Measuring the net circular polarization of the stochastic gravitational wave background with interferometers

Domcke, Garcia-Bellido, Peloso, Pieroni,
Ricciardone, LS & Tasinato 1910.08052, JCAP

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UMass Amherst

Motivation

How to disentangle stochastic GW background
of *primordial origin*
from that of *astrophysical origin?*

One possibility:
we do not expect astrophysical
stochastic GWs to have net chirality
(requires parity violation)

(But: lesson from BICEP + dust?)

Motivation

There are models with
parity violating dynamics
leading to net helicity
for primordial stochastic GW background

Parity-violating dynamics → (Helical gauge fields) → Helical SGWB

$\phi \epsilon_{\mu\nu\rho\lambda} R^{\mu\nu\alpha\beta} R^{\rho\lambda}{}_{\alpha\beta}$

$\phi\, \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}$

Stochastic GW background: power spectra

The transverse-traceless perturbation of the metric

$$h_{ij}(t,\, \vec{x}) = [g_{ij}(t,\, \vec{x}) - \eta_{ij}]^{TT}$$

is Fourier-transformed to

Polarization tensors in *chiral* basis

$$h_{ij}(t,\, \vec{x}) = \int d^3k\, e^{-2\pi i \vec{k}\cdot\vec{x}} \sum_\lambda e_{ij,\lambda}(\hat{k})\, h^\lambda(t, \vec{k})$$

Note the “cosmology” convention, instead of the GW convention

$$h_{ij}(t,\, \vec{x}) = \int_{-\infty}^{+\infty} df \int d^2\hat{k} e^{2\pi i f(t-\hat{k}\cdot\vec{x})} \sum_\lambda e_{ij,\lambda}(\hat{k}) h^\lambda(f,\, \hat{k})$$

Stochastic GW background: power spectra

The modes of the graviton have time-dependence

$$h^\lambda(t,\vec{k}) = A^\lambda_{\vec{k}} \cos(2\pi k\, t) + B^\lambda_{\vec{k}} \sin(2\pi k\, t)$$

Stochastic variables

Statistically homogeneous & isotropic power spectra

$$\langle A^\lambda_{\vec{k}}\, A^{\lambda'}_{\vec{k}'}\rangle = \langle B^\lambda_{\vec{k}}\, B^{\lambda'}_{\vec{k}'}\rangle = \frac{P_\lambda(k)}{4\pi k^3}\delta_{\lambda\lambda'}\delta(\vec{k}+\vec{k}')\,, \qquad \langle A^\lambda_{\vec{k}}\, B^{\lambda'}_{\vec{k}'}\rangle = 0$$

Equal time spectra

$$\langle h^\lambda(t,\vec{k}) h^{\lambda'}(t,\vec{k}')\rangle \equiv \frac{P_\lambda(k)}{4\pi k^3}\delta_{\lambda\lambda'}\delta(\vec{k}+\vec{k}')$$

