Features in the stochastic gravitational wave background from two-field inflationary models

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GW Primordial Cosmology Online Workshop

Features in the stochastic gravitational wave background from two-field inflationary models

Based on

- 2005.02895 MB, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar, A. A. Starobinsky (MB1)
- 2012.05821 MB, X. Chen, D. K. Hazra (MB2)

Outline

- The two-field model and its background evolution
- Amplification of curvature perturbations
- Signatures in the Stochastic Gravitational Wave Background (SGWB)
- Detectability of these features

A toy model of two-field inflation

General nonlinear sigma model

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\,R - \frac{1}{2}\,G_{IJ}\nabla^\mu\phi^I\nabla_\mu\phi^J - V(\phi)\right]$$

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Simple two-field toy model

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi_1)^2 - \frac{f^2(\phi_1)}{2}(\partial\phi_2)^2 - V(\phi_1,\phi_2)\right]$$

Starobinsky, Tsujikawa, Yokoyama 2001 - Di Marco, Finelli, Brandenberger 2002 - Lalak, Langlois, Pokorski, Turzynski 2007

A toy model of two-field inflation

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Assumptions:

- $V(\phi_1, \phi_2) = V(\phi_1) + U(\phi_2)$, with V and U slow-roll potentials
- $V \gg U$ Hierarchy of energy scales

(MB1)

A toy model of two-field inflation

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→ Two stages of inflation

Starobinsky, Polarski 1992

(MB1)

A toy model of two-field inflation

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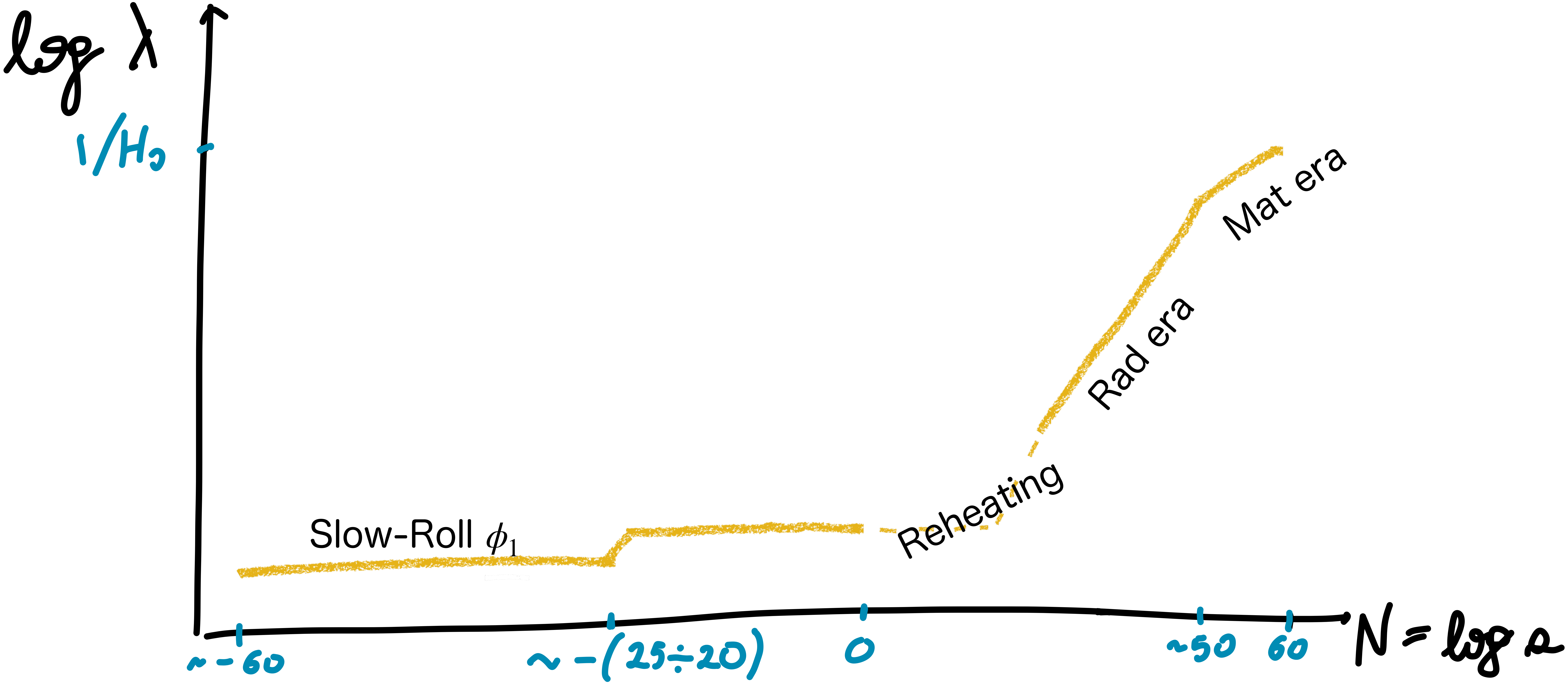
Two stages of inflation

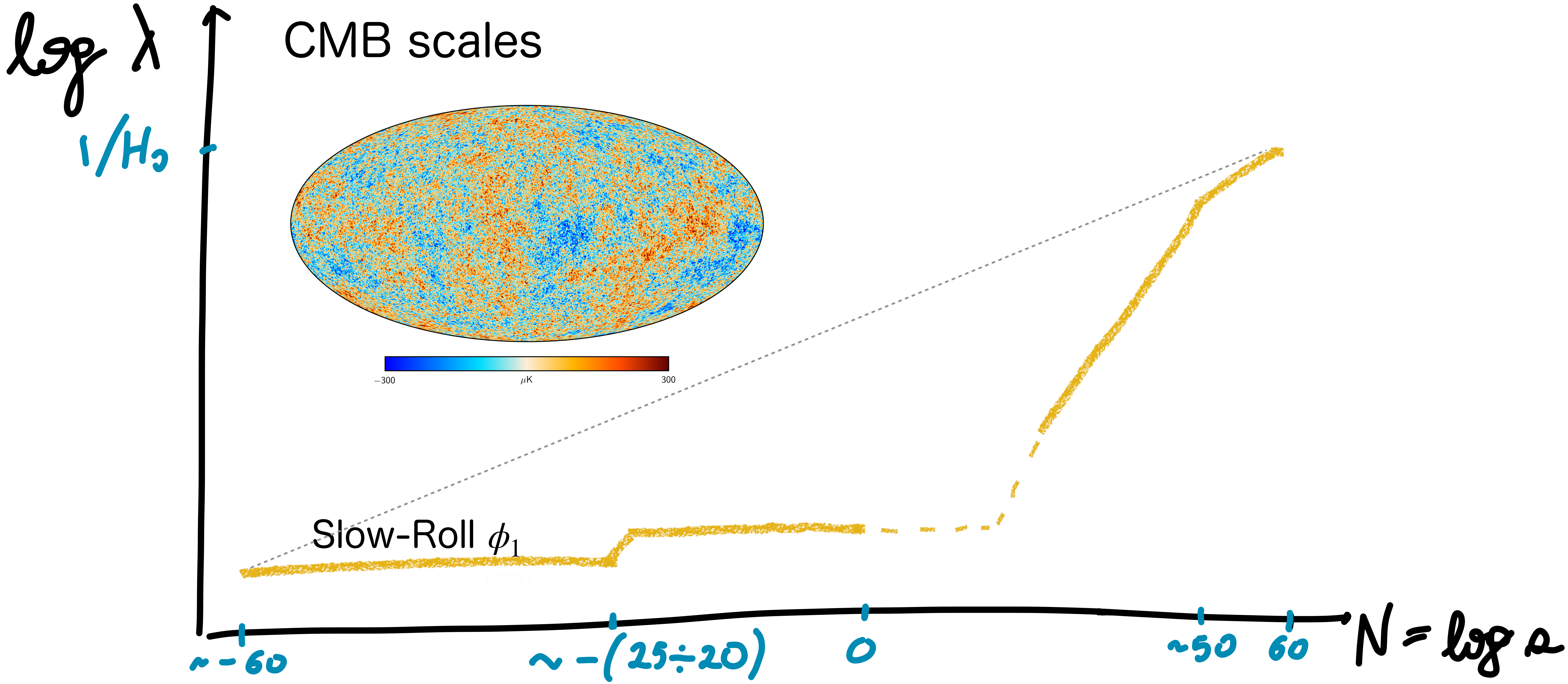
- ϕ_2 is kinetically coupled to ϕ_1

(MB1)

Two stages of slow-roll inflation



Two stages of slow-roll inflation



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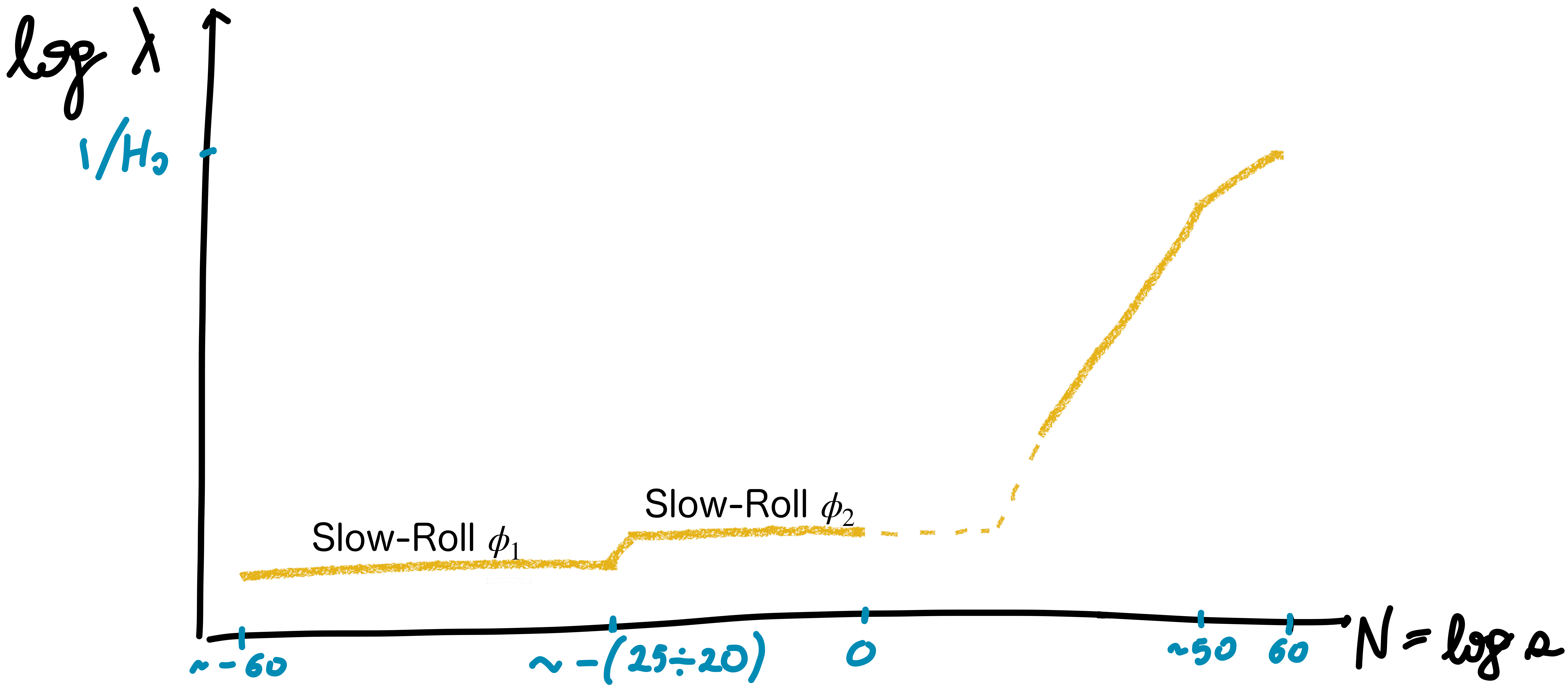

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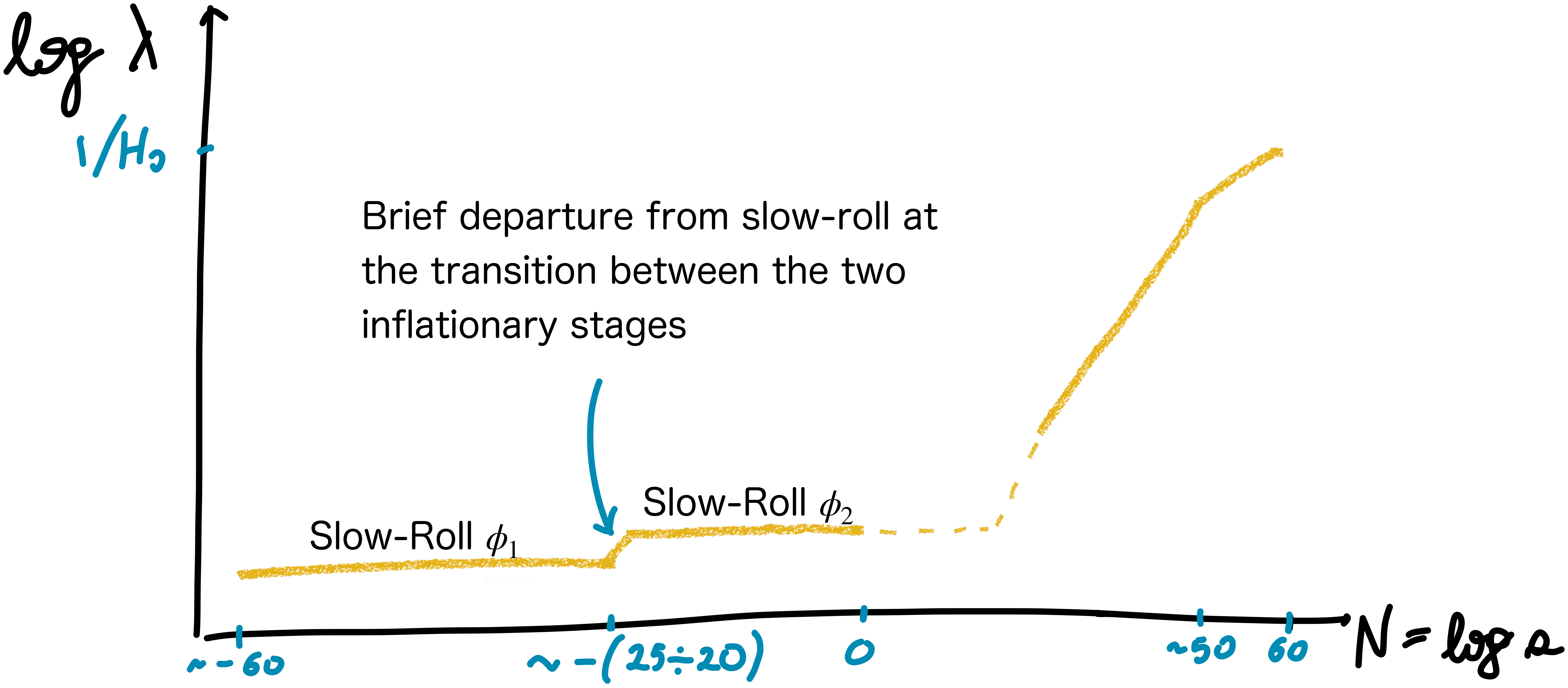

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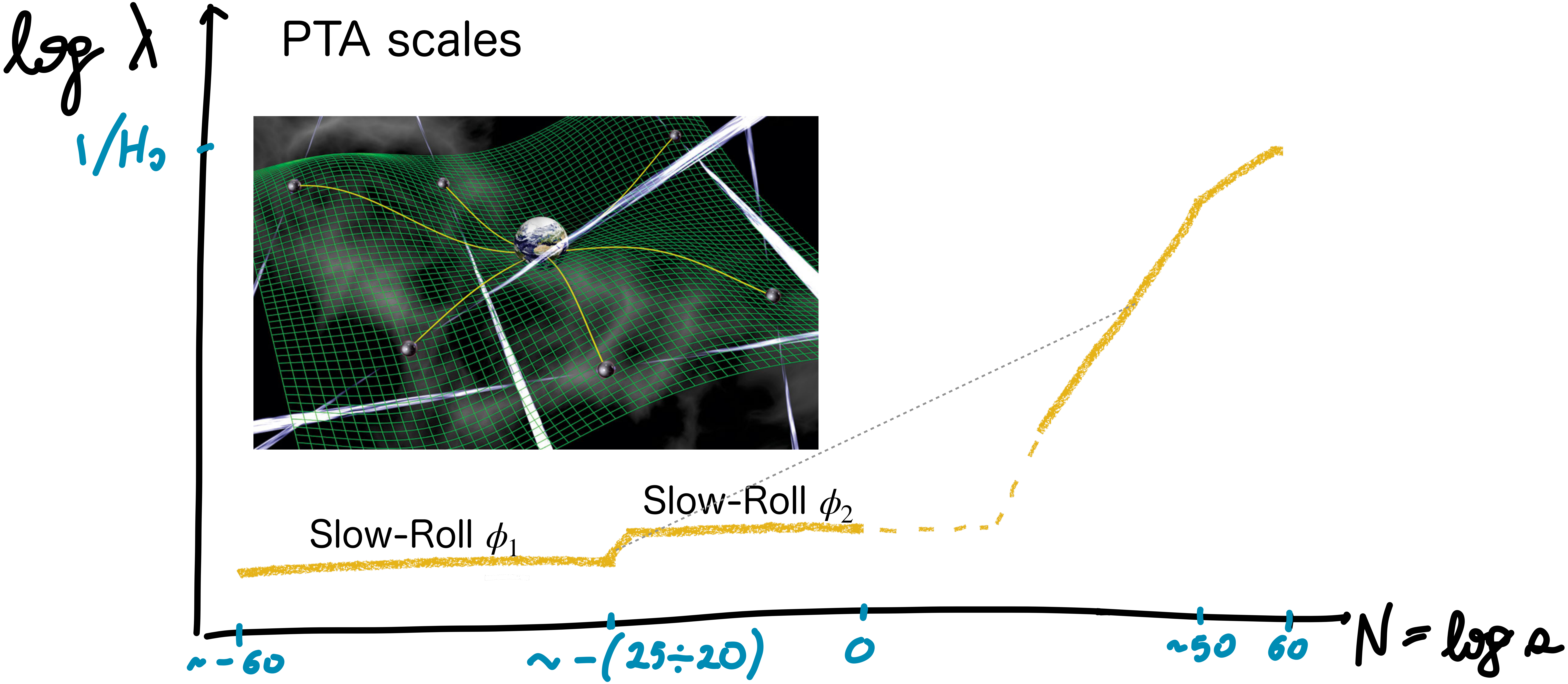

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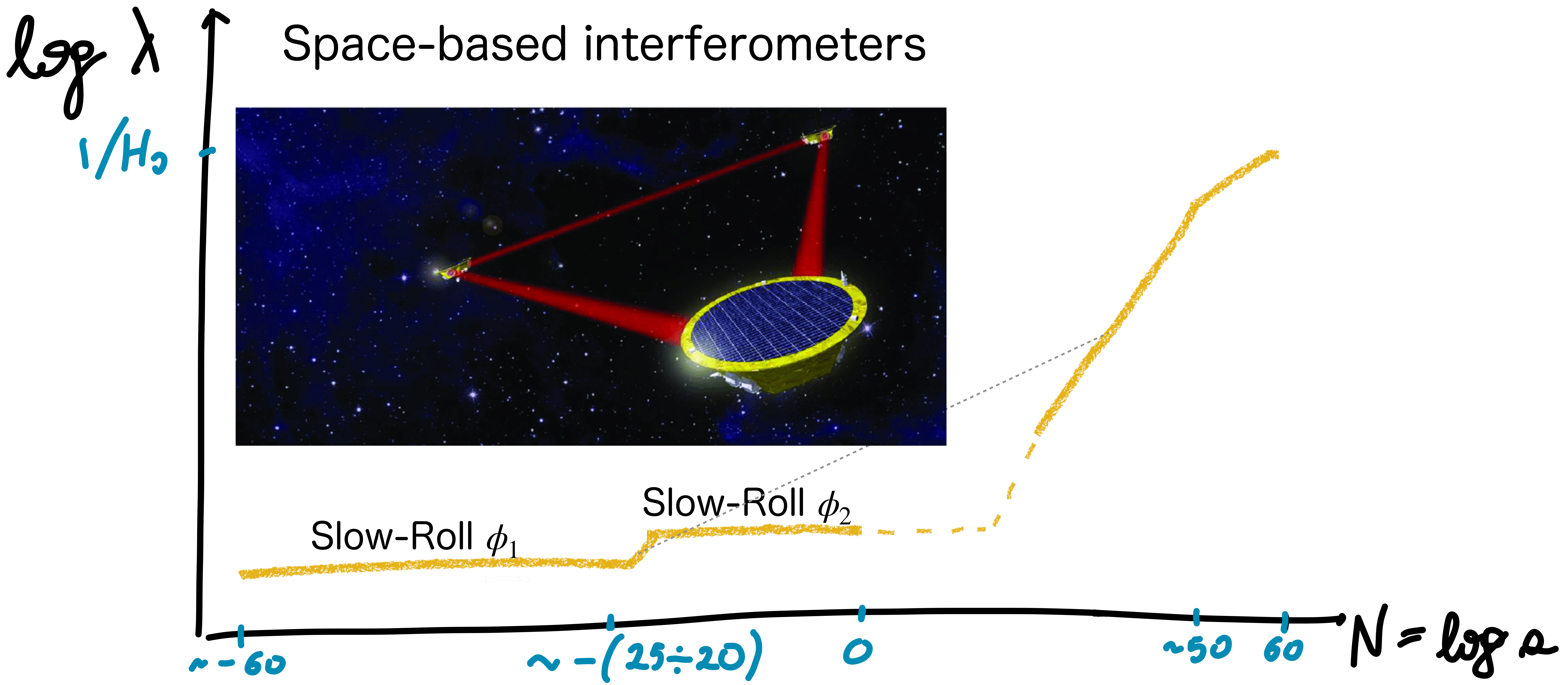

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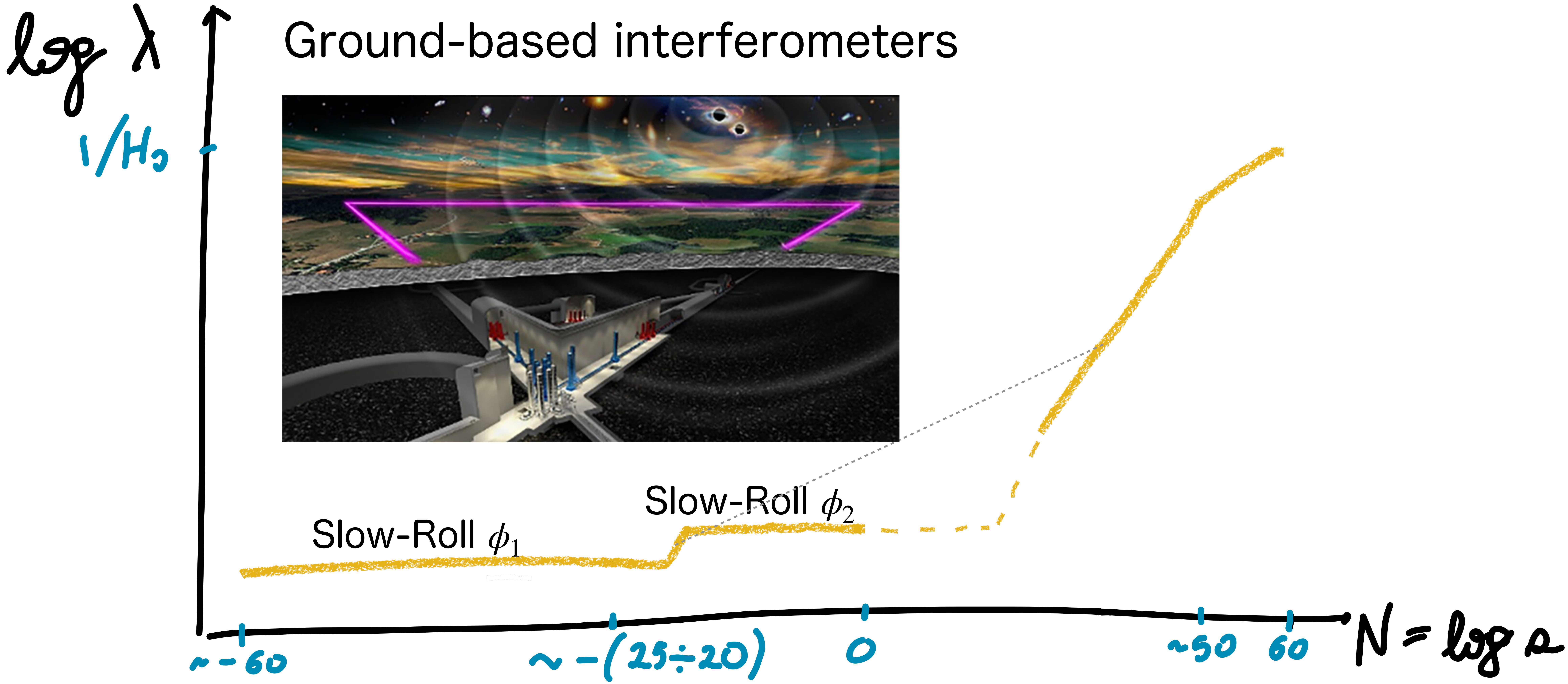
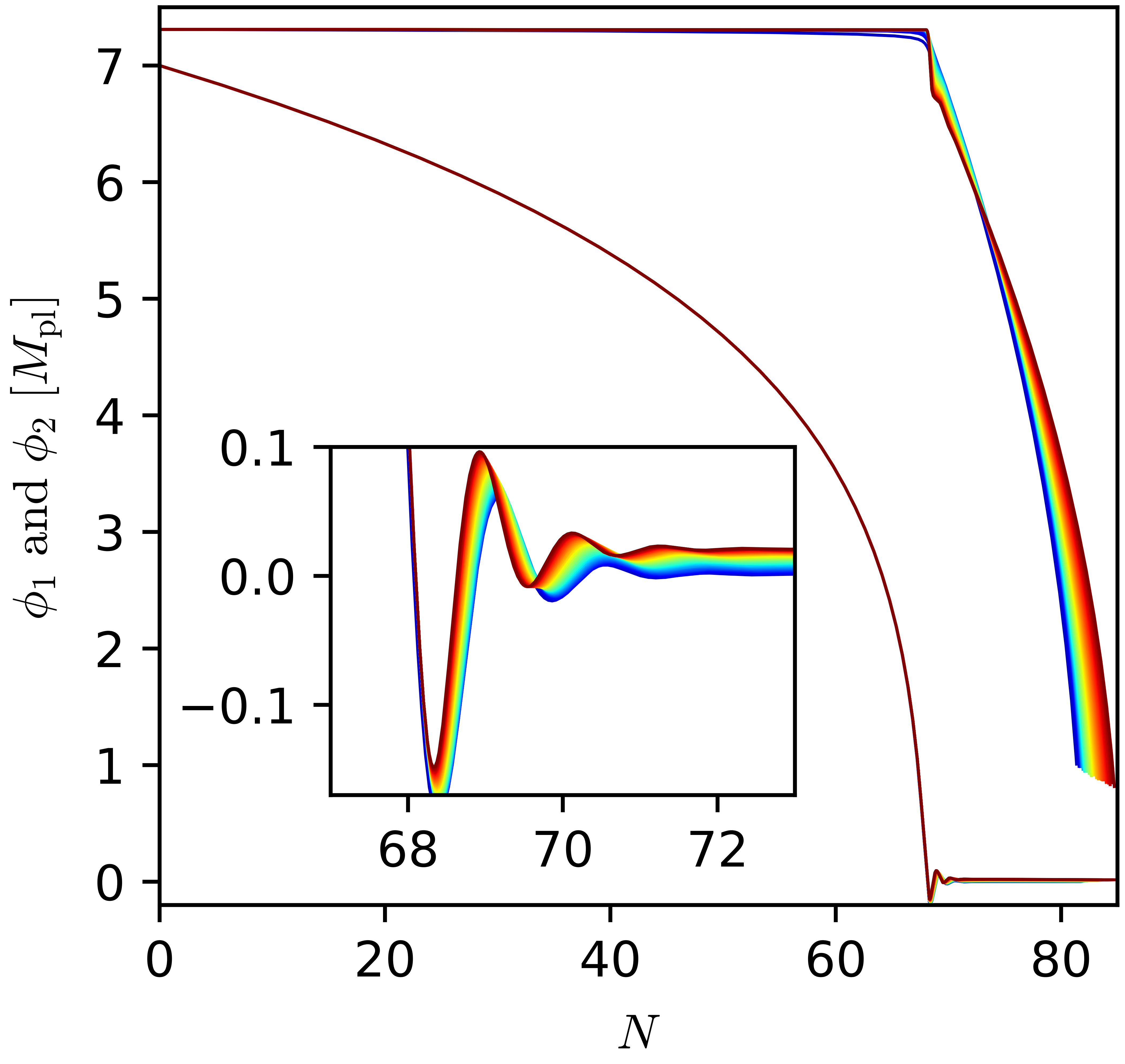


