

Testing inflation with small-scale anisotropies

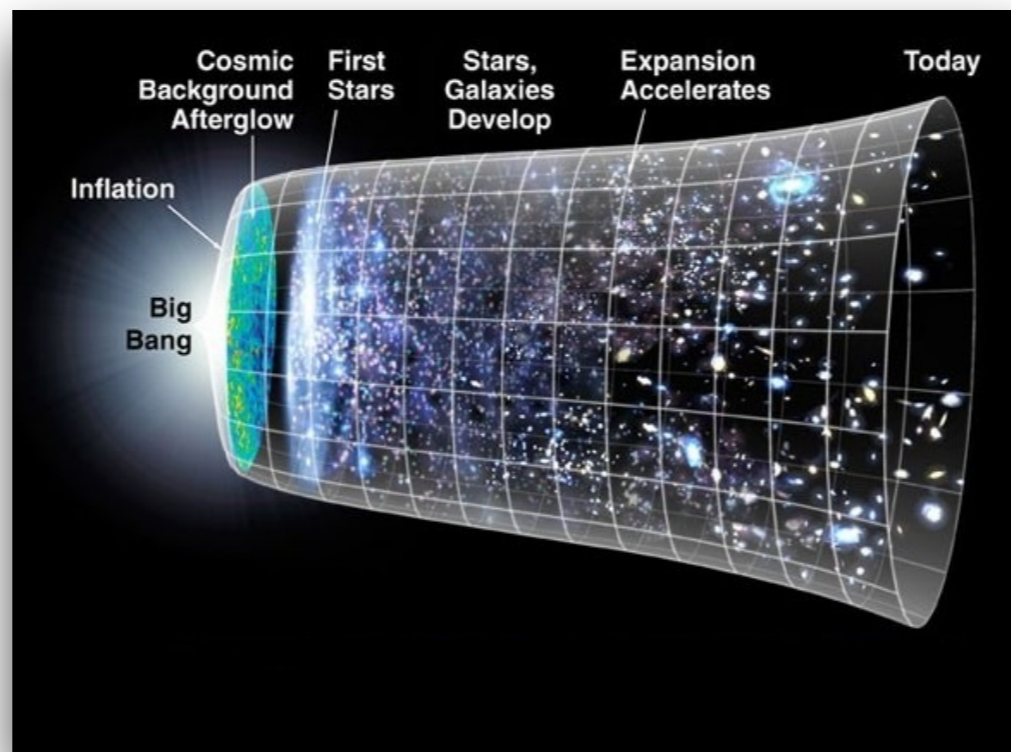
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Talk based on work in collaboration with:

Peter Adshead, Niayesh Afshordi, **Matteo Fasiello**, Tomohiro Fujita,
Marc Kamionkowski, Eugene Lim, **Ameek Malhotra**,
Maresuke Shiraishi, **Gianmassimo Tasinato**, David Wands

“Gravitational-Wave Primordial Cosmology” workshop - May 17th 2021



Inflation predicts a stochastic gravitational wave background

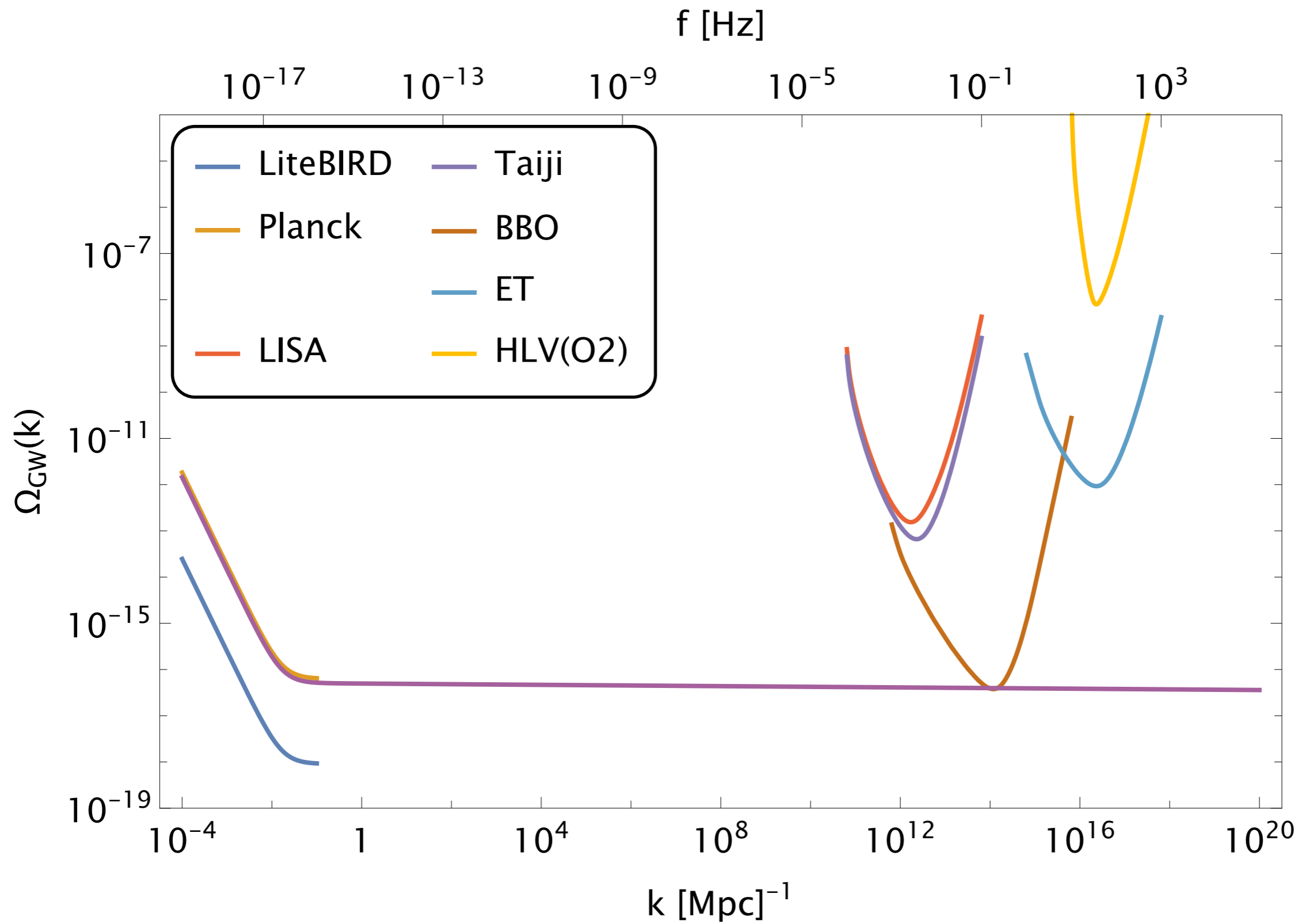
- How does it look like?
- What info does it provide on inflation?
- How do we **characterise** it?

