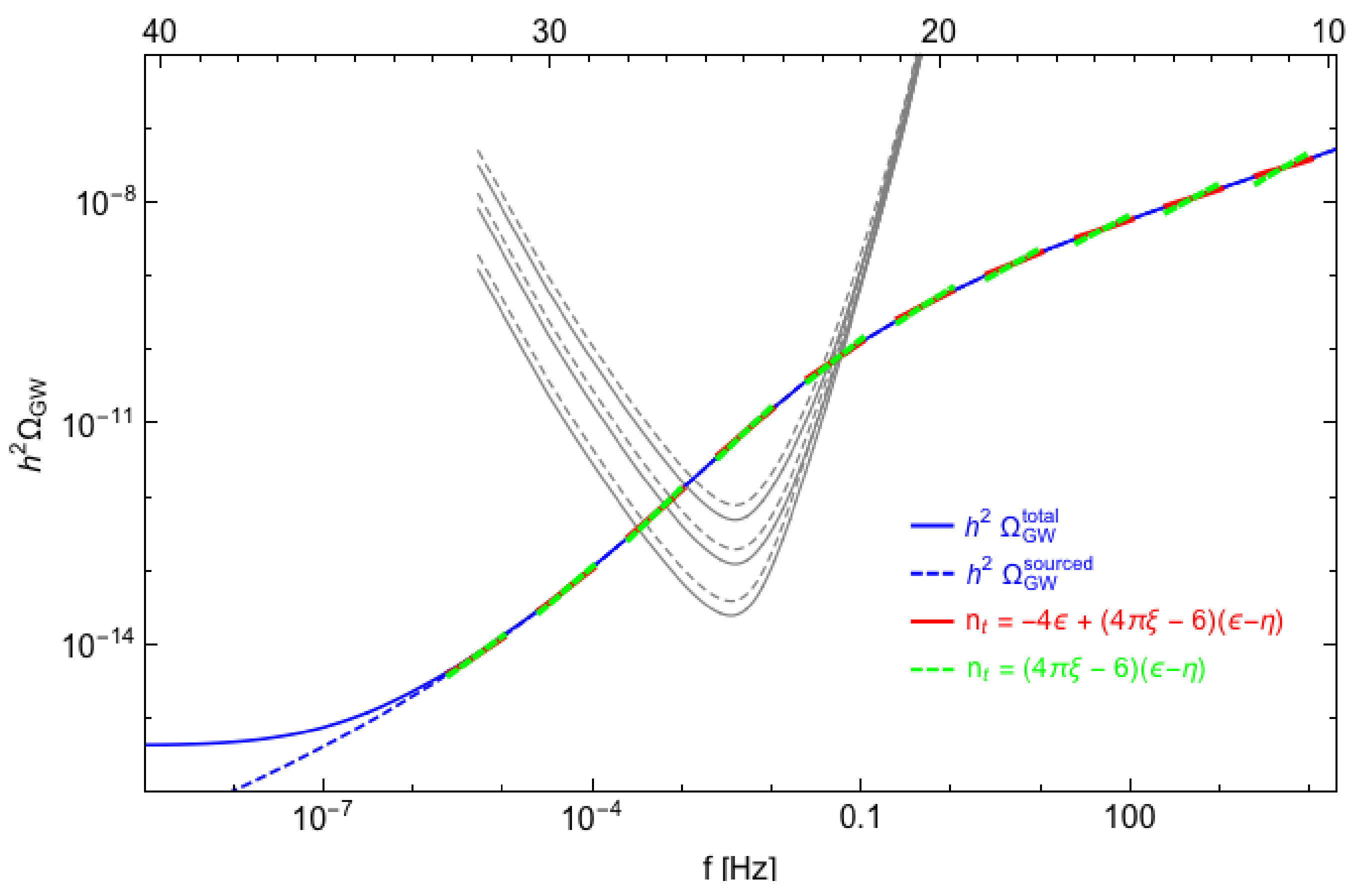
Paradigmatic example:

$$\phi\, F\, \tilde{F}$$

Transient instability of vectors feed tensor modes through anisotropic stress

$$\ddot{h}_{ij}\left(\mathbf{k},t\right) + 3H\,\dot{h}_{ij}\left(\mathbf{k},t\right) + k^2\,h_{ij}\left(\mathbf{k},t\right) = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{TT}\left(\mathbf{k},t\right)$$

↘ Model dependent



- Chiral spectrum
  L/R modes have different amplitude

LISA Cosmology Working Group

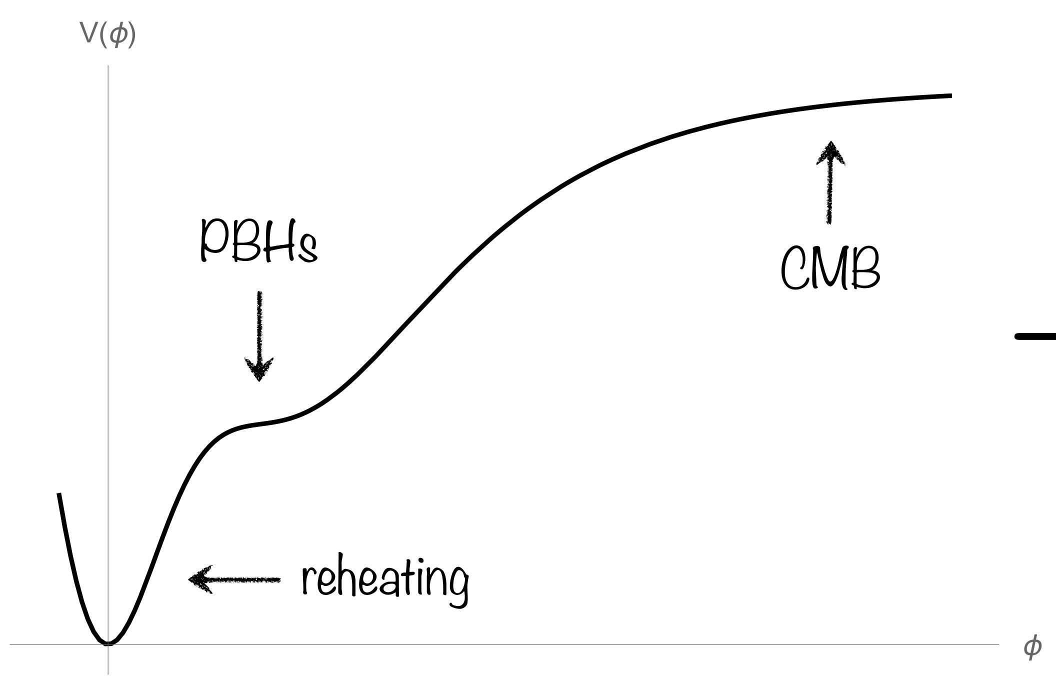
astro-ph.CO 1610.0648

First possibility: source terms

Secondary GWs produced during radiation domination

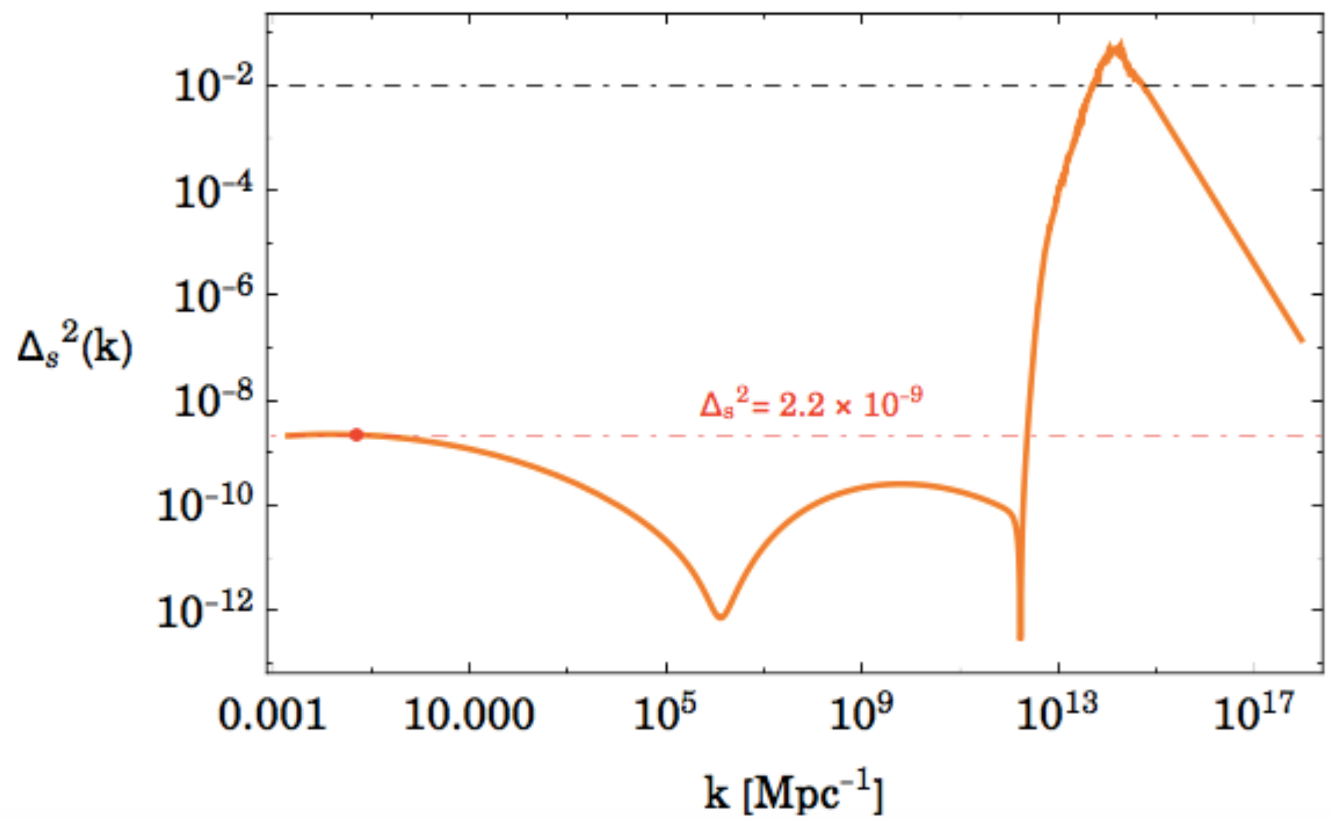
Tensor fluctuations $\Leftarrow$ (Scalar Fluctuations)^2

$\uparrow$



$\rightarrow$

Scalar power spectrum



[Kinney et al], [Garcia-Bellido et al]

First possibilty: interactions among fields during inflation

$\downarrow$

Secondary GWs produced during radiation domination

`Transient instabilities in scalar/vector sectors` **feed** `tensor modes at small scales`

$$h''(\boldsymbol{k},\eta) + \frac{2}{\eta} h'(\boldsymbol{k},\eta) + k^2 h(\boldsymbol{k},\eta) = \mathcal{S}(\boldsymbol{k},\eta),$$

[Ananda et al, Baumann et al]

$$\mathcal{S}(\boldsymbol{k},\tilde{\eta}) \;=\; \frac{q^{ij}(\boldsymbol{k})}{(2\pi)^{3/2}} \int d^3\tilde{k}\; \tilde{k}_i \tilde{k}_j \; \left\{ 12\Phi(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta})\Phi(\tilde{\boldsymbol{k}},\tilde{\eta}) + \left[\tilde{\eta}\Phi(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta}) + \frac{\tilde{\eta}^2}{2}\Phi'(\boldsymbol{k}-\tilde{\boldsymbol{k}},\tilde{\eta}) \right] \Phi'(\tilde{\boldsymbol{k}},\tilde{\eta}) \right\}.$$