Example of background evolution



Choice of the potential:

$$V(\phi_1) = V_0 C_1 \left[1 - \exp\left(-\phi_1^2/\phi_f^2\right)\right]$$

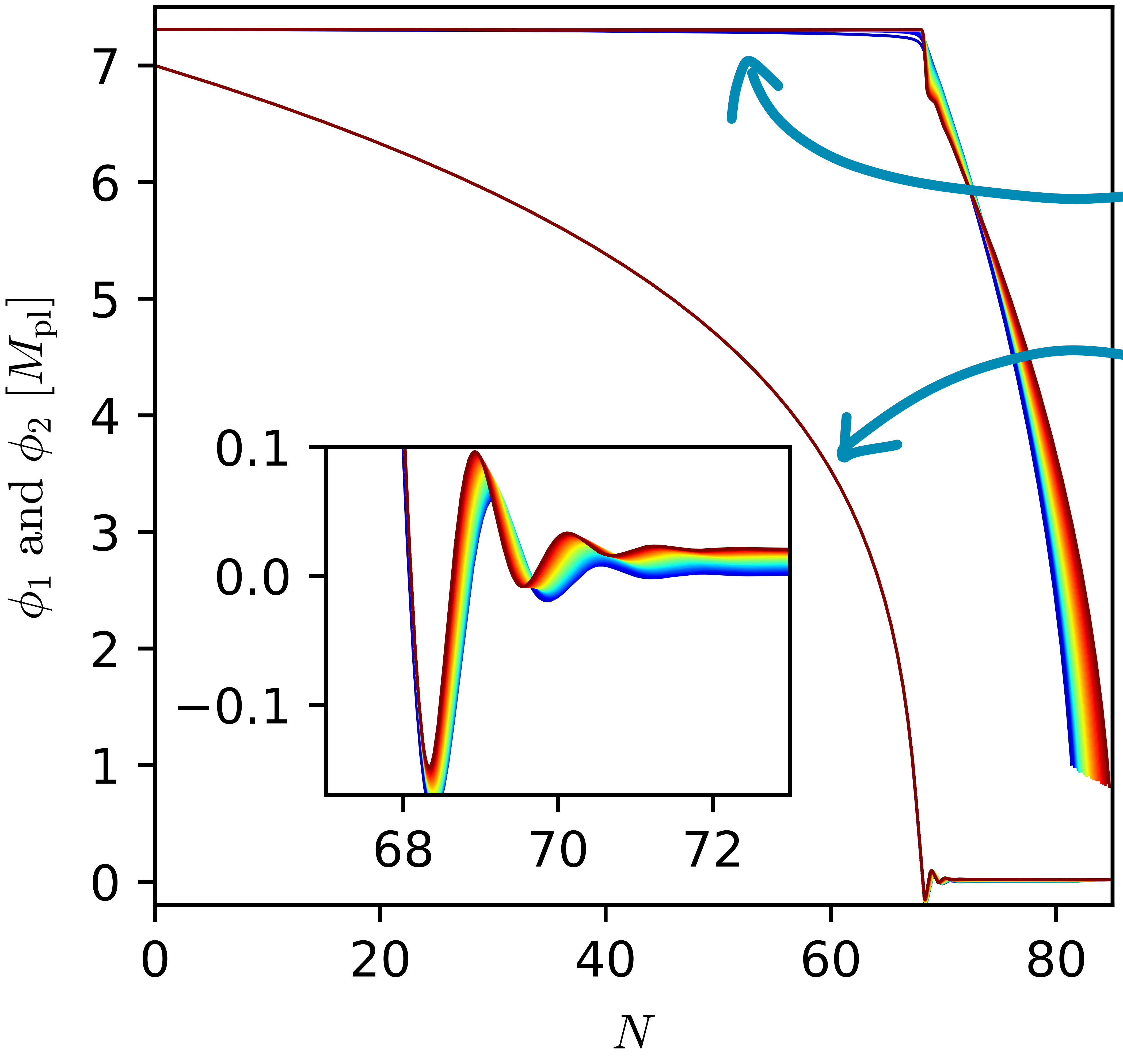
$$U(\phi_2) = V_0 \frac{m_2^2}{2} \phi_2^2$$

Choice of the kinetic coupling:

$$f(\phi_1) = \exp(b_1 \phi_1)$$

(MB1)

Example of background evolution



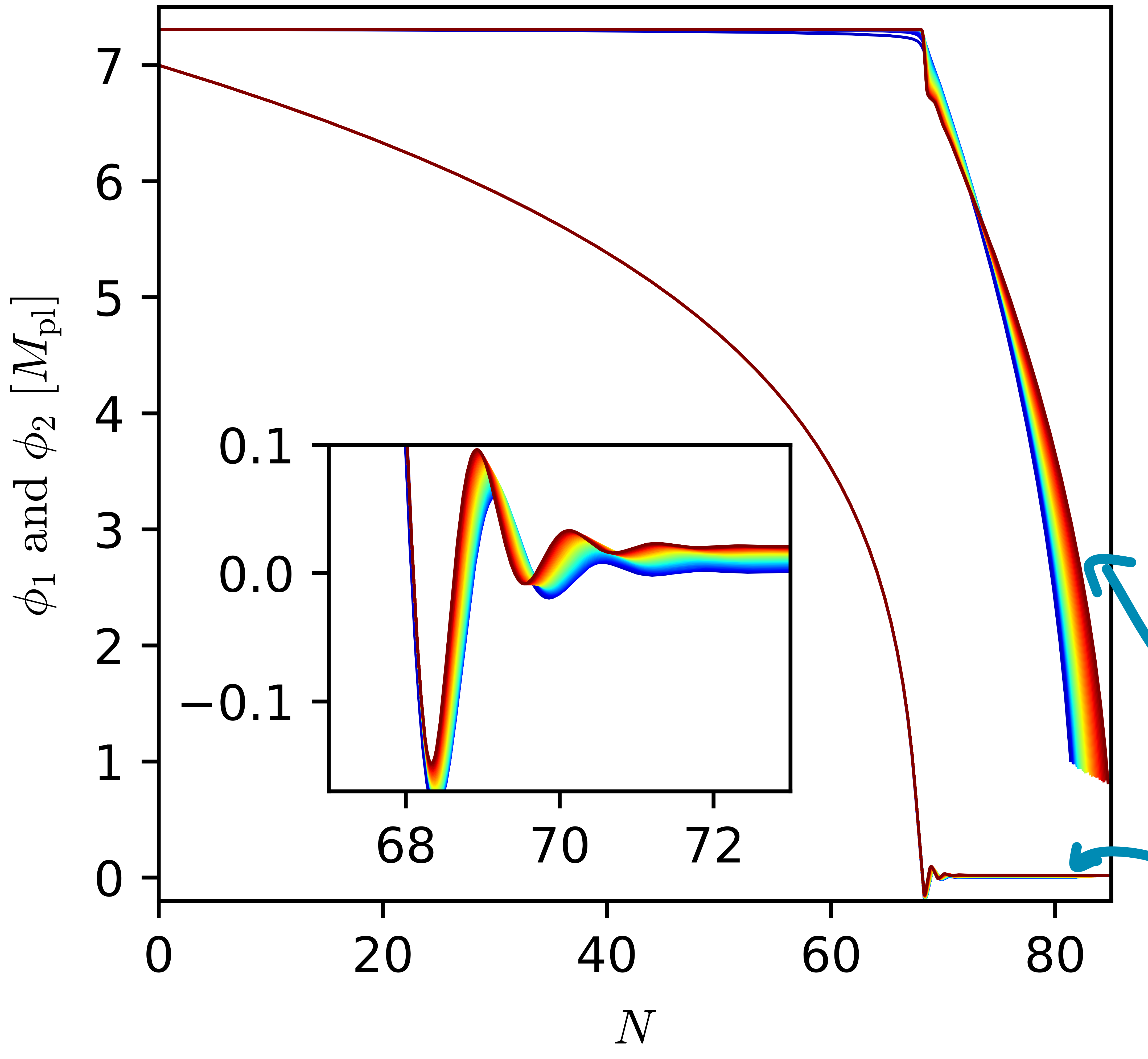
First stage of inflation:

$$\phi_2 = \mathrm{const} = \phi_{2,\,i}$$

$$\phi_1 = \sqrt{-\phi_f^2 + \sqrt{-8N\phi_f^4 + \left(\phi_i^2 + \phi_f^2\right)}}$$

(MB1)

Example of background evolution



First stage of inflation:

$$\phi_2 = \text{const} = \phi_{2,i}$$

$$\phi_1 = \text{slow} - \text{roll solution}$$

Second stage of inflation:

$$\phi_2 = \text{slow} - \text{roll solution}$$

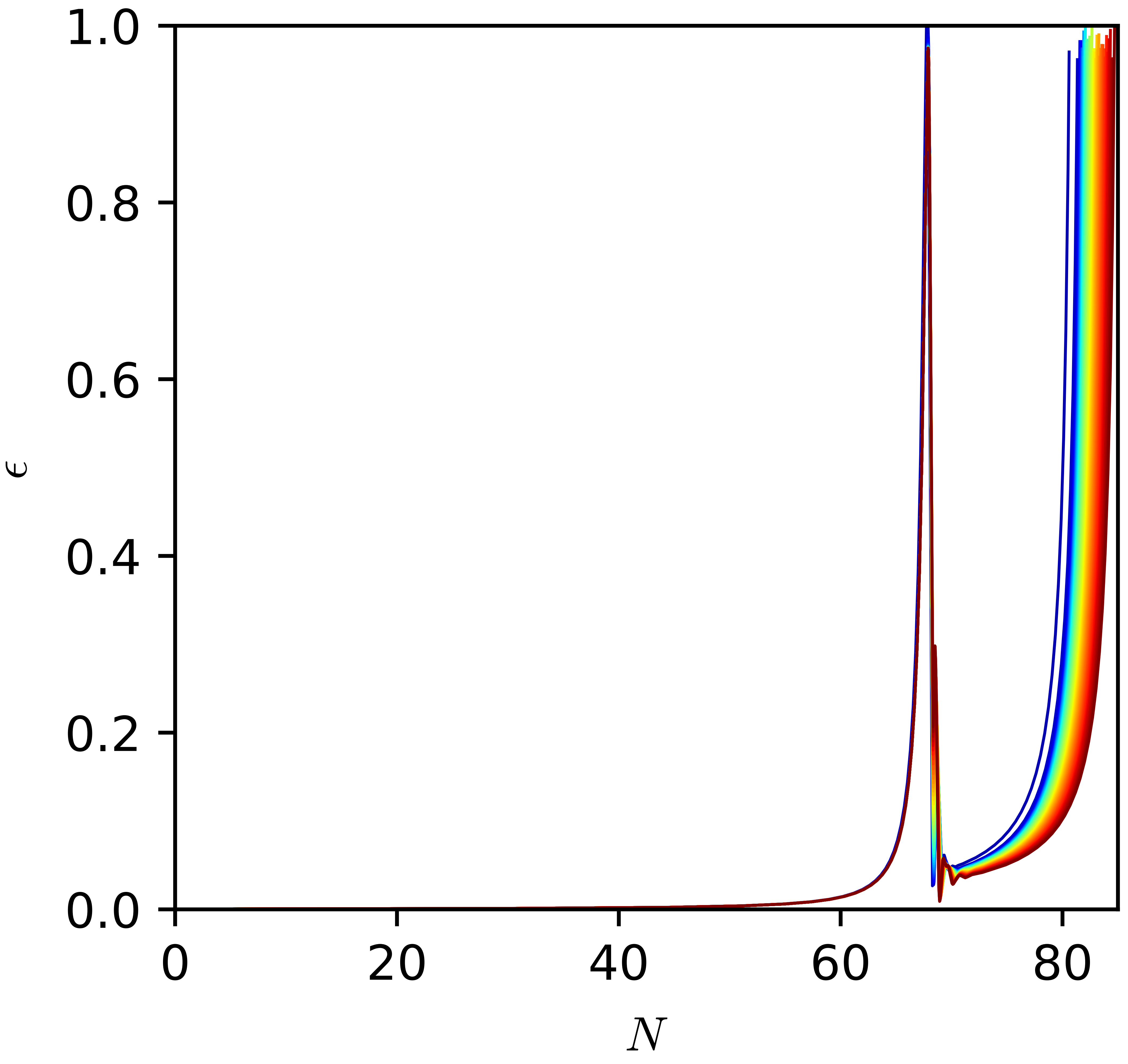
$$\phi_1 \simeq b_1 m_2^2 \phi_f^2 / 3$$

(MB1)

Example of background evolution

(MB1)

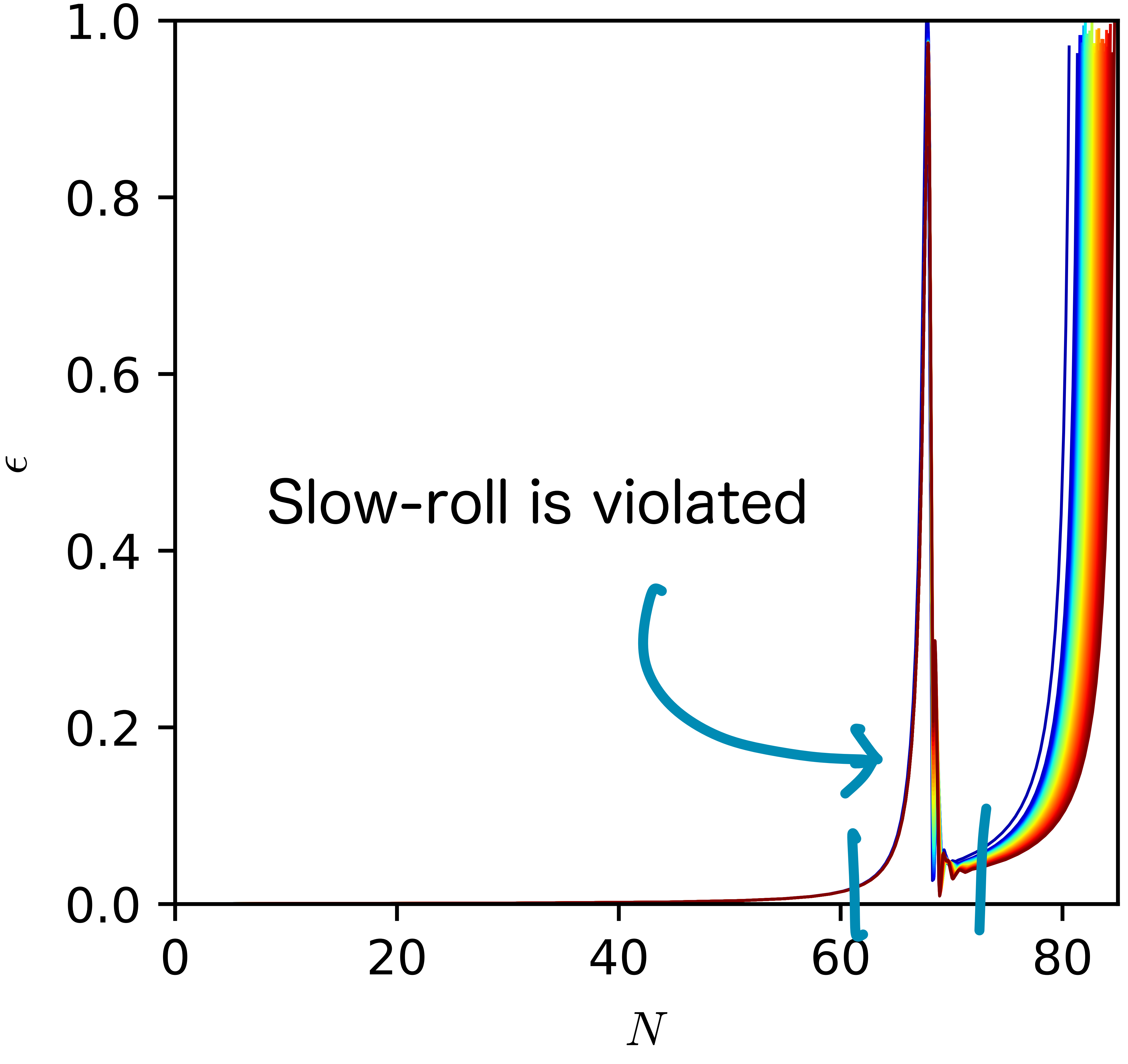
Example of background evolution



$$\epsilon = -\frac{\dot{H}}{H^2}$$

(MB1)

Example of background evolution



$$\epsilon = -\frac{\dot{H}}{H^2}$$

(MB1)

Example of background evolution

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot{\theta}^2 - \frac{1}{a^3 M_{\rm Pl}^2}\left(\frac{a^3\dot{\sigma}^2}{H}\right)^{\cdot} + b_\phi u(t)\right] Q_\sigma$$
$$= 2\left(\dot{\theta}\delta s\right)^{\cdot} - 2\left(\frac{\dot{H}}{H} + \frac{V_\sigma}{\dot{\sigma}}\right)\dot{\theta}\delta s + b_{\phi\phi}\dot{\sigma}^2 \sin 2\theta\delta s + 2b_\phi h(t)$$

$$\ddot{\delta s} + 3H\dot{\delta s} + \left[\frac{k^2}{a^2} + m_{\rm iso}^2\right]\delta s = 2\frac{V_s}{H}\left(\frac{H}{\dot{\sigma}}Q_\sigma\right)^{\cdot},$$

(MB1)

Example of background evolution

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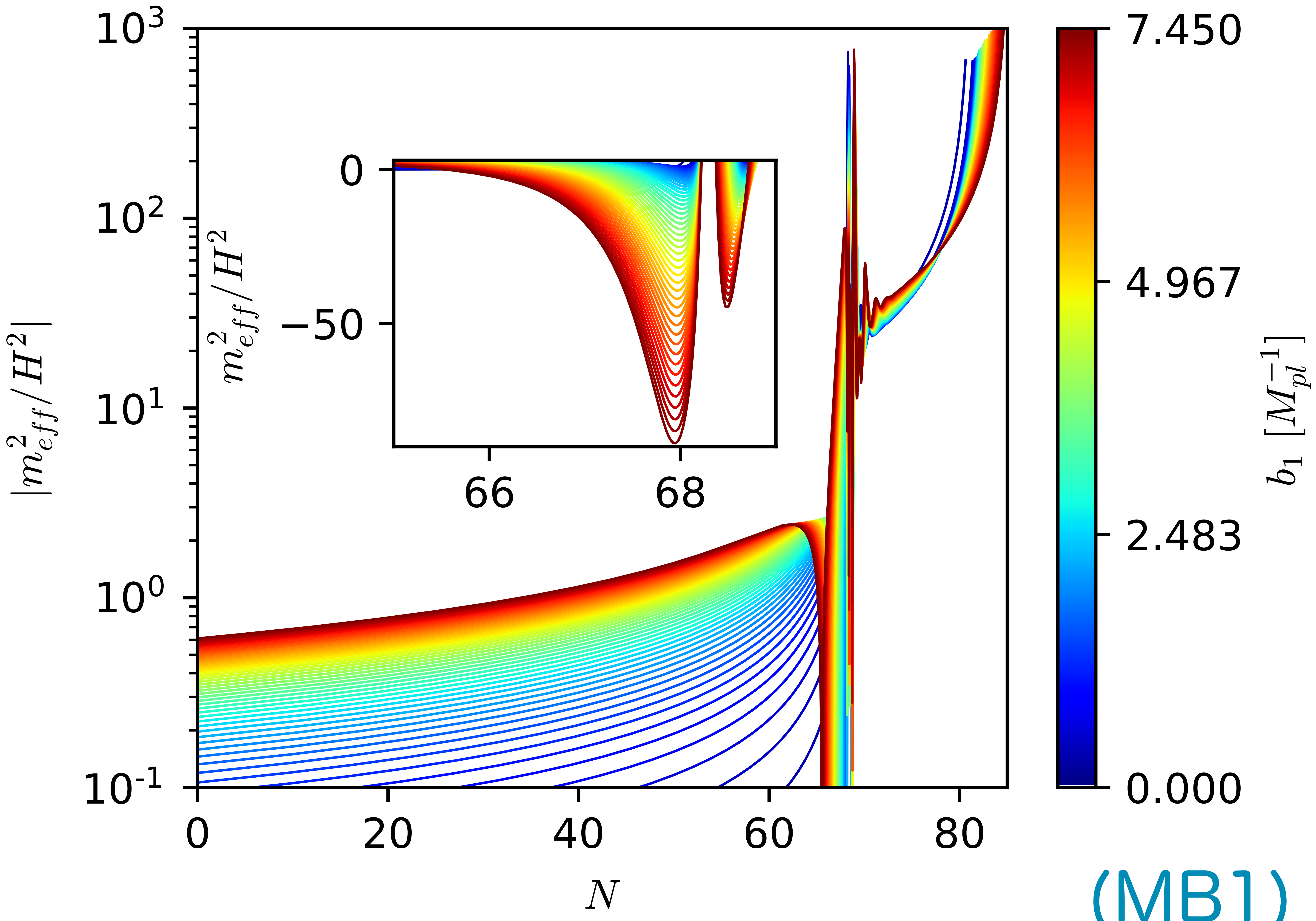


(MB1)

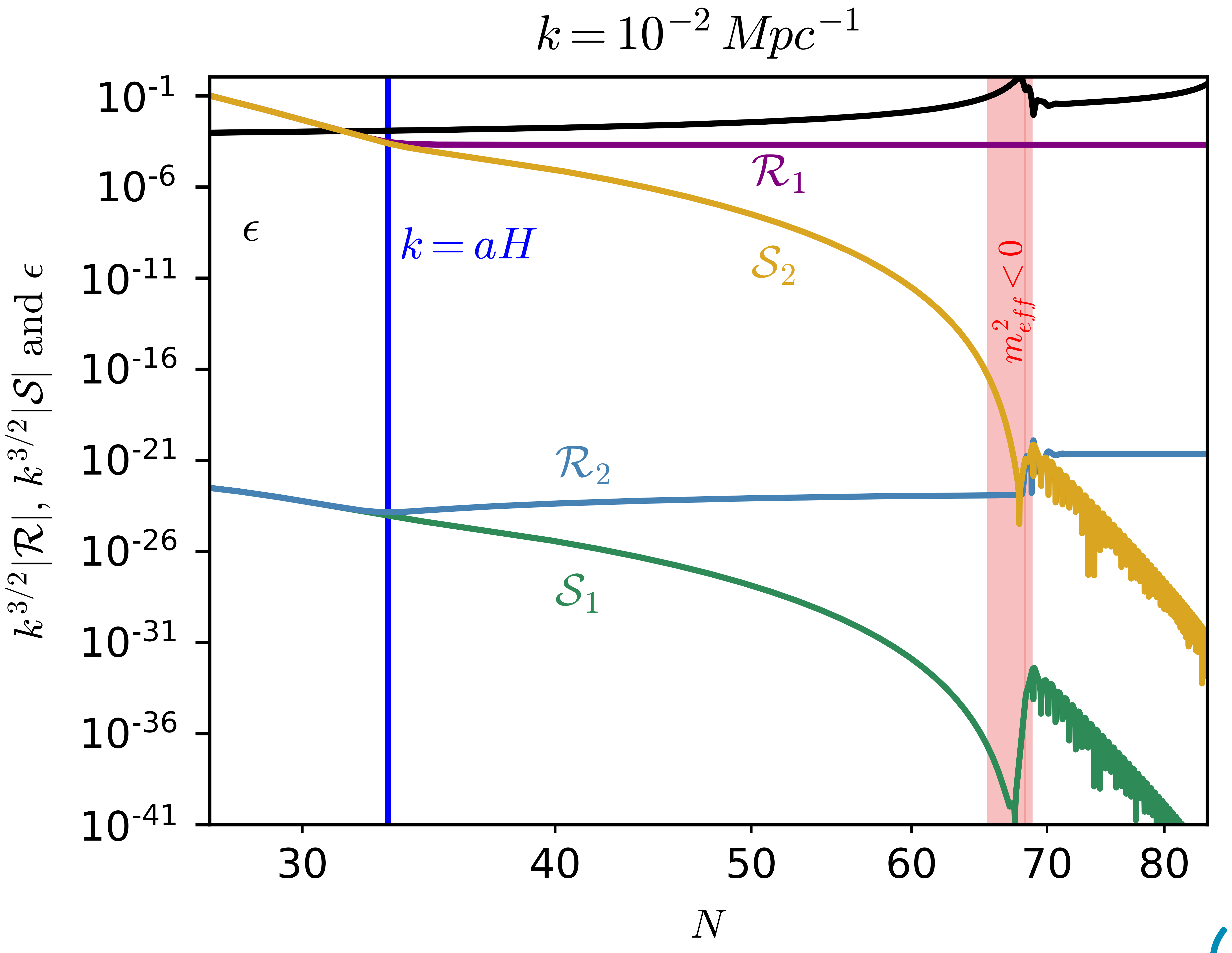
Example of background evolution

Evolution of adiabatic and isocurvature modes

Horizon crossing well before the transition:

Isocurvature modes decay

No amplification of curvature modes



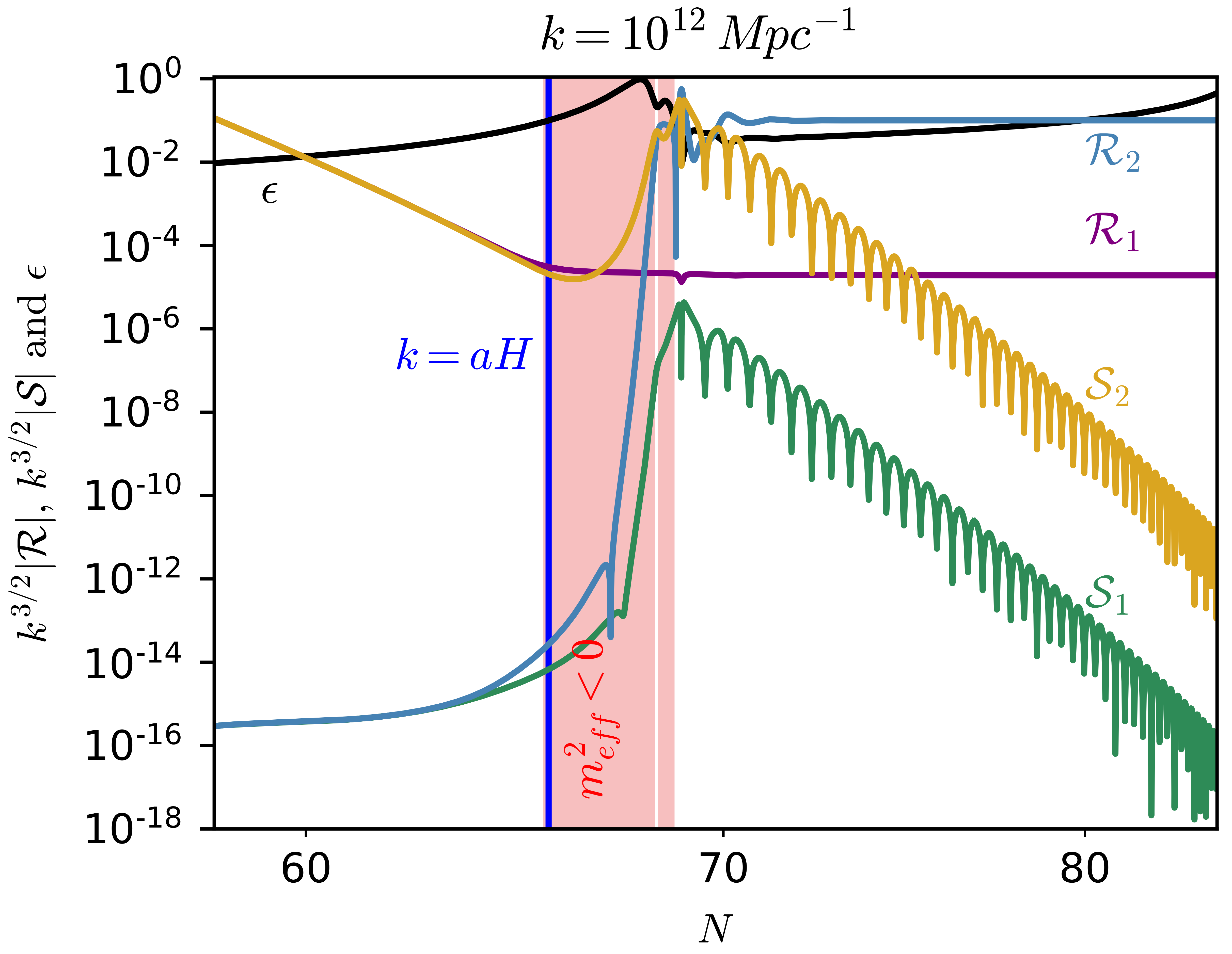
(MB1)

Evolution of adiabatic and isocurvature modes

Horizon crossing around the time of the transition:

Isocurvature modes experience a temporary growth

Sizeable amplification of curvature modes

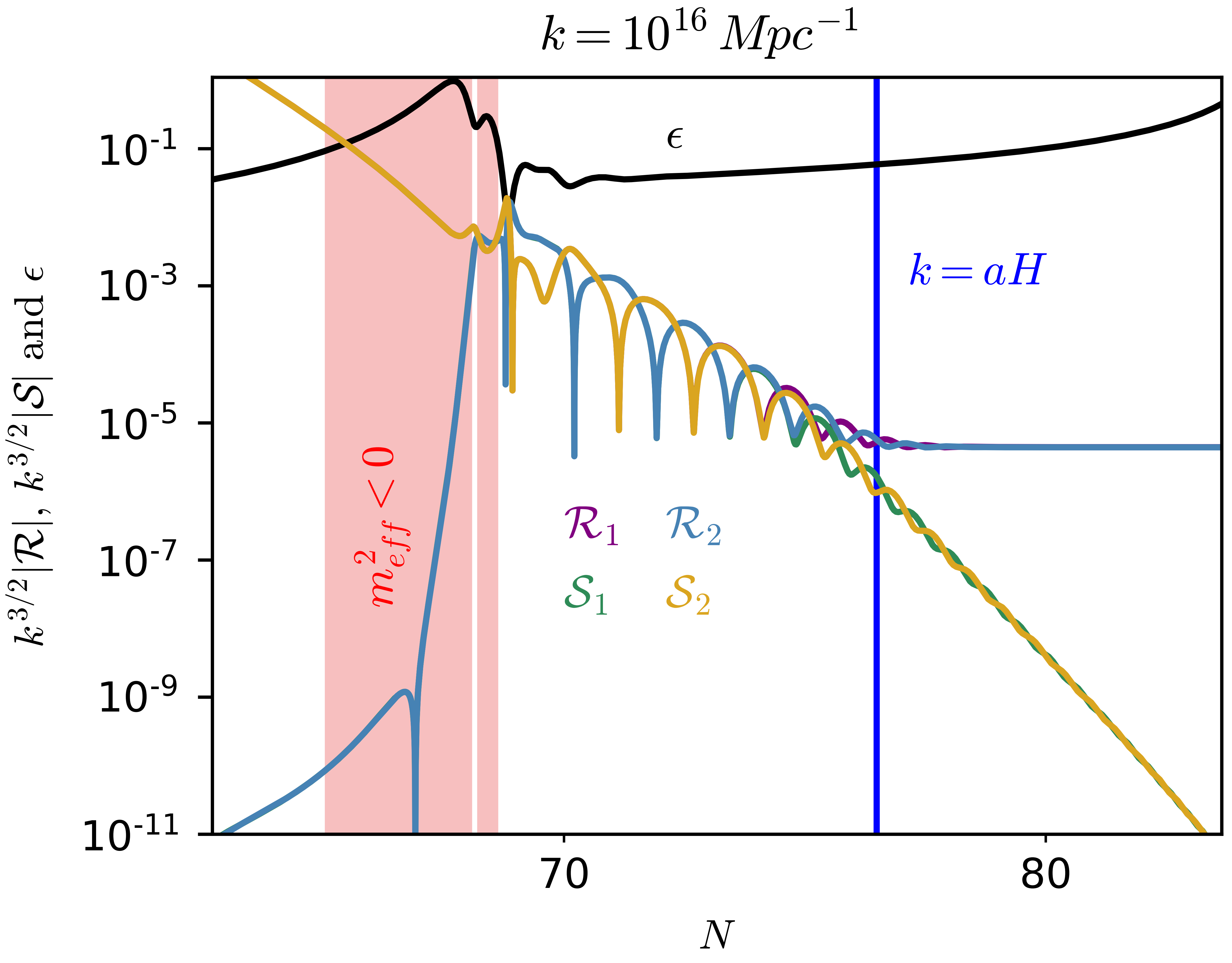


Evolution of adiabatic and isocurvature modes

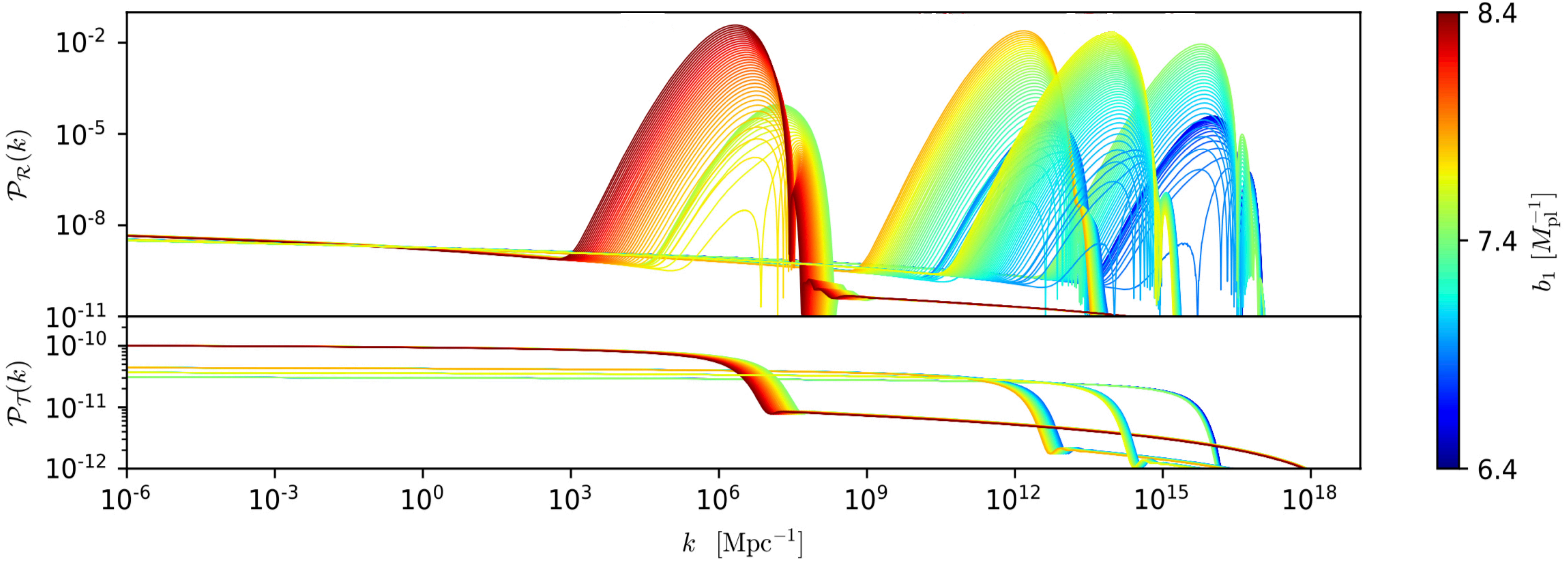
Horizon crossing long after the transition:

Isocurvature modes are inside the horizon when their mass becomes negative

No resulting amplification of curvature modes



Primordial power spectra



(MB1)

Second order SGWB

Large scalar perturbations act as a source of gravitational waves when they re-enter the horizon during radiation era

$$\Omega_{\rm GW} = \frac{\Omega_{r,0}}{36}\int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} ds \left[\frac{(d^2-1/3)(s^2-1/3)}{s^2-d^2}\right]^2 \mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s+d)\right)\mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s-d)\right)\left[\mathcal{I}_c(d,s)^2+\mathcal{I}_s(d,s)^2\right]$$

$$\mathcal{I}_c(x,y) = 4\int_0^\infty \mathrm{d}\tau\, \tau(-\sin\tau)\Big[2T(x\tau)T(y\tau) + \Big(T(x\tau)+x\tau\, T'(x\tau)\Big)\Big(T(y\tau)+y\tau\, T'(y\tau)\Big)\Big]$$

$$\mathcal{I}s(x,y) = 4\int_0^\infty \mathrm{d}\tau\, \tau(\cos\tau)\Big\{2T(x\tau)T(y\tau) + \Big[T(x\tau)+x\tau\, T'(x\tau)\Big]\Big[T(y\tau)+y\tau\, T'(y\tau)\Big]\Big\}$$

Acquaviva, Bartolo, Matarrese, Riotto 2003 - Ananda, Clarkson, Wands 2005 - Baumann, Steinhardt, Takahashi, Ichiki 2007

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Acquaviva, Bartolo, Matarrese, Riotto 2003 - Ananda, Clarkson, Wands 2005 - Baumann, Steinhardt, Takahashi, Ichiki 2007

Transfer functions that depend on the dominating fluid at the time of horizon re-entry

Domènech 2019, Domènech, Pi, Sasaki 2020

Second order SGWB

Large scalar perturbations act as a source of gravitational waves when they re-enter the horizon during radiation era

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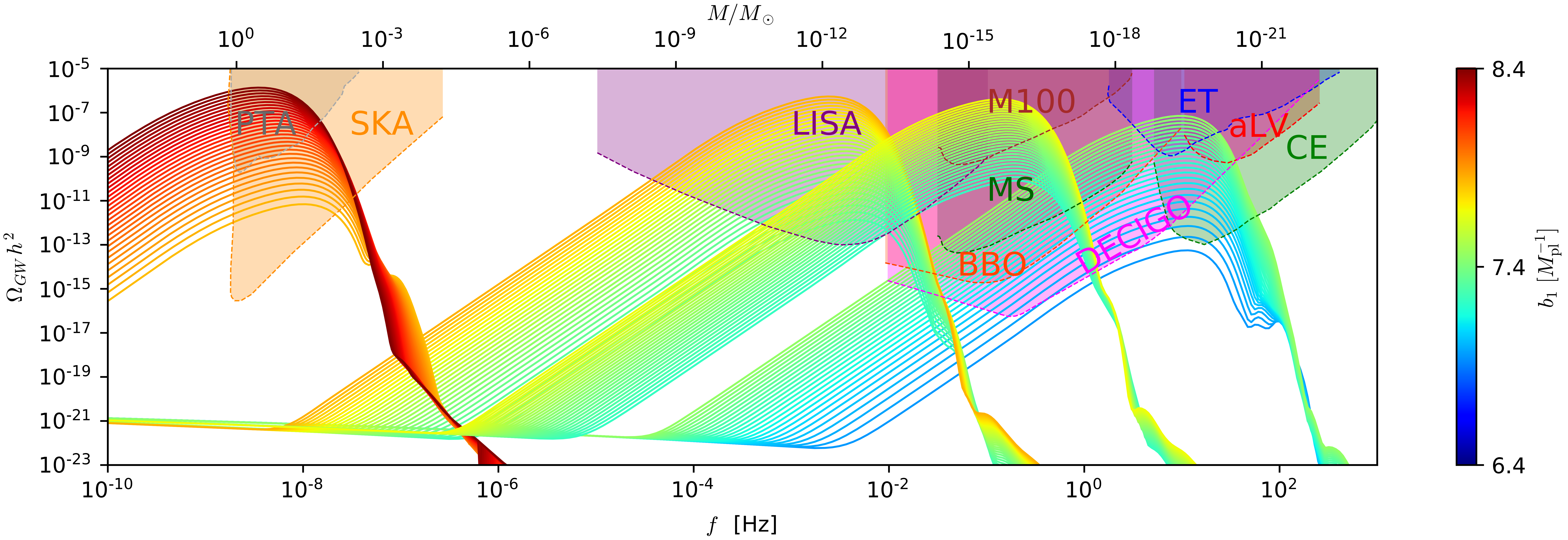
Acquaviva, Bartolo, Matarrese, Riotto 2003 - Ananda, Clarkson, Wands 2005 - Baumann, Steinhardt, Takahashi, Ichiki 2007

$$T(z) = \frac{9}{z^2}\left[\frac{\sin(z/\sqrt{3})}{z/\sqrt{3}} - \cos(z/\sqrt{3})\right]$$

Radiation dominated era

Second order SGWB



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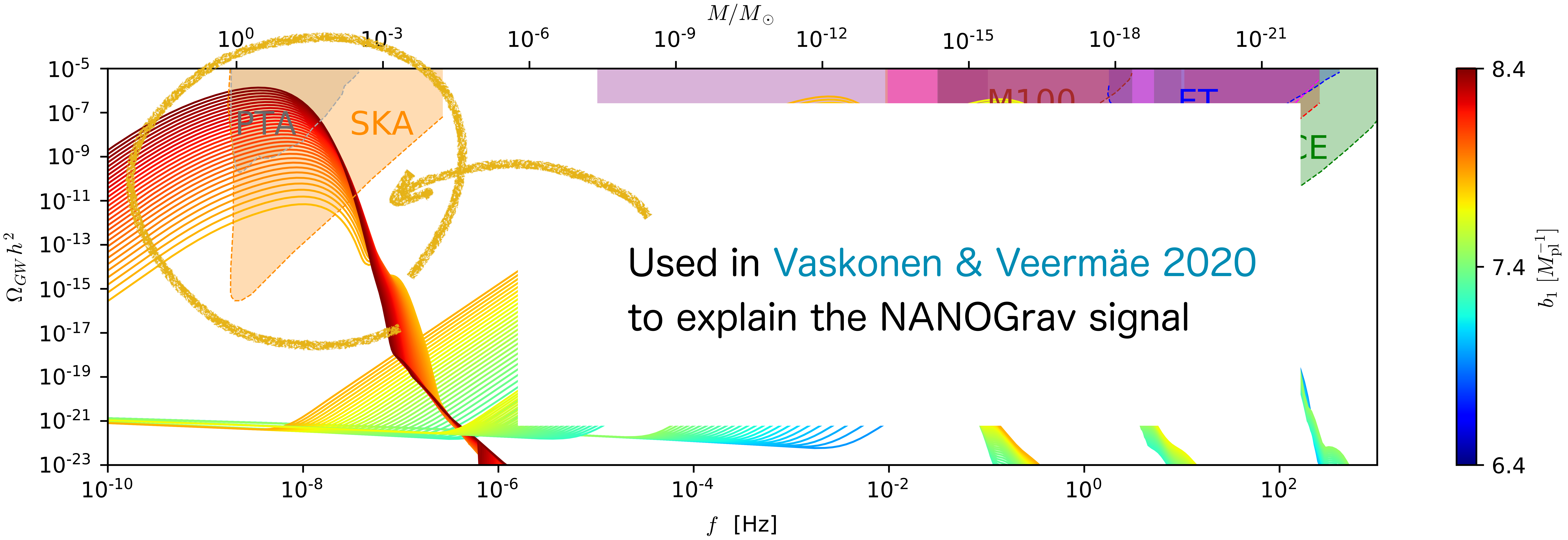
Second order SGWB



Used in Vaskonen & Veermäe 2020 to explain the NANOGrav signal

(See talk by Shi Pi for other interpretations)

NANOGrav 12.5-yr result and induced gravitational waves — *Shi Pi* — 14:00 - 14:30

Features in the scalar power spectrum

A SGWB with the shape of a bump or a broken power-law is expected in many other inflationary models, phase transitions etc.

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Is there any feature that can be used to distinguish between this model and other sources of GWs?

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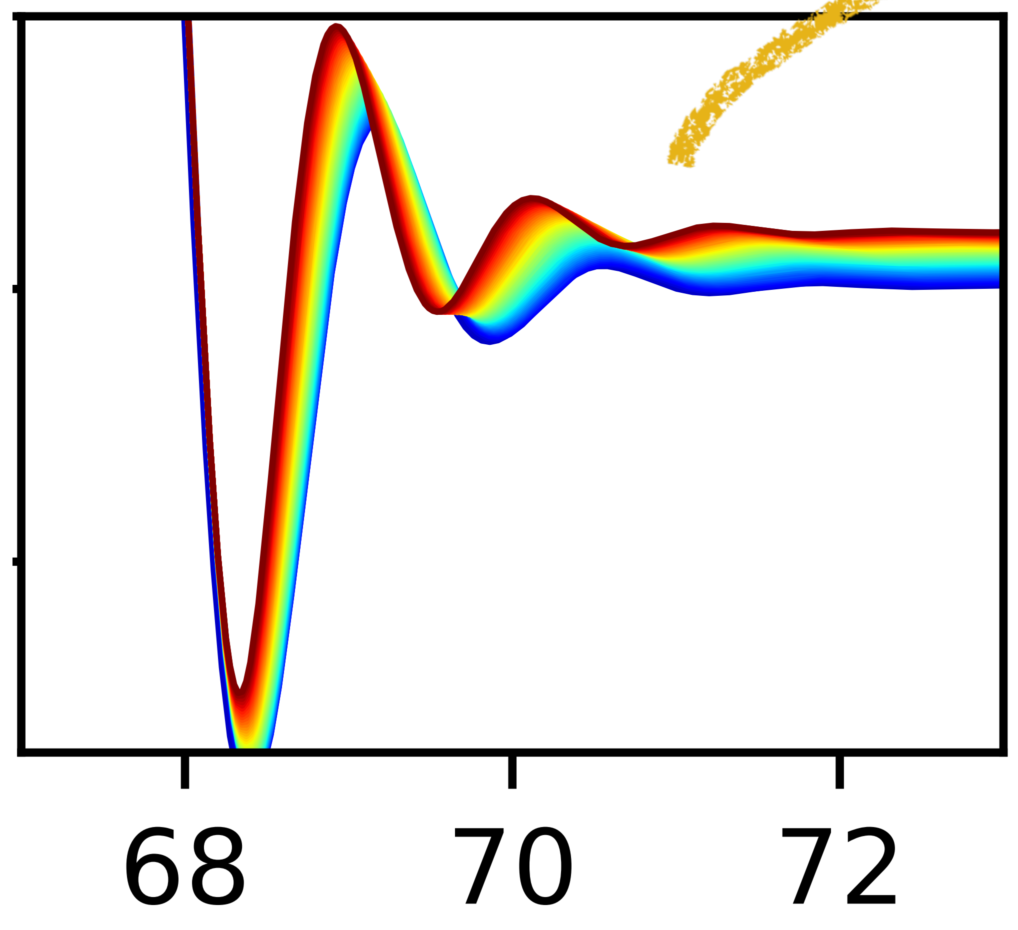
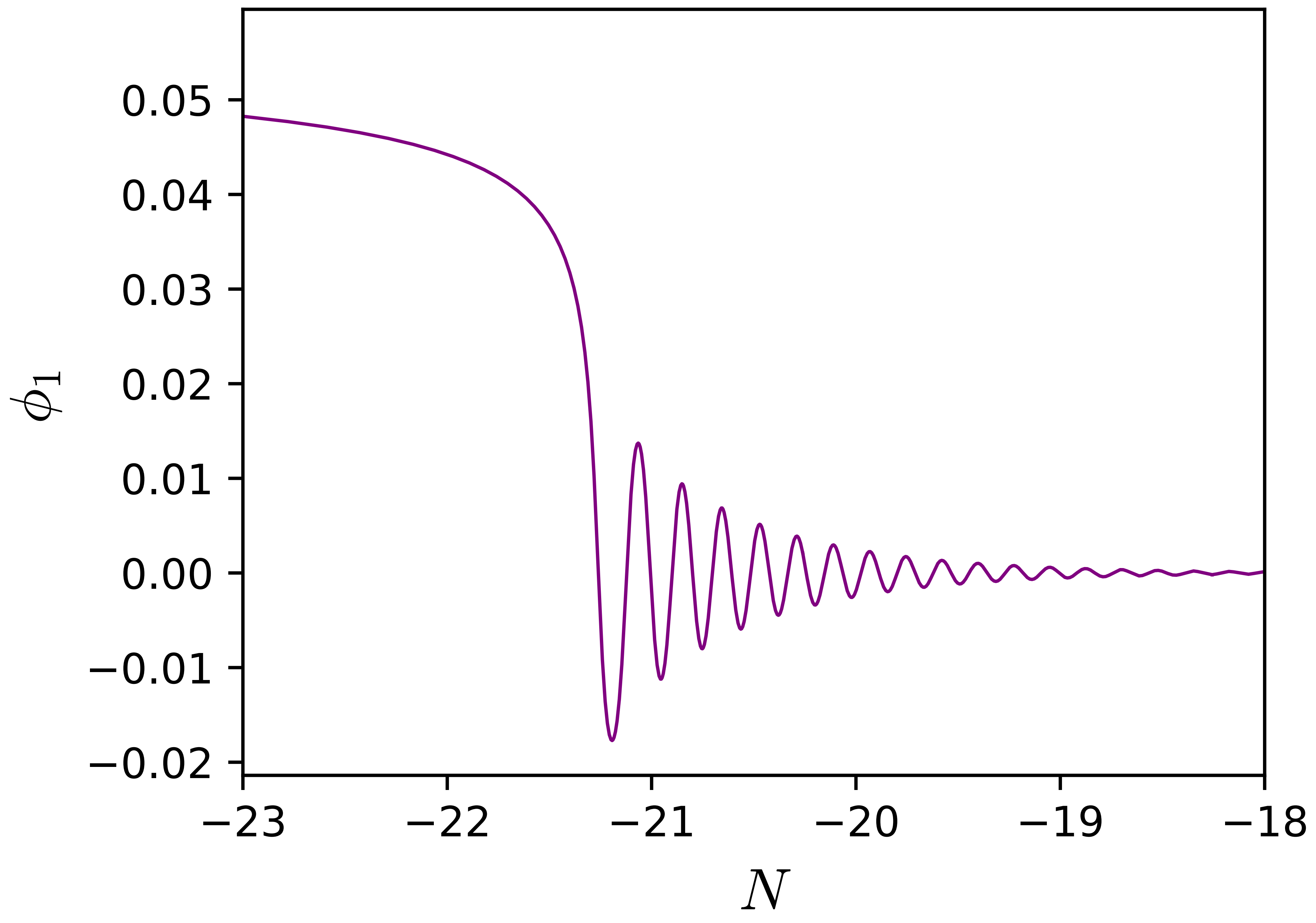