$P_+(k) \neq P_-(k) \Longrightarrow$ net circular polarization

Motivation

Q: can we detect a nonvanishing

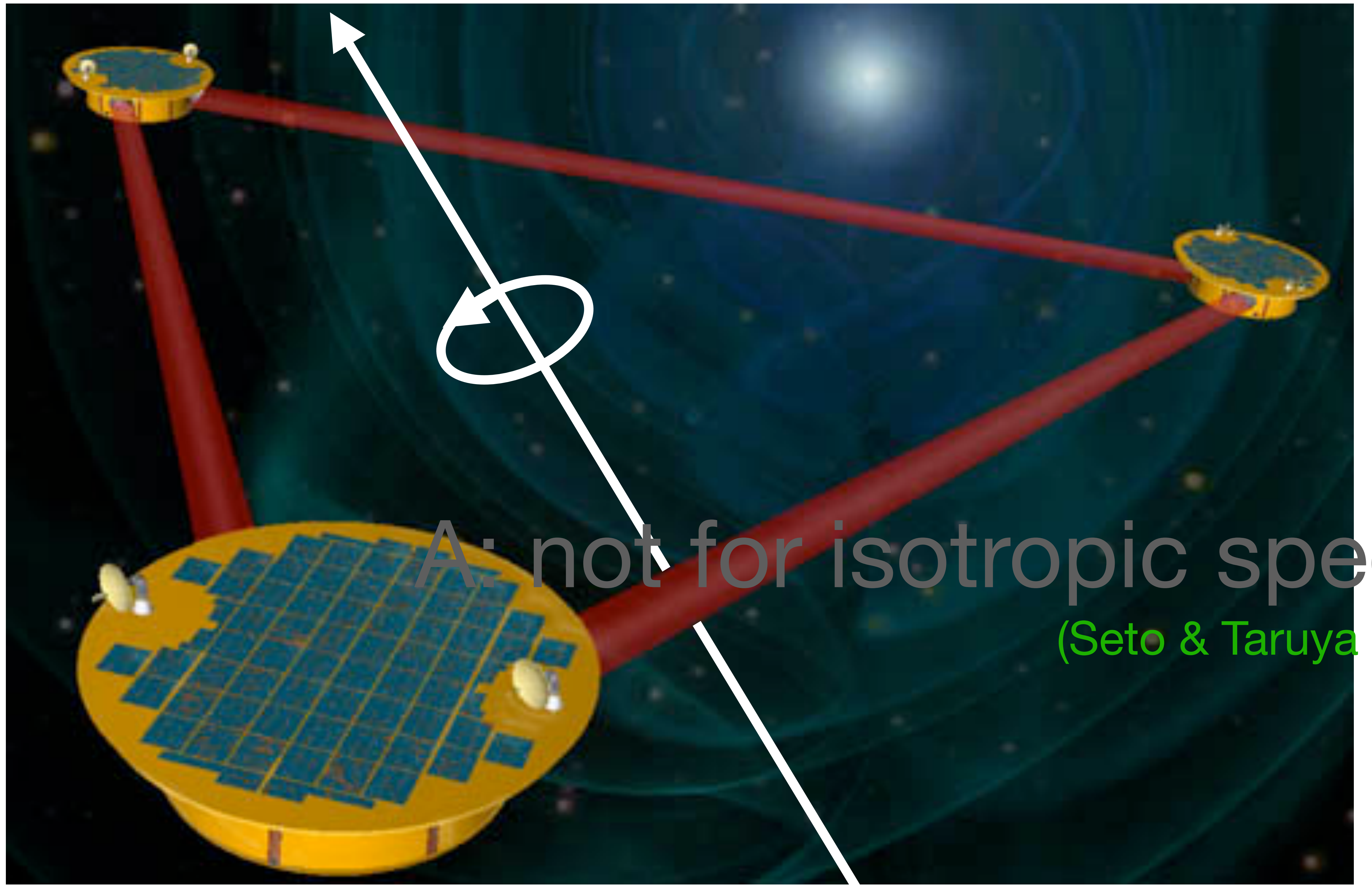
$$P_+(k)-P_-(k)$$

with interferometers?

A: not for isotropic spectra with one interferometer

(Seto & Taruya 07, Caldwell & Smith 16)