- Frequency profile
- Chirality
- **Non-Gaussianity**
- **Anisotropies**

Inflationary GW from vacuum fluctuations (SF SR)

- **Energy scale** of inflation: $V_{\text{inf}}^{1/4} \simeq 10^{16} \text{GeV} (r/0.01)^{1/4}$
 $H \simeq 2 \times 10^{13} \text{GeV} (r/0.01)^{1/2}$
- Red **tilt**: $n_T \simeq -2\epsilon = -r/8$
- Non-**chiral**: $P_L = P_R$
- Nearly **Gaussian**: $f_{\text{NL}} \ll 1$

Inflationary GW from vacuum fluctuations (SFSSR)



Gravitational waves

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G \Pi_{ij}^{TT}$$

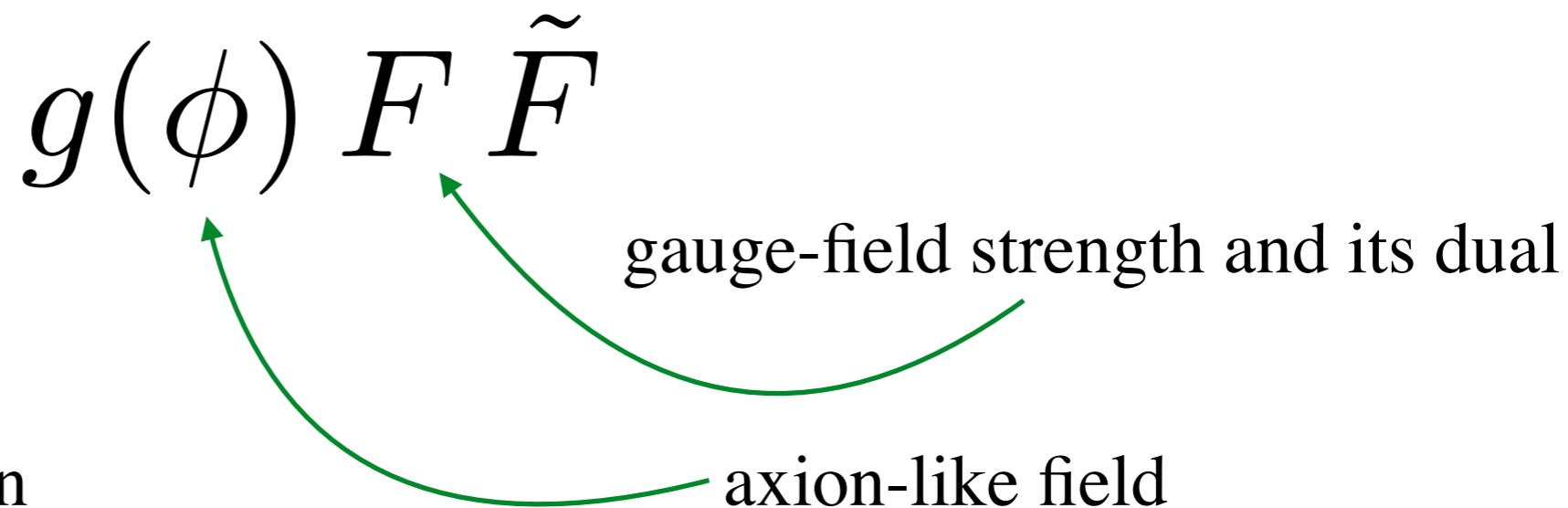
- **homogeneous** solution: GWs from **vacuum fluctuations**

Production of gravitons out of the vacuum
in an expanding universe!

- **inhomogeneous** solution: GWs from **sources**

$$\Pi_{ij}^{TT} \propto \{ \text{scalar fields, vector fields, fermions, tensors ...} \}$$

Axion-Gauge fields models: Chern-Simons coupling



- naturally light inflaton
- support reheating
- mechanism for baryogenesis
- primordial black holes formation
- **sourced chiral gravitational waves**

[Freese - Frieman - Olinto 1990, Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec - Wyman 2013, ED - Fasiello - Fujita 2016, Garcia-Bellido - Peloso - Unal 2016, Agrawal - Fujita - Komatsu 2017, Fujita - Namba - Obata 2018, Domcke - Mukaida 2018, Iarygina - Sfakianakis 2021, ...]

Axion-Gauge fields models: SU(2)

[Adshead - Wyman 2011]

$$\mathcal{L} = \mathcal{L}_{\text{inflaton}} - \underbrace{\left[\frac{1}{2} (\partial\chi)^2 - U(\chi) - \frac{1}{4} FF + \frac{\lambda\chi}{4f} F\tilde{F} \right]}_{\mathcal{L}_{\text{spectator}}}$$

\downarrow
 $P_{\gamma, \text{vacuum}} \qquad \mathcal{L}_{\text{spectator}} \rightarrow P_{\gamma, \text{sourced}}$

- Inflaton field dominates energy density of the universe
- Spectator sector contribution to curvature fluctuations negligible