Echoes of GWs from PBH production

Secondary GWs produced during radiation domination

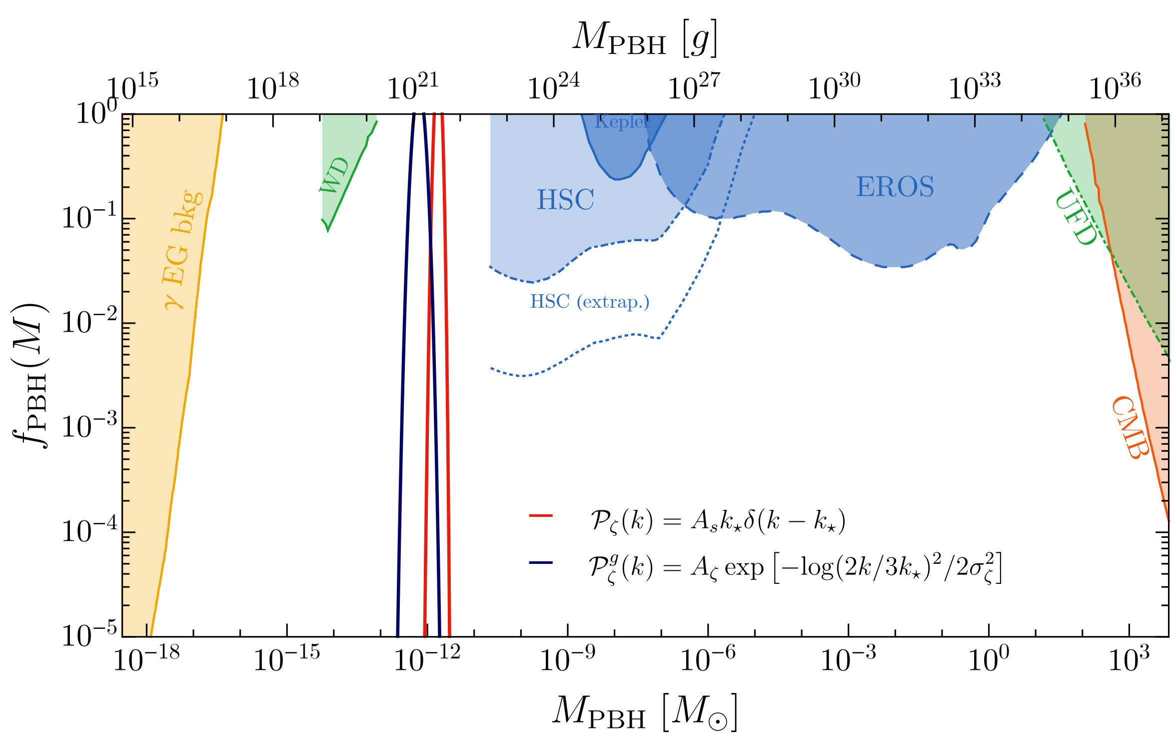
Transient instabilities in scalar/vector sectors **feed** tensor modes at small scales

Echoes of GWs from PBH production

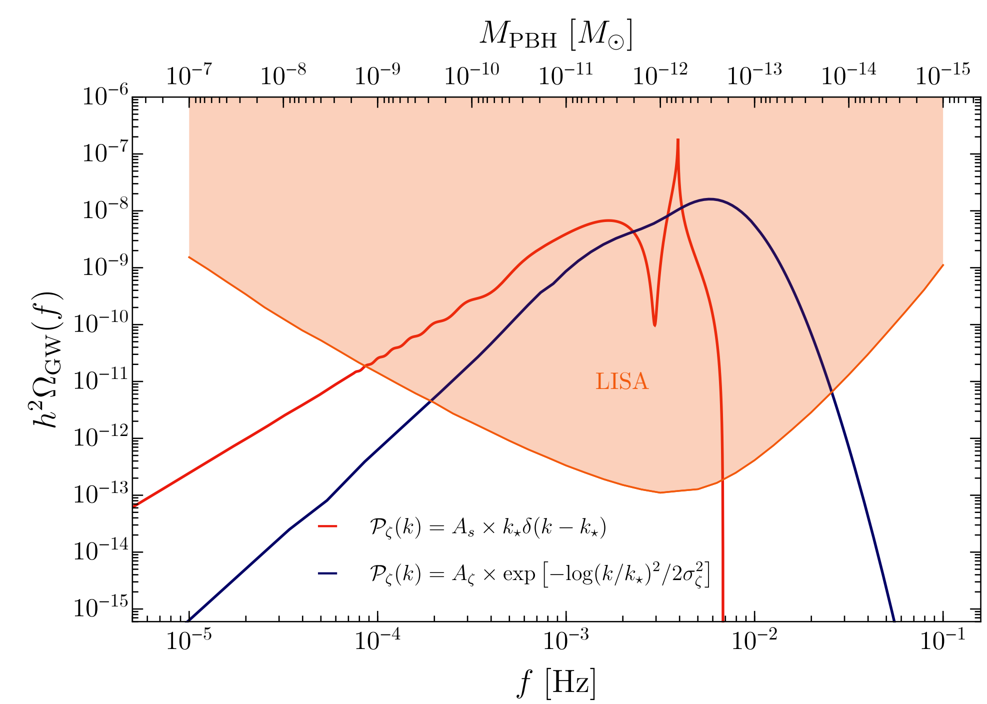
[Saito,Yokoyama]... [Bartolo et al]

$$\mathcal{P}_\psi(f) = \frac{\mathcal{A}}{\sqrt{2\pi}\Delta} \exp\left\{ -\frac{[\ln(f/f_\star)]^2}{2\Delta^2} \right\}$$

E.g. Lognormal spectrum
[Pi, Sasaki]