Motivation



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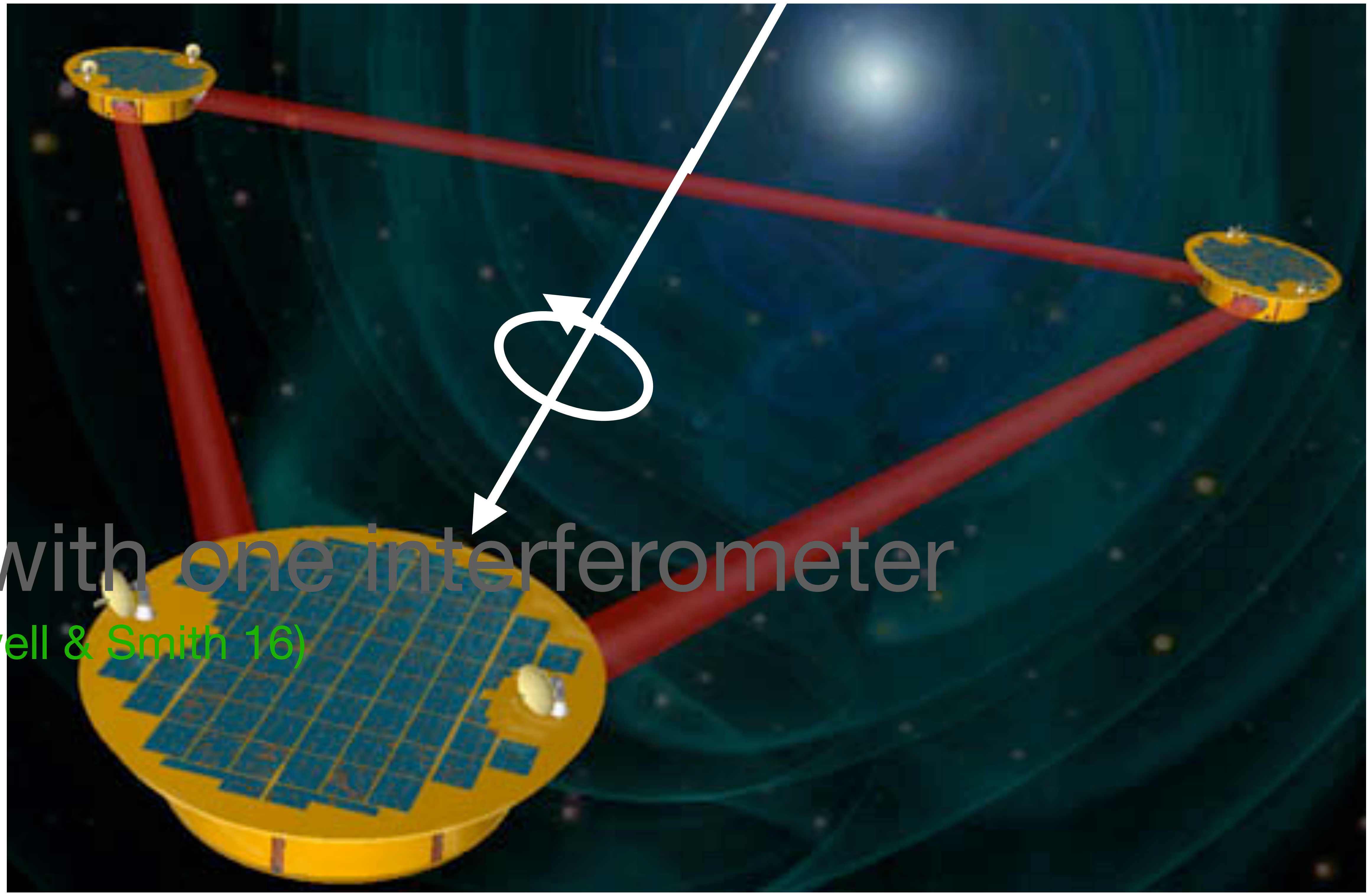


A: not for isotropic spectra with one interferometer

(Seto & Taruya 07, Caldwell & Smith 16)

Idea

Two possible ways out:

(1) assume SGWB is not isotropic (dipole)

(Seto 06, 07)

(2) use >1 non-coplanar detectors

(Seto & Taruya 07, 08, Crowder et al 12)

Domcke, Garcia-Bellido, Peloso, Pieroni, Ricciardone, LS & Tasinato 1910.08052

Completes and updates those analyses in various directions

(1) assume SGWB is not isotropic (dipole)

SGWB dipole with circular polarization

Start from isotropic spectrum and boost observer

$$\langle h^\lambda(\vec{k},\,t)\,h^{\lambda'}(\vec{k}\,',\,t')\rangle = \delta_{\lambda\lambda'}\,\frac{\delta^{(3)}(\vec{k}+\vec{k}\,')}{4\pi\,k^3}\left\{P^\lambda(k)\,\cos[2\pi k(t-t')] + i(\hat{k}\cdot\vec{v})\left[2P^\lambda(k) - k\,P^{\lambda\prime}(k)\right]\,\sin[2\pi k(t-t')]\right\}$$