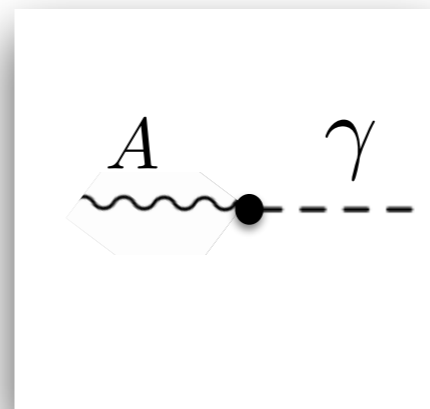
$$A_0^a = 0$$

$$A_i^a = aQ\delta_i^a$$

slow-roll background attractor solution

$$\delta A_i^a = t_{ai} + \dots$$

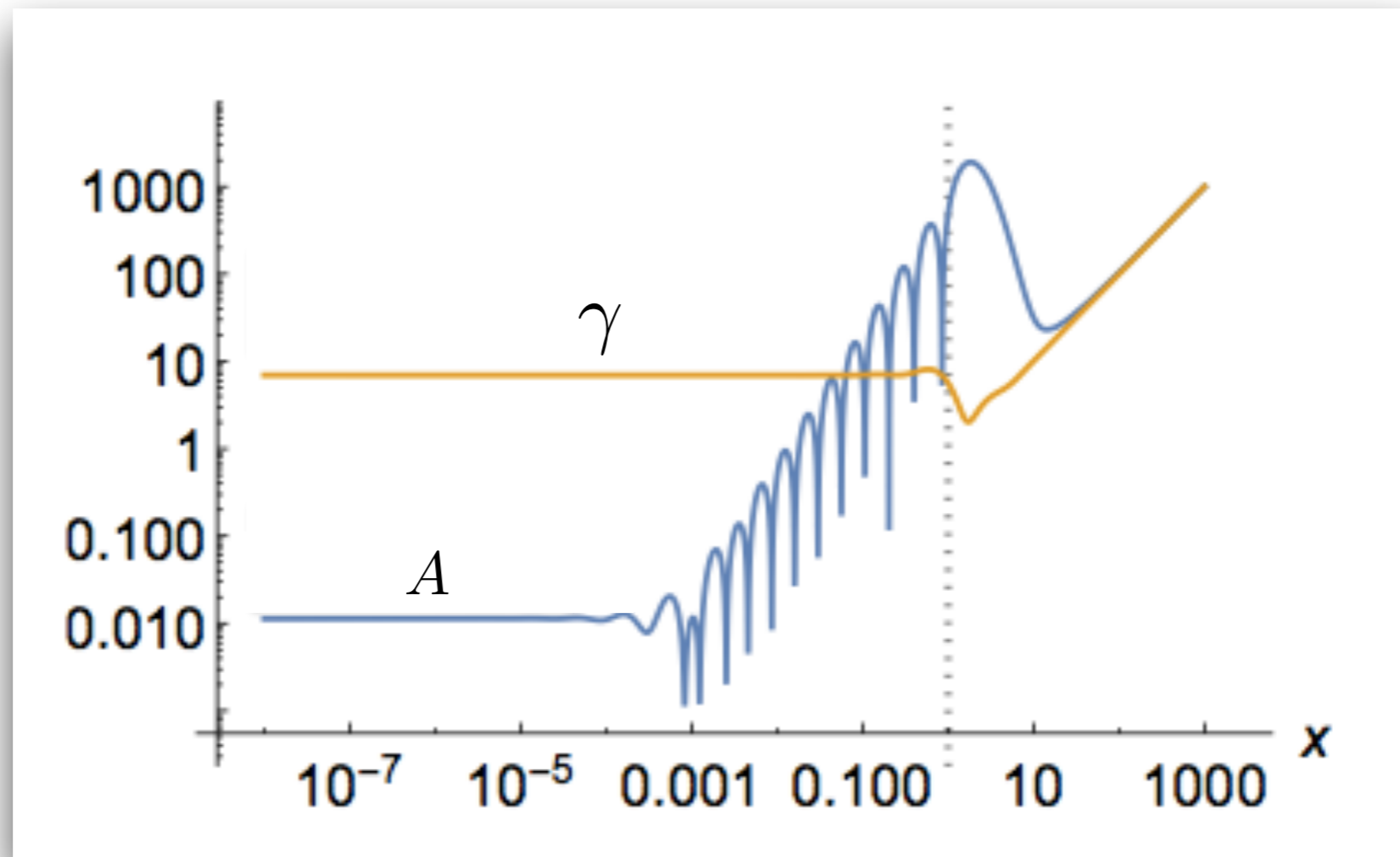
TT-component



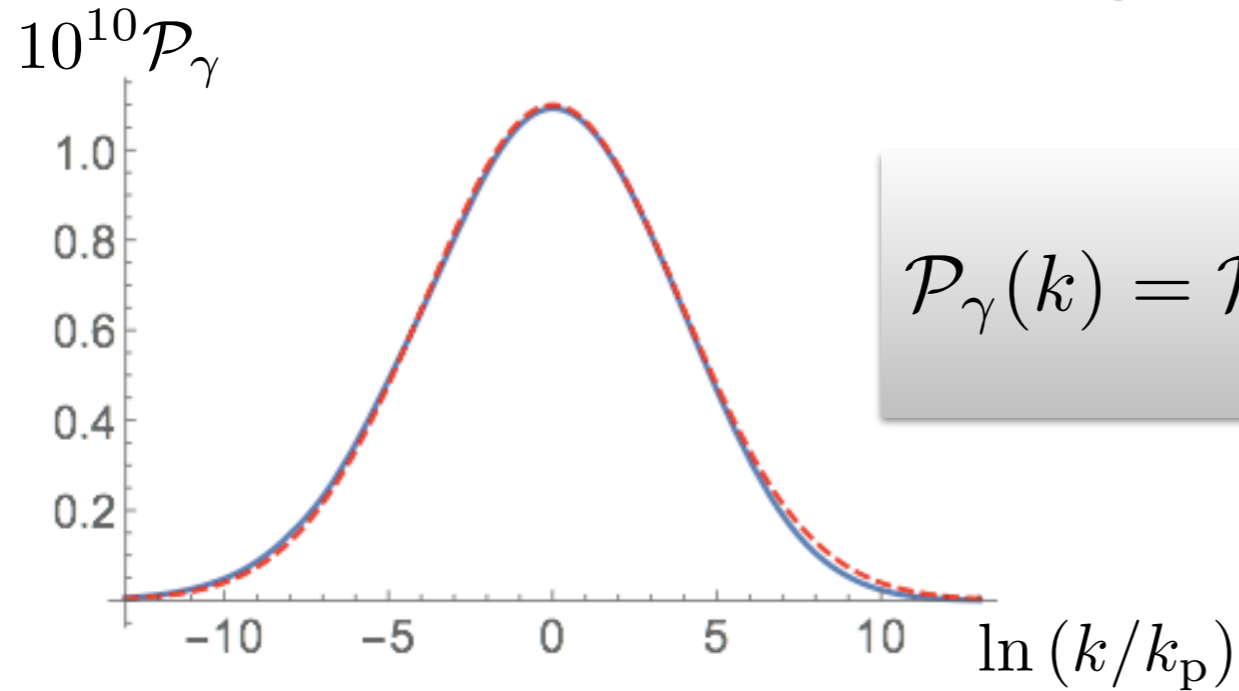
[ED-Fasiello-Fujita 2016]

Axion-Gauge fields models: SU(2)

One helicity of the gauge field fluctuations is amplified from coupling with axion \longrightarrow the same helicity of the tensor mode is amplified

