Observing primordial GWs from excited states

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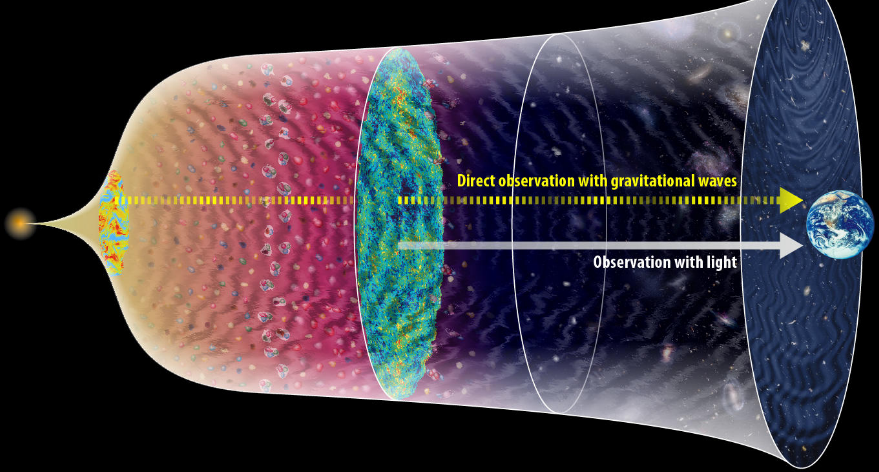


Gravitational-Wave Primordial Cosmology @ IAP

Based on

2106.xxxxx

in collaboration with:

Jacopo Fumagalli, Gonzalo Palma, Sébastien Renaux-Petel, Lukas Witkowski and Cristobal Zenteno

Inspired by PBHs in 2-inflation:

2004.06106, 2004.08369 (see also 2005.02895)

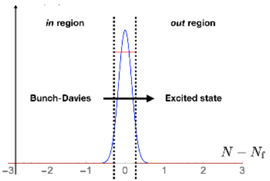
Related talks:

Lukas (2012.02761)

Matteo B. (2012.05821)

EFT perspective: sharp feature

preferred scale $\Rightarrow$ excited state



Imprints: bump/oscillations in P_ζ (CMB, PBHs)

$$\hat{\zeta}(\tau < \tau_{\rm f}) = \zeta_{\rm BD}(\tau)\hat{a}_\zeta + \zeta^*_{\rm BD}(\tau)\hat{a}^\dagger_\zeta$$

$$\hat{\zeta}(\tau > \tau_{\rm f}) = [\alpha_k \zeta_{\rm BD}(\tau) + \beta_k \zeta^*_{\rm BD}(\tau)]\,\hat{a}_\zeta + [\,]^* \hat{a}^\dagger_\zeta$$

UV completions

- ⋆ turn in 2-field inflation: $\mathcal{L} \supset \eta_\perp(\tau - \tau_\mathrm{f})\dot{\zeta}\psi$

  Achúcarro et al '10; Palma, SS, Zenteno; Fumagalli et al.; Braglia et al. '20
- ⋆ 2-stage inflation: $\epsilon(\tau - \tau_\mathrm{f})$ Pi et al '17
- ⋆ time dependent $c_s(\tau - \tau_\mathrm{f})$ Ballesteros, Jimenez, Pieroni '18
- ⋆ particle production: $m_\chi(\tau - \tau_\mathrm{f})$ Cook, Sorbo '11
- ⋆ PTs during inflation: $T(\tau - \tau_\mathrm{f})$ An et al '20
- ⋆ ...

Universal prediction

Oscillatory scalar spectrum

$$\hat{\zeta}(\tau > \tau_{\rm f}) = \left[\alpha_k \zeta_{\rm BD}(\tau) + \beta_k \zeta^*_{\rm BD}(\tau)\right] \hat{a}_\zeta + \left[\alpha_k \zeta_{\rm BD}(\tau) + \beta_k \zeta^*_{\rm BD}(\tau)\right]^* \hat{a}^\dagger_\zeta$$

$$\text{with } \zeta_{\rm BD}(\tau) = e^{-ik\tau}(1 + ik\tau)$$

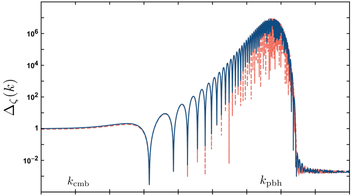
The Bogoliubov coefficients inherit a phase difference ($e^{\pm ik\tau_{\rm f}}$) from the BD states ($\tau < \tau_{\rm f} \equiv k_{\rm f}^{-1}$):

$$P_\zeta = |\alpha + \beta|^2 = |\alpha|^2 + |\beta|^2 + |\alpha||\beta| \cos \frac{2k}{k_{\rm f}}$$