Models of the Primordial Standard Clock

Chen, Namjoo, Wang 2014

Features in the scalar power spectrum

Models of the Primordial Standard Clock   Chen, Namjoo, Wang 2014

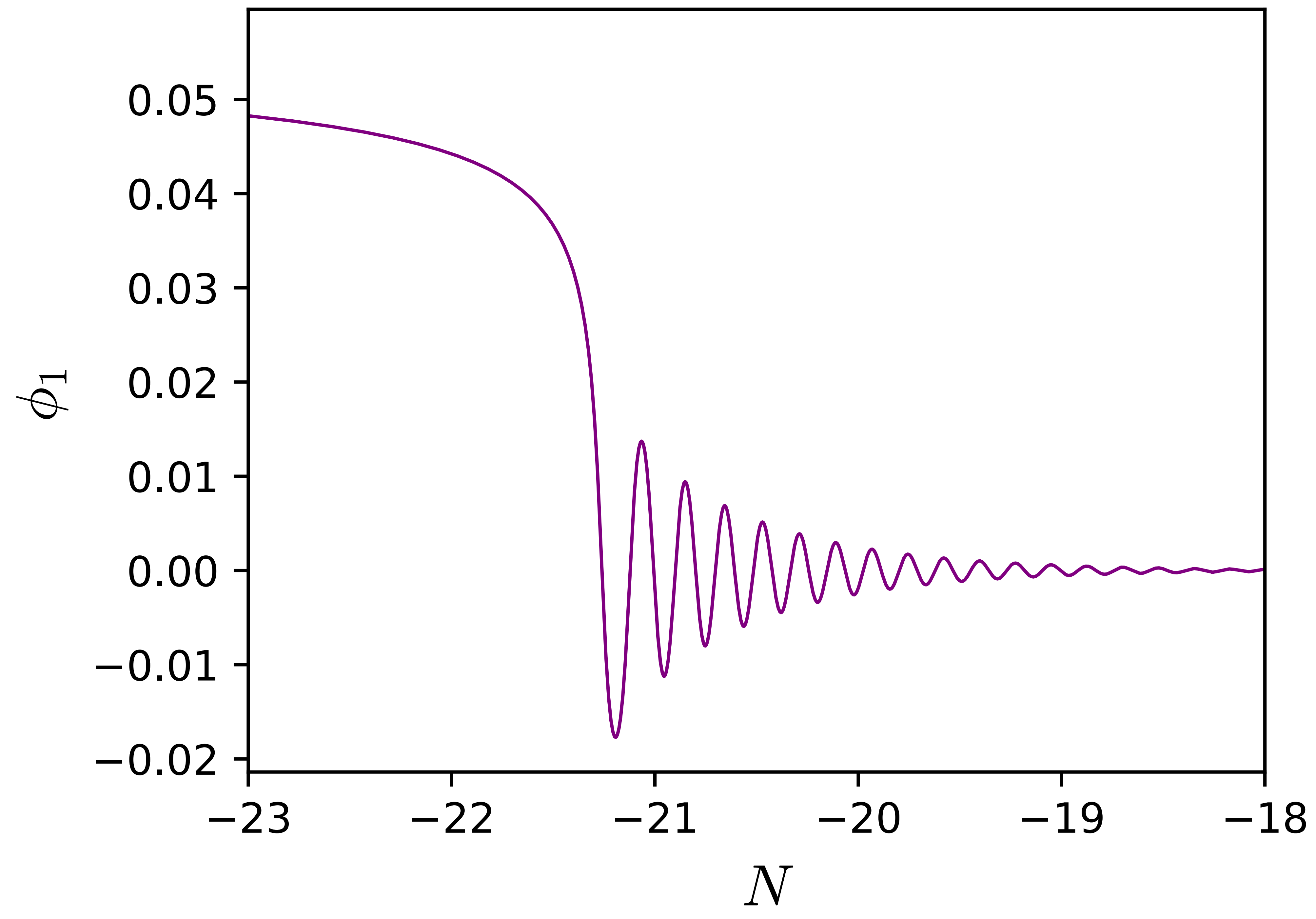


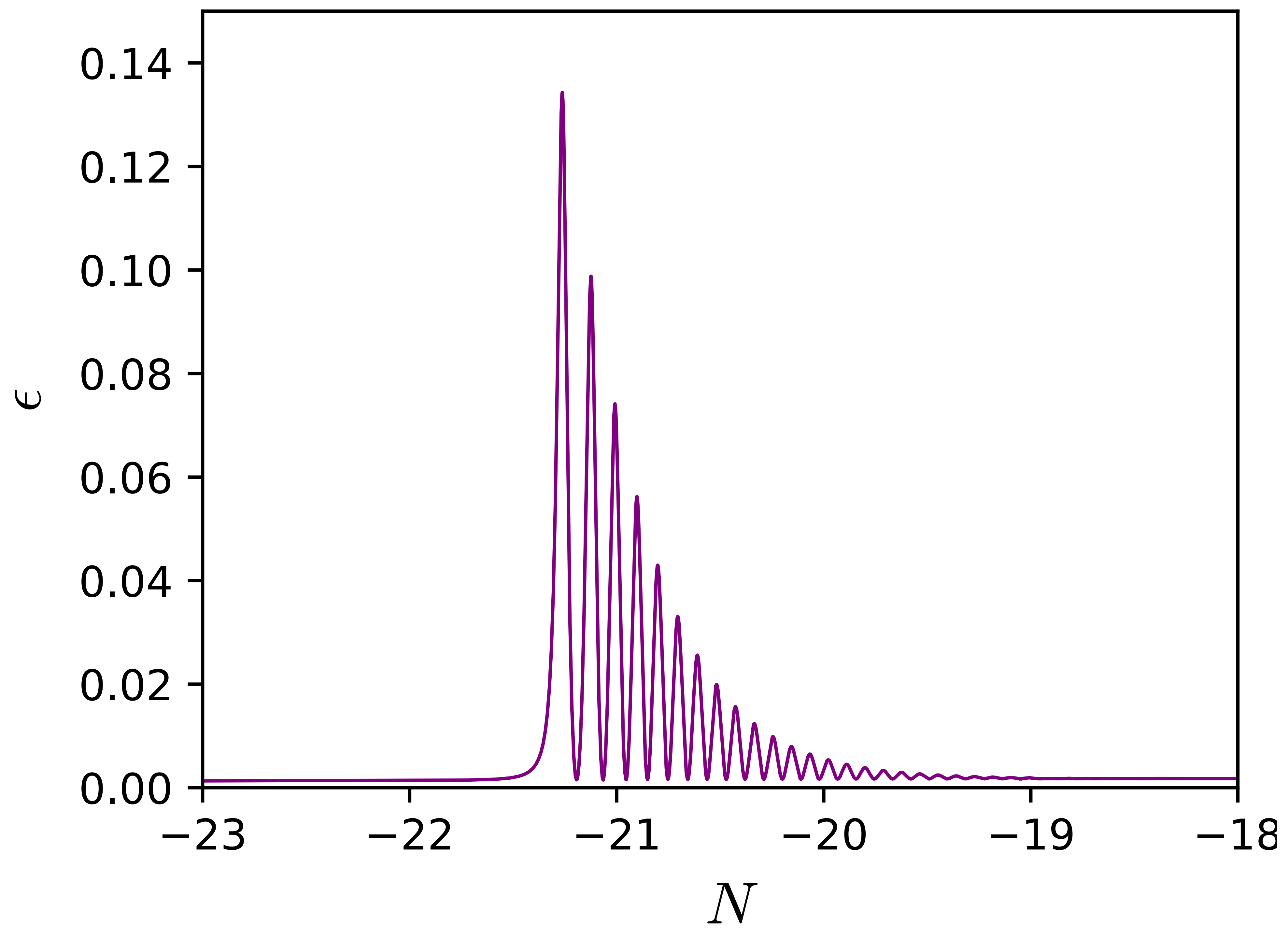
The frequency of the oscillations is proportional to

$$\frac{1}{H}\frac{d^2V}{d\phi_1^2}\bigg|_{\phi_1=0} \simeq \frac{\sqrt{6C_1}}{\phi_f}$$

Features in the scalar power spectrum

Models of the Primordial Standard Clock Chen, Namjoo, Wang 2014





Features in the scalar power spectrum

Models of the Primordial Standard Clock Chen, Namjoo, Wang 2014

The primordial standard clock signal was originally considered to produce features at CMB scales. The clock signal consists in a combination of:

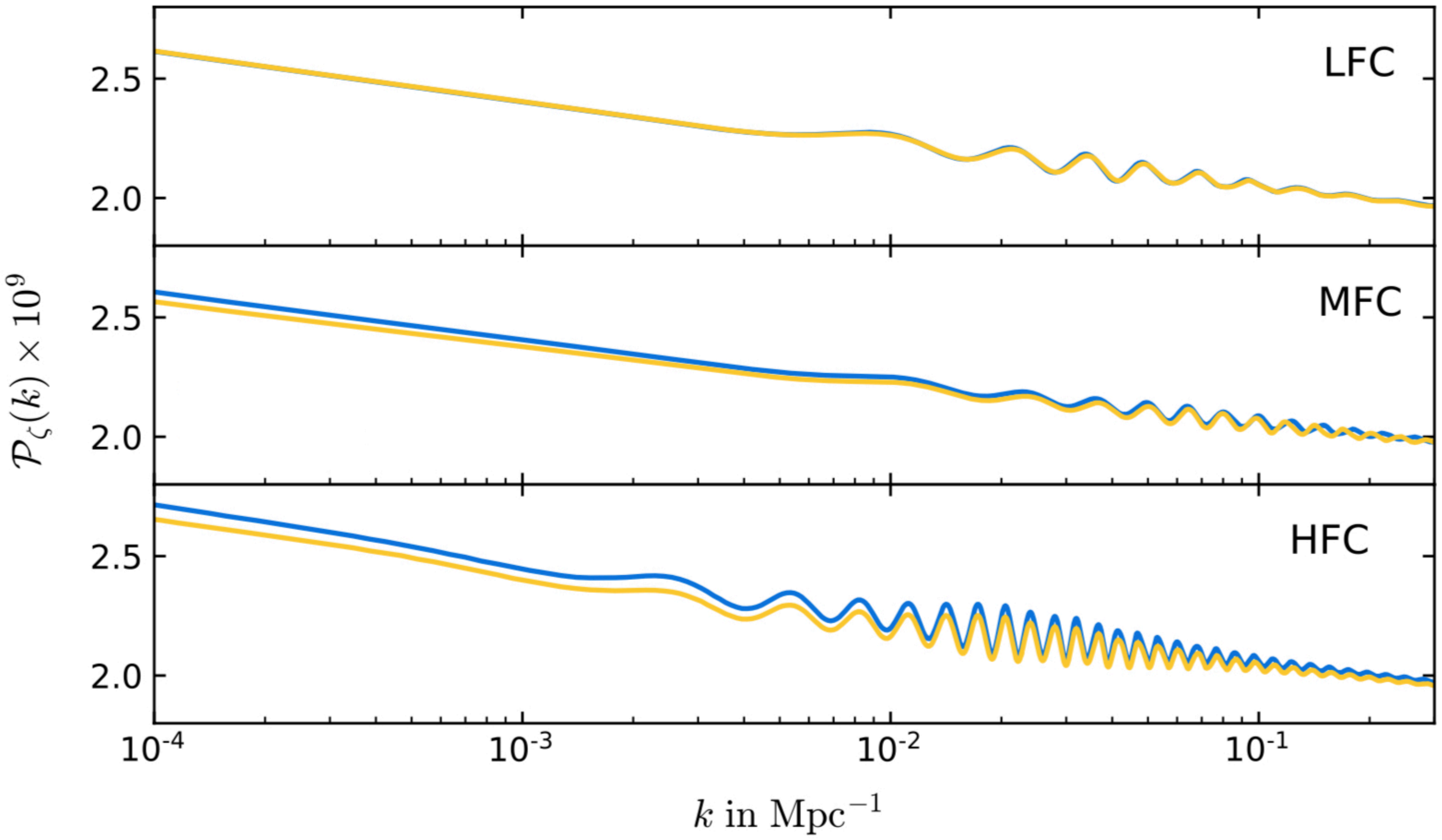
- Sharp feature signal: caused by the mechanism exciting the clock field oscillations. Oscillatory modulation of the PPS with sinusoidal dependence on the wavenumber.
- Resonance feature signal: generated by the classical oscillations of the massive clock field. Oscillatory modulation of the PPS with sinusoidal dependence on the logarithm of the wavenumber

(See also talk by Lukas)

Features in the scalar power spectrum

Models of the Primordial Standard Clock

Chen, Namjoo, Wang 2014



Plot from MB, Chen, Hazra 2021

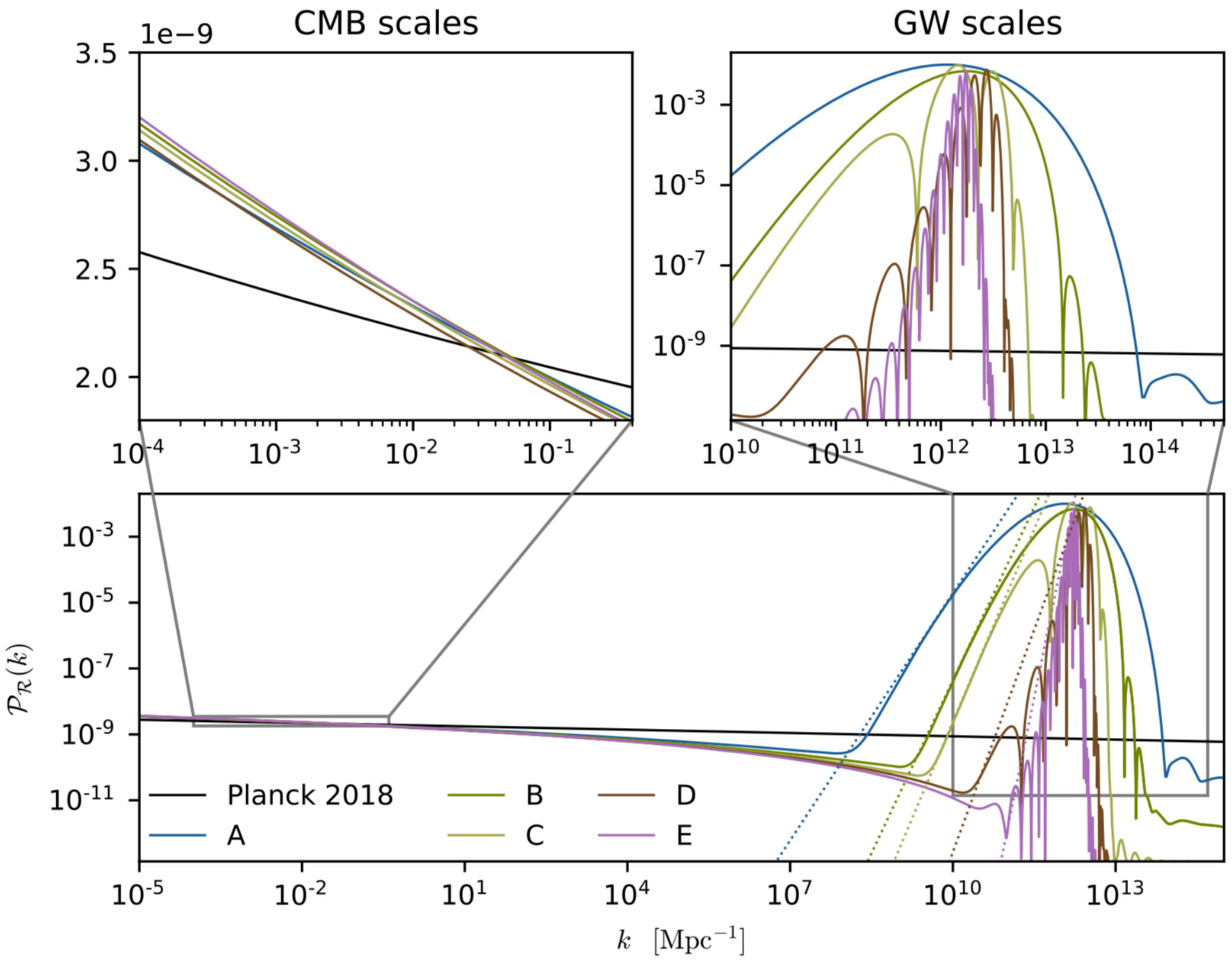
Features in the scalar power spectrum

Models of the Primordial Standard Clock Chen, Namjoo, Wang 2014

The primordial standard clock signal was originally considered to produce features at CMB scales.

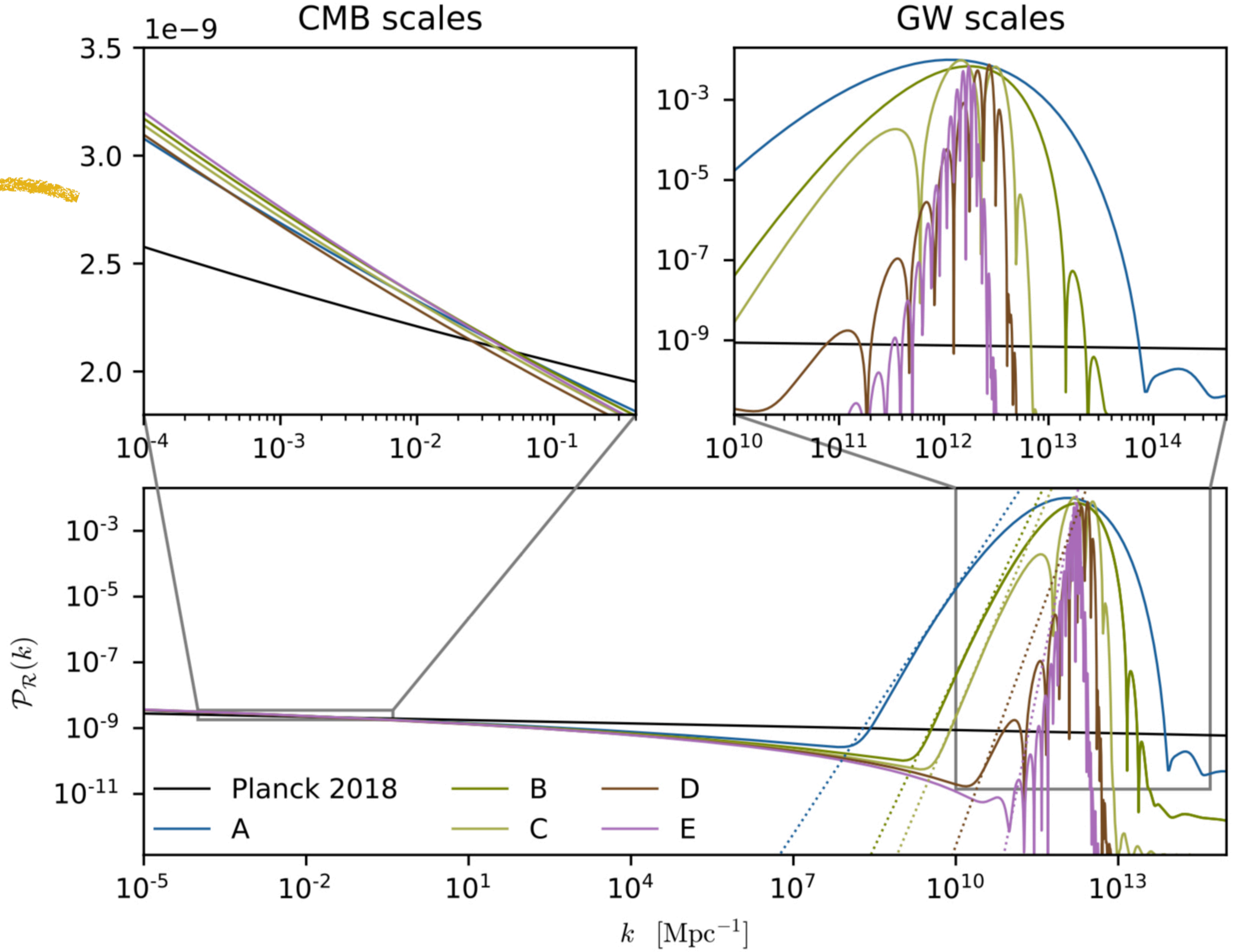
Can we observe these features in the Stochastic Gravitational Wave Background?

Models of the primordial standard clock (1)



(MB2)

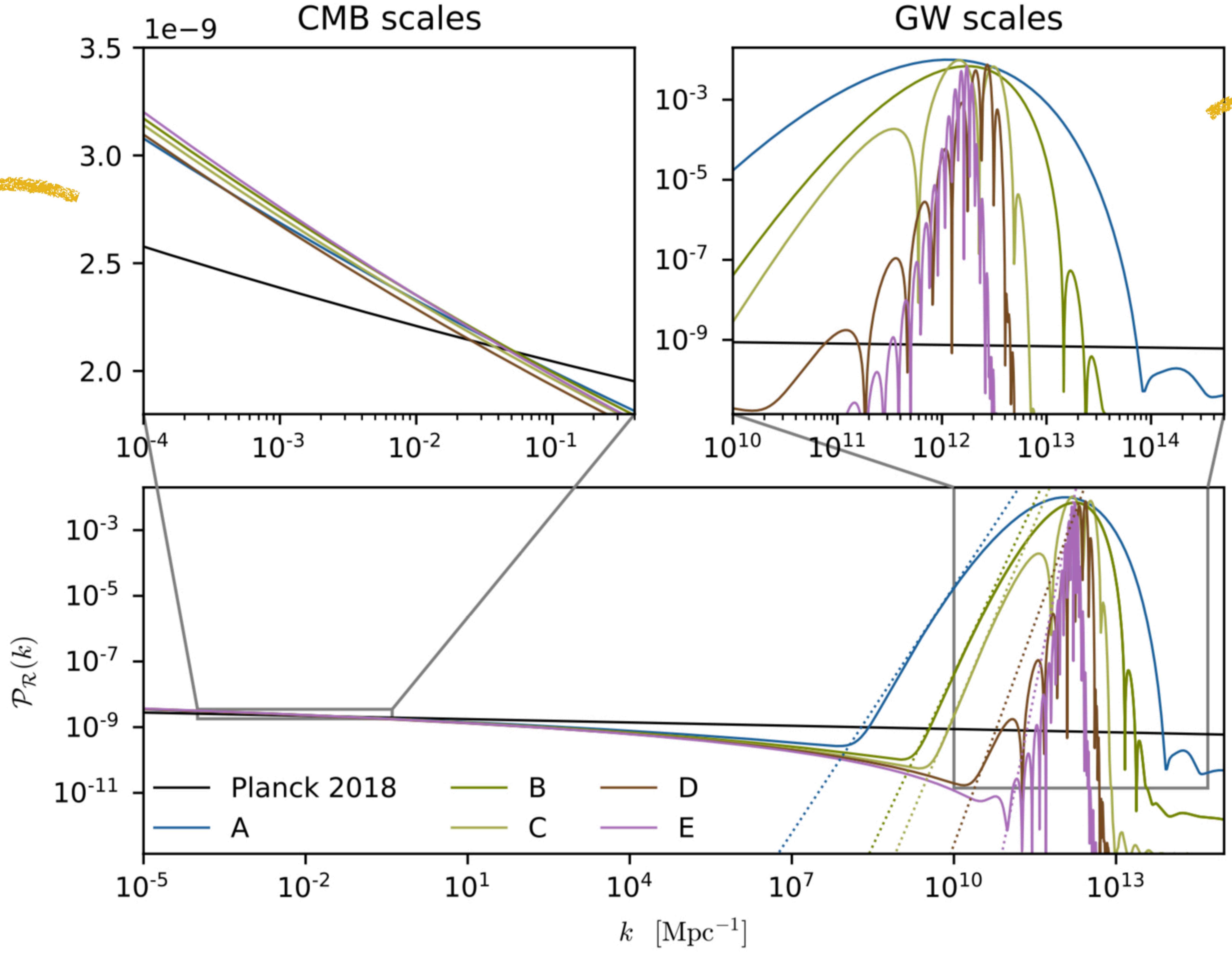
Models of the primordial standard clock (1)



Some tension with Planck

(MB2)

Models of the primordial standard clock (1)



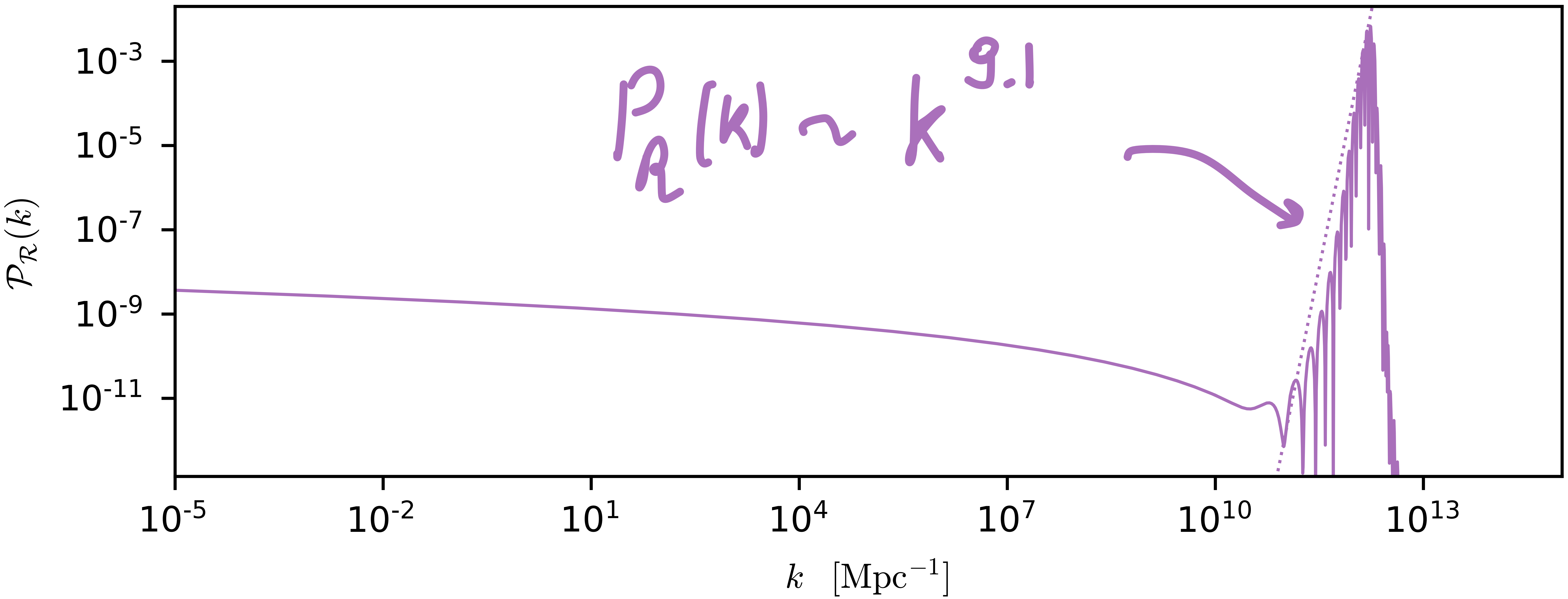
Some tension with Planck

Increasing m_{ϕ_1}/H the peak becomes more narrow and shows oscillations

(MB2)

Models of the primordial standard clock (1)

The PPS can grow faster than in single field inflation for which the steepest possible growth is k^4



Byrnes, Cole, Patil 2018
Carrillho, Malik, Mulryne 2019,
Özsoy, Tasinato 2019,
Tasinato 2020

(MB2)

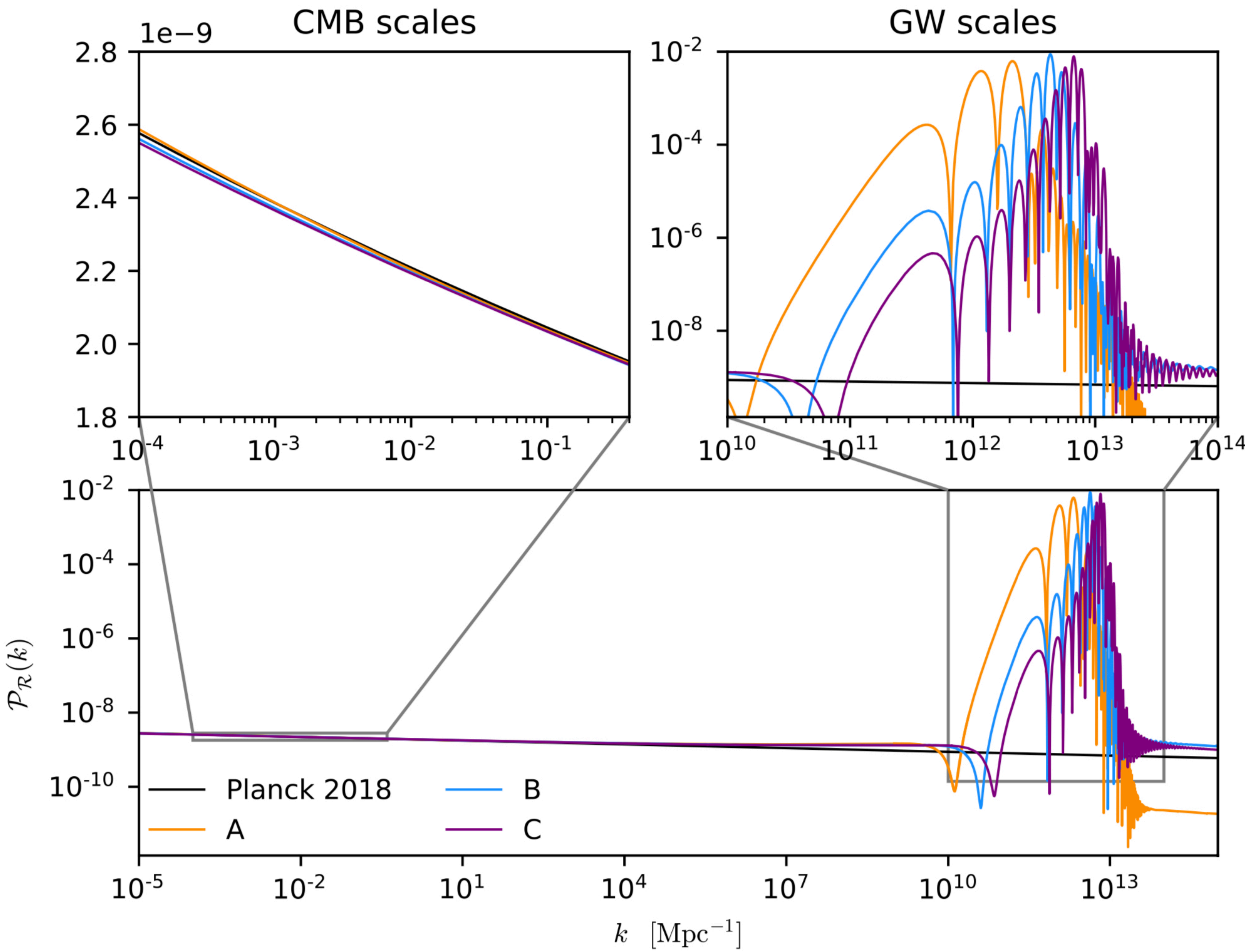
Models of the primordial standard clock (2)