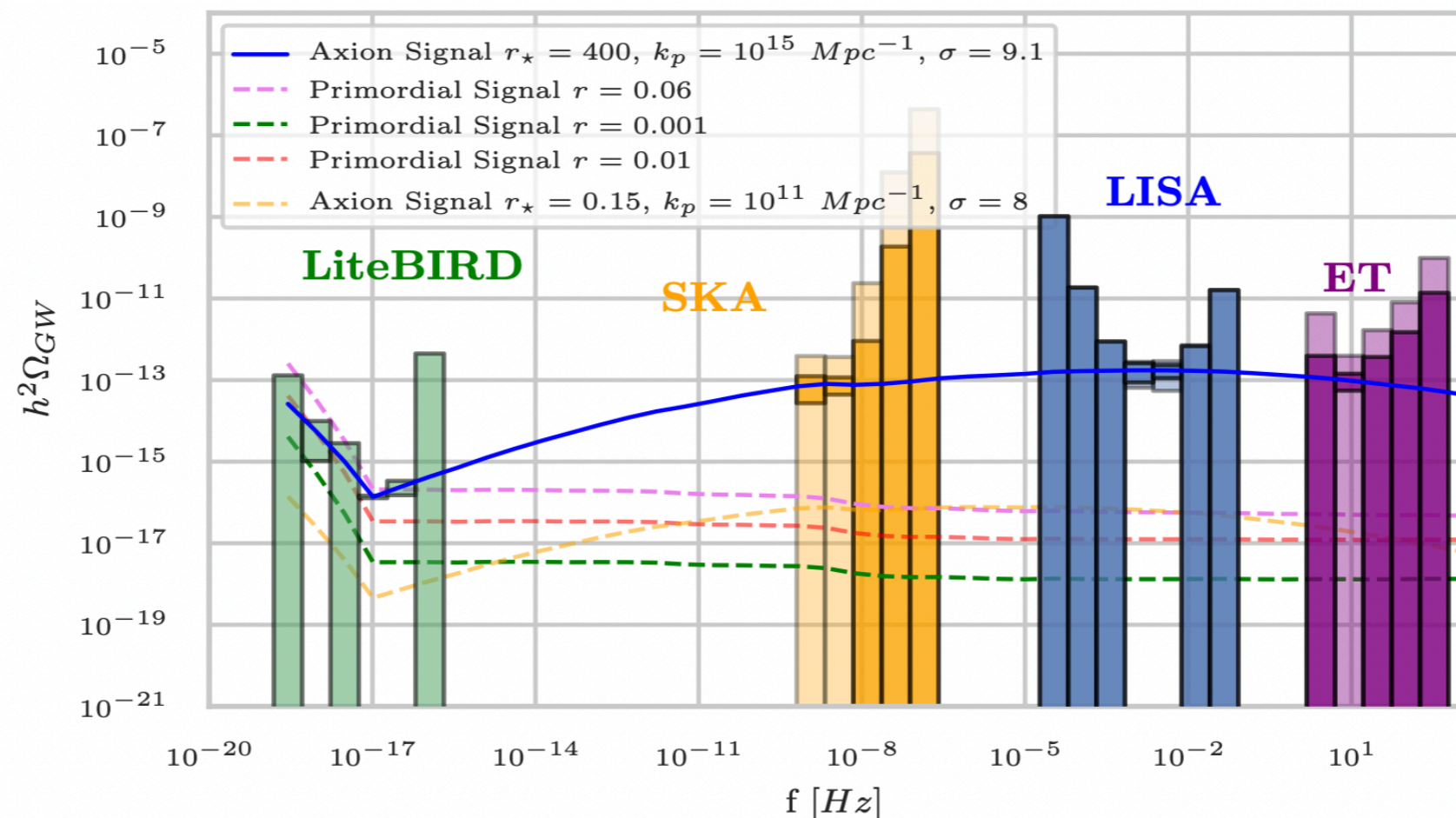


Axion-Gauge fields models: SU(2)



$$\mathcal{P}_\gamma(k) = \mathcal{P}_{\gamma, L}^{(\text{sourced})}(k) = r_* \mathcal{P}_\zeta(k) e^{-\frac{1}{2\sigma^2} \ln^2(k/k_p)}$$

[ED-Fasiello-Fujita, 2016 — Thorne et al, 2017]



[Campeti et al, 2020]

Inflationary GW from vacuum fluctuations (SFSSR)

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 $H \simeq 2 \times 10^{13} \text{GeV} (r/0.01)^{1/2}$
- ~~Red tilt~~: $n_T \simeq -2\epsilon = -r/8$
- ~~Non-chiral~~: $P_L = P_R$
- Nearly **Gaussian**: $f_{\text{NL}} \ll 1$

How do we detect chirality?

CMB angular power spectra & chirality

$$C_{\ell}^{XY} = \int dk \Delta_{\ell}^X(k, \eta_0) \Delta_{\ell}^Y(k, \eta_0) [\mathcal{P}_{\gamma}^R(k) + \epsilon \cdot \mathcal{P}_{\gamma}^L(k)]$$

$$X, Y = T, E, B$$

$$\epsilon = \begin{cases} 1 & \text{for TT, EE, BB, TE} \\ -1 & \text{for TB, EB} \end{cases}$$

For parity-conserving theories $\langle TB \rangle, \langle EB \rangle = 0$

For parity-violating theories $\langle TB \rangle, \langle EB \rangle \neq 0$

Axion-Gauge fields models: SU(2)

axion-gauge field model

$$\gamma_L \neq \gamma_R$$

chiral



$$\langle TB \rangle, \langle EB \rangle \neq 0$$

Detectable at 2σ by LiteBIRD for $r > 0.03$

[Thorne et al, 2017]

Constraining chirality at high frequencies (interferometers)

A **planar** detector cannot distinguish L from R
for an isotropic SGWB

Lorenzo's talk

An 'effective' **non-planar** geometry can be realised by:

- using different (non co-planar) detectors at once → monopole
- exploiting the motion of a detector → higher multiples

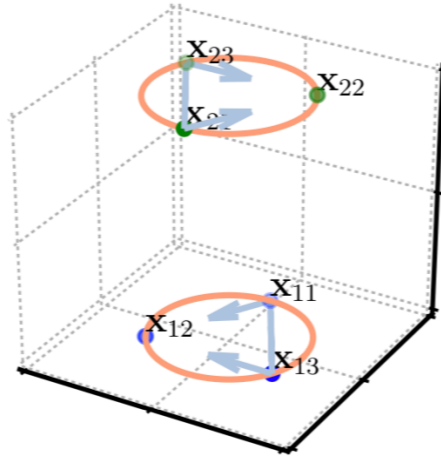
* For (networks of) space-based interferometers see: [Domcke et al, 2020](#) - [Orlando et al., 2021](#)

* For ground-based networks see, e.g. : [Seto-Taruya, 2007](#)— [Smith-Caldwell, 2017](#)

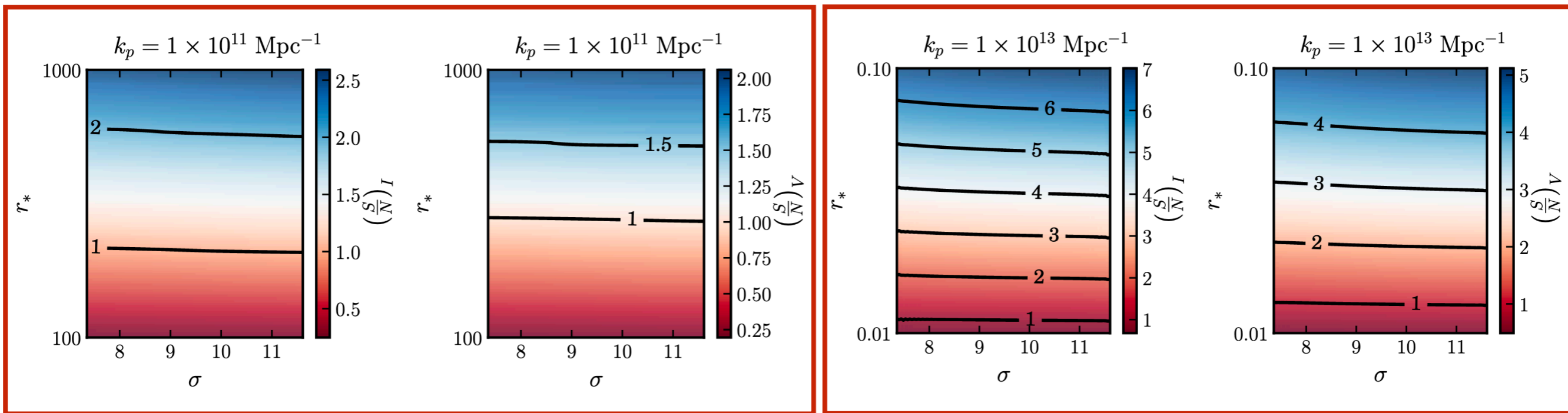
* For PTA: [Belgacem-Kamionkowski, 2020](#)

[See also [Seto 2006-2007](#)]

Cross-correlating signal from different detectors



Forecasts for axion-gauge field model [Komatsu et al 2017]