observer's velocity

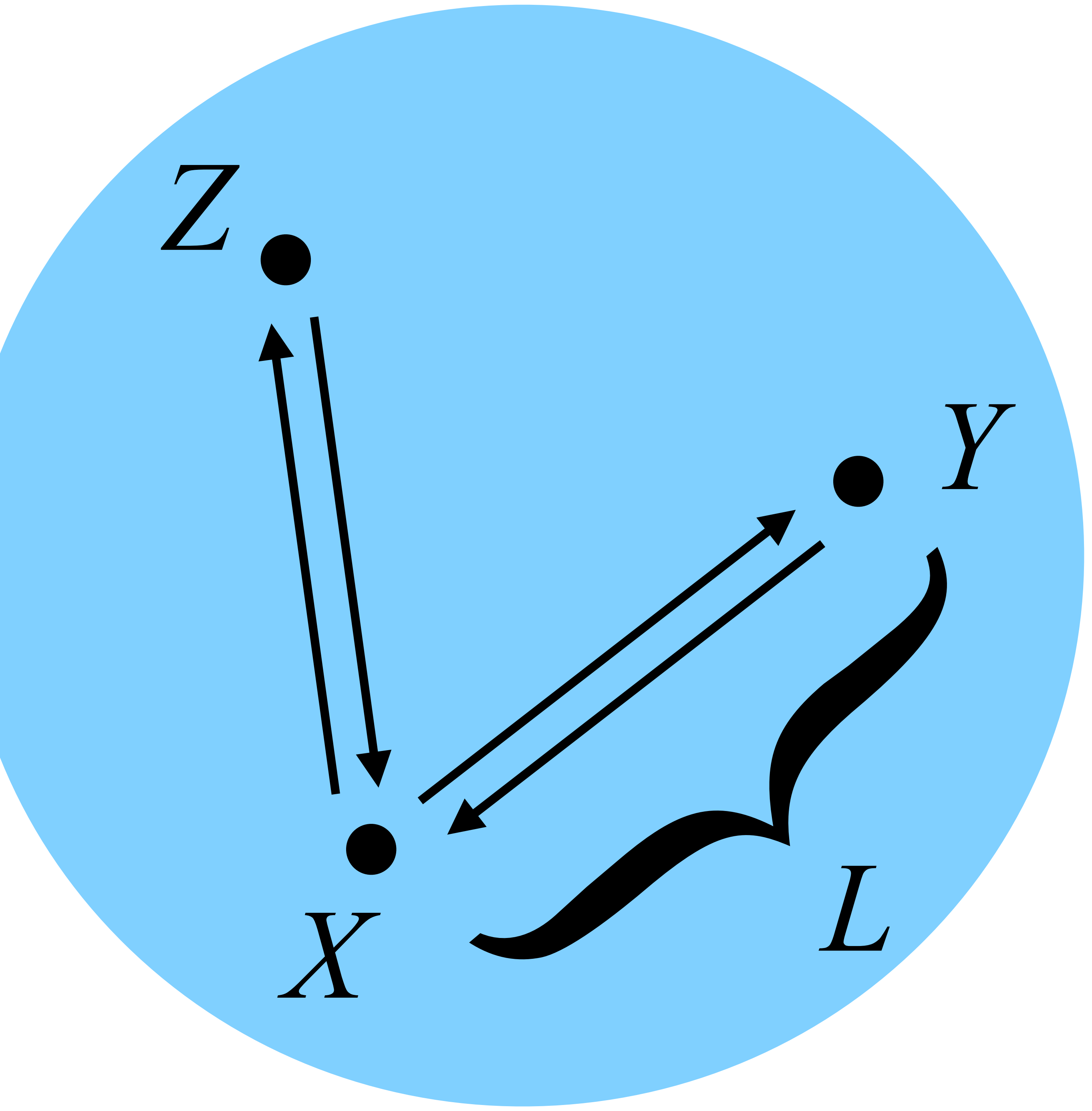
$$+O(v^2)$$

[First correction =0 at equal times]

LISA observables

Basic quantity: time delay δt_i , due to GWs, between photons on two-way trips along two arms ending at vertex $i \in \{X, Y, Z\}$