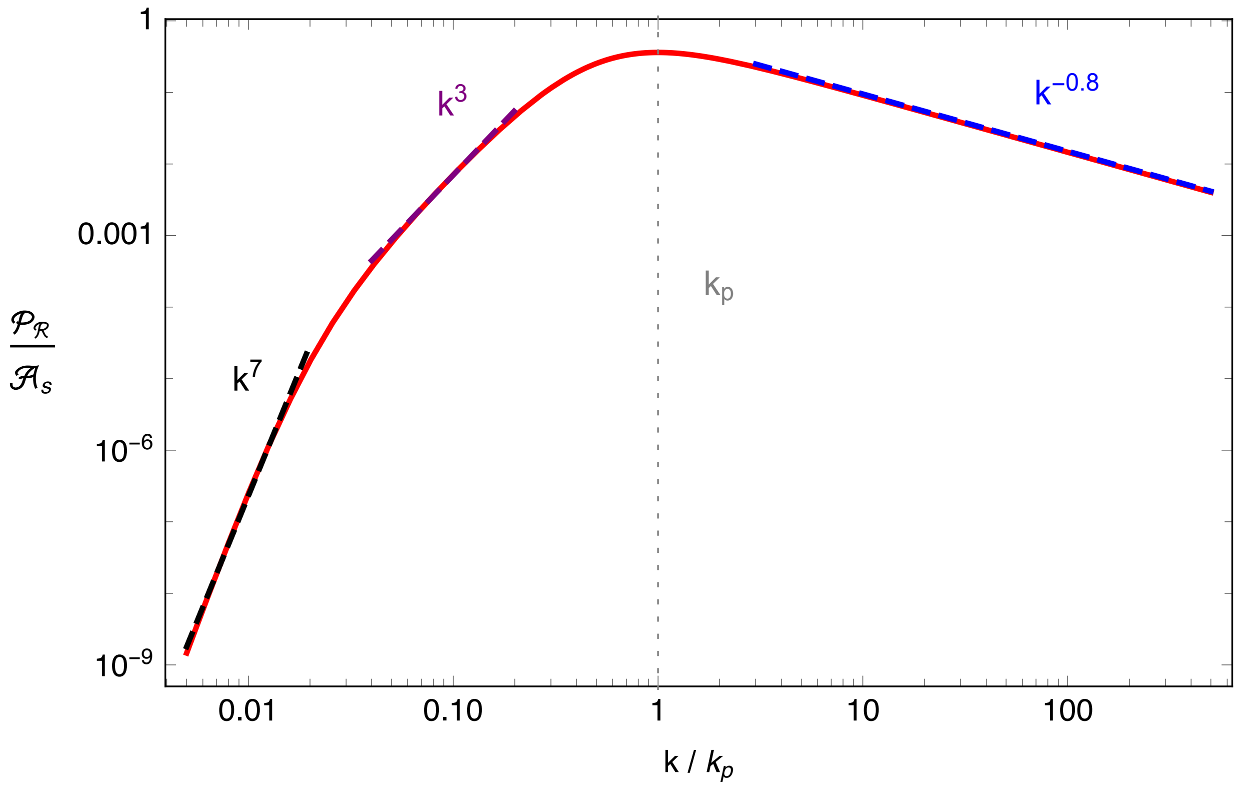


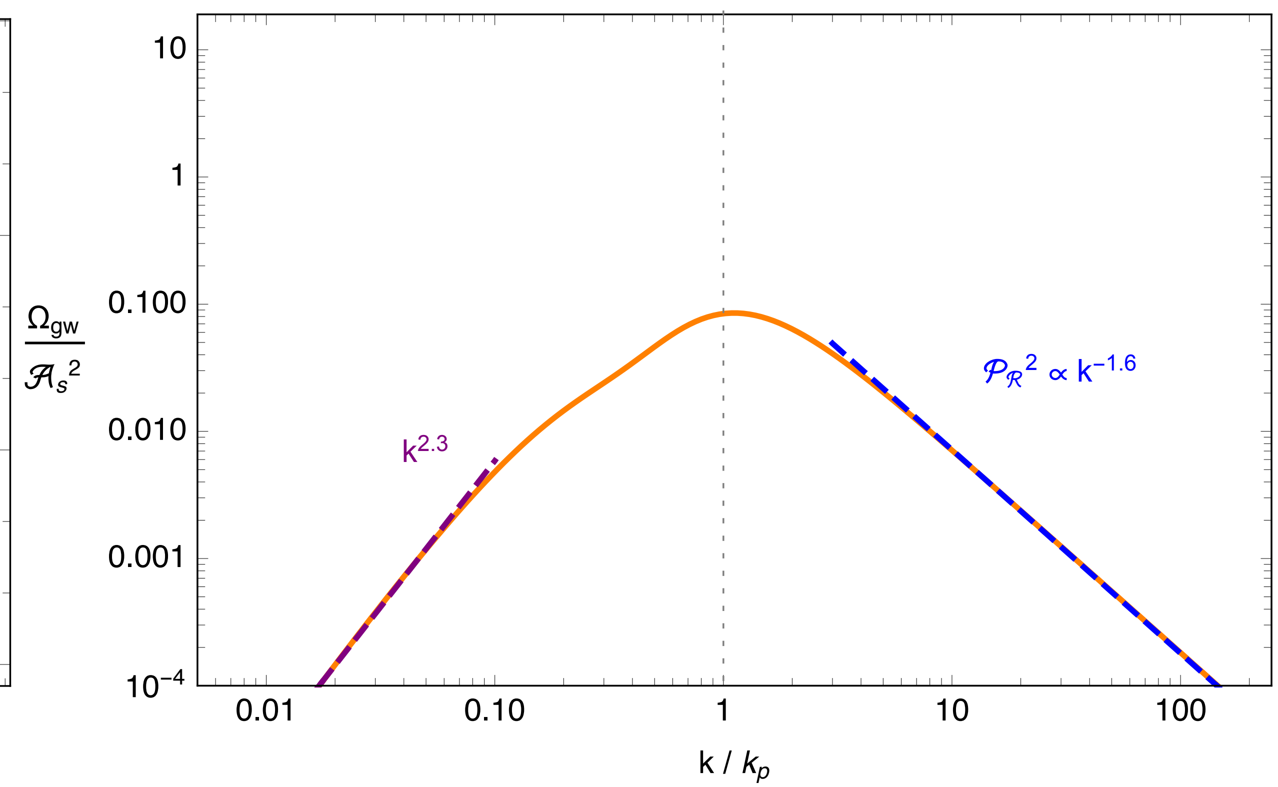
PBH production



Induced SGWB
at LISA frequencies

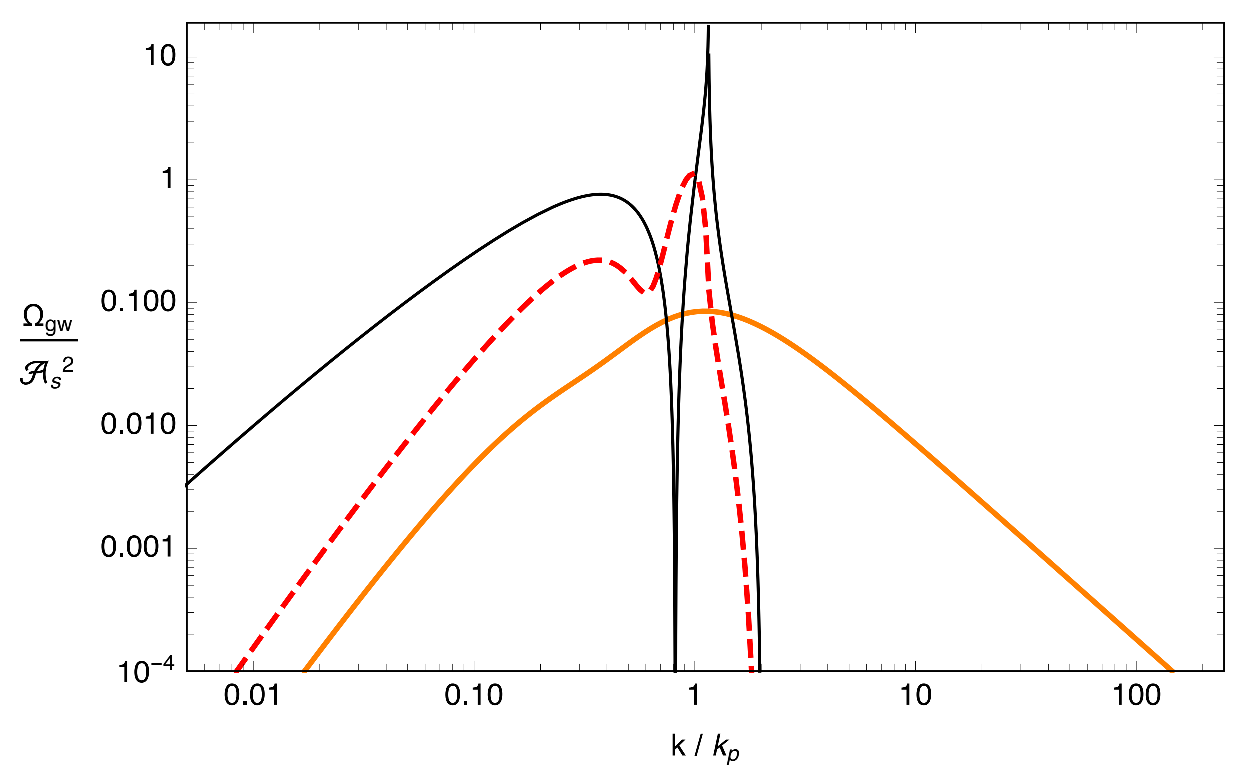
The amplitude of SGWB has a characteristic profile as function of the frequency, which also depends on the scalar spectrum that sources it





Detailed analysis
in single-field inflation

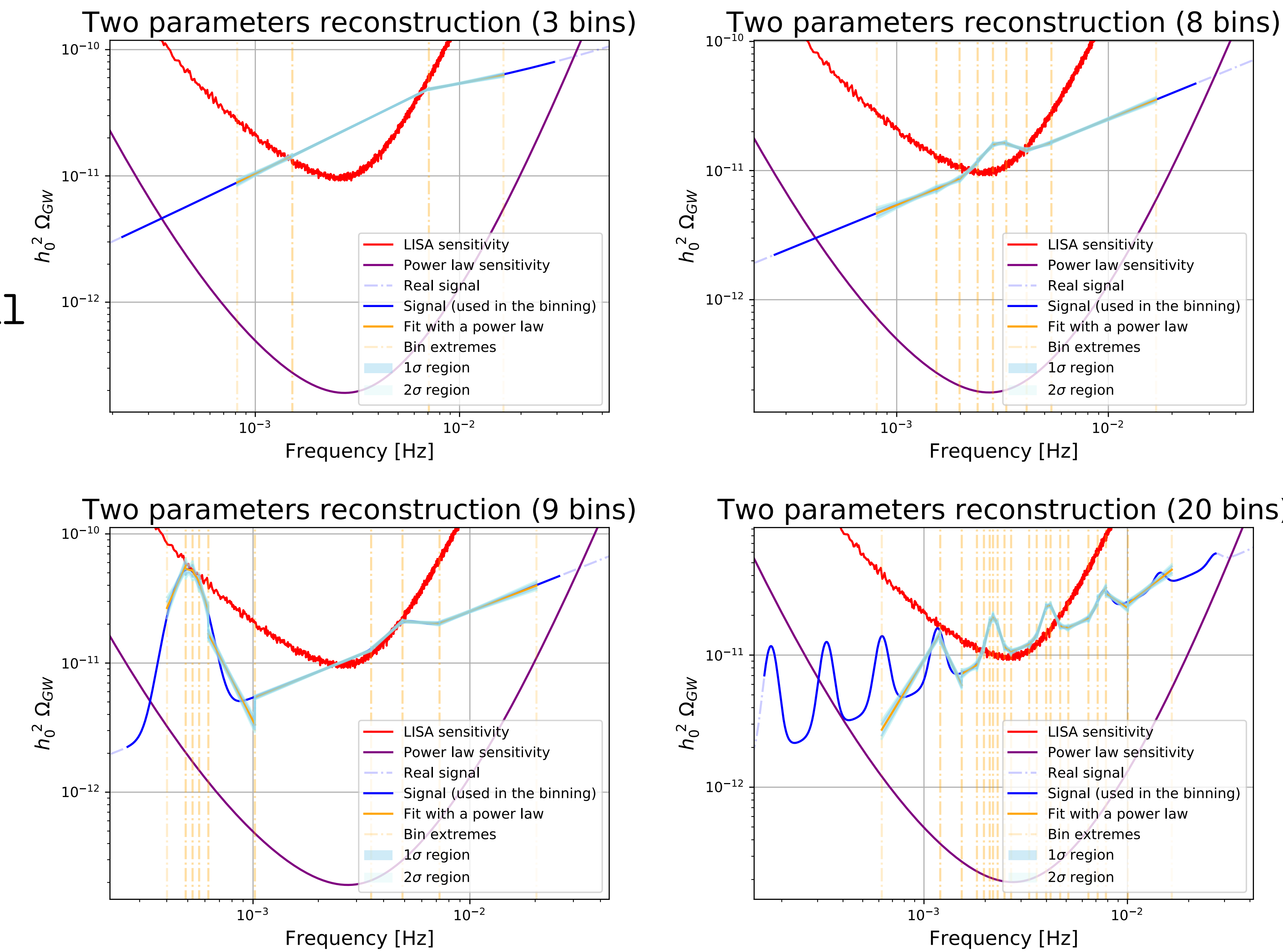
[Byrnes et al]
[Cai et al]
[Özsoy et al]



How to distinguish among different SGWB profiles?

GWbinner code: divide LISA band in small intervals reconstructing signal in each bin

arXiv:1906.09244



building from notion of Power-Law Sensitivity Curve

[Thrane, Romano]

The SGWB is anisotropic, and non-Gaussian

[Contaldi], [Bartolo et al][3]

$$\delta_{\rm GW} \equiv \frac{\omega_{\rm GW}(\vec{x},\, q,\, \hat{n}) - \bar{\Omega}_{\rm GW}(q)}{\bar{\Omega}_{\rm GW}(q)} = \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}\,(q)}{\partial \ln\, q}\right] \Gamma\,(\eta_0,\, \vec{x},\, q,\, \hat{n})$$

Anisotropies of SGWB

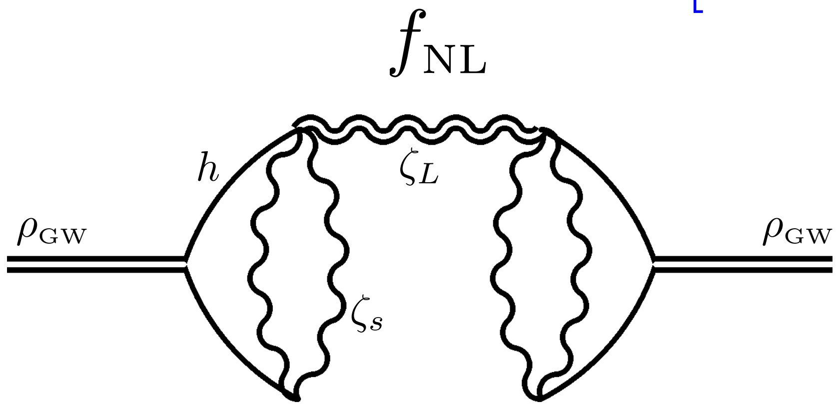
Primordial origin
(for example, induced by primordial nonG)

$$\Gamma\left(\eta,\, \vec{k},\, q,\, \hat{n}\right) = {\rm e}^{ik\mu(\eta_{\rm in}-\eta)}\, \Gamma\left(\eta_{\rm in},\, \vec{k},\, q,\, \hat{n}\right) + \int_{\eta_{\rm in}}^{\eta} d\eta'\, {\rm e}^{ik\mu(\eta'-\eta)} \left[\frac{d\Psi\left(\eta',\, \vec{k}\right)}{d\eta'} - ik\mu\Phi\left(\eta',\, \vec{k}\right) - \frac{1}{2} n^i n^j \frac{\partial \chi_{ij}\left(\eta',\, \vec{k}\right)}{\partial \eta'}\right] .$$

Induced by propagation effects
in a perturbed background

The SGWB is anisotropic, and non-Gaussian

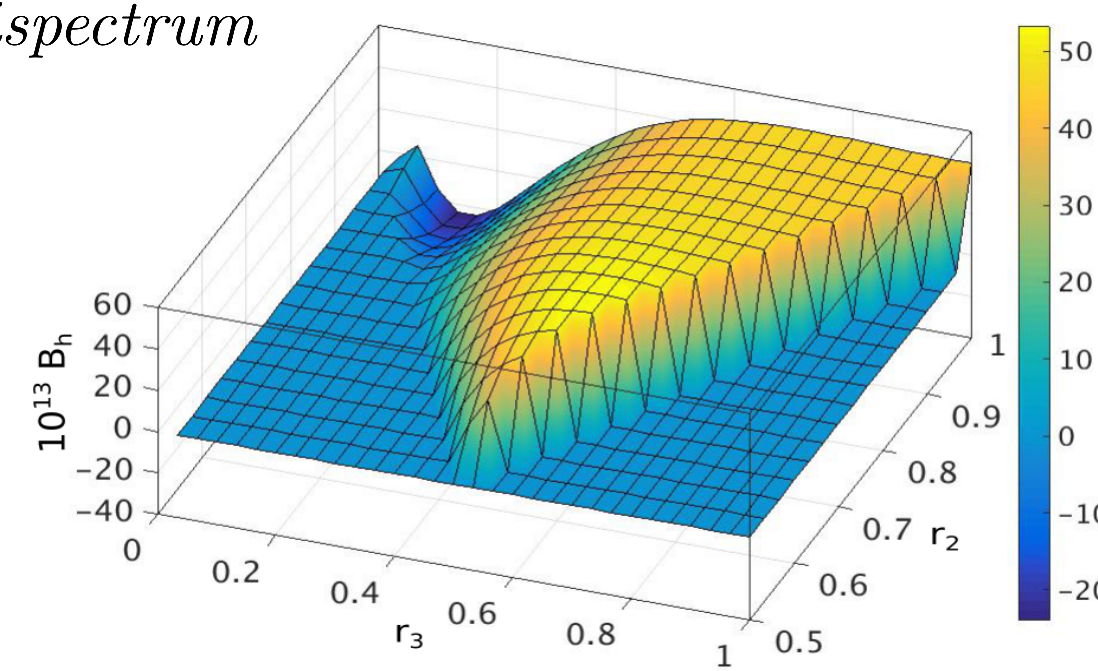
[Contaldi], [Bartolo et al][3]



2-pt function for the SGWB energy density

$$\left\langle \Gamma_{\ell_1 m_1, I+S}\left(k\right) \Gamma^*_{\ell_2 m_2, I+S}\left(k\right)\right\rangle = \delta_{\ell_1 \ell_2} \delta_{m_1 m_2} C_{\ell, I+S}\left(k\right)$$