Small field potential for ϕ_2

$$U(\phi_2) = V_0\left(1 - \frac{m_2^2}{2}\phi_2^2\right)$$

(MB2)

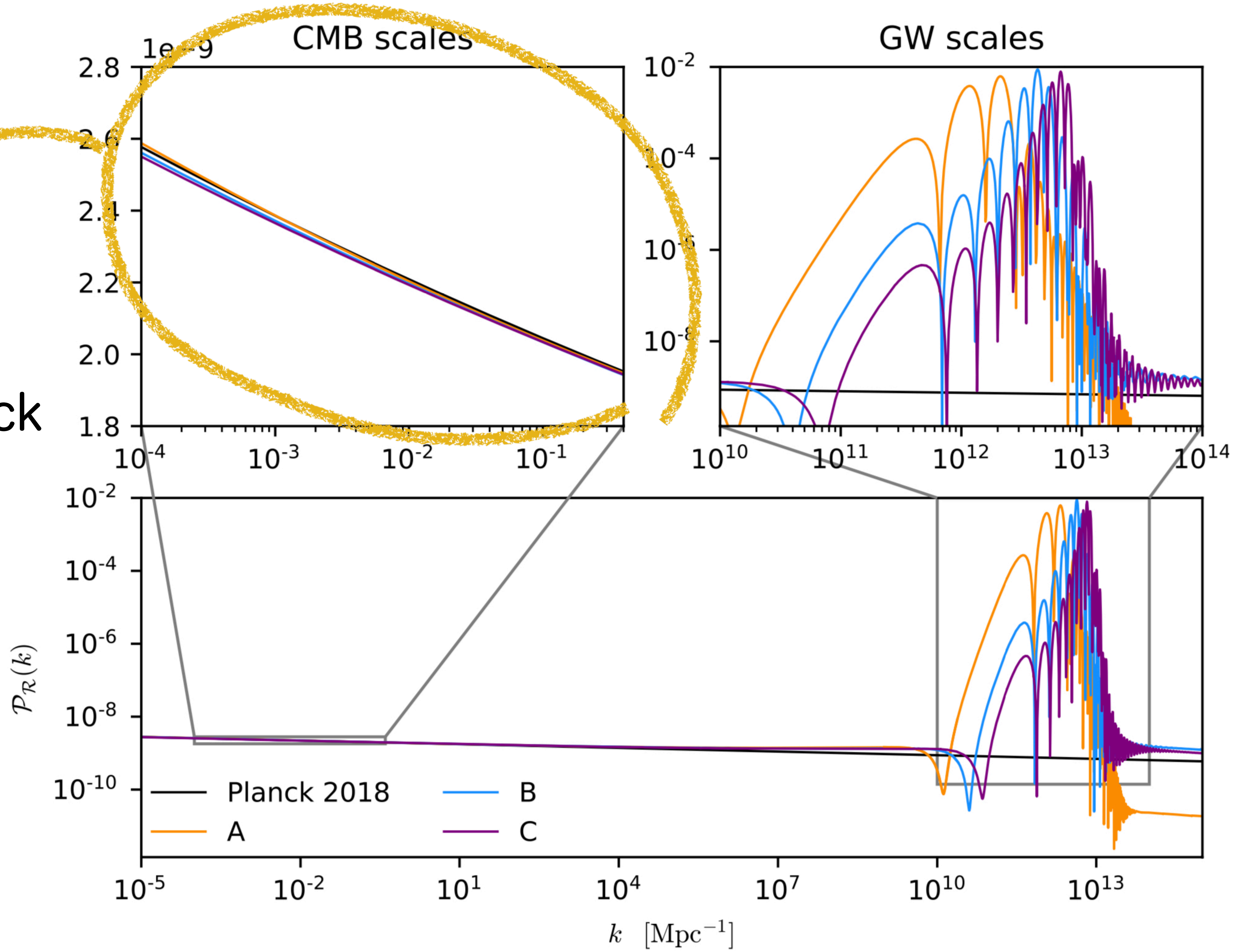
Models of the primordial standard clock (2)



(MB2)

Models of the primordial standard clock (2)

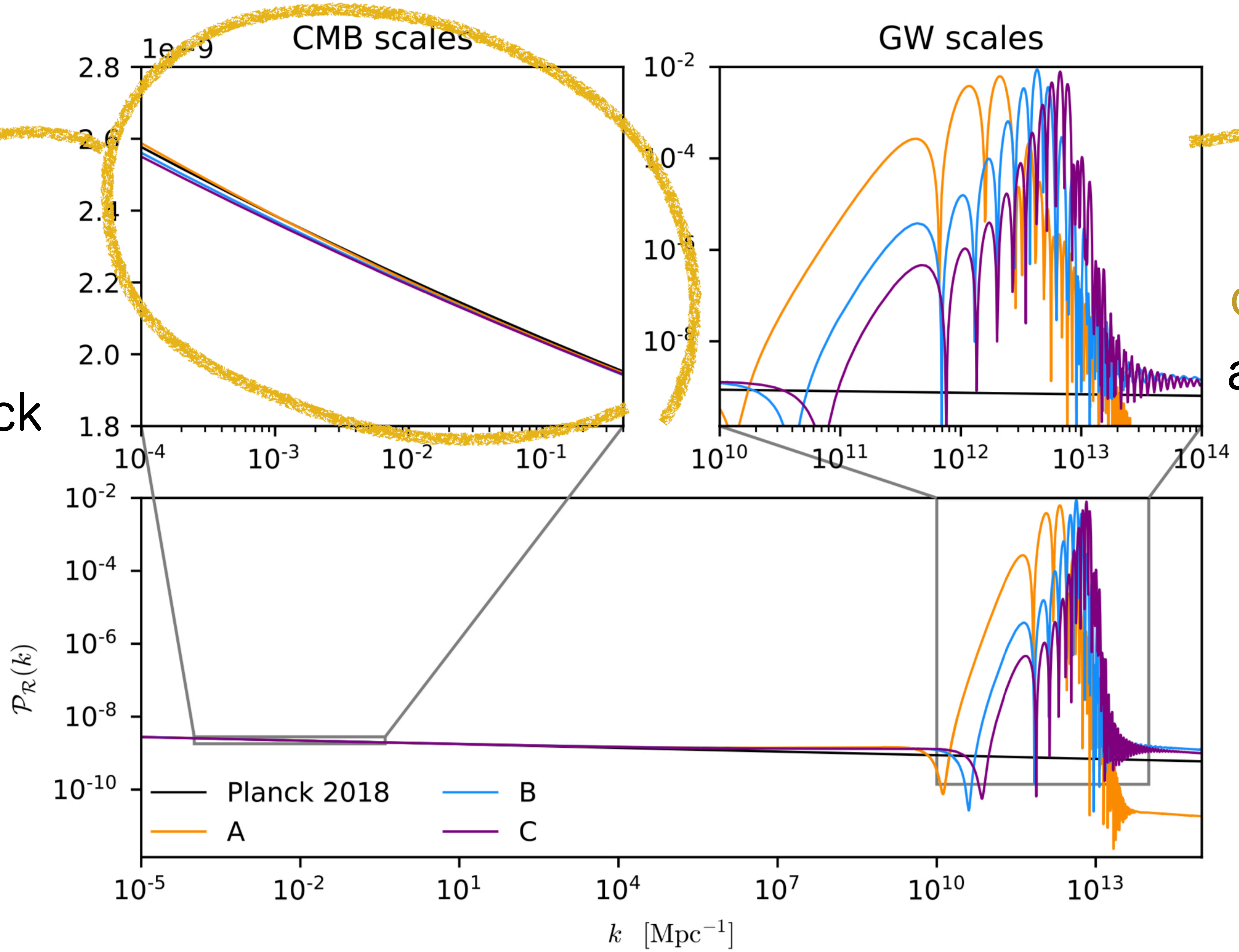
Agreement with Planck



(MB2)

Models of the primordial standard clock (2)

Agreement with Planck



Different pattern of oscillations before and around/after the peak. Mixture of sinusoidal and logarithmic oscillations.

(MB2)

Models of the primordial standard clock (3)

Small field potential for ϕ_2

$$U(\phi_2) = V_0 \left(1 - \frac{m_2^2}{2} \phi_2^2 \right)$$

Additional mass term for ϕ_1

$$V(\phi_1) = V_0 C_1 \left[1 - \exp\left(-\phi_1^2 / \phi_f^2 \right) \right] + V_0 \frac{m_0^2}{2} \phi_1^2$$

(MB2)

Models of the primordial standard clock (3)

Additional mass term for ϕ_1

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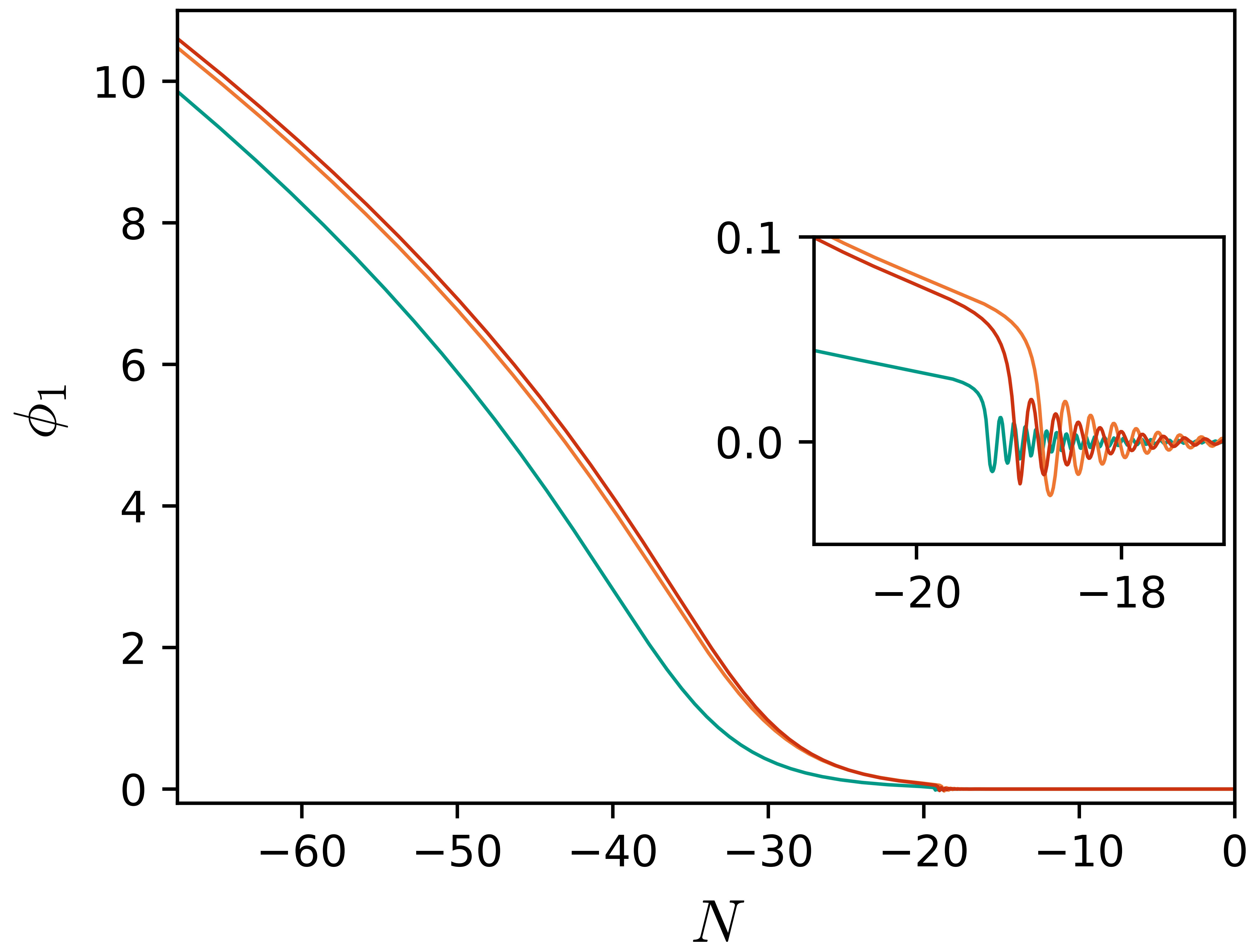
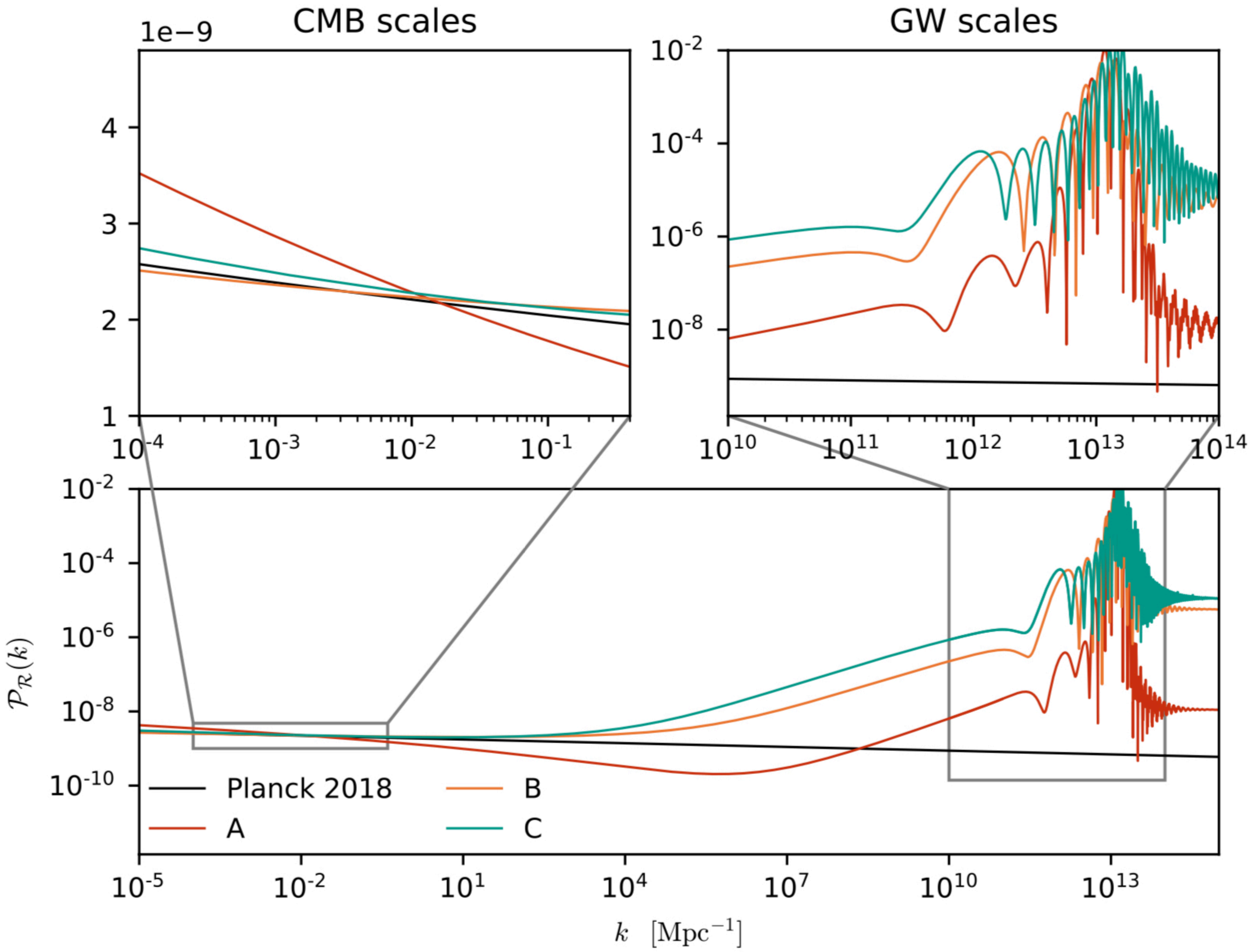


(MB2)

Models of the primordial standard clock (3)

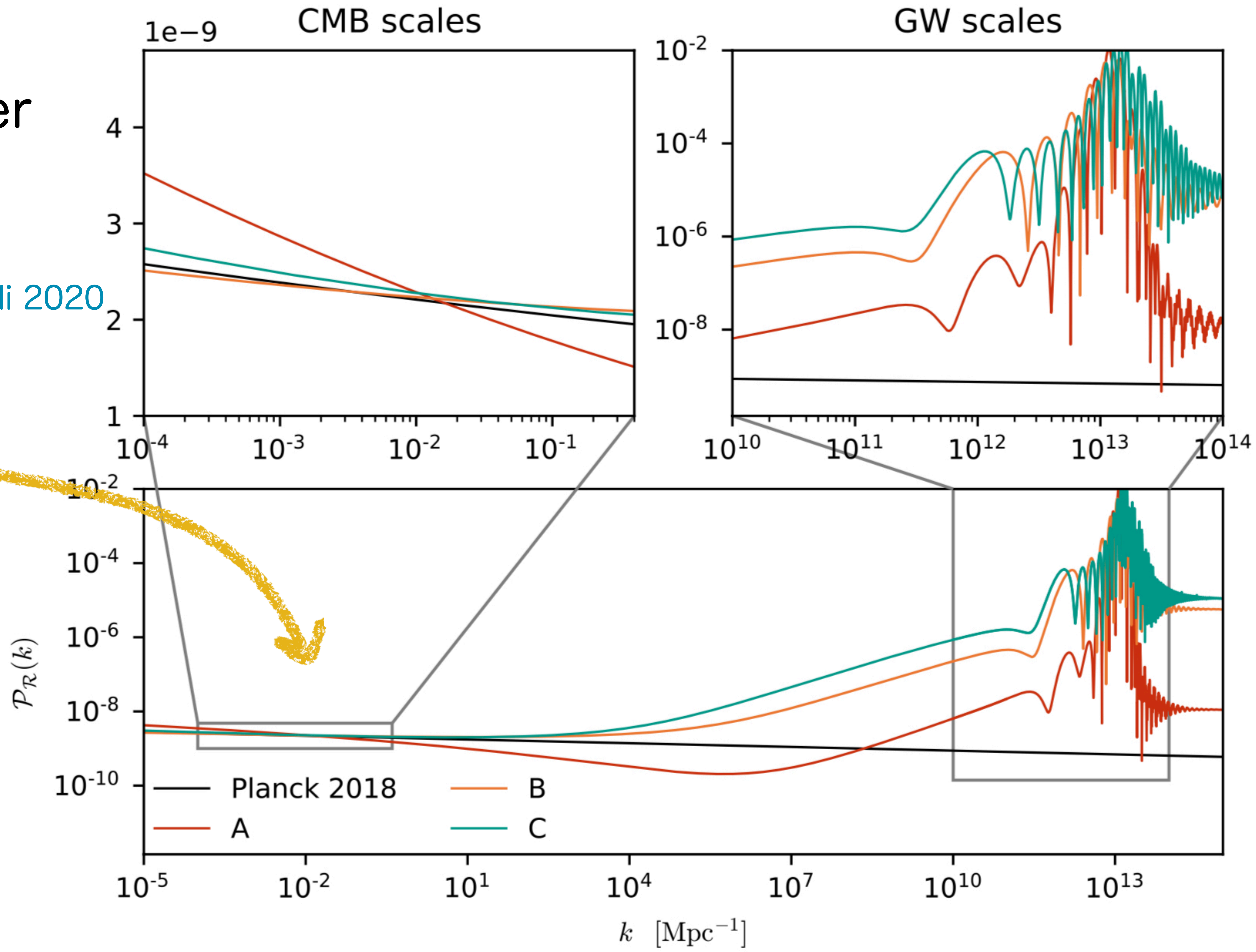


(MB2)

Models of the primordial standard clock (3)

Suppression of power at large scales

Chen, Namjoo, Wang 2014
MB, Hazra, Sriramkumar, Finelli 2020

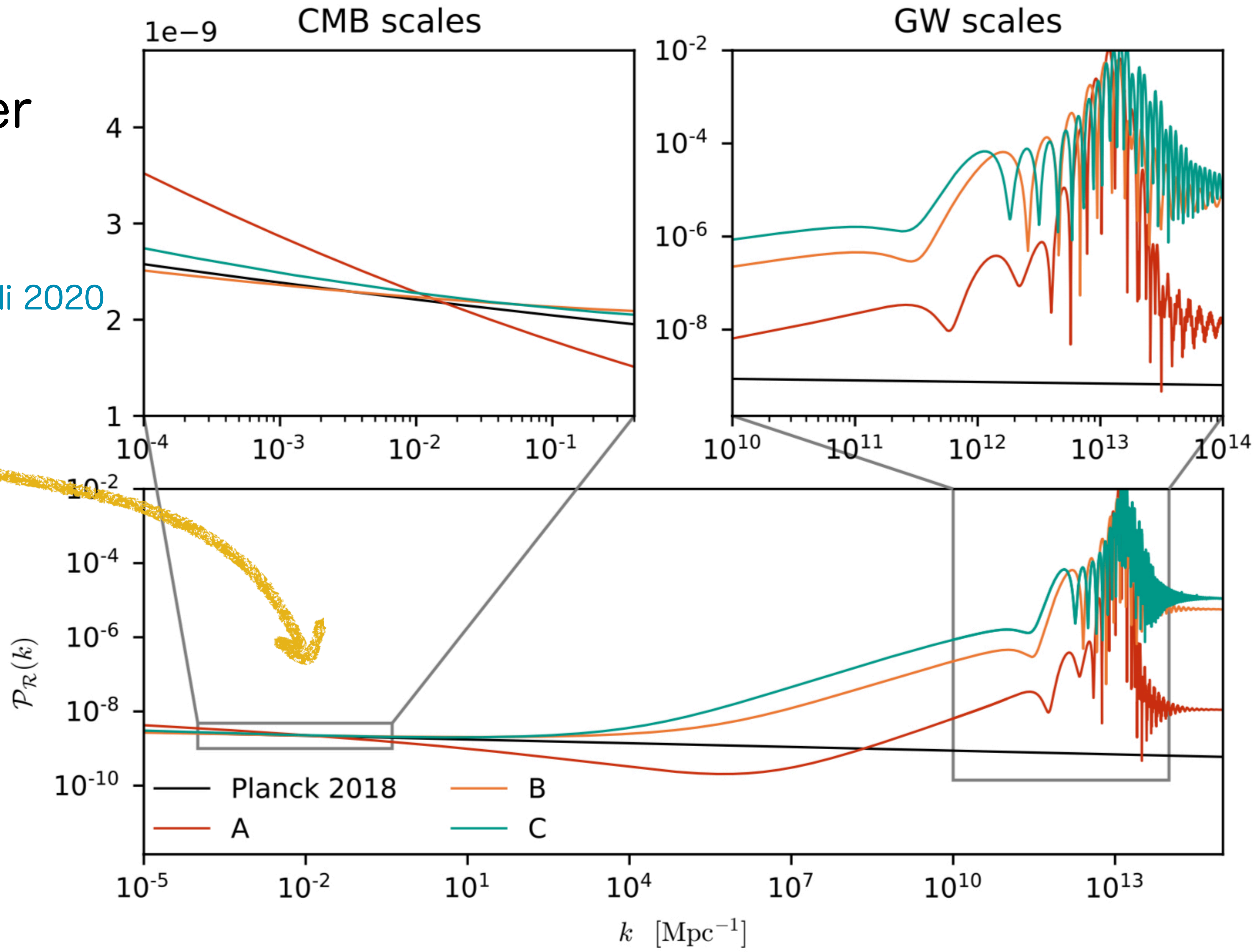


(MB2)

Models of the primordial standard clock (3)