Time Delay Interferometry LISA channels $\{A, E, T\}$



$$\Sigma_A = \frac{2\,\delta t_X - \delta t_Y - \delta t_Z}{6L}$$

$$\Sigma_E = \frac{\delta t_Z - \delta t_Y}{2\sqrt{3}\,L}$$

$$\Sigma_T = \frac{\delta t_X + \delta t_Y + \delta t_Z}{6L}$$

Depends on GW momentum and detector geometry

$$\boxed{\Sigma_O(t) = \sum_\lambda \int d^3k\, h_\lambda(\vec{k}, t-L)\, e_{ab,\lambda}(\hat{k})\, \mathcal{Q}^O_{ab}(\vec{k}; \{\hat{x}_j\})}$$

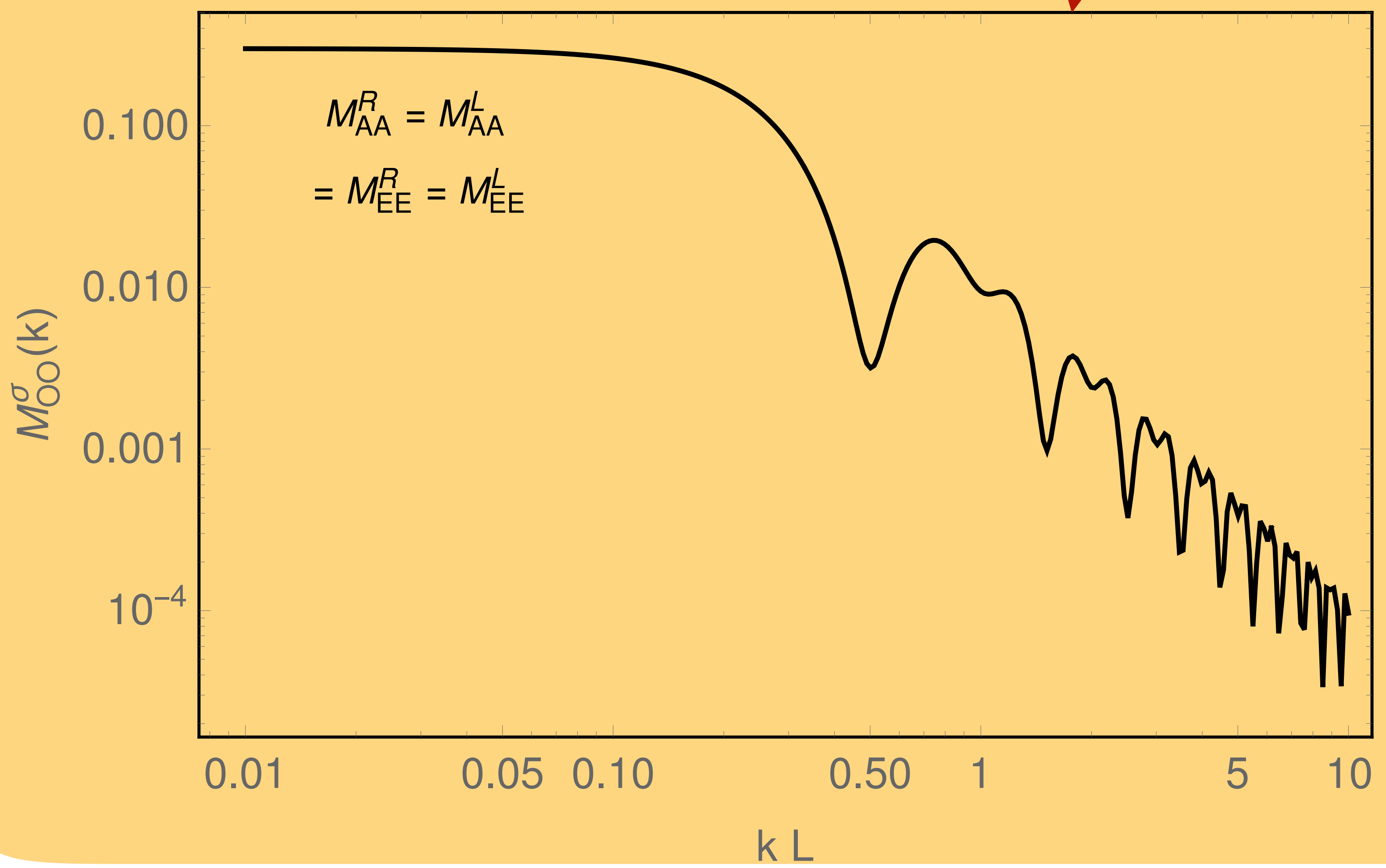
LISA observables and dipole