3-pt function: equilateral shape for GW bispectrum



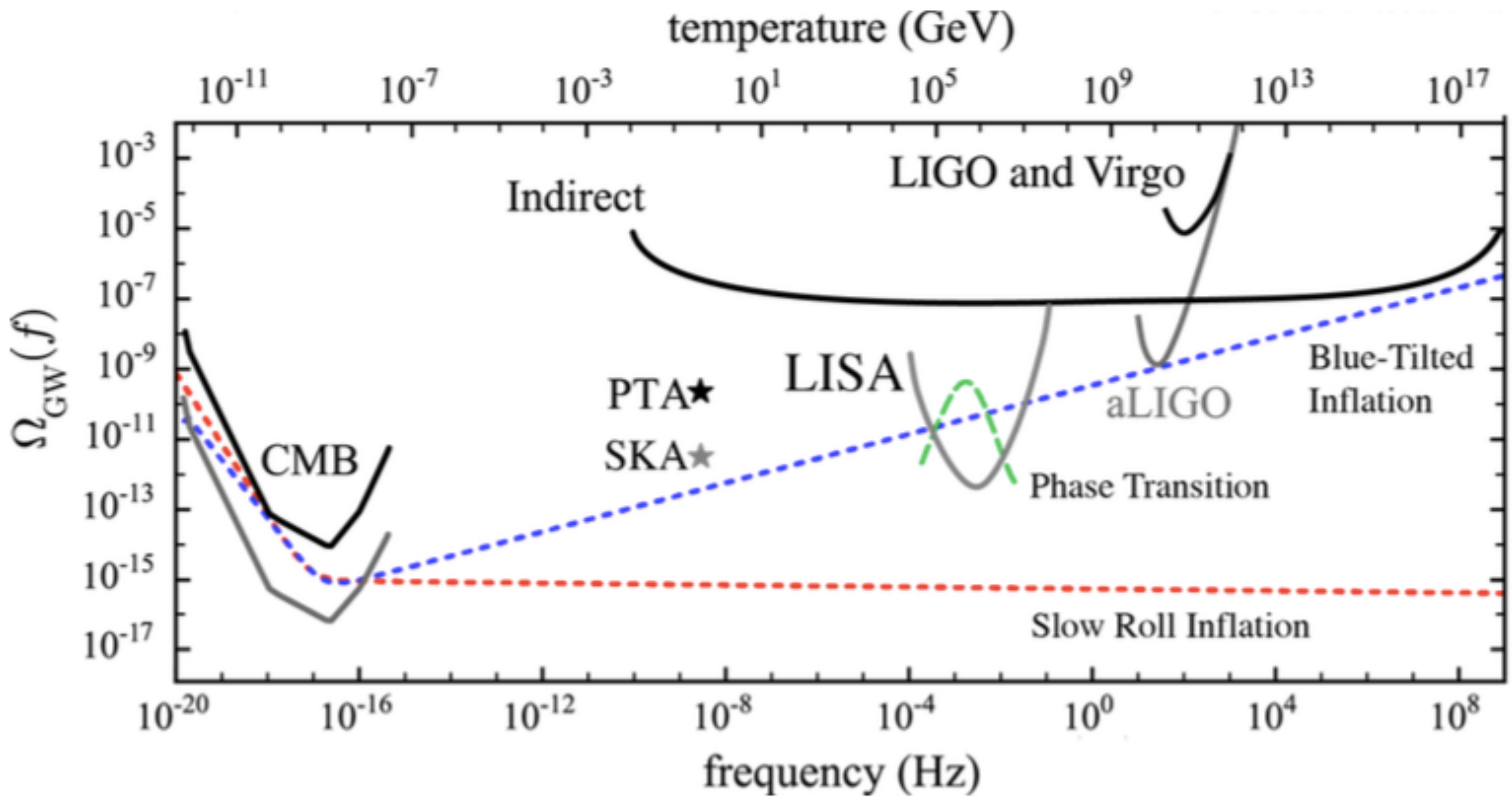
Second possibility: break space-time symmetries during inflation

$$\cancel{t \to t + \xi(t,x)}\,, \qquad \cancel{x^i \to x^i + \xi(t,x)}$$

$$S^T = \frac{1}{4}\int dt\, d^3x\, a^3\, M_{Pl}^2 \left[\frac{1}{2}\,\dot{h}_{ij}^2 - \frac{1}{2\,a^2}\,(\partial_k\, h_{ij})^2 - \frac{m^2}{2}\,h_{ij}^2\right]$$

Allowed by (absence) of symmetries
in the system

$n_T > 0$:
blue spectrum of tensor modes (amplitude increase at small scales)

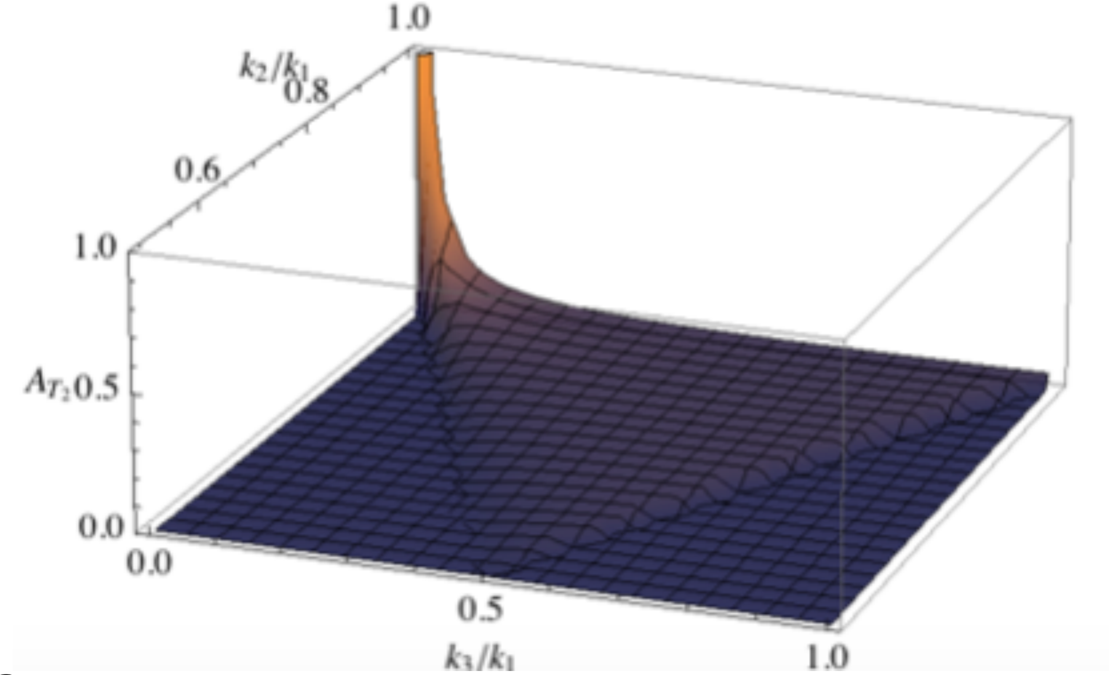

temperature (GeV)
10^-11 10^-7 10^-3 10^1 10^5 10^9 10^13 10^17
Ω_GW(f)
10^-3 10^-5 10^-7 10^-9 10^-11 10^-13 10^-15 10^-17
Indirect
LIGO and Virgo
LISA
PTA★
SKA★
CMB
aLIGO
Blue-Tilted Inflation
Phase Transition
Slow Roll Inflation
10^-20 10^-16 10^-12 10^-8 10^-4 10^0 10^4 10^8
frequency (Hz)

Caldwell

The SGWB is anisotropic, and non-Gaussian

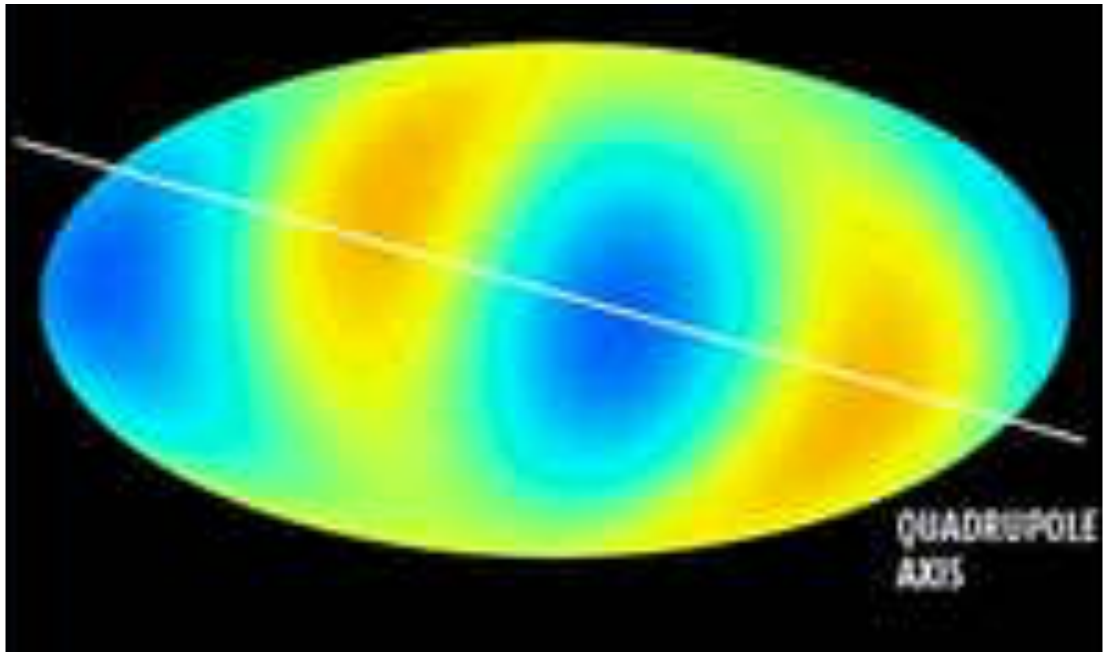
[Ricciardone, GT]

[Dimastrogiovanni et al]

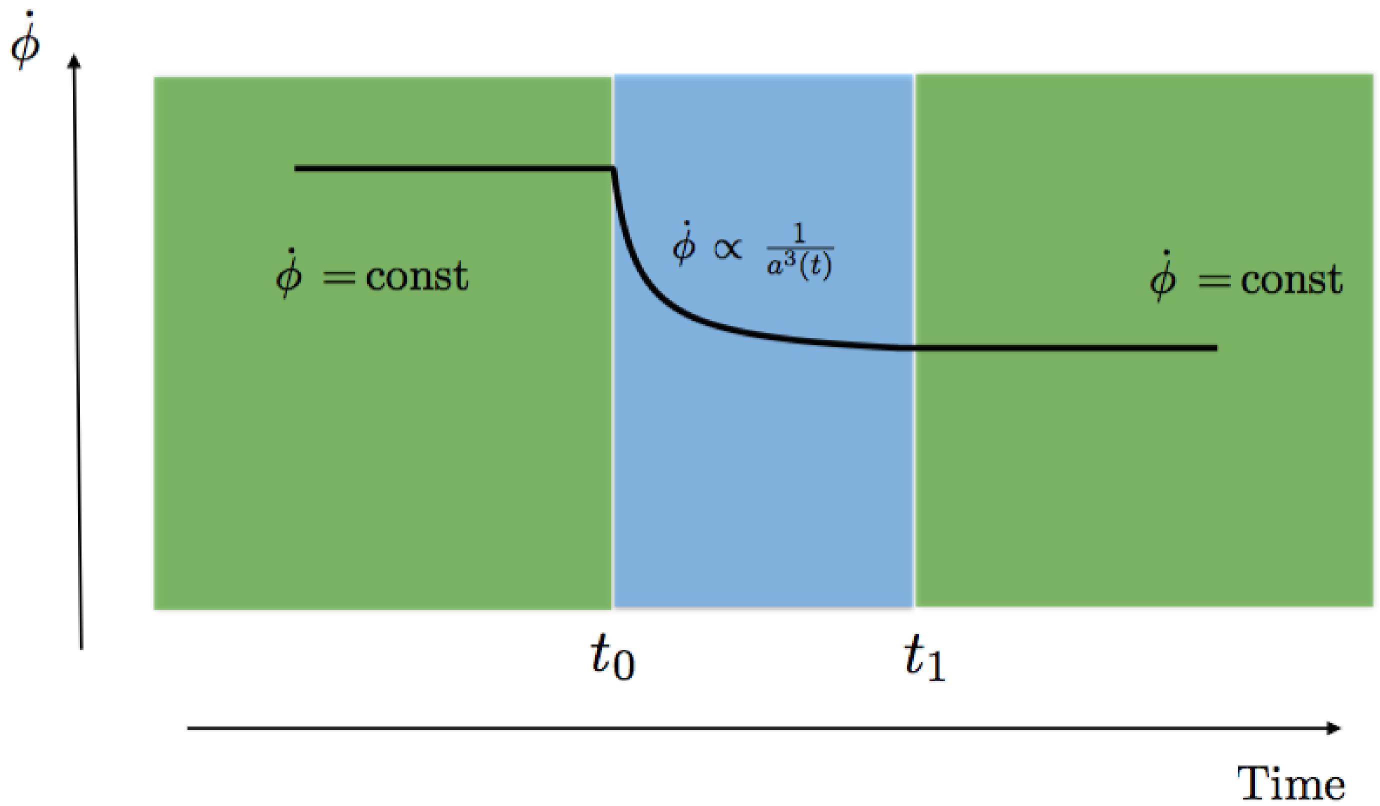


GW 3-pt function enhanced in squeezed configurations

...leads to quadrupolar anisotropy in the GW 2-pt function



Third possibilty: strong (but brief) violation of slow-roll conditions



Single-field inflation

$$
\begin{aligned}
\mathcal{L}_{\rm tot} &= \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\,, \\
\mathcal{L}_2 &= G_2\,, \\
\mathcal{L}_3 &= -G_3\,\Box\phi\,, \\
\mathcal{L}_4 &= G_4\,R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]
\end{aligned}
$$

$$
\mathcal{L}_5 = G_5\,G_{\mu\nu}\,\nabla^\mu\nabla^\nu\,\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]
$$