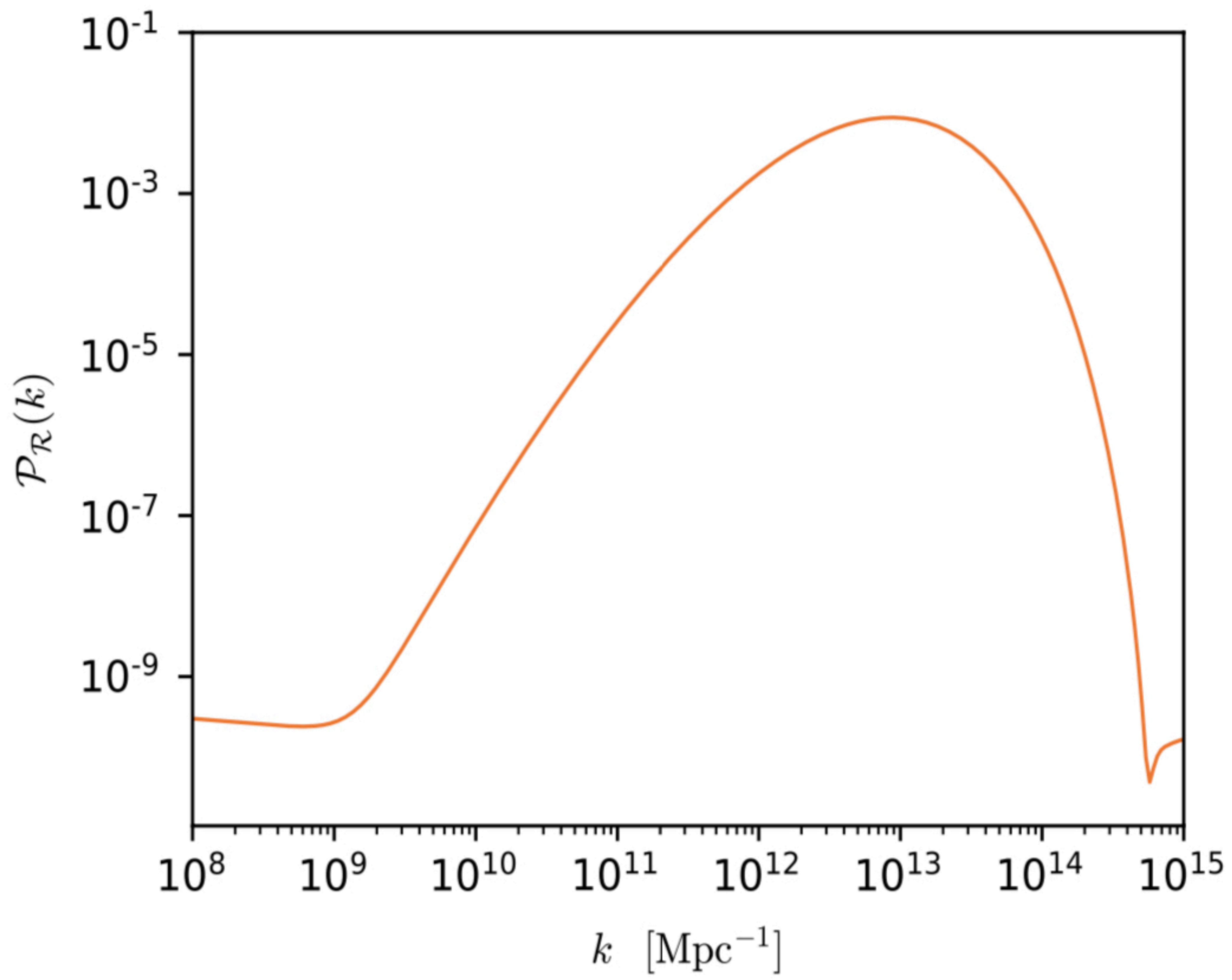
Suppression of power at large scales

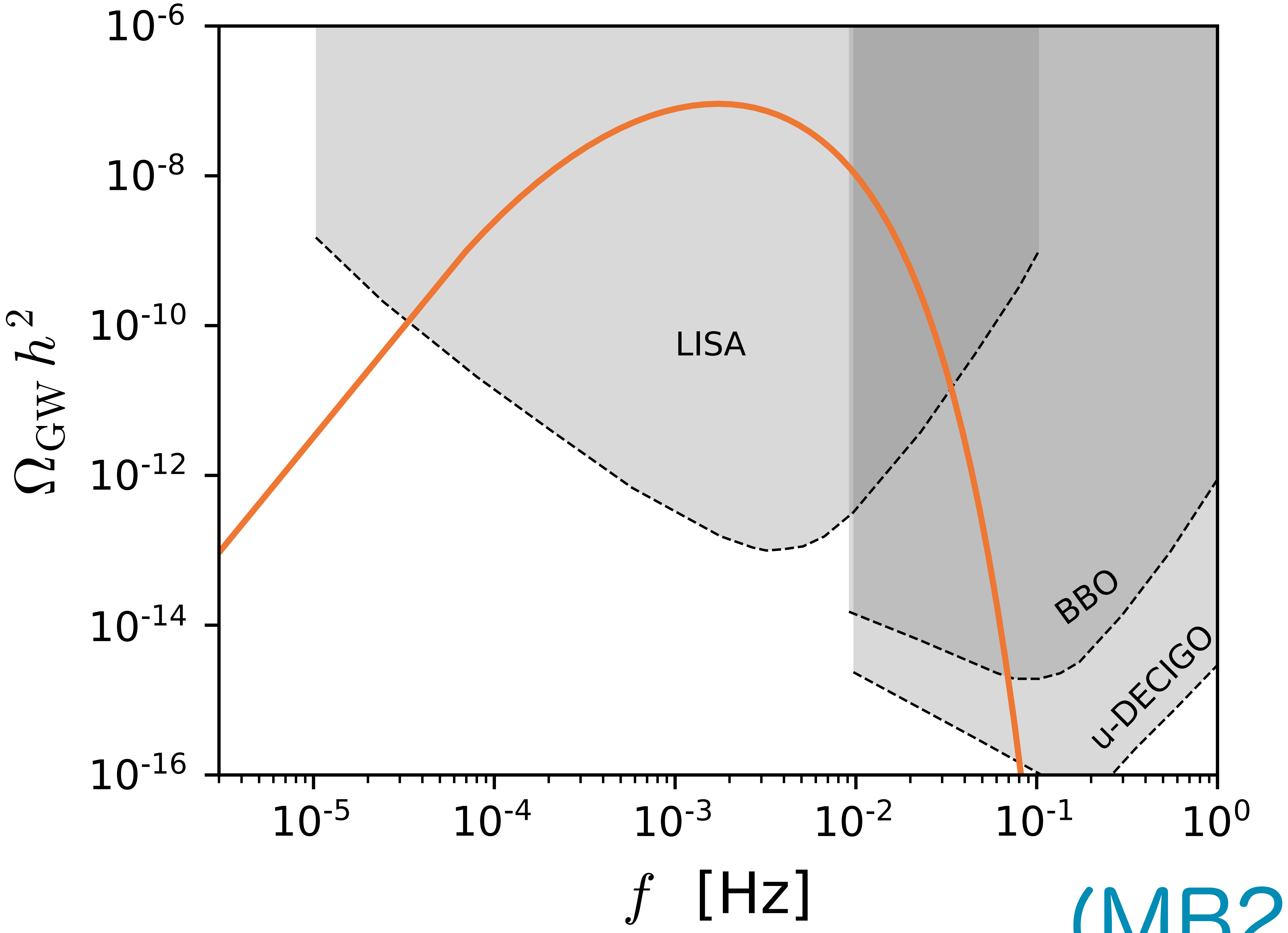
Chen, Namjoo, Wang 2014
MB, Hazra, Sriramkumar, Finelli 2020



(MB2)

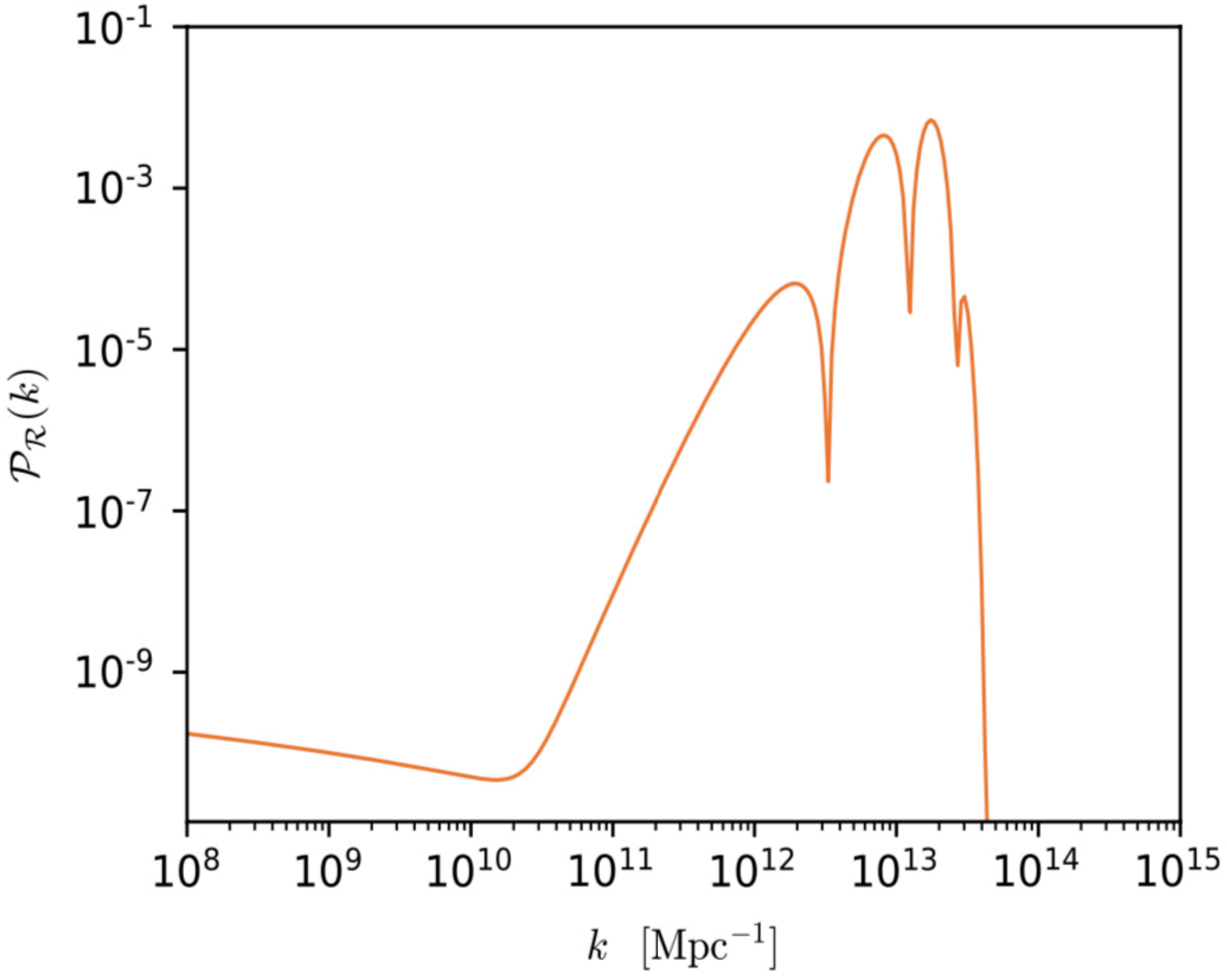
Features in the SGWB

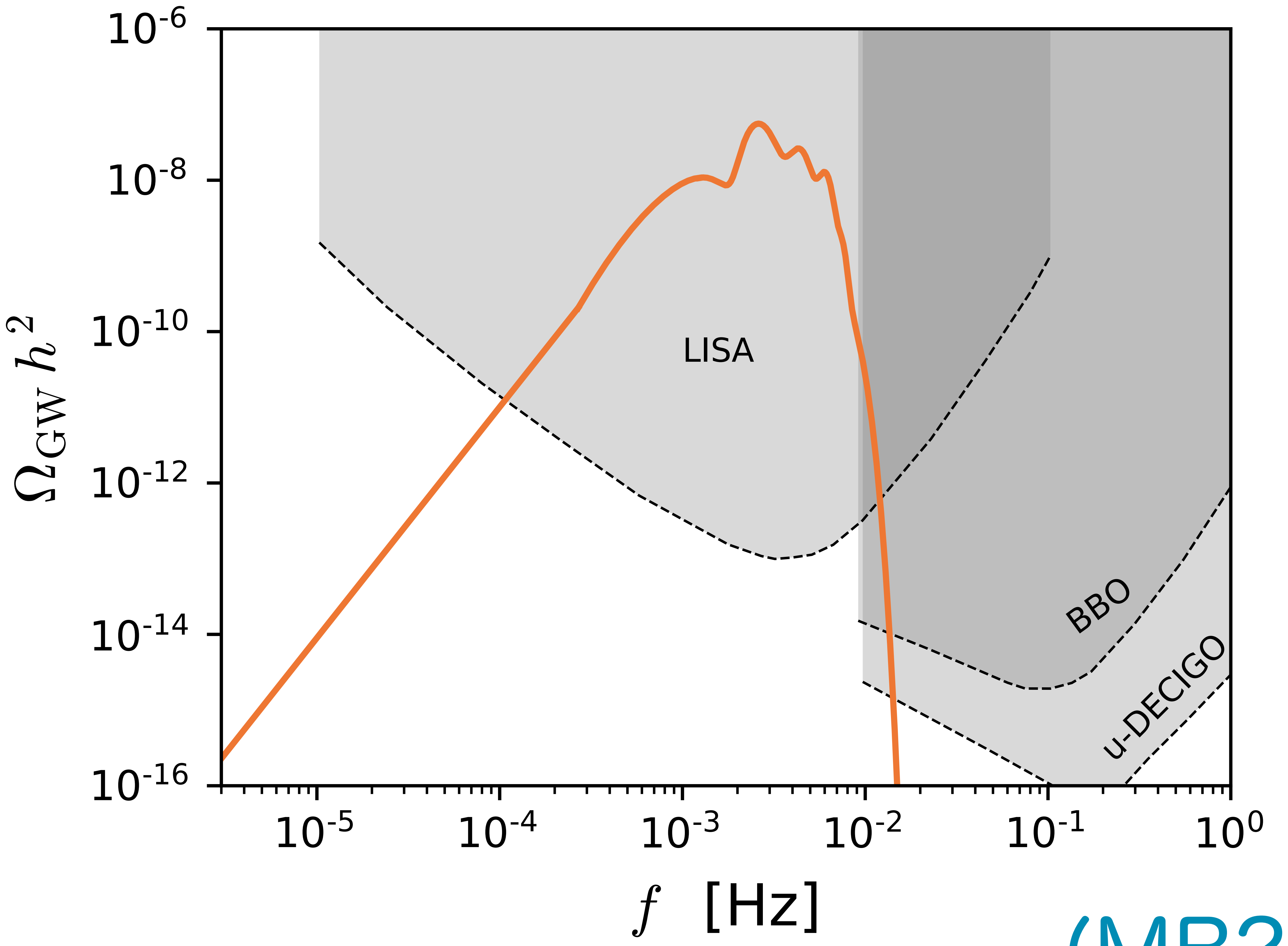




(MB2)

Features in the SGWB





(MB2)

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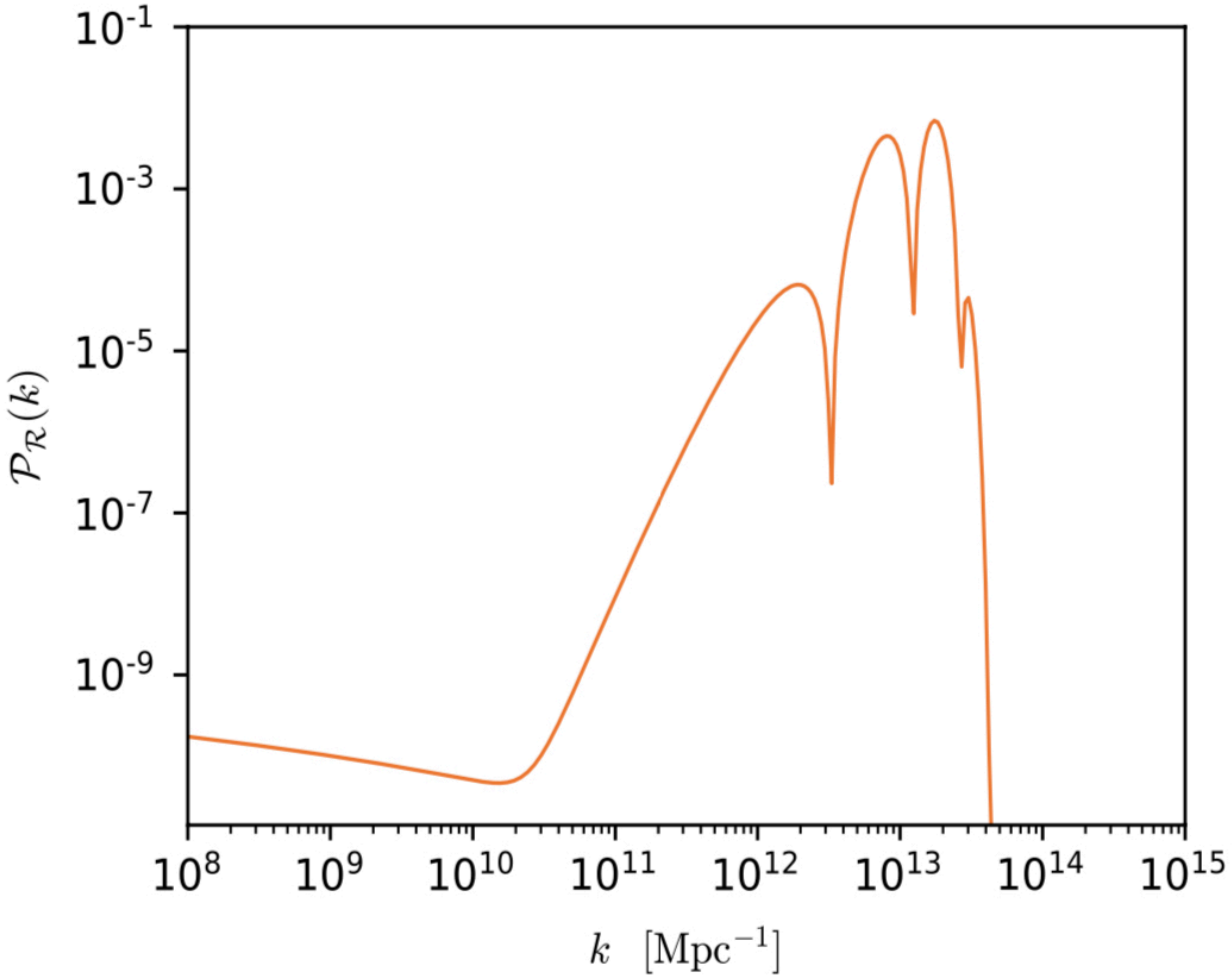

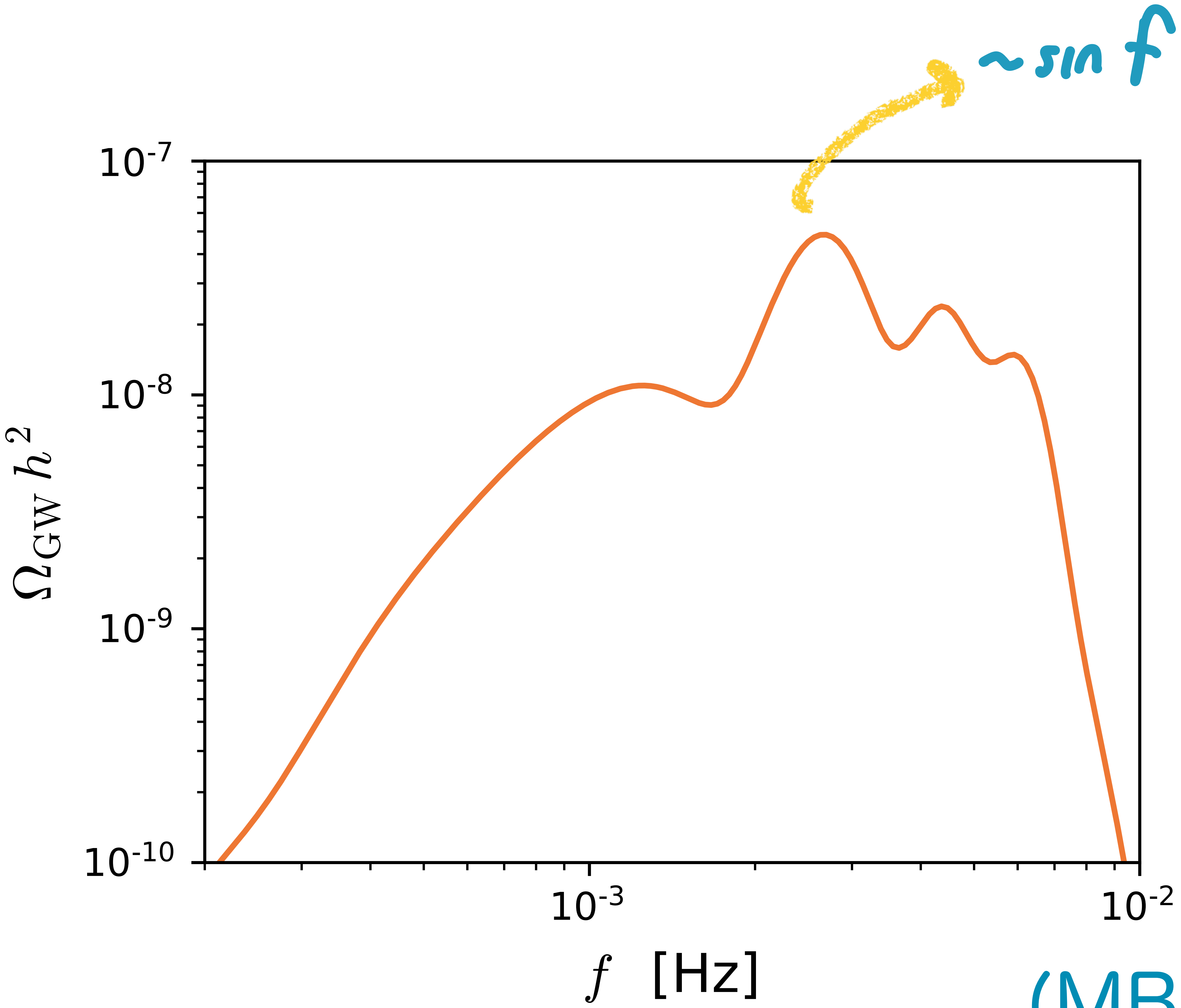

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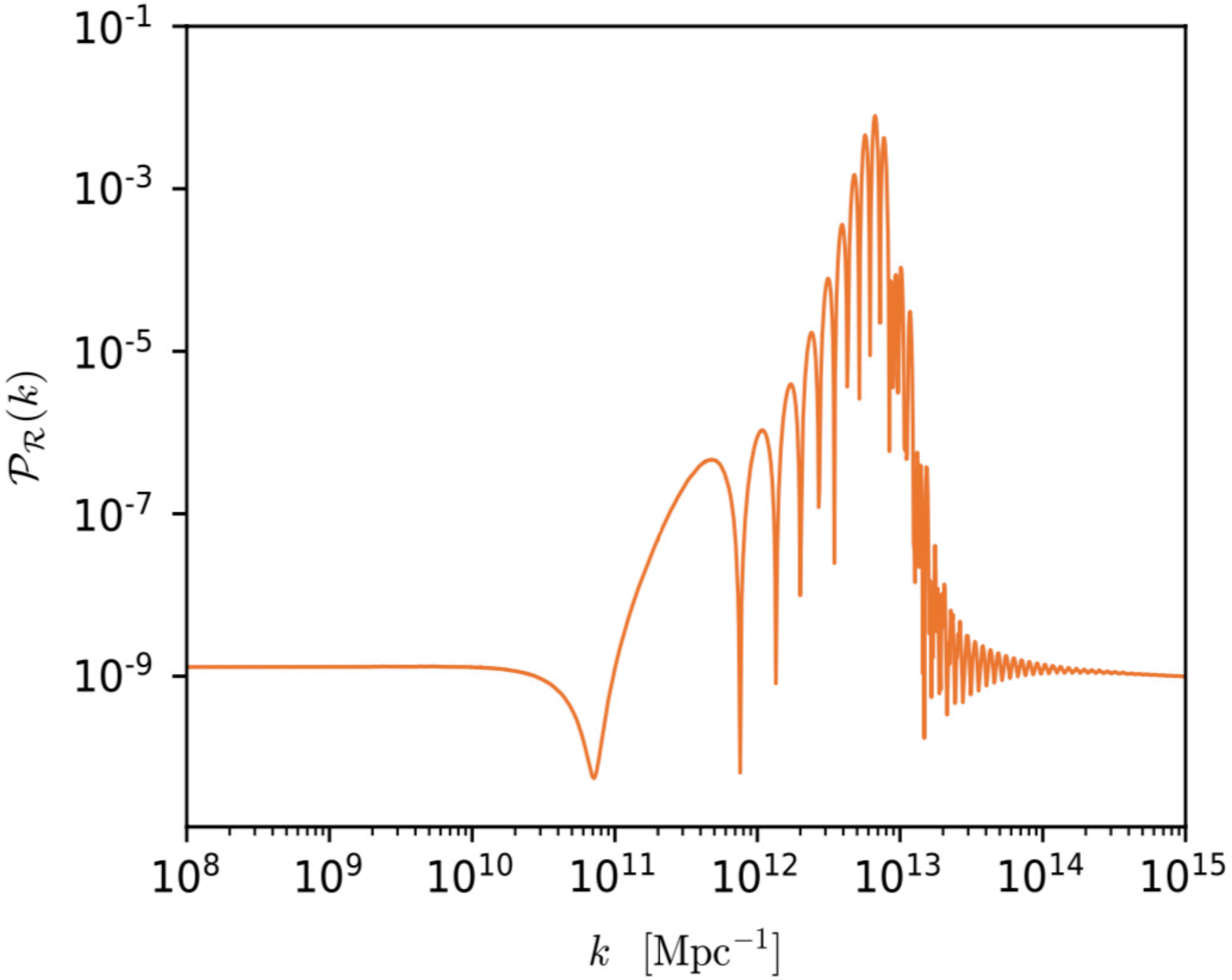