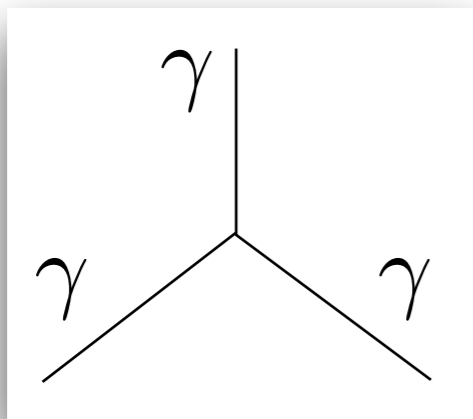


LISA-like

BBO-like

Tensor non-Gaussianity

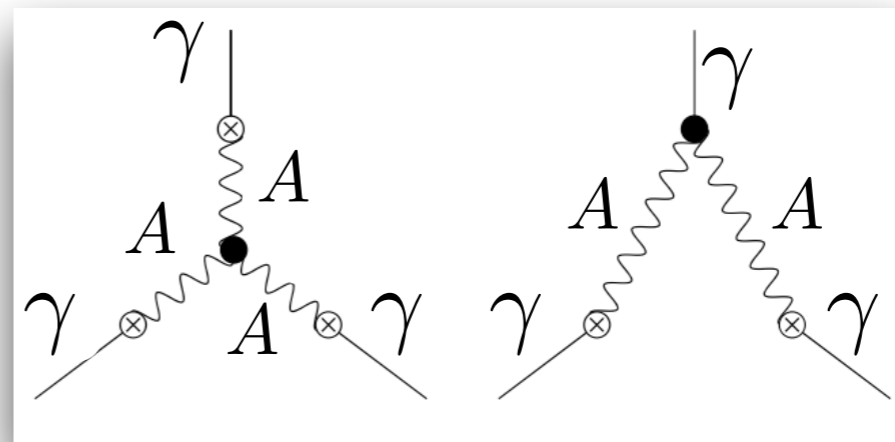
basic single-field inflation



$$f_{NL} = \mathcal{O}(r^2)$$

too small for detection

axion-gauge fields models

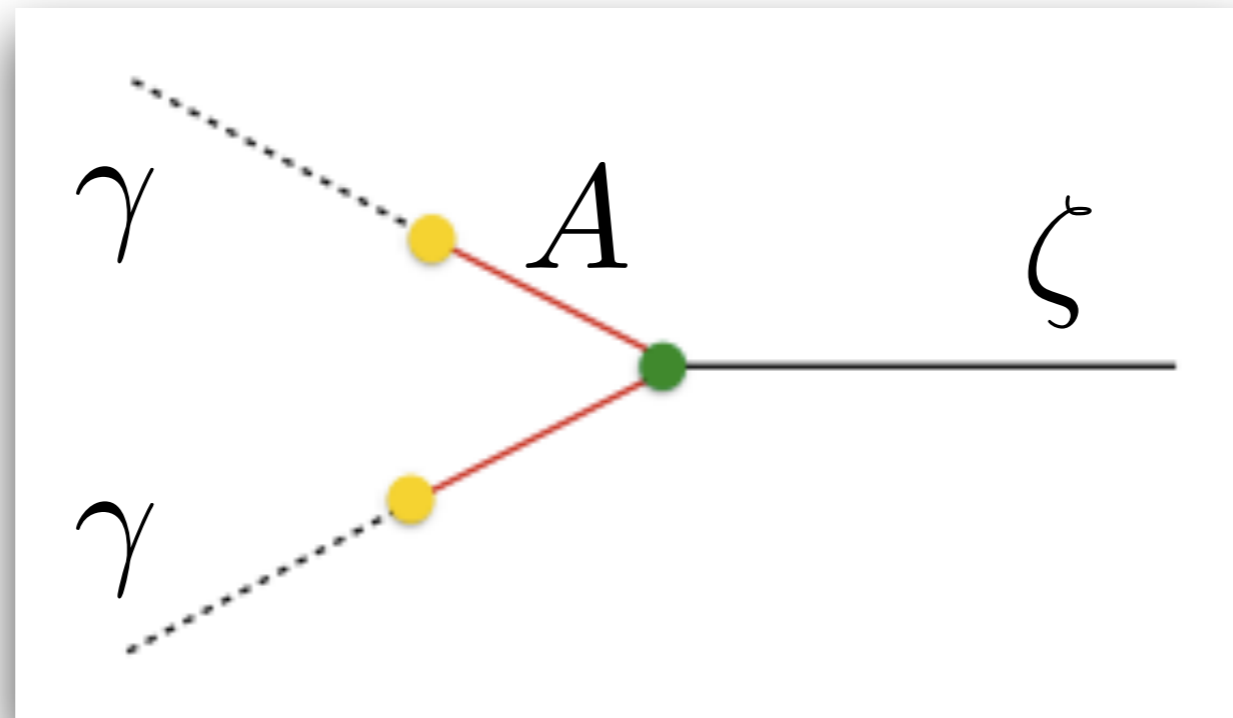


$$f_{NL} = r^2 \cdot \frac{25}{\Omega_A} \gtrsim \mathcal{O}(r^2 \cdot 10^6)$$

- detectable by upcoming CMB space missions
[Agrawal - Fujita - Komatsu 2017]

Mixed (scalar-tensor) non-Gaussianity

testing interactions of tensors and matter fields



Mixed (scalar-tensor) non-Gaussianity

basic single-field inflation
(tensor-scalar-scalar)

$$f_{NL} = \mathcal{O}(r)$$

too small for detection

axion-gauge fields models
(tensor-scalar-scalar)

$$f_{NL} \gtrsim 10^5 \cdot r$$

[ED - Fasiello - Hardwick - Koyama - Wands 2018]

potentially observable with CMBS4

[CMBS4 science book, 2016]

Non-Gaussianity at interferometers

Shapiro time delay:

$$\gamma'' + 2\mathcal{H}\gamma' - [1 + (12/5)\zeta] \gamma_{,kk} = 0$$

GW propagating in FRW background
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij} e^{ik\tau + ik \cdot 2 \int^\tau d\tau' \zeta[\tau', (\tau' - \tau_0)\hat{k}]}$$

GW from different directions
undergo different phase shift
due to intervening structure

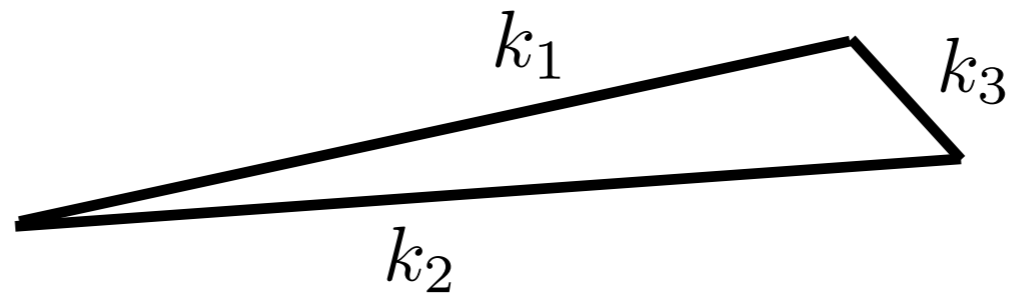
→ decorrelation → cannot measure bispectrum directly with interferometers

[Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

Note: signal measured by an interferometer arises from the superposition of signals from a large number of Hubble patches (CLT)

[Adshhead, Lim 2009 — Caprini, Figueroa 2018 — Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