Third possibilty: strong (but brief) violation of slow-roll conditions

General action for quadratic fluctuations

$$S_T \;=\; \frac{1}{8}\int dt\, d^3x\, a^3(t)\,\left[\mathcal{G}_T(t)\,\left(\partial_t h_{ij}\right)^2-\frac{\mathcal{F}_T(t)}{a^2(t)}\left(\vec{\nabla} h_{ij}\right)^2\right]\,,$$

- **Idea:** Determine a non-attractor regime which enhance the would be decaying mode

[Mylova et al]

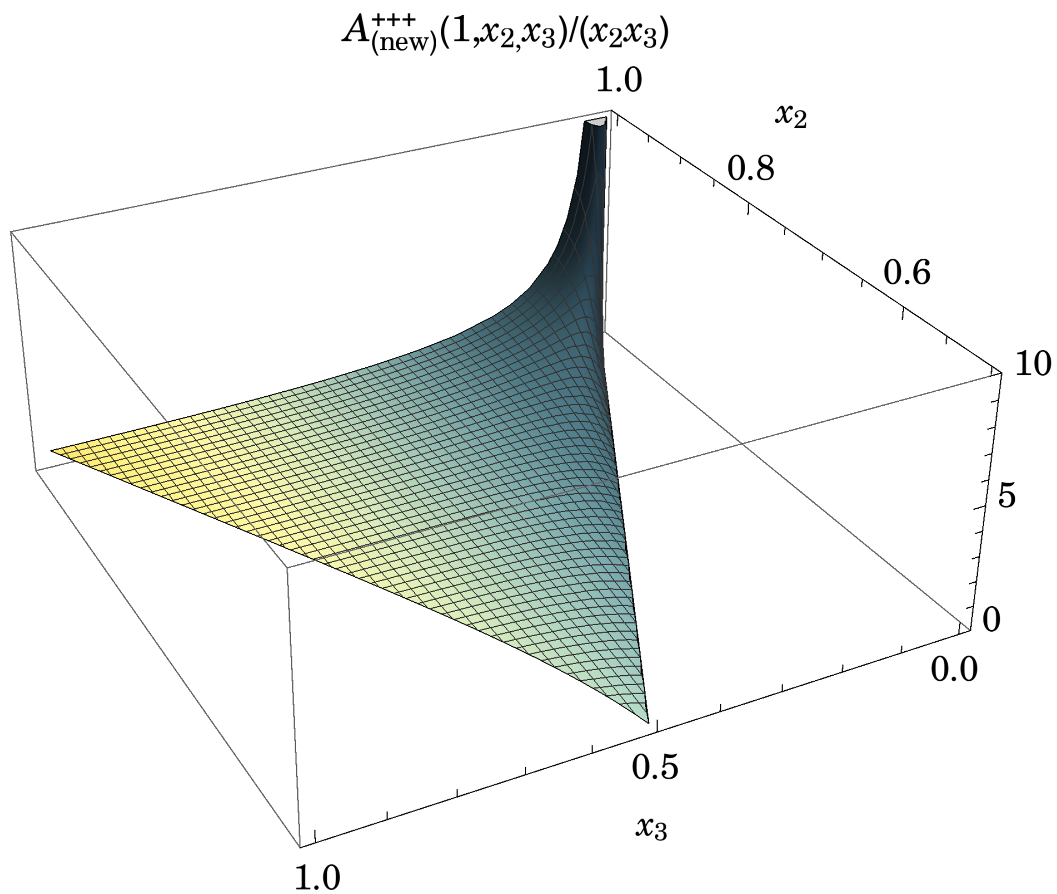
[Özsoy et al]

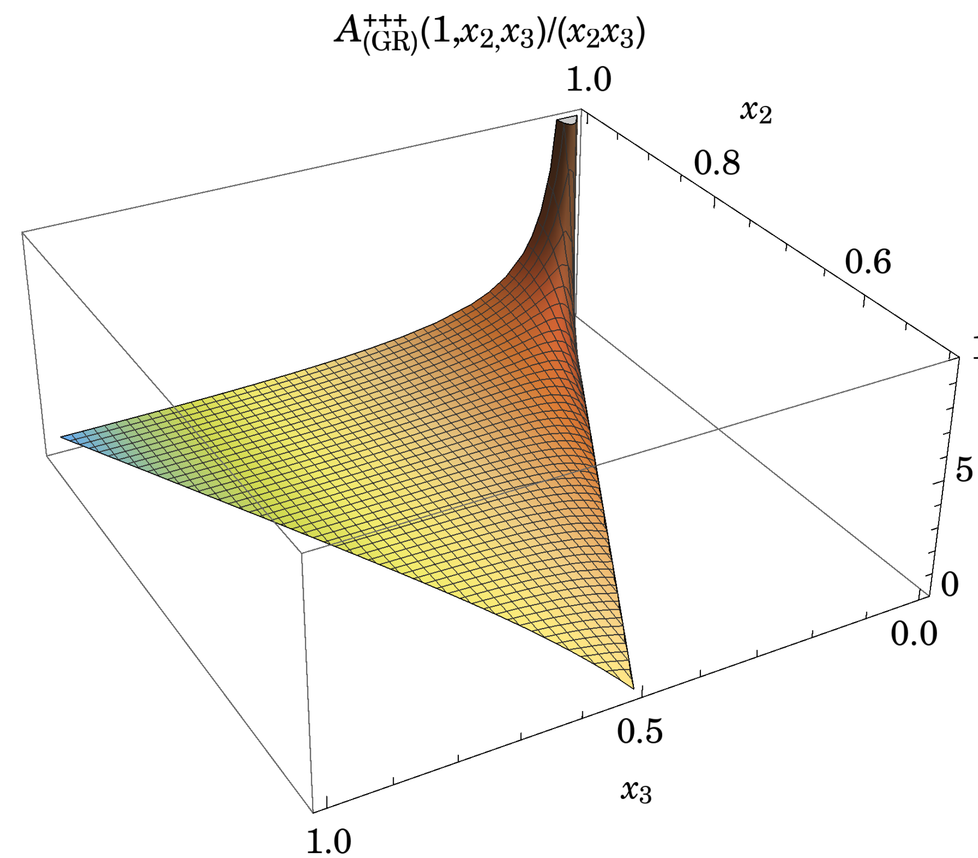
$$h_{ij} \;=\; \mathcal{C}_1+\mathcal{C}_2\int\frac{dt}{a^3\,\mathcal{F}_T\,\mathcal{G}_T}$$

$\searrow$

Decaying mode can grow if $\mathcal{F}_T\,\mathcal{G}_T$ decreases fast in time

The SGWB is anisotropic, and non-Gaussian





Parametrically enhanced sqz nG: `violates Maldacena's consistency relation`

$$S_T^{(3)} = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left[\frac{\mathcal{F}_T}{4a^2} \left(h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl} \right) \partial_k \partial_l h_{ij} + \frac{\dot{\phi} X G_{5X}}{12} \dot{h}_{ij} \dot{h}_{jk} \dot{h}_{ki} \right],$$

Moral

There are plenty of theoretical mechanisms
($\pm$ theoretically well motivated)
to enhance tensor spectrum at small scales.

Even if $r \leq 10^{-3}$ at CMB scales

...and many possible 'smoking guns' of cosmological origin of a SGWB

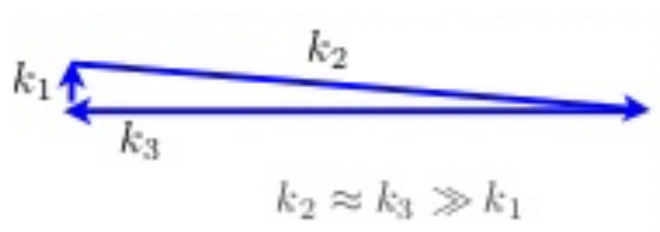
- Rich profile of GW power spectrum
- Non-Gaussian signal (`astro signal is Gaussian`)
- Chirality
- `Anisotropies` → Intrinsic
  → Induced

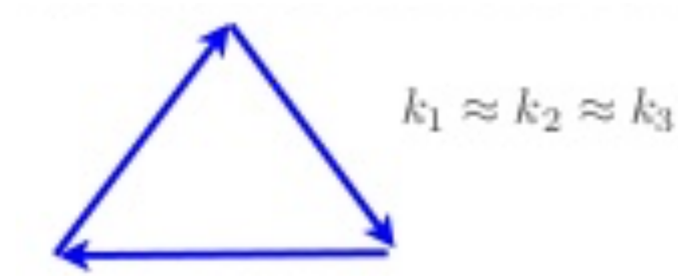
Non-Gaussianity of the SGWB

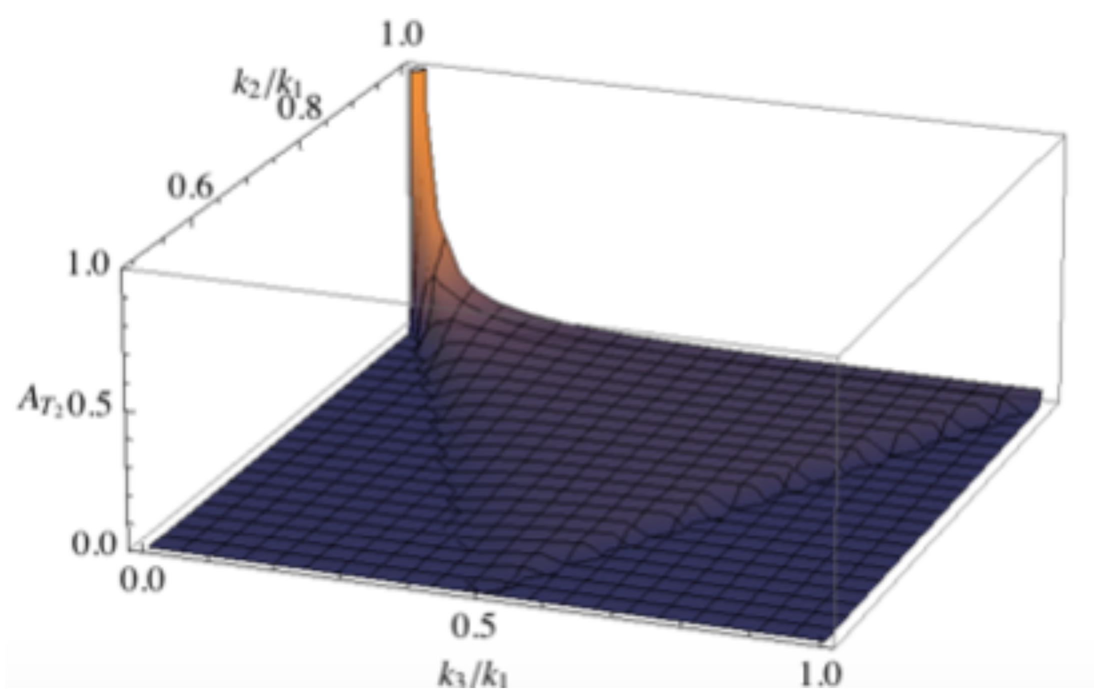
$$\langle h^{(s_1)}_{i_1 j_1}\, h^{(s_2)}_{i_2 j_2}\, h^{(s_3)}_{i_3 j_3} \rangle$$

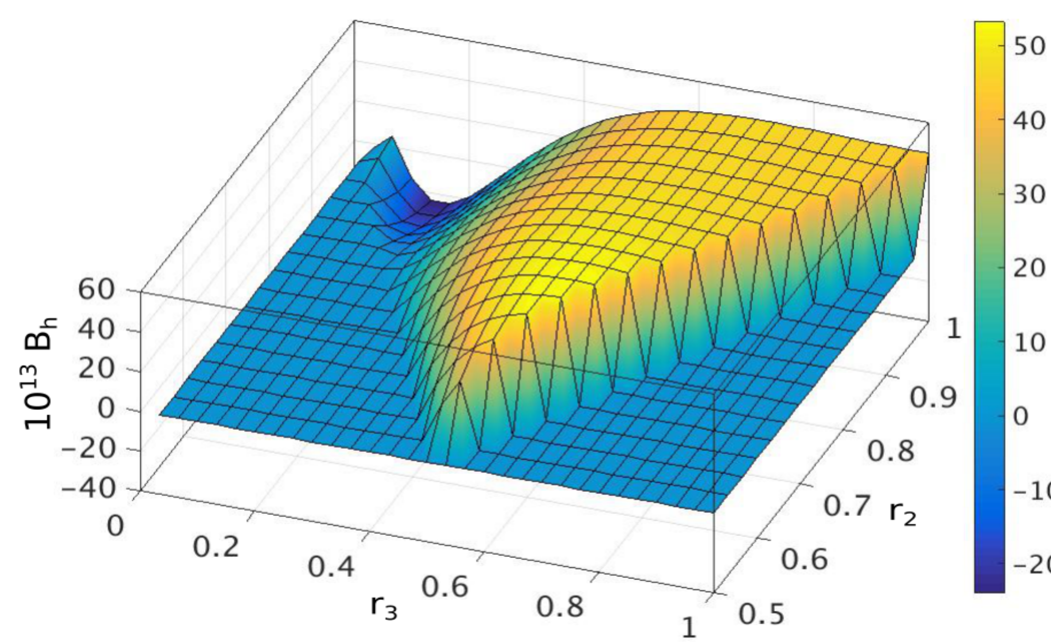
Broken
space reparameterizations

Interactions
with additional fields









...one can build 3-point response functions for interferometers...