How *excited* and how *observable* ($|\beta|, k_{\rm f}$) depends on UV

Example: PBHs in multifield inflation

$$\frac{P_\zeta}{H^2/\epsilon} \sim \begin{cases} 1 + 4\delta\theta^2 & \text{if } k \ll k_\star \quad \text{CMB} \\ \frac{1}{4} e^{2\delta\theta\sqrt{\lambda k - k^2}} & \text{if } k \sim k_\star \quad \text{PBH} \\ 1 & \text{if } k \gg k_\star \end{cases}$$



analytics vs numerics

Scope: how do these affect $\Omega_{\rm GW}^{\rm infl}$ and $\Omega_{\rm GW}^{\rm rad}$? (Lukas/Matteo)

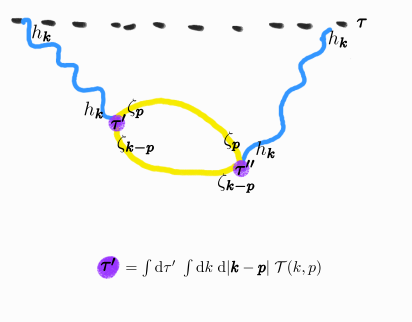
Suppose that the amplitude of the feature in the scalar PS is significant: 2^{nd} order source for tensors

$$h = h_0 + h_S$$

$$h_{\boldsymbol{k}}^{\lambda\,''}(\tau) + 2\mathcal{H} h_{\boldsymbol{k}}^{\lambda\,'}(\tau) + k^2 h_{\boldsymbol{k}}^{\lambda}(\tau) = S_{\boldsymbol{k}}^{\lambda}(\tau)$$

$$\hat{S}_{\boldsymbol{k}}^{\lambda}(\tau) = \int \frac{\mathrm{d}^3 \boldsymbol{p}}{(2\pi)^3} \varepsilon^{\lambda}(\boldsymbol{k}, \boldsymbol{p}) \left(2\epsilon \hat{\zeta}_{\boldsymbol{p}}(\tau) \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|}(\tau) + \underbrace{\hat{\psi}_{\boldsymbol{p}}(\tau) \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|}(\tau)}_{2-\text{field generalisation}} \right)$$

$$\left\langle \hat{h}_{\boldsymbol{k}}^{\lambda} \hat{h}_{\boldsymbol{k}'}^{\sigma} \right\rangle (\tau) = \int_{\tau_{\mathrm{f}}}^{\tau} \mathrm{d}\tau' \; G_{\boldsymbol{k}}(\tau, \tau') \int_{\tau_{\mathrm{f}}}^{\tau} \mathrm{d}\tau'' \; G_{\boldsymbol{k}'}(\tau, \tau'') \left\langle \hat{S}_{\boldsymbol{k}}^{\lambda}(\tau') \hat{S}_{\boldsymbol{k}'}^{\sigma}(\tau'') \right\rangle_{\beta}$$



$$\langle h_k h_k \rangle (\tau) = \int \mathrm{d}\tau' \int \mathrm{d}\tau'' \left\langle e^{-i\hat{h}\partial\hat{\zeta}\partial\hat{\zeta}(\tau')} \hat{h}\hat{h}(\tau) e^{i\hat{h}\partial\hat{\zeta}\partial\hat{\zeta}(\tau'')} \right\rangle_{\beta}$$

with excited ζ:

$$\hat{\zeta} = (\alpha_k \zeta_{\mathrm{BD}} + \beta_k \zeta^*_{\mathrm{BD}})\, \hat{a}_{\zeta} + (\alpha_k \zeta_{\mathrm{BD}} + \beta_k \zeta^*_{\mathrm{BD}})^*\, \hat{a}^{\dagger}_{\zeta}$$

$$P_h(k,\tau) = \frac{H^4}{16\pi^4}\int_1^\infty \mathrm{d}y \int_{|1-y|}^{1+y} \mathrm{d}x\, \mathcal{T}(x,y) \times$$