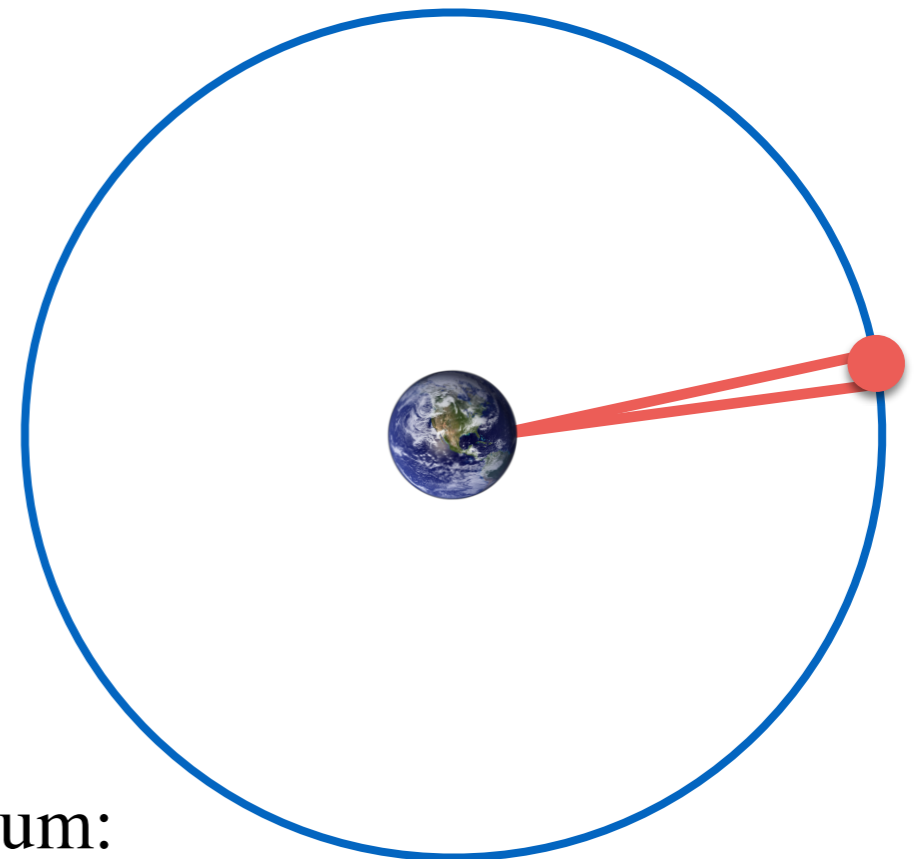
Ultra squeezed non-Gaussianity



Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode: the latter has not undergone propagation!

Signals originate from the same patch!

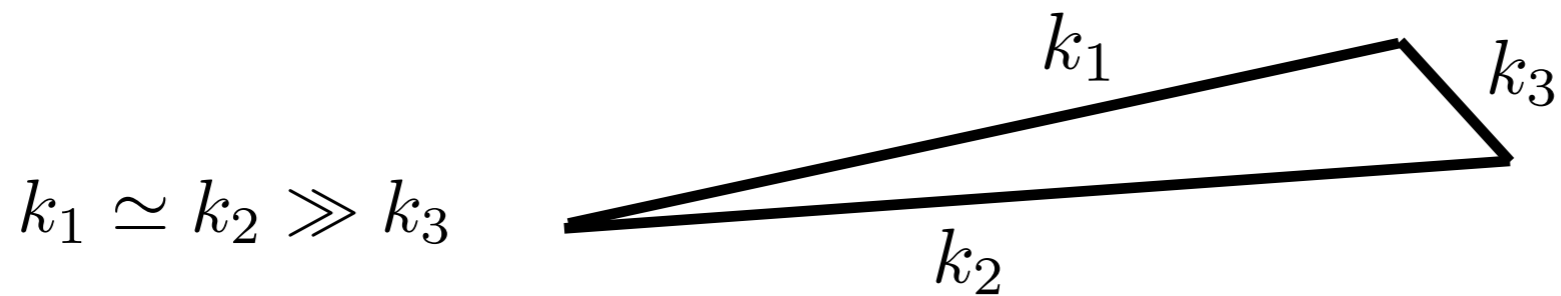
How do we constrain this ultra-squeezed bispectrum:



Look for anisotropies in the SGWB!

$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[1 + \frac{1}{4\pi} \int d^2\hat{n} \delta_{\text{GW}}(k, \hat{n}) \right]$$

Soft limits and 'fossils'



long wavelength modes introduces a modulation
in the primordial power spectrum of the short wavelength modes

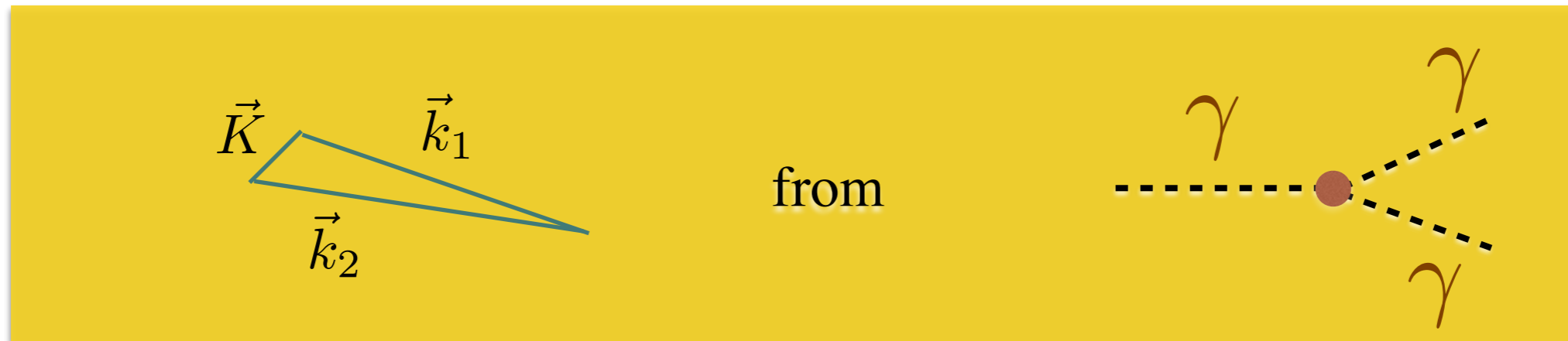
$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L}$$

$$\delta \langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)} \cdot F_L^*$$

$f_{\text{NL}}^{F\gamma\gamma}$

$$\langle \gamma_S \gamma_S \rangle'_{\text{total}} = P_\gamma(k_1) \left(1 + f_{\text{NL}}^{F\gamma\gamma} \cdot F_L^* \right)$$

Soft limits and fossils



$$P_{\gamma}^{\text{mod}}(\mathbf{k}, \mathbf{x}) = P_{\gamma}(k) [1 + \mathcal{Q}_{\ell m}(\mathbf{k}, \mathbf{x}) \hat{n}_{\ell} \hat{n}_m]$$

$$\int \frac{d^3 q}{(2\pi)^3} e^{i\mathbf{q} \cdot \mathbf{x}} \sum_{\lambda_3} h_{\ell m}^{\lambda_3}(\mathbf{q}) F_{\text{NL}}^{\text{ttt}}(\mathbf{k}, \mathbf{q})$$

$$\delta_{\text{GW}}(k, \hat{n}) = \mathcal{Q}_{\ell m}(\mathbf{k}, \mathbf{d}) \hat{n}_{\ell} \hat{n}_m$$

$$\mathbf{d} = -(\eta_0 - \eta_{\text{in}}) \hat{n}$$

$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[1 + \frac{1}{4\pi} \int d^2 \hat{n} \delta_{\text{GW}}(k, \hat{n}) \right]$$

Soft limits and fossils



$$\delta_{\text{GW}}(k, \hat{n}) = \int \frac{d^3 q}{(2\pi)^3} e^{-i d \hat{n} \cdot \mathbf{q}} \zeta(\mathbf{q}) F_{\text{NL}}^{\text{stt}}(\mathbf{k}, \mathbf{q})$$

$$\Omega_{\text{GW}}(k) = \bar{\Omega}_{\text{GW}}(k) \left[1 + \frac{1}{4\pi} \int d^2 \hat{n} \delta_{\text{GW}}(k, \hat{n}) \right] \quad \mathbf{d} = -(\eta_0 - \eta_{\text{in}}) \hat{n}$$

Soft limits in inflation

- *Extra fields / superhorizon evolution*

[Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013, ED - Fasiello - Kamionkowski 2015, ...]