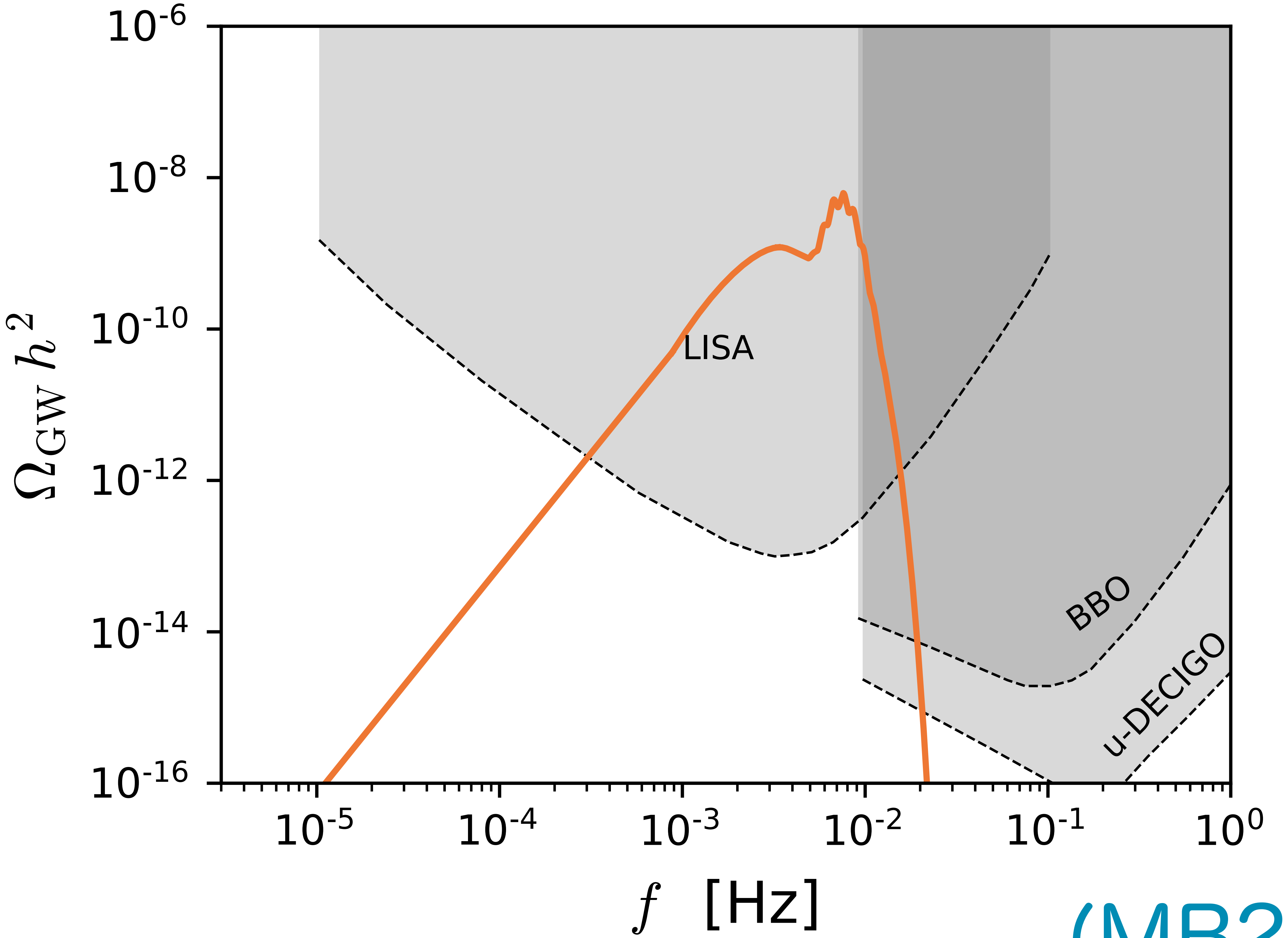

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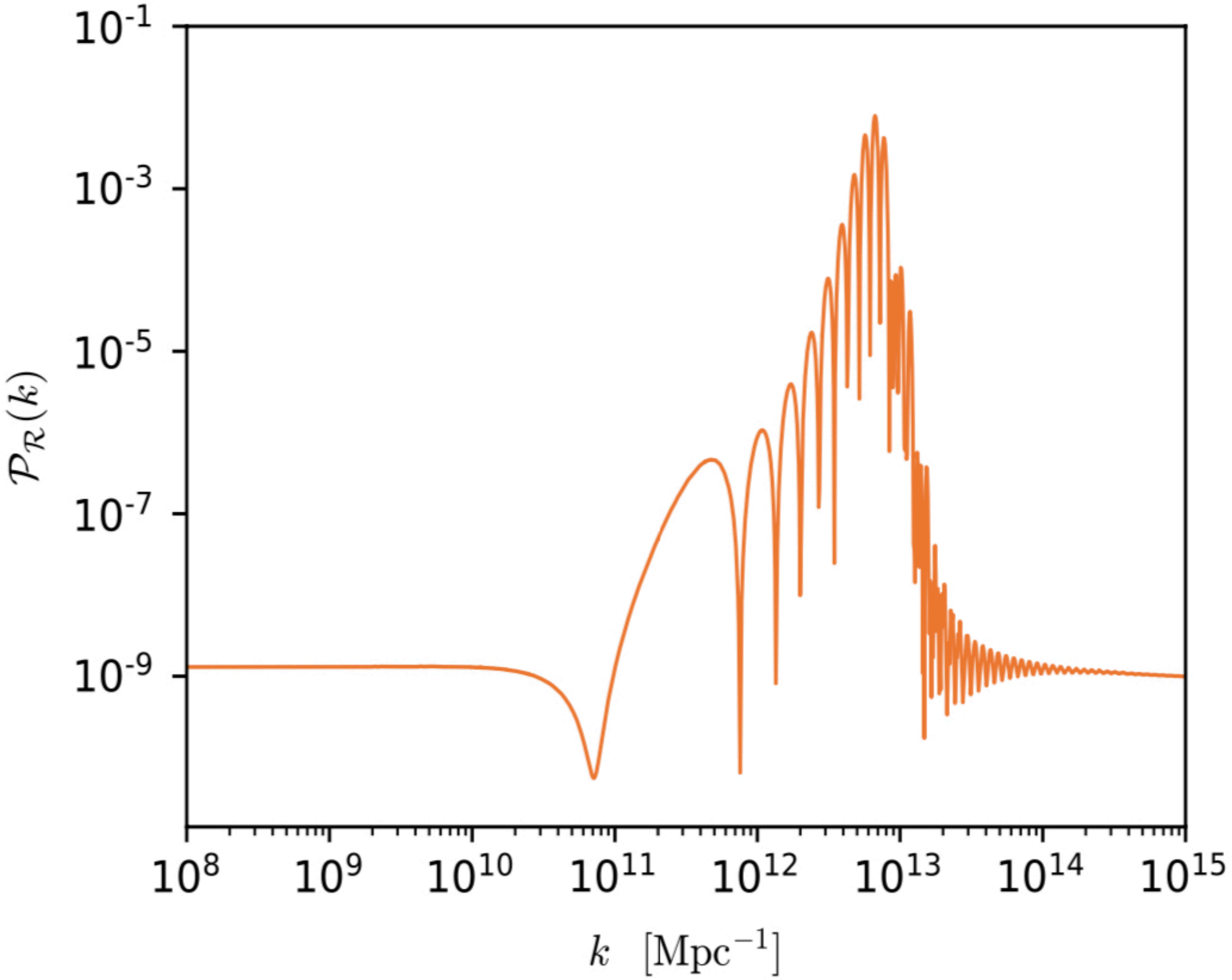

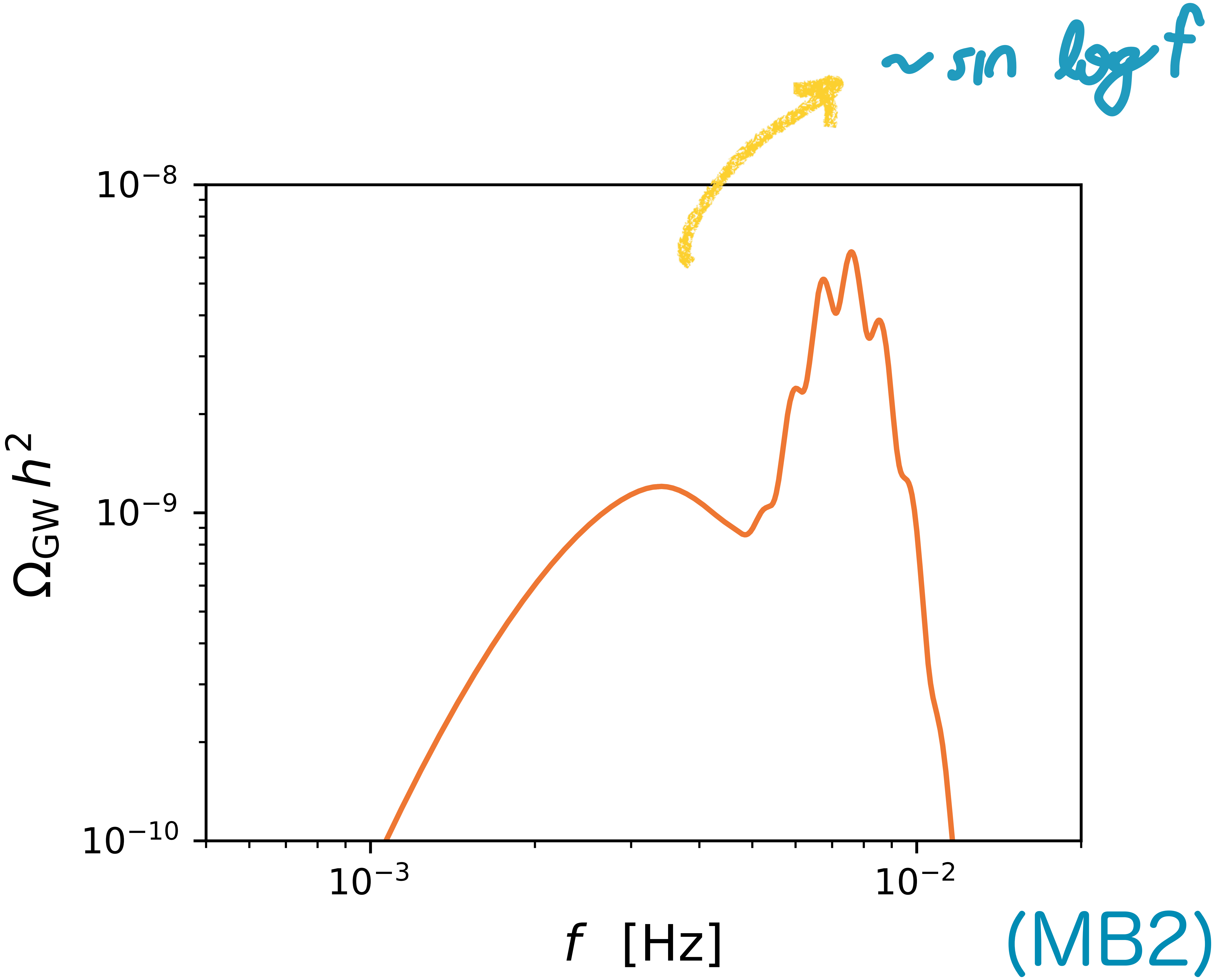

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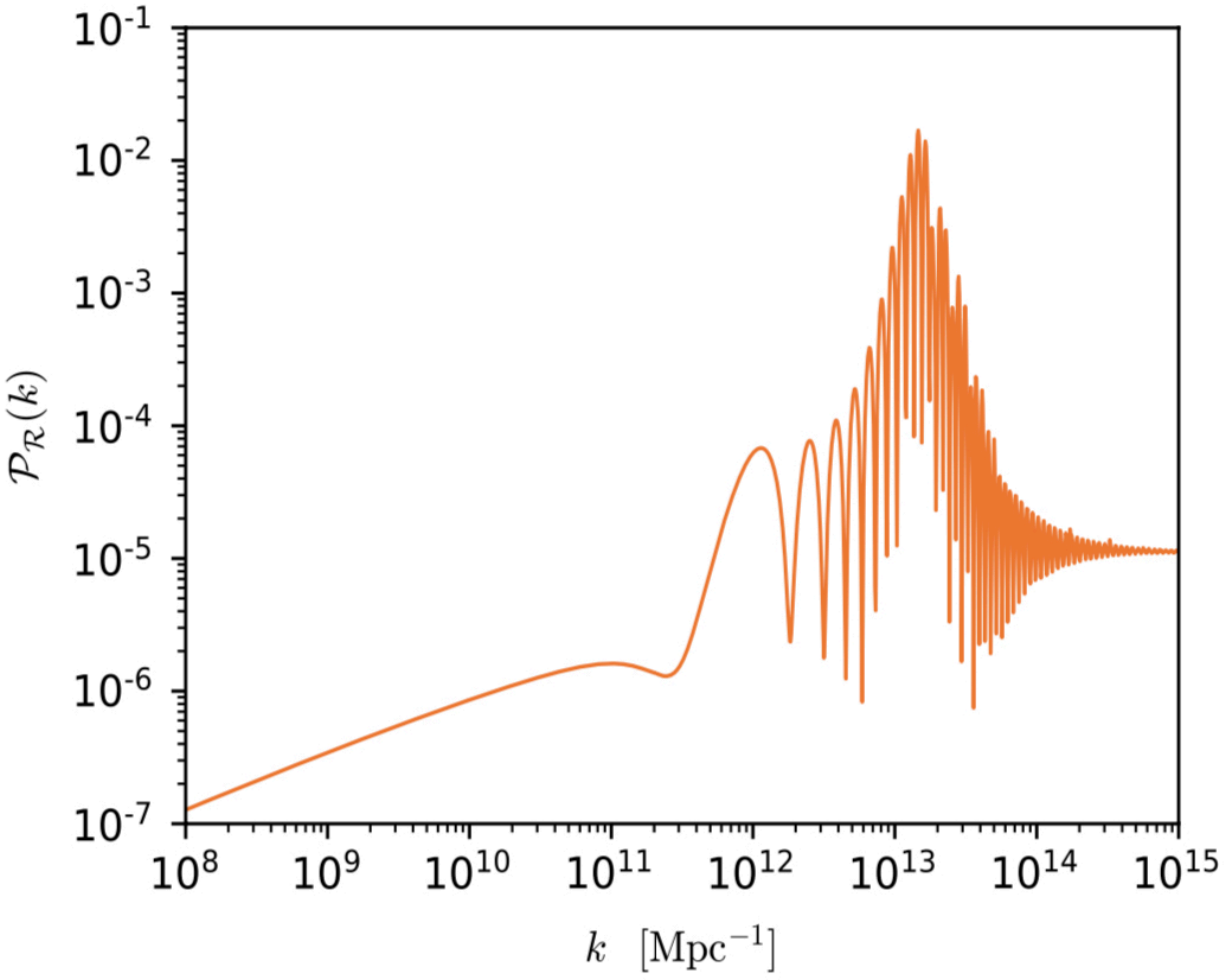
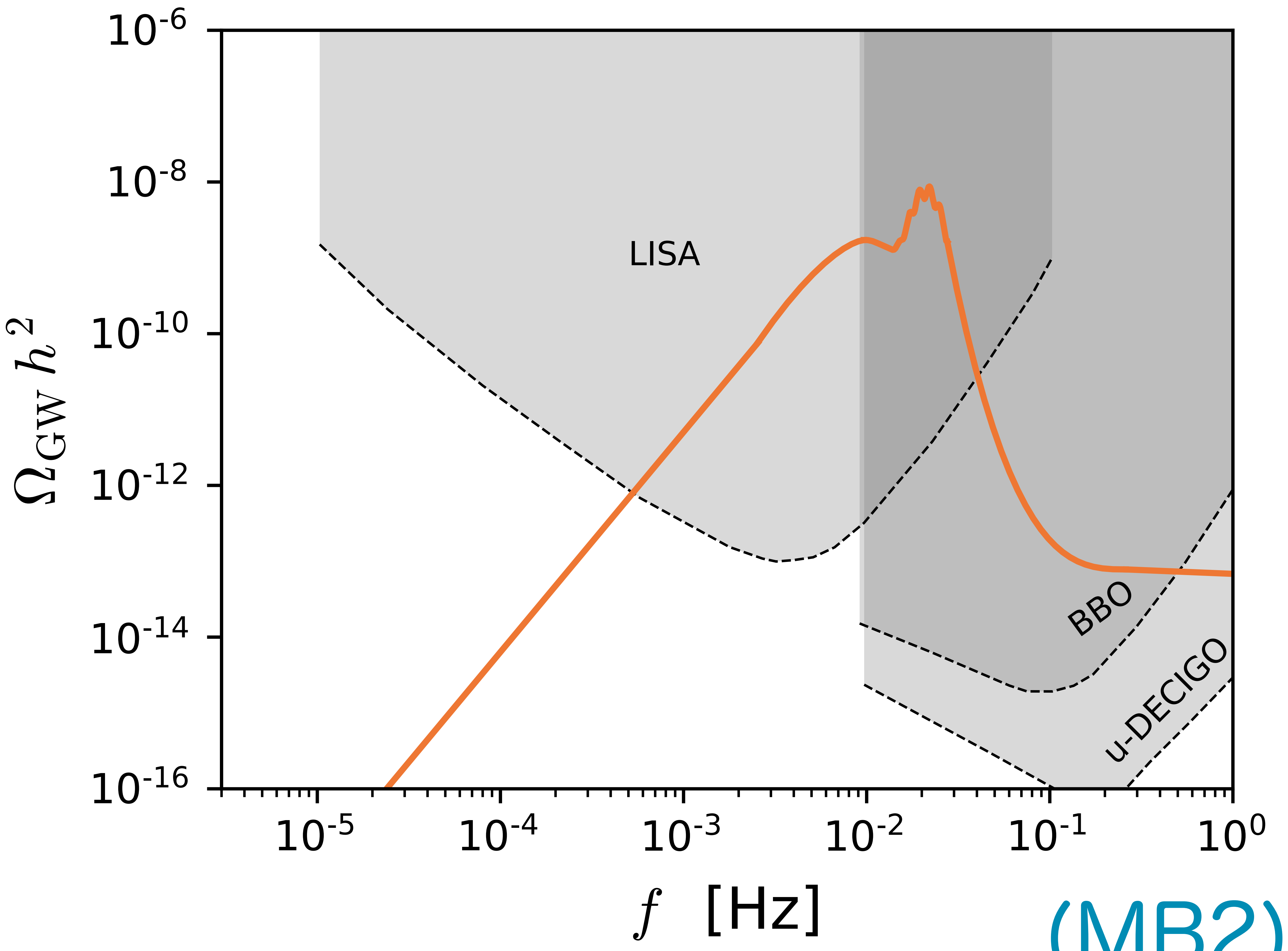
Features in the SGWB





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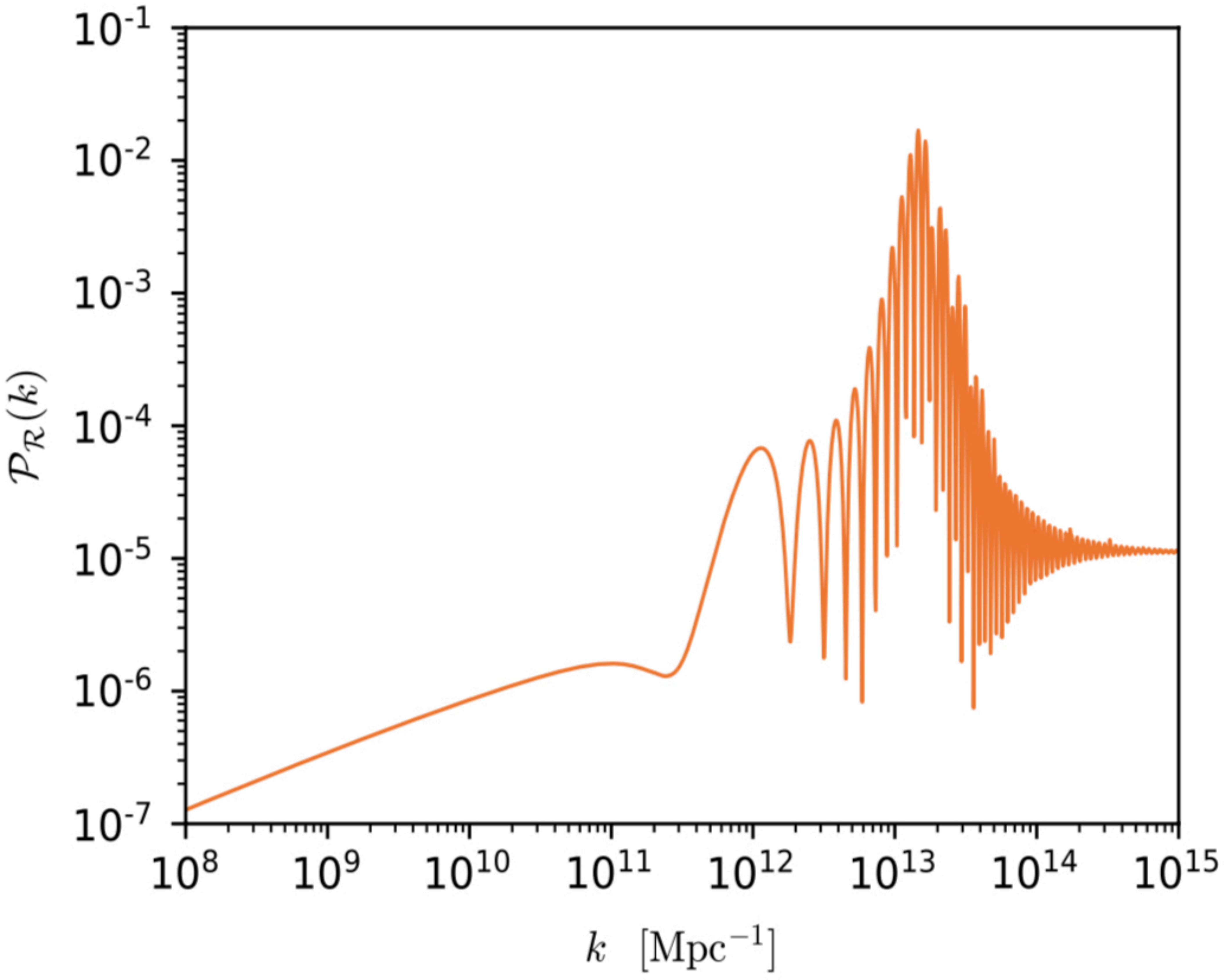

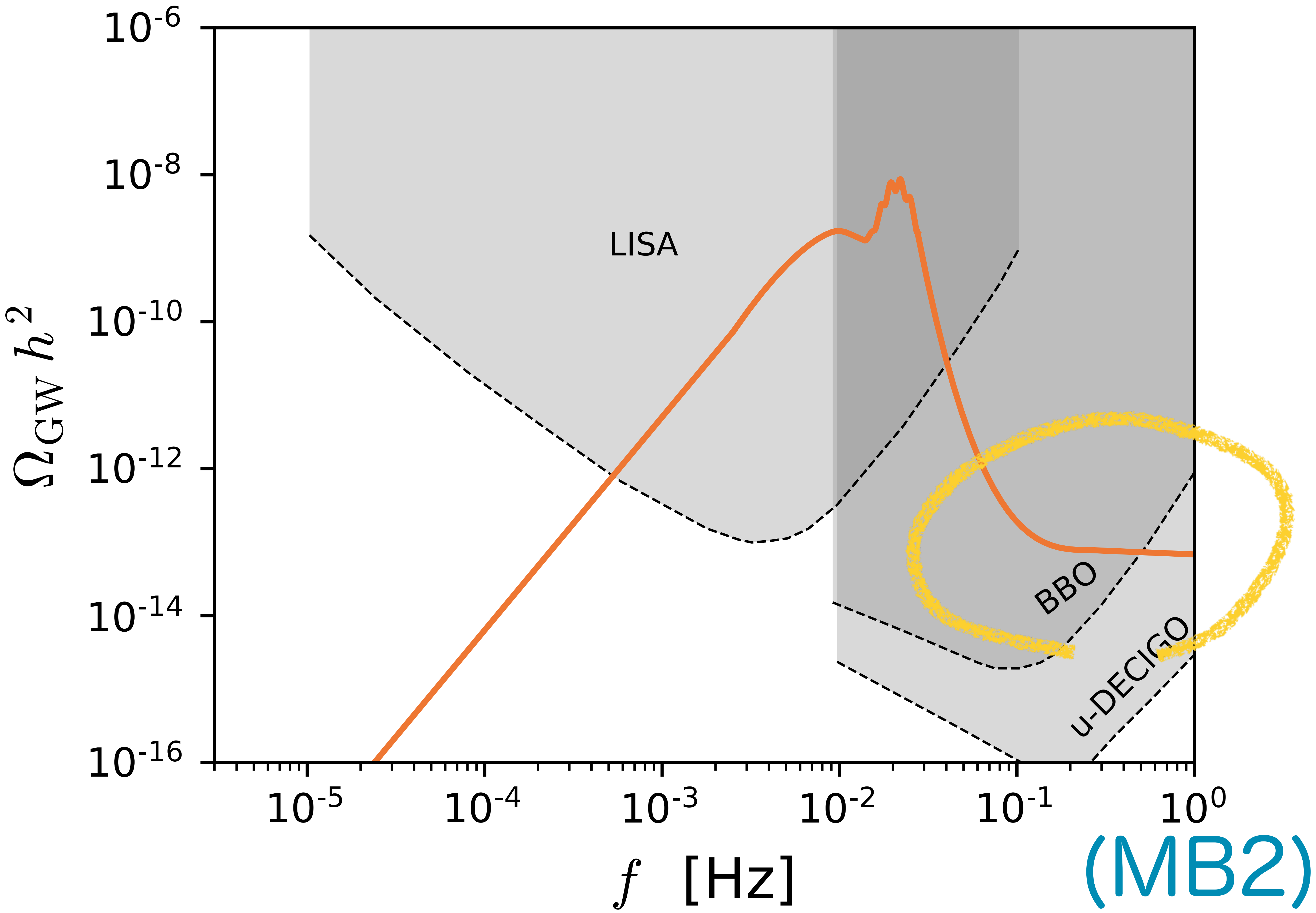

(MB2)

Templates for the features in the SGWB

$$\Omega_{\mathrm{GW}}(f) = \text{broad peak} + \text{narrow peak}\ (1 + \text{oscillatory feature})$$

(MB2)

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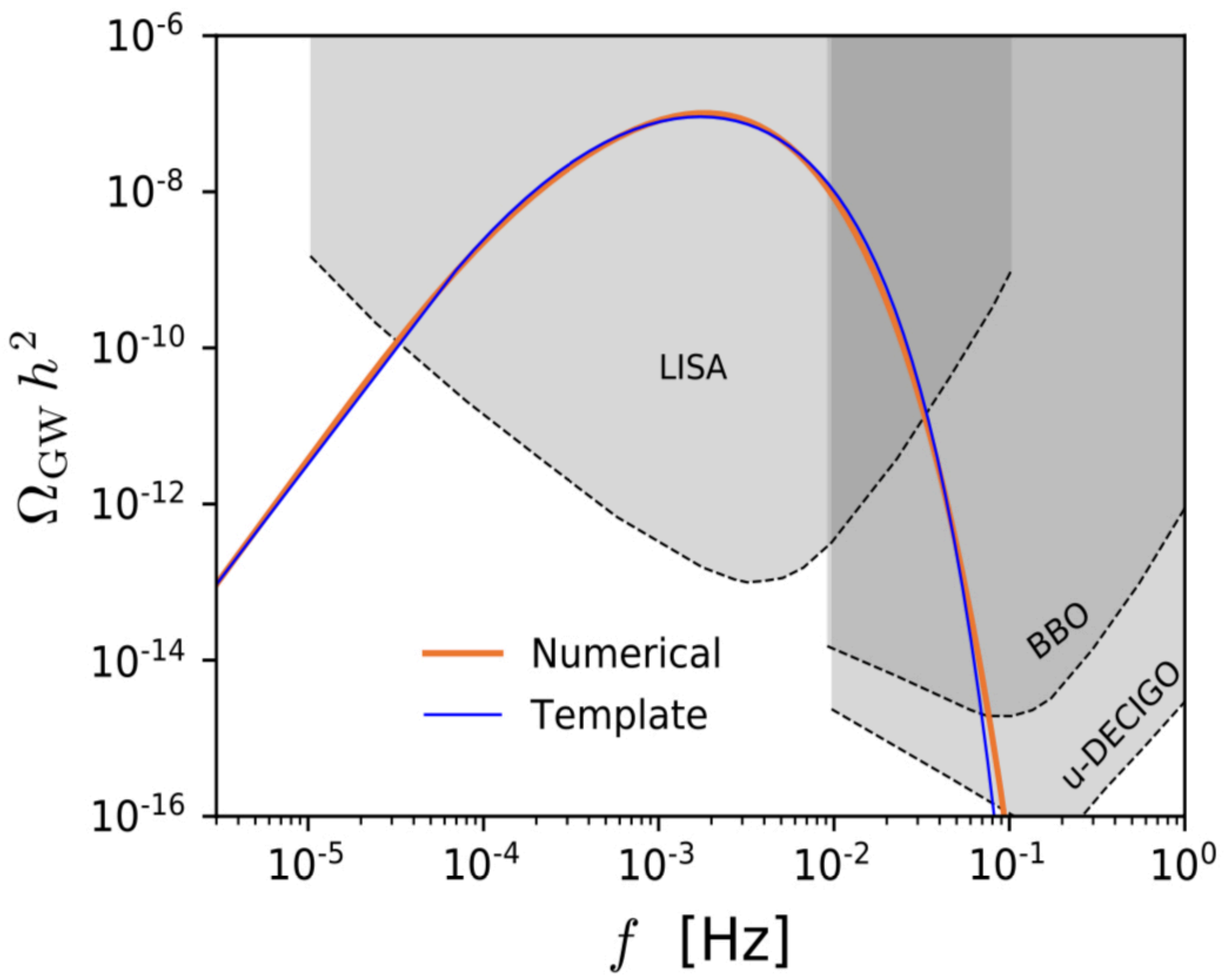
$$\Omega_{\rm GW}(f) = \text{broad peak} + \text{narrow peak}\ (1 + \text{oscillatory feature})$$