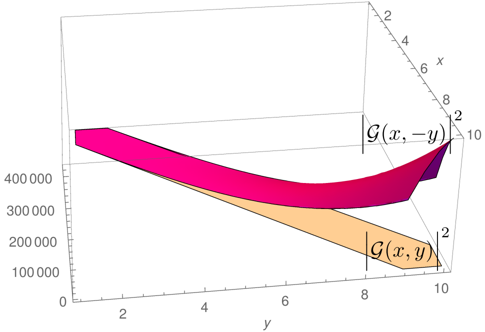
$$\sum_{ij}\left|\sum_{X\pm} A^{\pm}_{Xi}(xk)A^{\pm}_{Xj}(yk)\mathcal{G}(\pm x,\pm y;k\tau_{\mathrm{f}})\right|^2$$

$$\underbrace{A^{+}\equiv\alpha,\ A^{-}\equiv\beta}_{model\ \ dependence};\ \zeta^{*}_{\mathrm{BD}}(k\tau)=\zeta_{\mathrm{BD}}(-k\tau)$$

$$\mathcal{G}(\pm x,\pm y;k\tau_{\mathrm{f}}) \equiv \int_{\tau_{\mathrm{f}}}^{0} \mathrm{d}\tau'\, \zeta_{\mathrm{BD}}(\pm xk\tau')\zeta_{\mathrm{BD}}(\pm yk\tau')G_k(\tau',0)$$

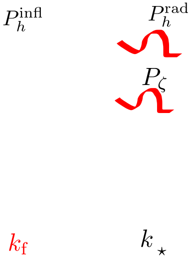
Generalisation of Biagetti, Fasiello, Riotto '13 to excited states (and n-fields)

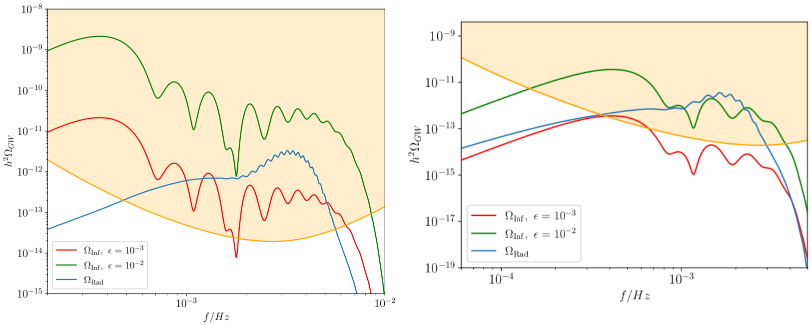
$$P_h(k) \supset \int_1^\infty \mathrm{d}y \int_{|1-y|}^{1+y} \mathrm{d}x\, \mathcal{T}(x,y)\, P_\zeta(xk) P_\zeta(yk) \times \left(|\mathcal{G}(x,y;k\tau_\mathrm{f})|^2 + |\mathcal{G}(x,-y;k\tau_\mathrm{f})|^2 \right)$$



$$P_h^{\rm infl} = \epsilon^2 P_\zeta(k_\star) P_\zeta(k_\star)\, \mathcal{G}(k_{\rm f}) \quad \Big| \quad P_h^{\rm rad} = P_\zeta(k_\star) P_\zeta(k_\star)\ T_{\rm rad}$$

$$\mathcal{G}(x, -y; k\tau_{\rm f}) \supset e^{-i(1+x-y)k\tau_{\rm f}} \mathcal{F}(x, -y; k\tau_{\rm f}) \quad \Big| \quad \beta = \alpha e^{2ik\tau_{\rm f}}$$





Observables: left/right peak positions, amplitude, osc frequency

$$f_{\rm left}\,,\, f_{\rm right}\,,\, \frac{\Omega_{\rm left}}{\Omega_{\rm right}}\,,\, \omega_{\rm right}$$

Params: $\epsilon, M/H, k_f$

Message: Fumagalli et al. to appear

smoking gun for excited states:

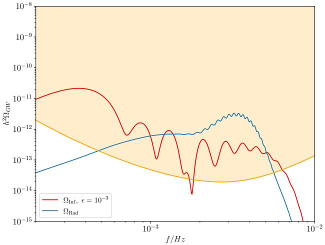
bump at low f vs bump/oscillations at high f

EFT diagnostics:

- ⋆ $f_{\rm left} \sim \omega_{\rm right}$
- ⋆ $\dfrac{f_{\rm right}}{f_{\rm left}} \sim \dfrac{M^2}{H^2}$
- ⋆ spectral feature $(\delta\tau)$
- ⋆ PBHs?