- *Non-Bunch Davies* initial states

[Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, Aravind - Lorschbough - Paban 2013...]

- *Broken space diffs*

(e.g. space-dependent background)

[Endlich et al. 2013, Ricciardone - Tasinato 2016, Celoria - Comelli - Pilo - Rollo 2021, ...]

probe for (extra) fields, pre-inflationary dynamics,
(non-standard) symmetry patterns

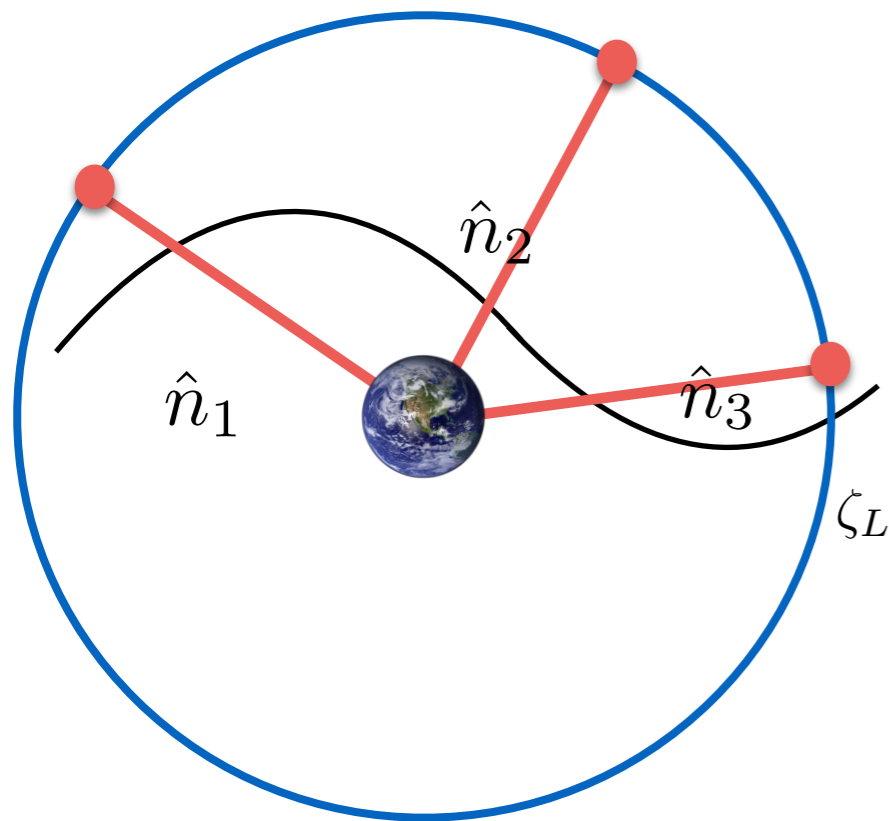
[Maldacena 2003 — Creminelli, Zaldarriaga 2004 — Pajer, Schmidt, Zaldarriaga 2013]

Any other kinds of anisotropies expected in the SGWB?

GW propagate through the perturbed universe, so they are subject to Sachs-Wolfe / integrated Sachs-Wolfe, ... , just like CMB photons!

Simplified treatment in **[Alba, Maldacena 2015]**:

large-scale gravitational potential \longrightarrow SW dominates



$$\frac{\delta f(\hat{n})}{f} = \frac{1}{5} [\zeta_L(\text{today}) - \zeta_L(\hat{n} \cdot \eta_0)]$$

Gravitational redshift/blueshift of gravitons

$$\zeta_L(\hat{n}_1 \cdot \eta_0) \neq \zeta_L(\hat{n}_2 \cdot \eta_0)$$



Direction-dependent frequency shift



Anisotropy in the GW energy density $\delta_{\text{GW}}(f, \hat{n}) \simeq \frac{\alpha}{5} \cdot \zeta_L(\hat{n} \cdot \eta_0)$

$$\bar{\Omega}_{\text{GW}}(f) \sim \left(\frac{f}{f_0}\right)^\alpha$$

[See also: Contaldi, 2017 — Bartolo, Bertacca, Matarrese, Peloso, Ricciardone, Riotto, Tasinato, 2019 — for full Boltzmann treatment of GW anisotropies]

Cross-correlations of GW and CMB anisotropies



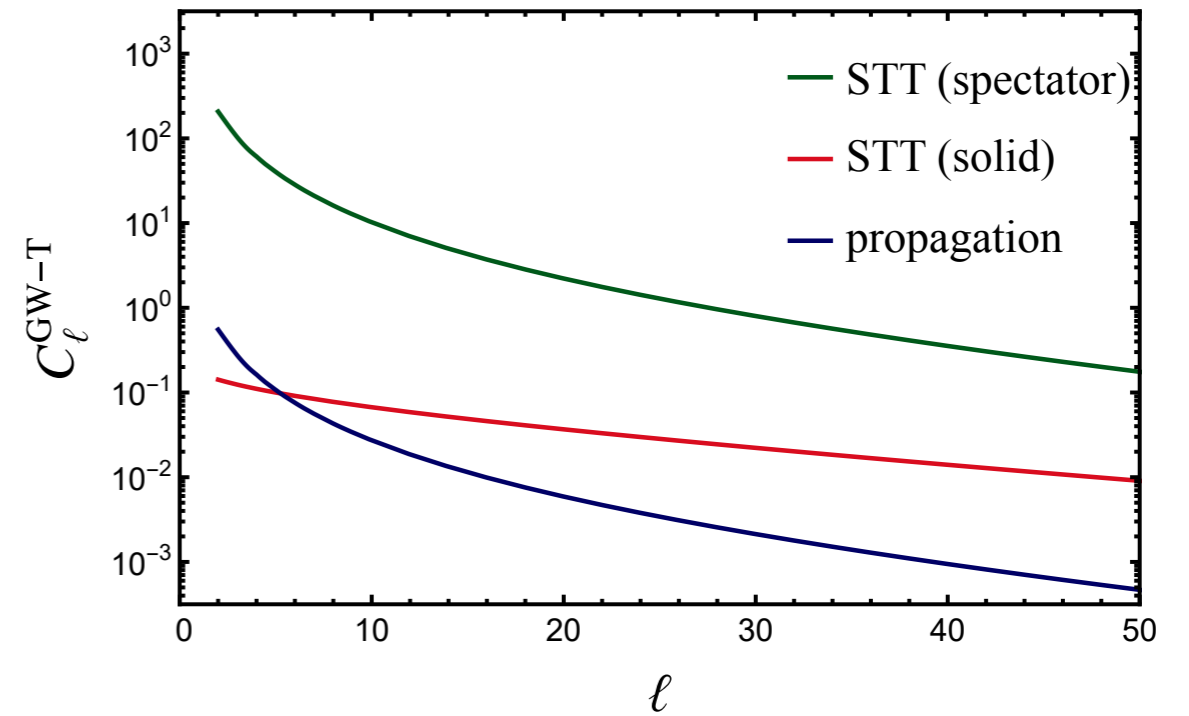
$$\delta_{\text{GW}}^{\text{propagation}} \sim \zeta_L$$

$$\delta_{\text{GW}}^{\text{stt}} \sim F_{\text{NL}}^{\text{stt}} \cdot \zeta_L$$

$$\delta_{\text{GW}}^{\text{stt}} \sim F_{\text{NL}}^{\text{stt}} \cdot \zeta_L$$