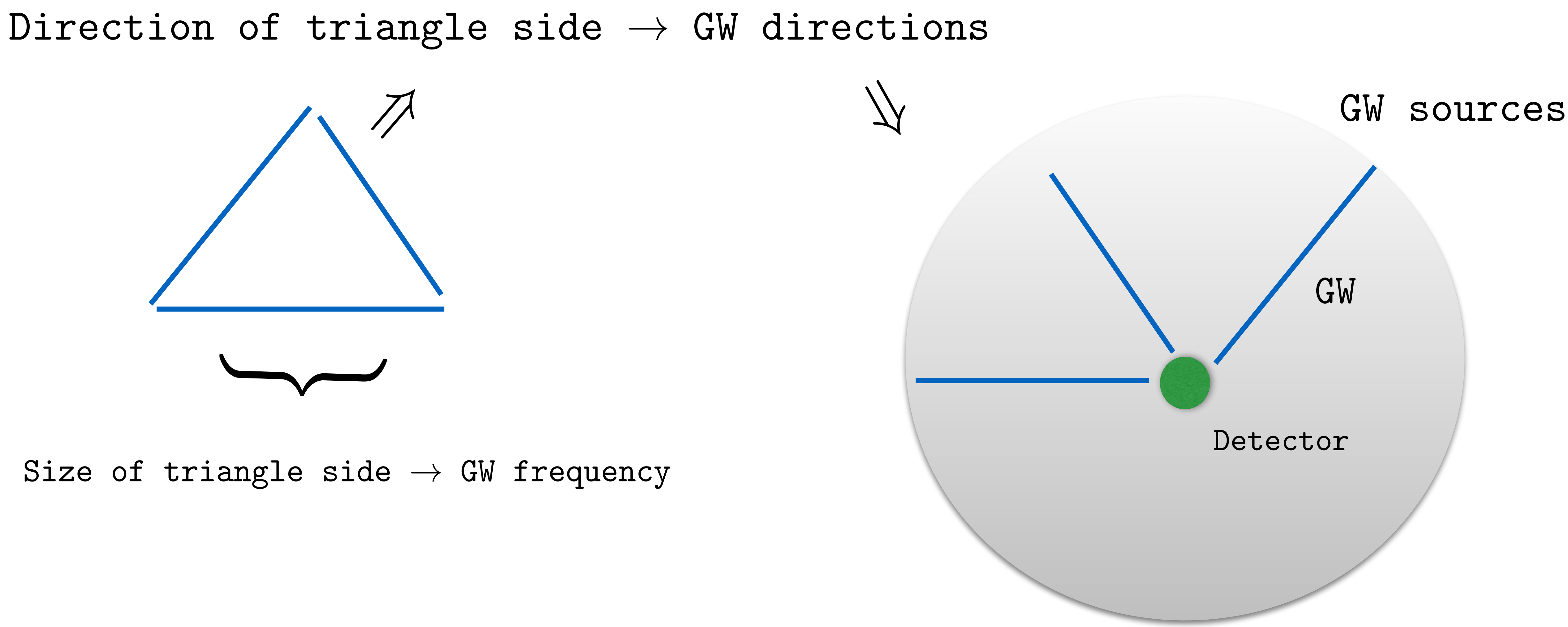
Depends on phase correlations

$\Downarrow$

$$\langle s_a(t) s_b(t) s_c(t) \rangle \Leftrightarrow R_{abc}^{(\lambda_1 \lambda_2 \lambda_3)} \langle h_{im}^{(\lambda_1)}(t) h_{mj}^{(\lambda_2)}(t) h_{ji}^{(\lambda_3)}(t) \rangle$$

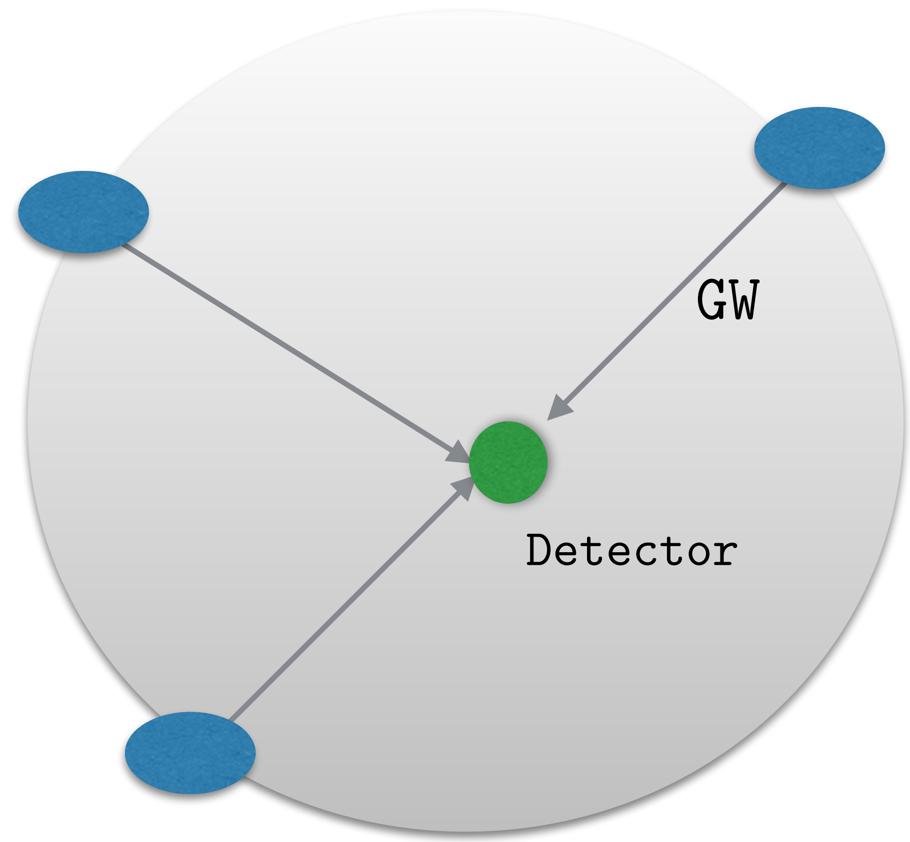
Quantifies how 3-pt correlators of the measured signal $\Leftrightarrow$ Depends on the intrinsic 3-pt correlator of GWs

Momentum conservation implies that GW momenta form a closed triangle



Problem:
GWs from different directions tend to 'Gaussianize' the signal

[Allen], [Adshead, Lim] [Bartolo et al.]



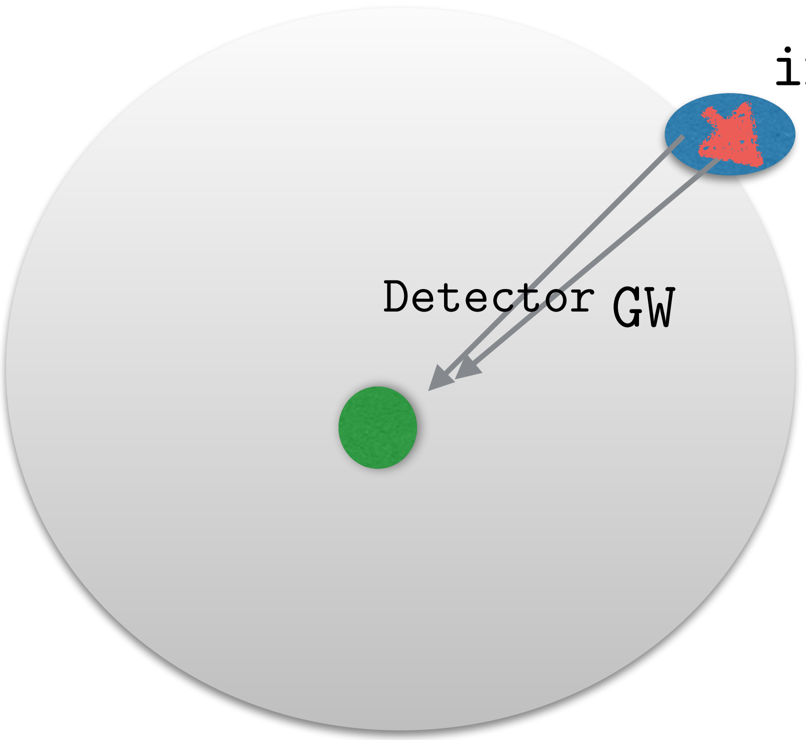
GW emitted from different patches of sky

Phase decorrelations due to propagation in a perturbed universe

[Bartolo et al.]

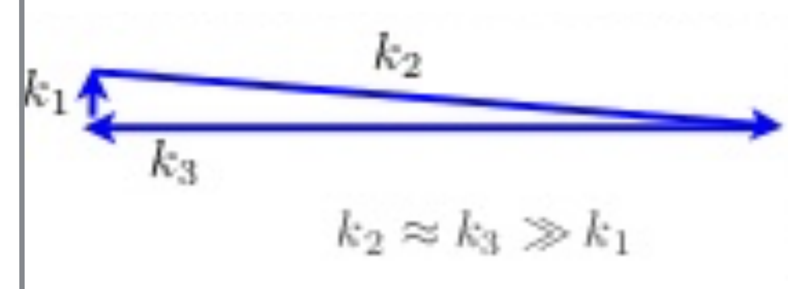
Way out: measure correlators of GWs originating from the same patch

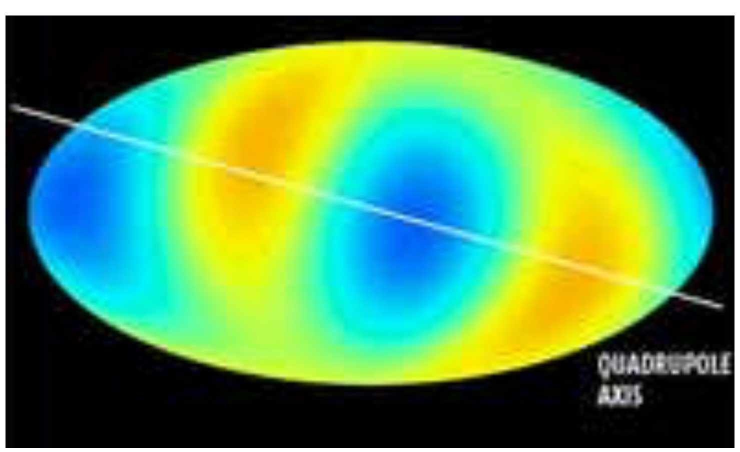
Small-momentum mode
induces correlations