UV diagnostic:

- ⋆ $\dfrac{f^3_{\rm left}}{f^3_{\rm right}}\sqrt{\dfrac{\Omega_{\rm left}}{\Omega_{\rm right}}} \sim \epsilon(\tau_{\rm f})$



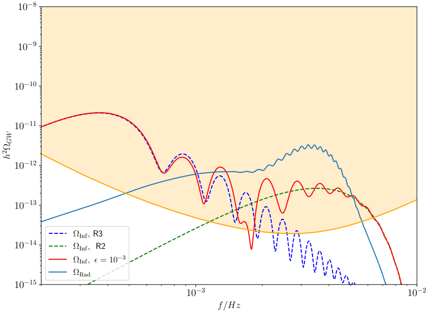
Caveats: observability 😄

Merci!

$$\mathcal{G}(x,-y;k\tau_{\rm f}) \equiv \int_{\tau_{\rm f}}^{0} \mathrm{d}\tau' \, \zeta_{\rm BD}(xk\tau')\zeta_{\rm BD}(-yk\tau') G_k(\tau',0)$$

but if $\delta\tau \neq 0$ (and it is not)

$$\int_{\tau_{\rm f}}^{0} \mathrm{d}\tau' = \int_{-\infty}^{\tau_{\rm f}-\delta\tau} \mathrm{d}\tau' + \int_{\tau_{\rm f}-\delta\tau}^{\tau_{\rm f}+\delta\tau} \mathrm{d}\tau' + \int_{\tau_{\rm f}+\delta\tau}^{0} \mathrm{d}\tau'$$



$$\hat{\zeta} = \left[\alpha_\zeta^\zeta \zeta_{\rm BD} + \beta_\zeta^\zeta \zeta^*_{\rm BD}\right] \hat{a}_\zeta + \left[\alpha_\zeta^\psi \zeta_{\rm BD} + \beta_\zeta^\psi \zeta^*_{\rm BD}\right] \hat{a}_\psi$$
$$+ \left[\alpha_\zeta^\zeta \zeta_{\rm BD} + \beta_\zeta^\zeta \zeta^*_{\rm BD}\right]^* \hat{a}_\zeta^\dagger + \left[\alpha_\zeta^\psi \zeta_{\rm BD} + \beta_\zeta^\psi \zeta^*_{\rm BD}\right]^* \hat{a}_\psi^\dagger$$
$$\hat{\psi} = \left[\alpha_\psi^\zeta \zeta_{\rm BD} + \beta_\psi^\zeta \zeta^*_{\rm BD}\right] \hat{a}_\zeta + \left[\alpha_\psi^\psi \zeta_{\rm BD} + \beta_\psi^\psi \zeta^*_{\rm BD}\right] \hat{a}_\psi$$
$$+ \left[\alpha_\psi^\zeta \zeta_{\rm BD} + \beta_\psi^\zeta \zeta^*_{\rm BD}\right]^* \hat{a}_\zeta^\dagger + \left[\alpha_\psi^\psi \zeta_{\rm BD} + \beta_\psi^\psi \zeta^*_{\rm BD}\right]^* \hat{a}_\psi^\dagger$$

$$\left\langle \left(2\epsilon \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} + \hat{\psi}_p \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|}\right) \left(2\epsilon \hat{\zeta}_q \hat{\zeta}_{|\boldsymbol{k'}-\boldsymbol{q}|} + \hat{\psi}_q \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|}\right) \right\rangle =$$
$$4\epsilon^2 \left\langle \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\zeta}_q \hat{\zeta}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle + 4\epsilon \,\mathrm{Re} \left\langle \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle + \left\langle \hat{\psi}_p \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle$$

Noting that $[\hat{a}(\boldsymbol{k} - \boldsymbol{p}), \hat{a}^\dagger(\boldsymbol{p})] = 0$, $k \neq 0$:

$$\left\langle \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\zeta}_q \hat{\zeta}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle = \left\langle \hat{\zeta}_p \hat{\zeta}_q \right\rangle \left\langle \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\zeta}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle + \left\langle \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle \left\langle \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\zeta}_q \right\rangle$$
$$\left\langle \hat{\zeta}_p \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle = \left\langle \hat{\zeta}_p \hat{\psi}_q \right\rangle \left\langle \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle + \left\langle \hat{\zeta}_p \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle \left\langle \hat{\zeta}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \right\rangle$$
$$\left\langle \hat{\psi}_p \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle = \left\langle \hat{\psi}_p \hat{\psi}_q \right\rangle \left\langle \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle + \left\langle \hat{\psi}_p \hat{\psi}_{|\boldsymbol{k'}-\boldsymbol{q}|} \right\rangle \left\langle \hat{\psi}_{|\boldsymbol{k}-\boldsymbol{p}|} \hat{\psi}_q \right\rangle$$