Inserting the expression for power spectrum

$$\langle \Sigma_O(t)\Sigma_{O'}(t')\rangle = \frac{1}{4}\sum_\lambda \int \frac{dk}{k}\Big[\mathcal{M}^\lambda_{OO'}(k)P_\lambda(k)\cos\left[2\pi k(t-t')\right]$$
$$+ v\,\mathcal{D}^\lambda_{OO'}\left(2P_\lambda(k) - kP'_\lambda(k)\right)\sin\left[2\pi k(t-t')\right]\Big]$$

monopole response function

dipole response function



$$\mathcal{M}^\lambda_{AE} = 0$$

LISA observables and dipole

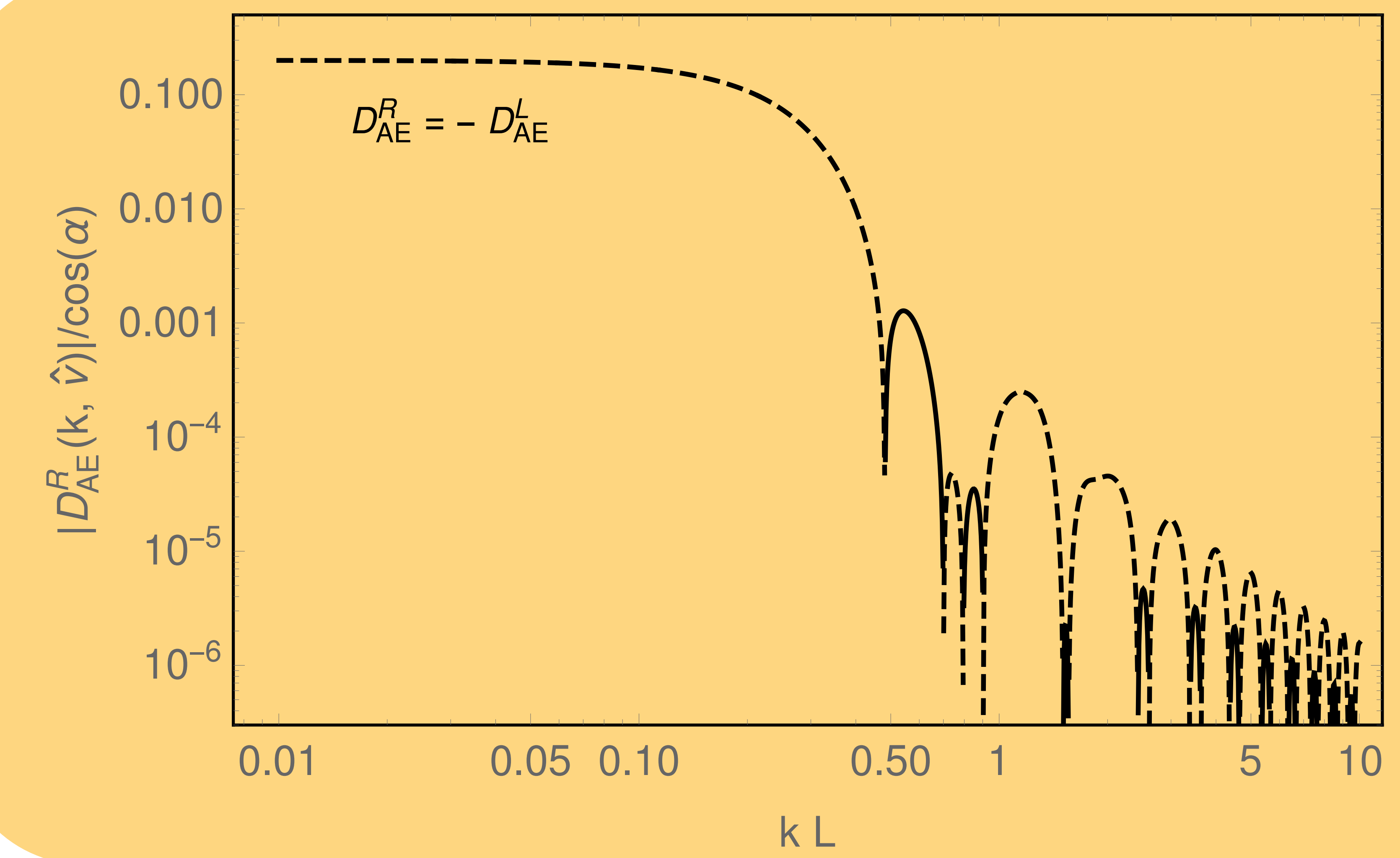
Inserting the expression for power spectrum