First

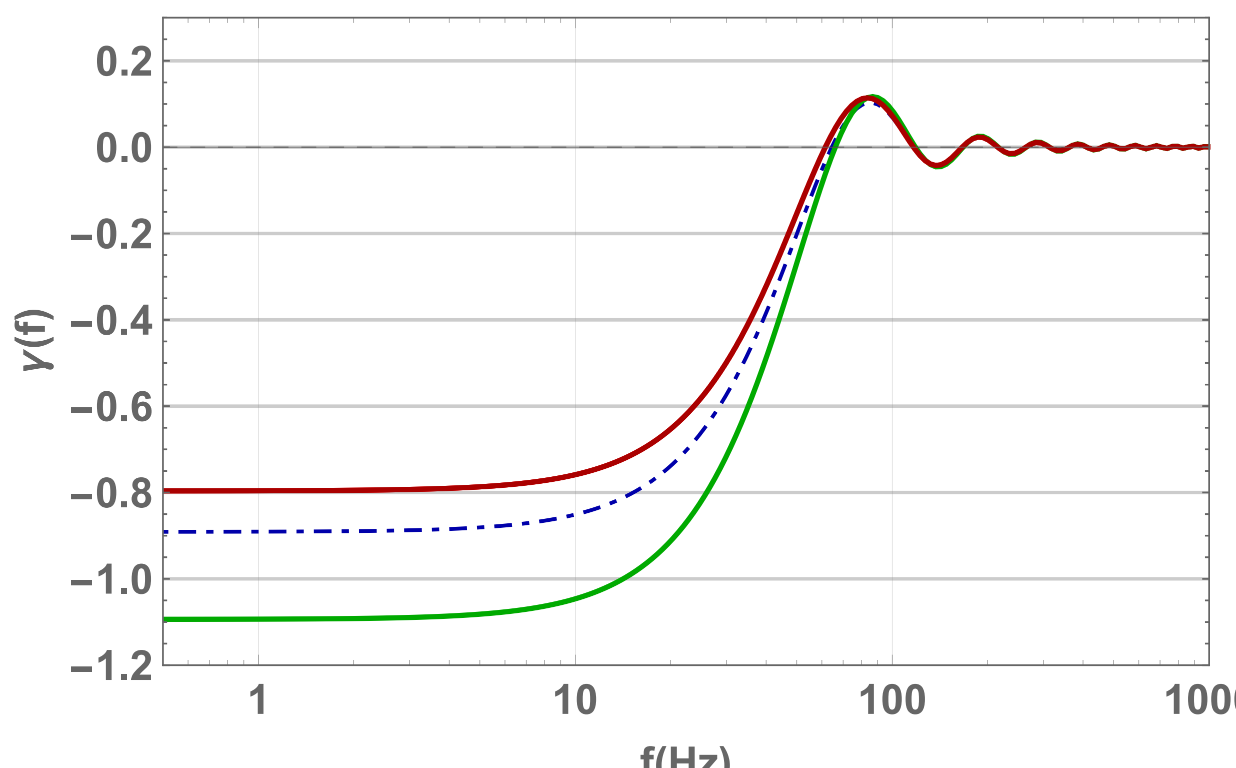
Squeezed non-Gaussianity



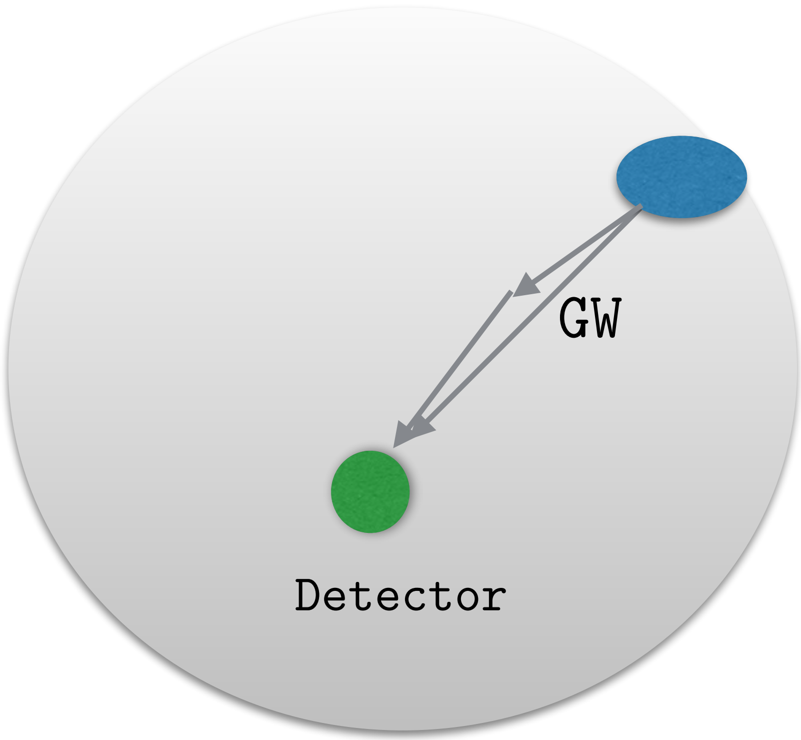


[Dimastrogiovanni et al]

Quadrupolar modulation
of detector overlap function

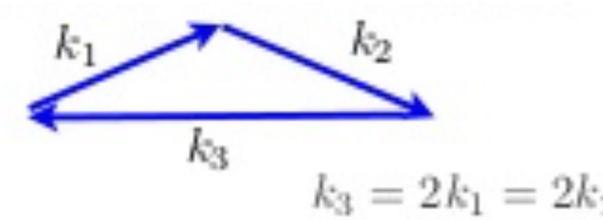


Way out: measure correlators of GWs originating from the same patch



Second

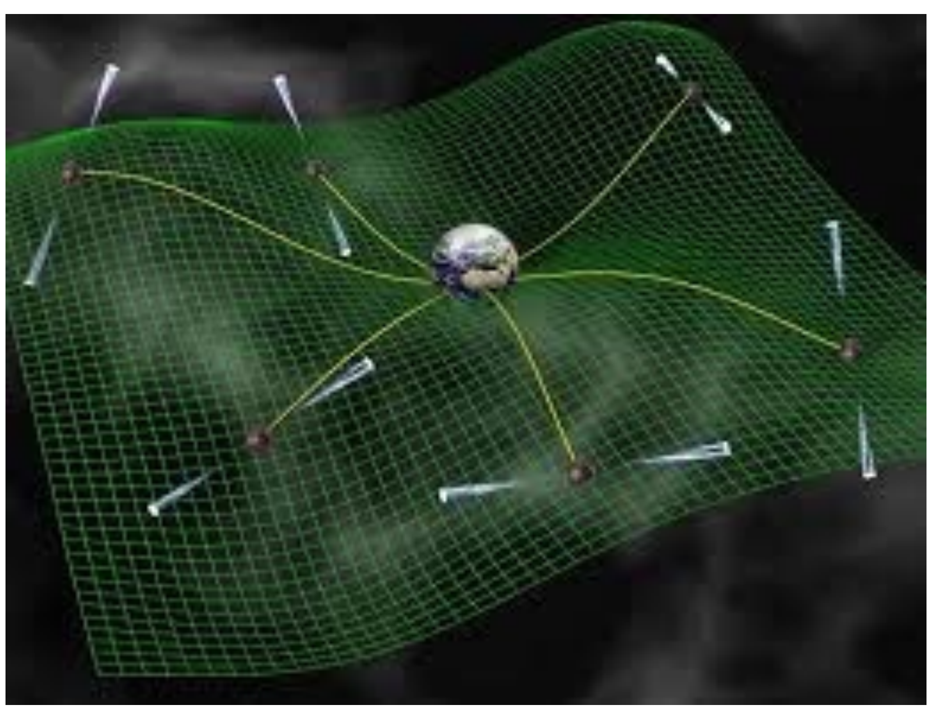
Folded non-Gaussianity

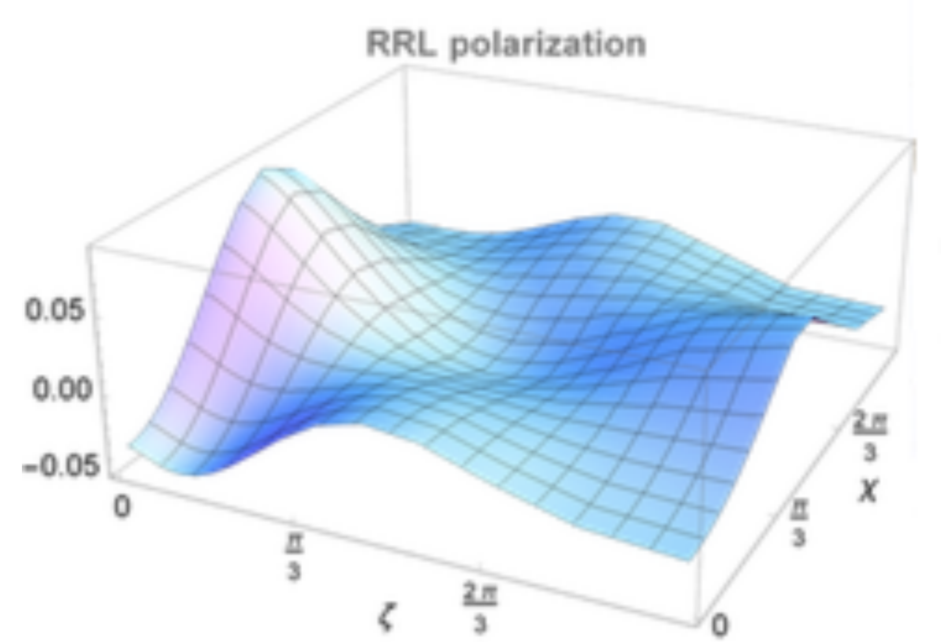


non-Bunch-Davies vacuum

?