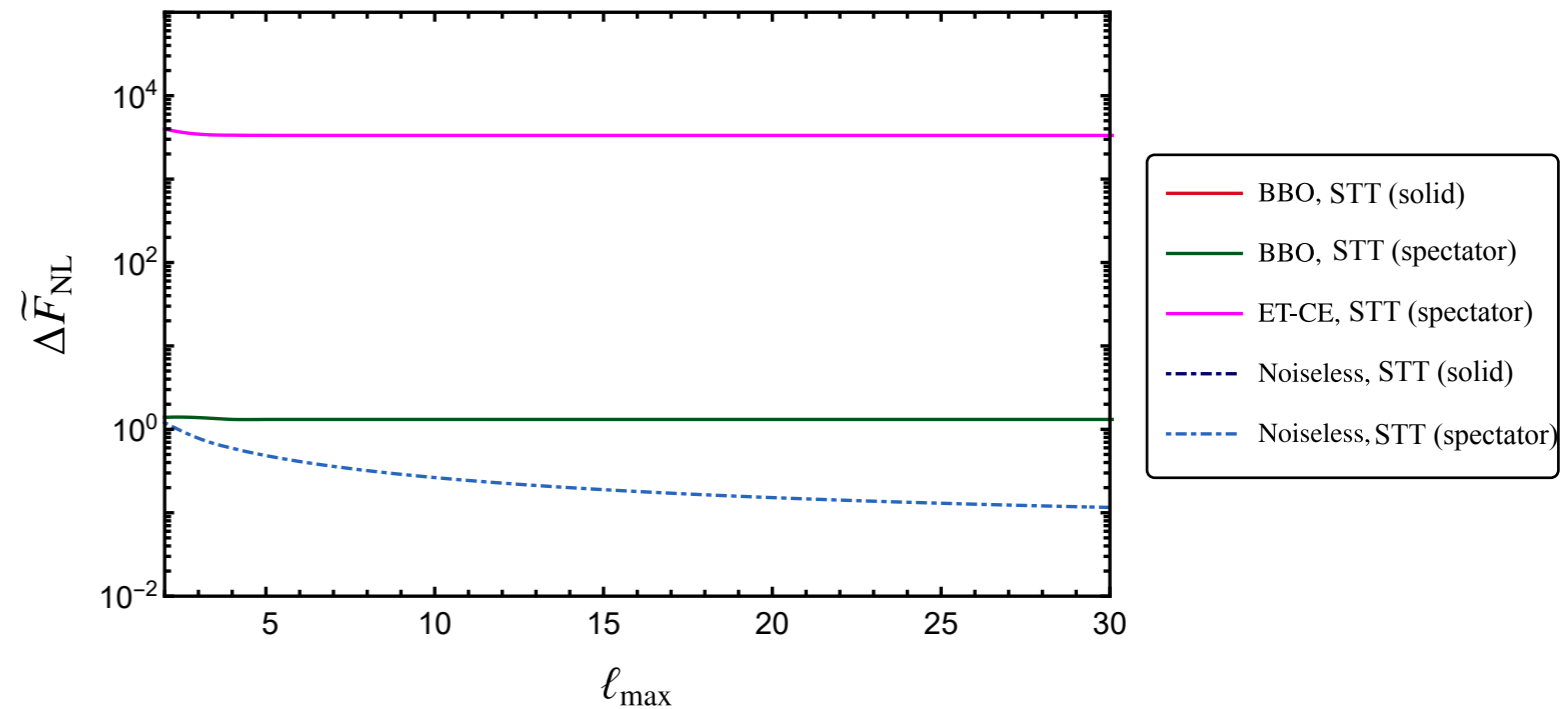
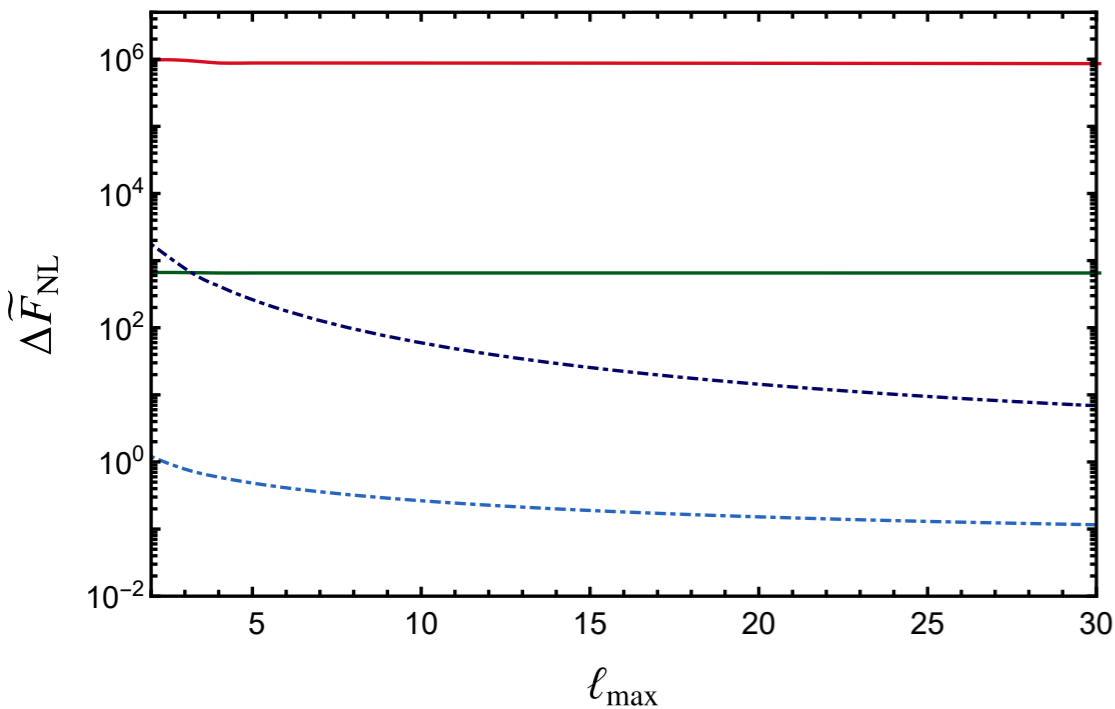
$$\frac{\Delta T}{T} \sim \zeta_L$$

$$\left. \begin{array}{l} \delta_{\text{GW}}^{\text{stt}} \sim F_{\text{NL}}^{\text{stt}} \cdot \zeta_L \\ \frac{\Delta T}{T} \sim \zeta_L \end{array} \right\} C_{\ell}^{\text{GW-T}} \sim F_{\text{NL}}^{\text{stt}} \cdot C_{\ell}^{\text{TT}}$$



$r = 0.07, n_T = 0.08$

$r = 0.07, n_T = 0.27$



Gravitational waves

- Extremely useful in testing inflation, also at interferometer scales
- A variety of classes of models (beyond the vanilla scenario) generate interesting gravitational waves signatures
- Crucial for disentangling inflationary SGWB from the one due to other cosmological sources and from the astrophysical SGWB:
 - spectral shape
 - chirality
 - non-Gaussianity and anisotropies