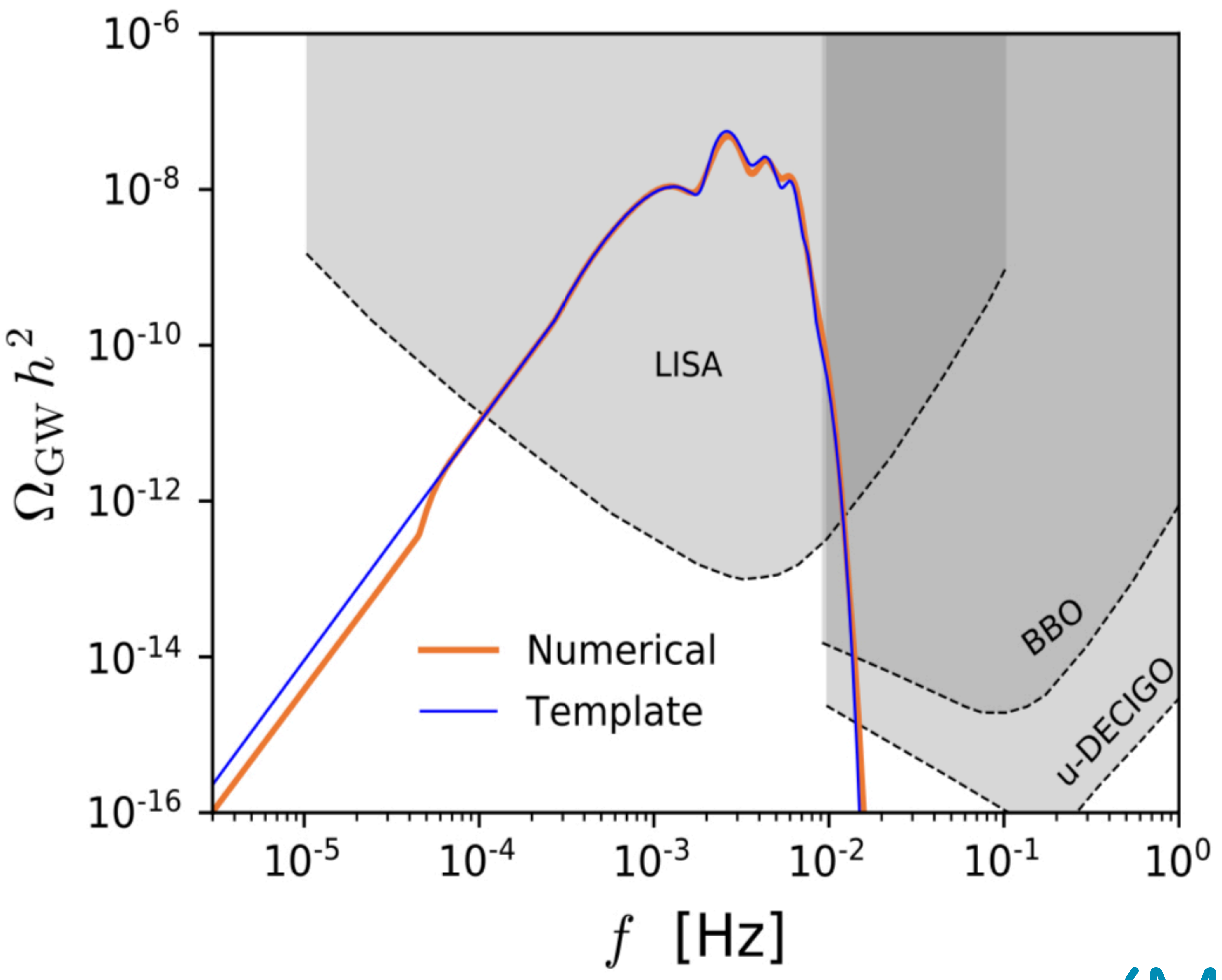
$$h^2\Omega_{\rm GW}(x = f/f_p) = \begin{cases} A_1 \left(\frac{f}{f_1}\right)^{\alpha} & \text{for } f \le f_1 \\ A_2 \exp\left[-b_1(-\ln x - B_1)^{\beta}\right] & \text{for } f_1 < f < f_2 \\ A_3 \Big\{ \exp\left[-d_1 \ln x - \sum_{i=2}^{3} d_i\,(\ln x - D_i)^{\delta_i} + d_4\, x\right] + & \\ A_4 \exp\left[-g_1 \ln x - \sum_{i=2}^{3} g_i\,(\ln x - G_i)^{\gamma_i}\right] g(x) \Big\} & \text{for } f_2 < f < f_3 \\ A_5 \exp\left[\left(-\ln \frac{f}{f_4}\right)^{\kappa}\right] & \text{for } f_3 < f < f_4 \\ A_5 \left(\frac{f}{f_4}\right)^{\iota}, & \end{cases}$$

$$g(x) = \begin{cases} 0 & \text{for bump features} \\ \left[1 + l \sin\left(\omega\,(f - f_p) + \phi\right)\right] & \text{for sinusoidal features} \\ \left[1 + l \sin\left(\omega \ln x + \phi\right)\right] & \text{for resonant features.} \end{cases}$$

(MB2)

Templates for the features in the SGWB





(MB2)

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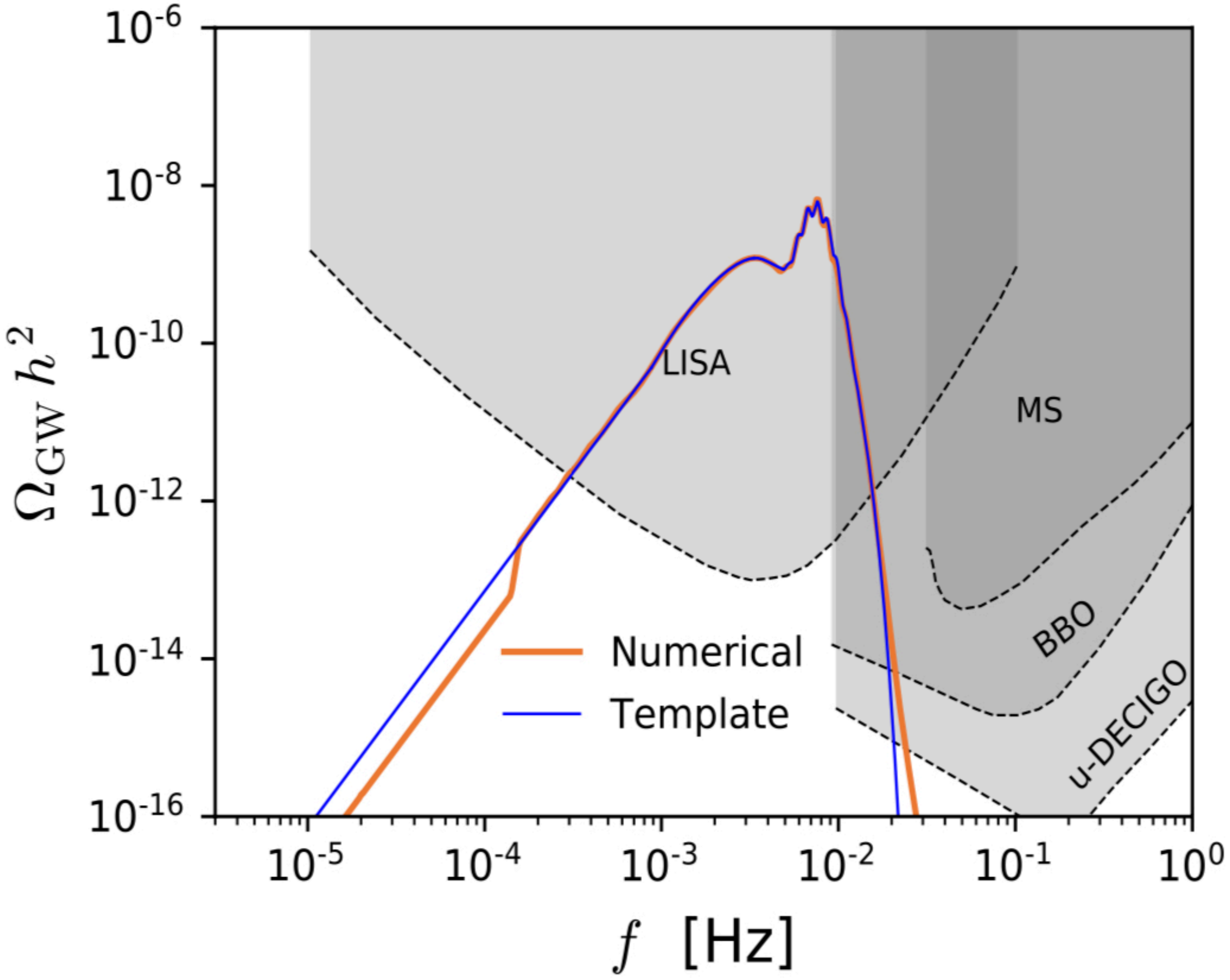
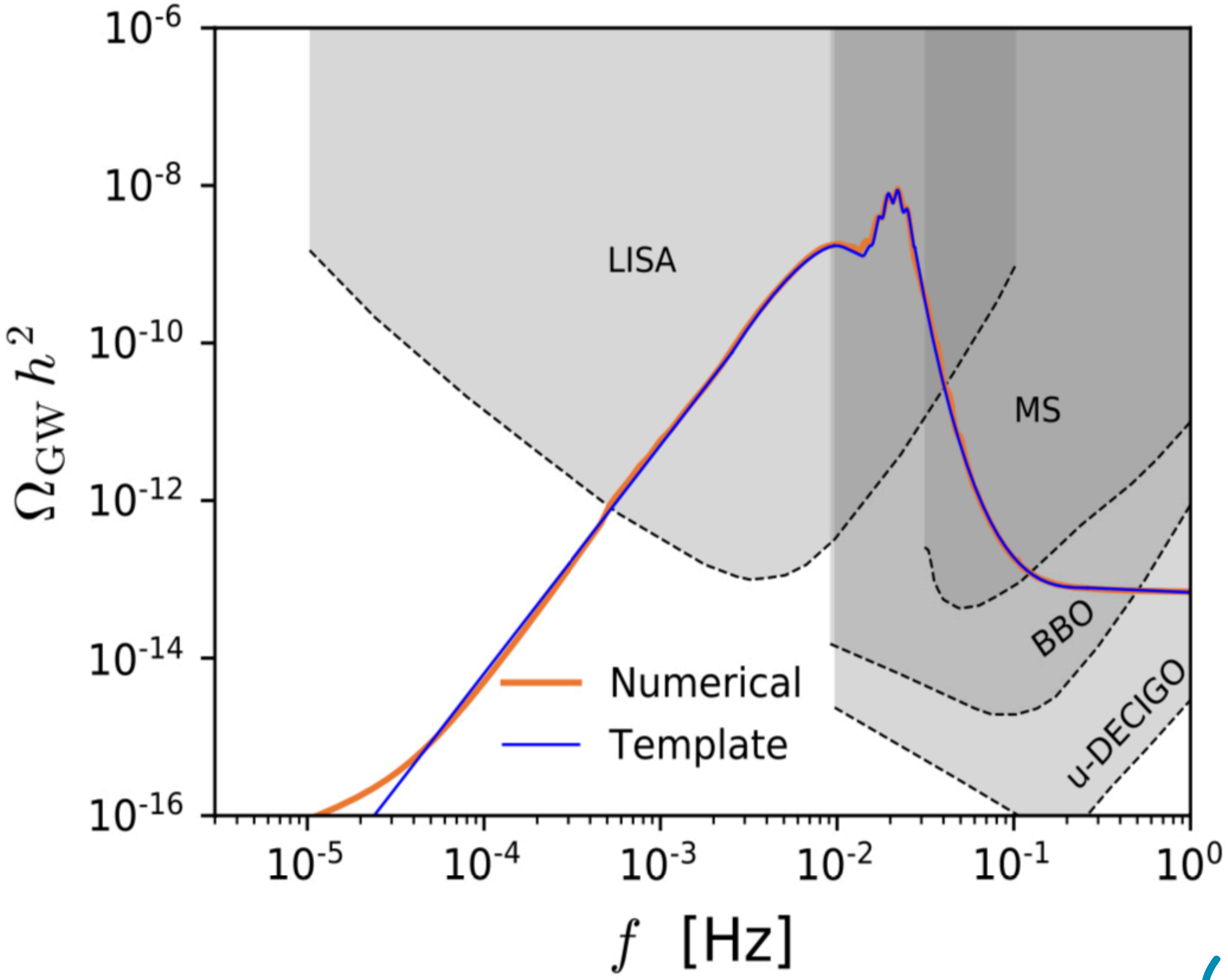
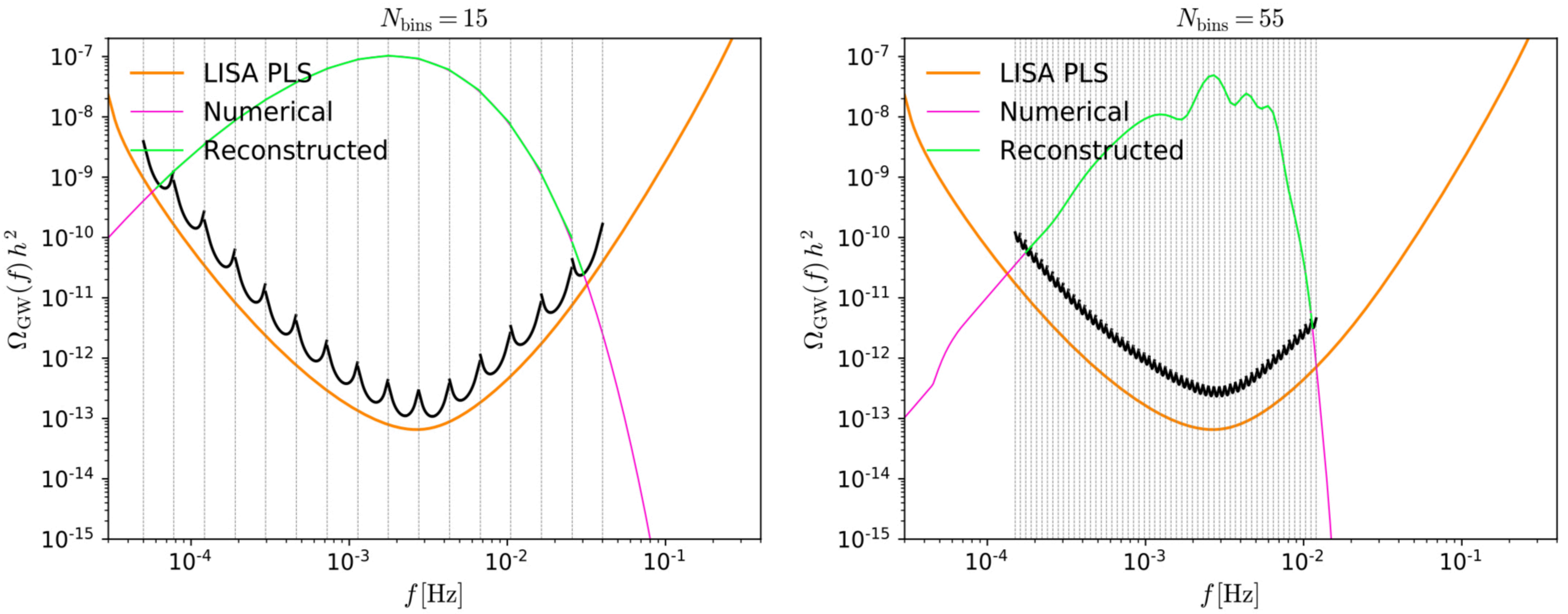




(MB2)

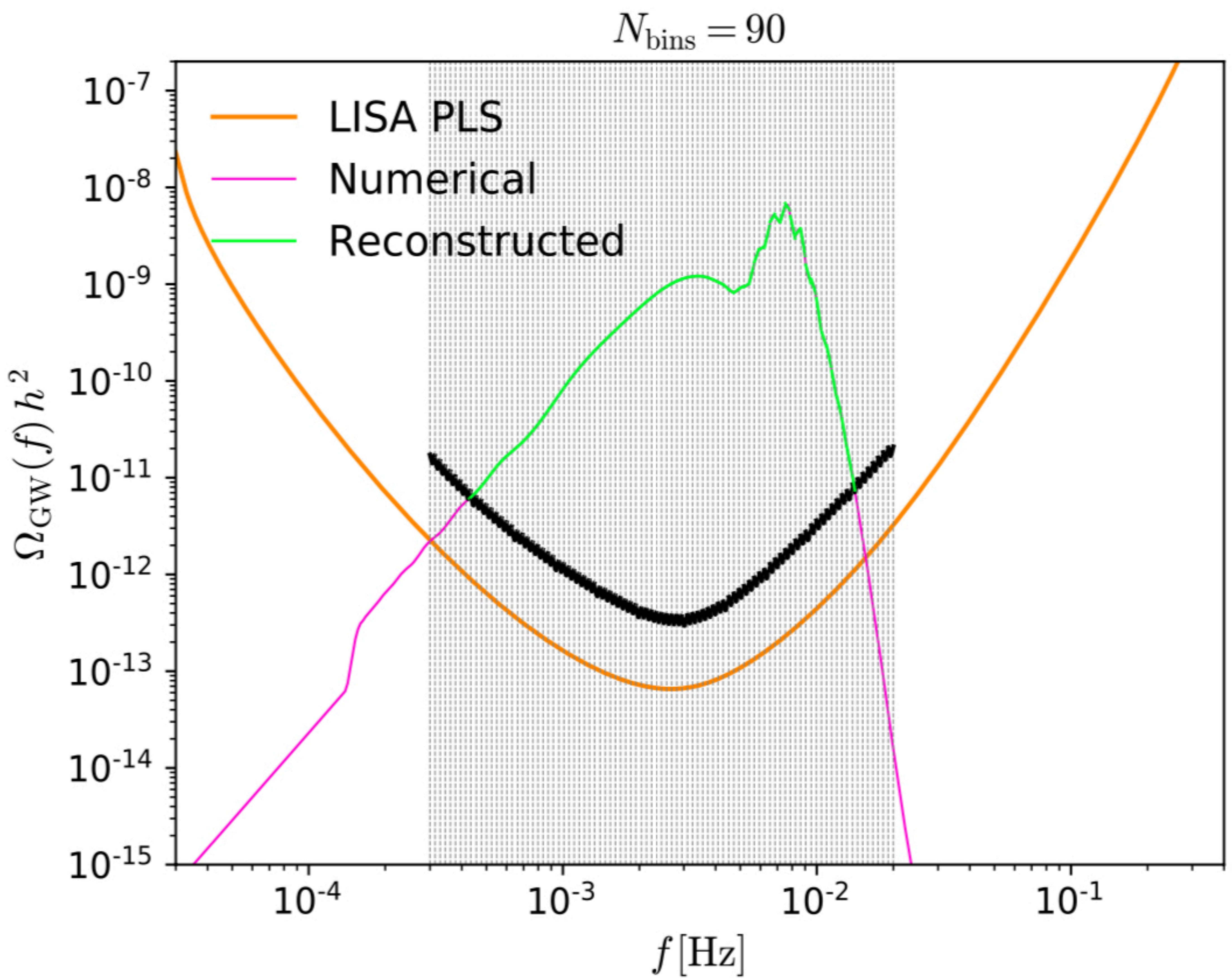
Detectability: LISA binned PLS

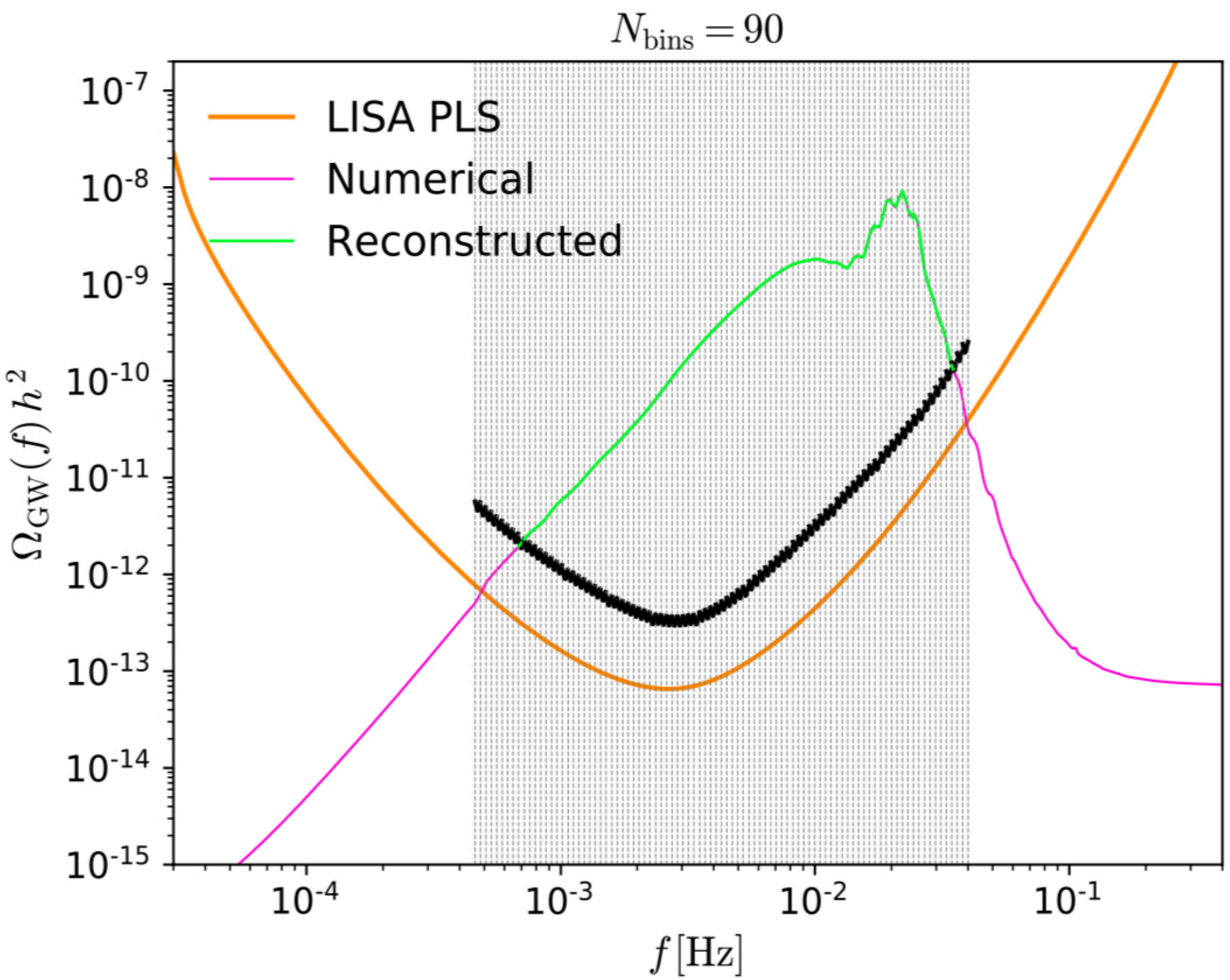


(See also Mauro Pieroni's talk)

(MB2)

Detectability: LISA binned PLS





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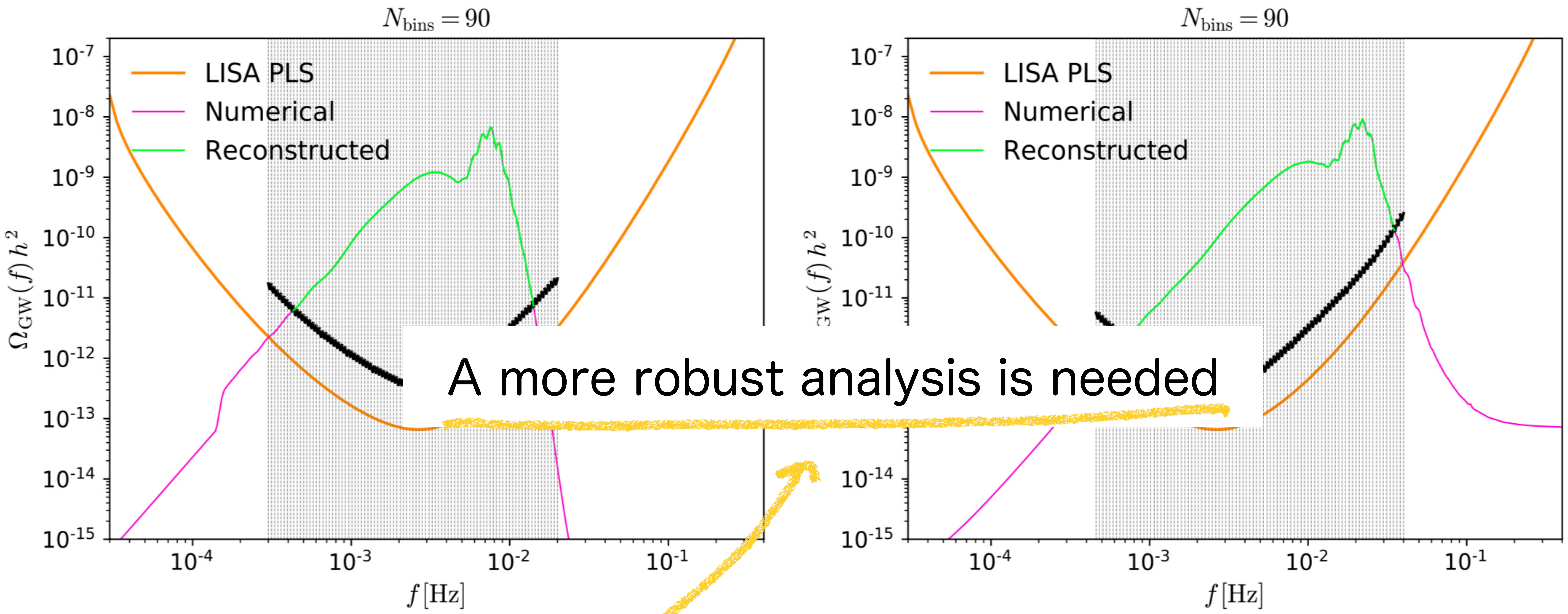
Detectability: LISA binned PLS



A more robust analysis is needed

(See also Mauro Pieroni's talk)

(MB2)

Conclusions

- The model offers unique signatures in the form of oscillations in $\Omega_{\rm GW}$. These specific oscillatory patterns are not produced by other models.

(See also talk by Lukas)

Oscillations in the stochastic gravitational wave background from small-scale features *Lukas Witkowski*

14:30 - 15:00

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Oscillations in the stochastic gravitational wave background from small-scale features *Lukas Witkowski*
14:30 - 15:00

However... !

- GWs during inflation were neglected

(See talk by Spyros)

Observing primordial GWs from excited states *Spyros Sypsas*
15:00 - 15:30

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Oscillations in the stochastic gravitational wave background from small-scale features *Lukas Witkowski* 14:30 - 15:00

However... !

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(See talk by Spyros)

Observing primordial GWs from excited states *Spyros Sypsas* 15:00 - 15:30

- Non-Gaussian contributions were neglected too

(See talk by Caner)

Imprints of Primordial Non-Gaussianity on Gravitational Wave Spectrum *Caner Ünal* 17:00 - 17:30

Conclusions

- The model offers unique signatures in the form of oscillations in $\Omega_{\rm GW}$. These specific oscillatory patterns are not produced by other models.

(See also talk by Lukas)

Oscillations in the stochastic gravitational wave background from small-scale features	*Lukas Witkowski*
	14:30 - 15:00

However... !

- Although this is a phenomenological possibility, some fine tuning is needed and further model building is required.