[Powell, GT] **Use PTA detectors**

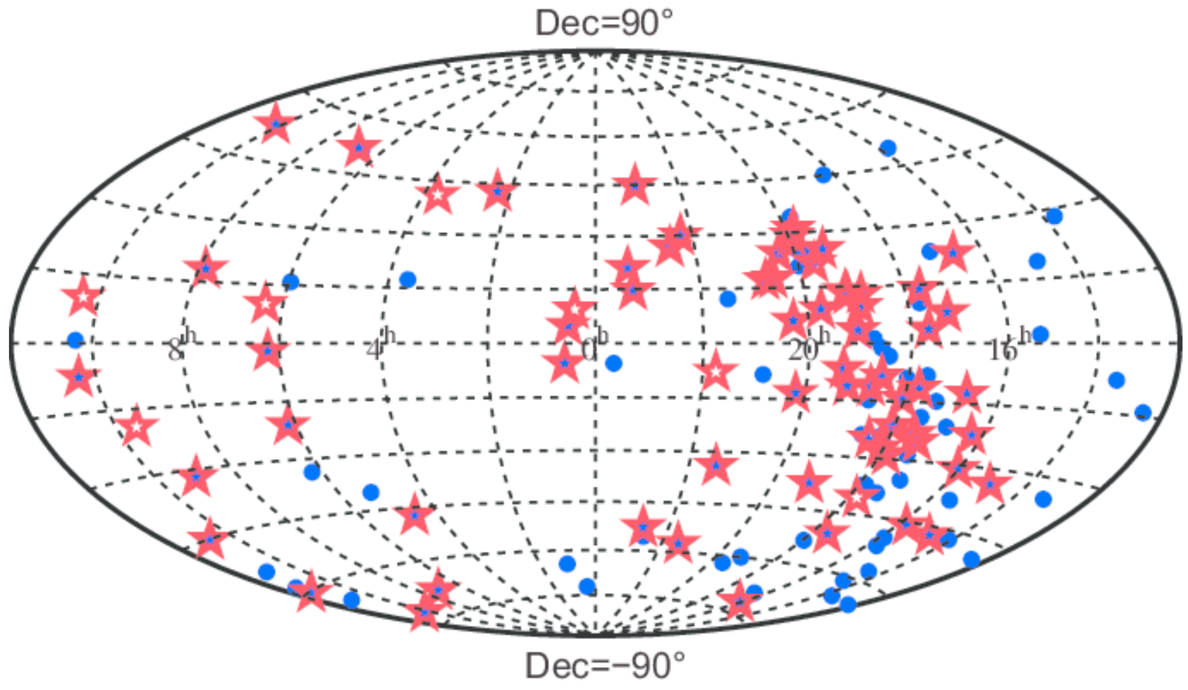




Use pulsar positions from IPTA collaboration

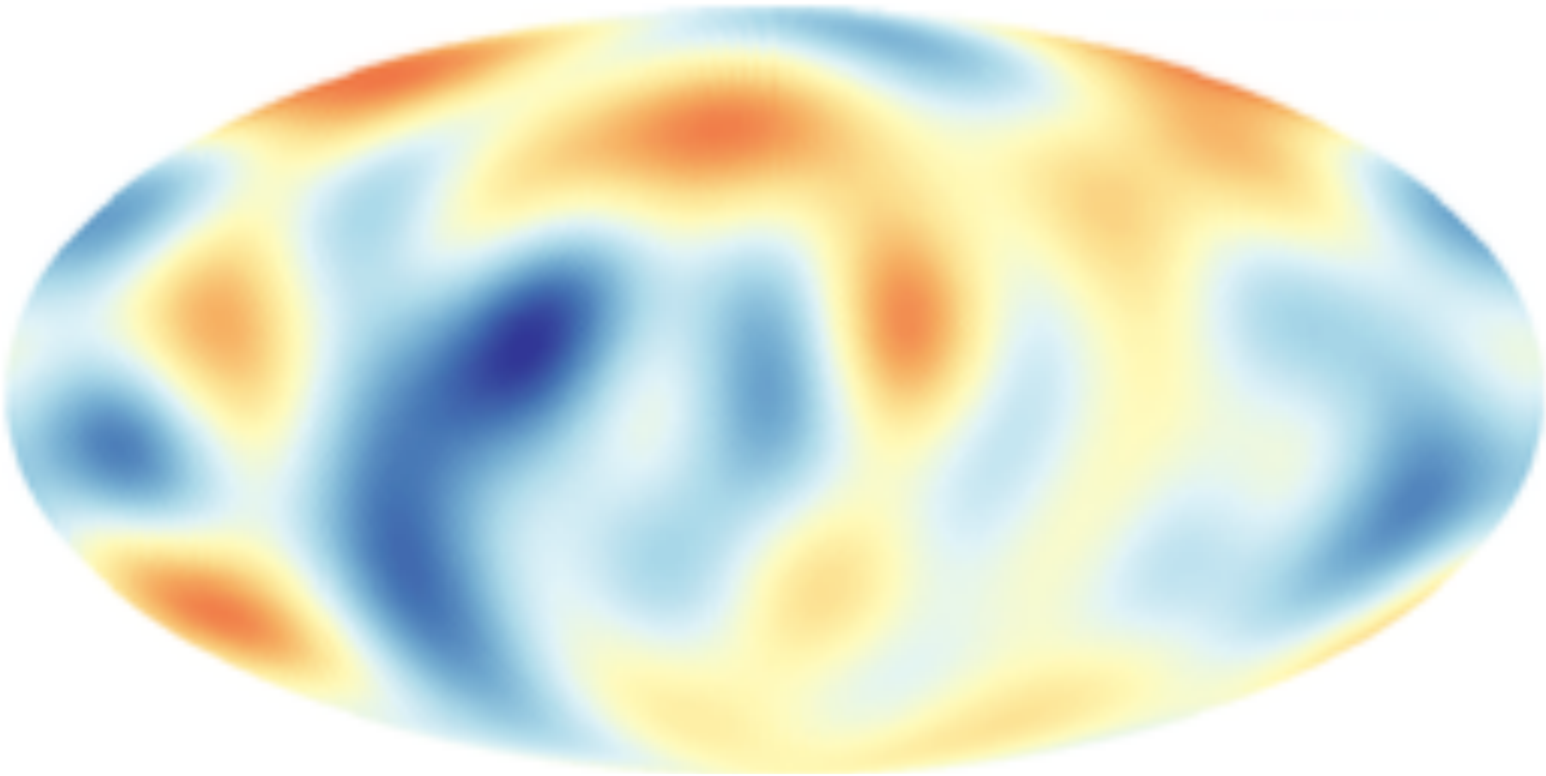
$$\mathrm{SNR_{opt}} = \sqrt{4T}\left[\sum_{\lambda_1\lambda_2\lambda_3} \frac{\int_0^\infty df_A\, df_B \left(\mathbf{r}^{\lambda_1\lambda_2\lambda_3} B^{\lambda_1\lambda_2\lambda_3}(f_A, f_B, \hat{n}_\star)\right)^2}{S_n^3}\right]^{\frac{1}{2}}$$

	Case 1	Case 2
$\mathbf{r}^{RRR}$	20.17	0.58
$\mathbf{r}^{RRL}$	19.58	0.99
$\mathbf{r}^{RLR}$	19.58	0.99
$\mathbf{r}^{STT}$	38.11	0.28
$\mathbf{r}^{SST}$	119.49	1.96
$\mathbf{r}^{SSS}$	168.99	9.12



Another way out: study SGWB anisotropies...

[Renzini et al]

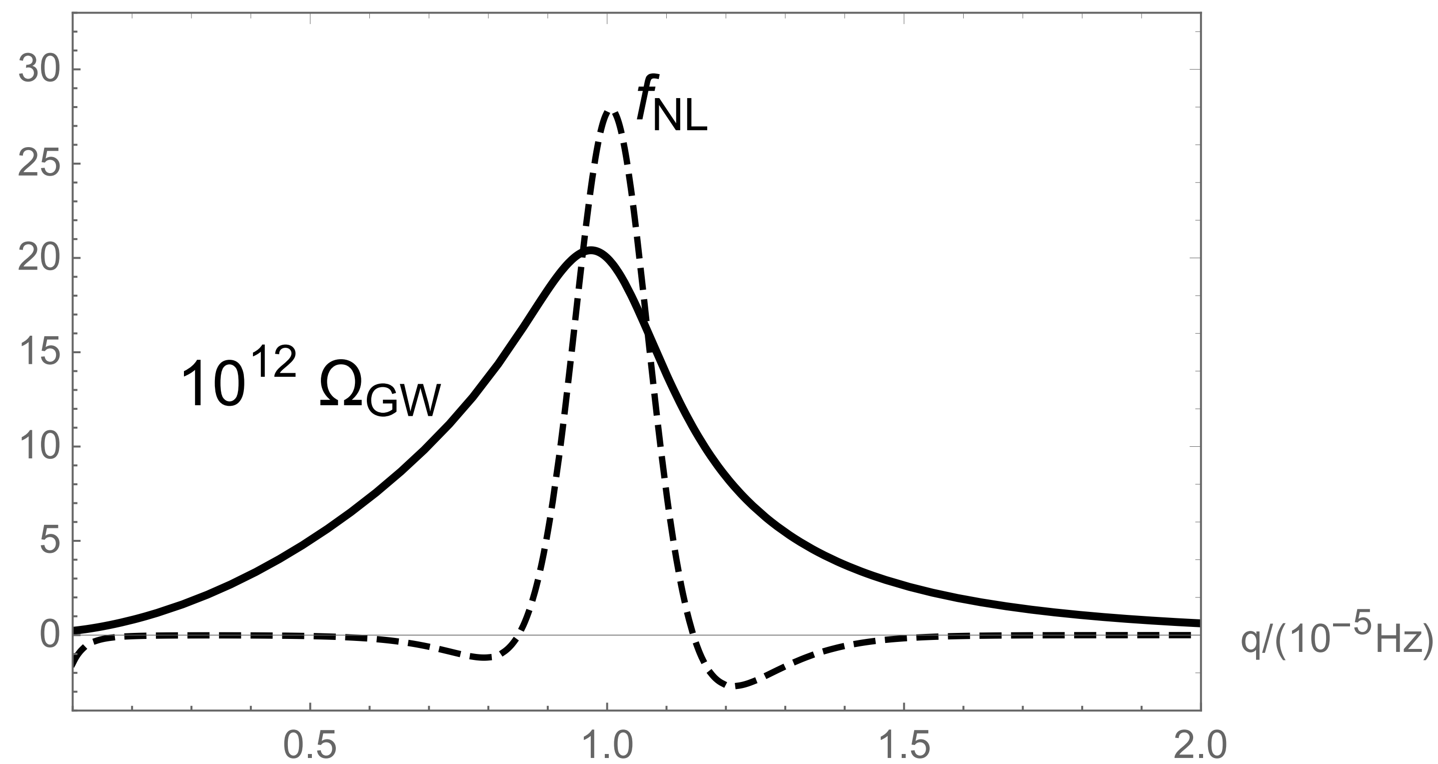


...where issues of phase decorrelations do not apply

[Bartolo et al]

$$\left\langle \prod_{i=1}^{3} \Gamma_{\ell_i m_i, I}(q) \right\rangle = \mathcal{G}_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \int_0^\infty dr\, r^2 \prod_{i=1}^{3} \left[\frac{2}{\pi} \int dk_i\, k_i^2 j_{\ell_i}\left(k_i\left(\eta_0 - \eta_{\rm in}\right)\right) j_{\ell_i}\left(k_i\, r\right) \right] B_I\left(q,\, k,\, k',\, k''\right)$$

Example: squeezed limit of SGWB anisotropies (single-field inflation)

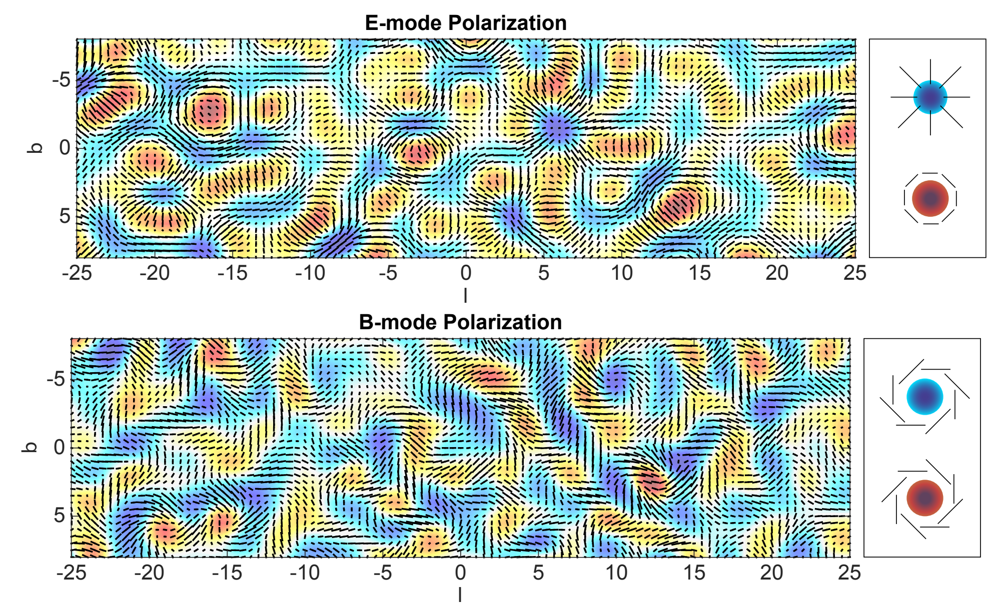


$$\lim_{\vec{k}_3 \to 0} \left\langle \delta_{\rm GW}(\vec{k}_1)\delta_{\rm GW}(\vec{k}_2)\delta_{\rm GW}(\vec{k}_3)\right\rangle = f_{\rm NL}^{\delta_{\rm GW}} \left(\frac{4\pi^4}{k_1^3\, k_3^3}\right) P_{\delta_{\rm GW}}(k_1)\, P_\zeta(k_3)$$

$$f_{\rm NL}^{\delta_{\rm GW}} = -\frac{\partial \ln \bar{f}(q)}{\partial\, \ln q}\, T_S(\eta,\, k_3,\, \mu_3) \left[2\,\frac{\partial \ln P_\zeta}{\partial \ln k_1} + 2\beta_q(\eta)\, \frac{\partial \ln q}{\partial \ln \bar{f}(q)}\, \frac{\partial^2 \ln \bar{f}(q)}{\partial\, (\ln q)^2} + \right.$$
$$\left. +\epsilon(\eta)\, \frac{\partial \ln |\, T_S|^2}{\partial \eta} + \frac{\partial \ln |\, T_S|^2}{\partial \ln k_1} + \beta_n(\eta)\, \frac{\partial \ln |\, T_S|^2}{\partial \ln \mu_1} \right],$$

GW from inflation – a challenge for observational cosmology

> The inflationary paradigm predicts the existence of a stochastic background of GWs from inflation
> but many of its properties depend on the inflationary model



- Most of the effort concentrates on CMB B-modes...

- ... but there're arising well motivated theoretical scenarios predicting primordial GWs at interferometer scales – a possibility worth exploring!