$$\langle \Sigma_O(t)\Sigma_{O'}(t')\rangle = \frac{1}{4}\sum_\lambda \int \frac{dk}{k}\Big[\mathcal{M}^\lambda_{OO'}(k)P_\lambda(k)\cos\left[2\pi k(t-t')\right]$$

$$+\, v\,\mathcal{D}^\lambda_{OO'}\big(2P_\lambda(k)-kP'_\lambda(k)\big)\sin\left[2\pi k(t-t')\right]\Big]$$

monopole response function

dipole response function



$$\mathcal{D}^\lambda_{OO'} \propto \hat{v}\cdot\hat{n}$$

normal to LISA plane

$$\mathcal{D}^\lambda_{AA} = \mathcal{D}^\lambda_{EE} = 0$$

LISA observables and dipole

Inserting the expression for power spectrum

$$\langle \Sigma_O(t)\Sigma_{O'}(t')\rangle = \frac{1}{4}\sum_\lambda \int \frac{dk}{k}\left[\mathcal{M}^\lambda_{OO'}(k) P_\lambda(k)\cos\left[2\pi k(t-t')\right]\right.$$

monopole response function

$$\left. + \, v\,\mathcal{D}^\lambda_{OO'}\left(2P_\lambda(k) - kP'_\lambda(k)\right)\sin\left[2\pi k(t-t')\right]\right]$$

dipole response function

$$\mathcal{D}^+_{AE} = -\mathcal{D}^-_{AE} \quad \Rightarrow \quad \langle \Sigma_A\, \Sigma_E\rangle = 0$$

if $P_+(k)=P_-(k)$

SNR for LISA & parity-odd dipole

Observable:

$$\langle \Sigma_A(f)\, \Sigma_E(f') \rangle$$

Optimal SNR for LISA
(assuming scale-invariant Ω_{GW})

yearly modulation

$$SNR \simeq 8.5 \times 10^{13} v \left| \sum_\lambda \lambda\, \Omega^\lambda_{GW} h^2 \right| \sqrt{\int_0^{\frac{T}{1\ \mathrm{year}}} (\hat{v} \cdot \hat{n}(t))^2\, dt}$$

SNR for LISA & parity-odd dipole

Observable:

$$\langle \Sigma_A(f)\, \Sigma_E(f') \rangle$$

Optimal SNR for LISA
(assuming scale-invariant Ω_{GW})

$$SNR \simeq 2000 \left(\frac{v}{10^{-3}} \right) \frac{\left| \sum_\lambda \lambda \Omega_{GW}^\lambda \right|}{6 \times 10^{-8}} \sqrt{\frac{T}{3 \text{ years}}}$$

(Normalized to current LIGO upper bound on Ω_{GW})

SNR for LISA & parity-odd dipole

Observable:

$$\langle \Sigma_A(f)\, \Sigma_E(f') \rangle$$

Optimal SNR for LISA
(assuming scale-invariant Ω_{GW})

$$\boxed{SNR \simeq \left(\frac{v}{10^{-3}}\right) \frac{\left|\sum_\lambda \lambda \Omega^\lambda_{GW}\right|}{3 \times 10^{-11}} \sqrt{\frac{T}{3 \text{ years}}}}$$

(Compare to LISA sensitivity to $\Omega_{GW} \simeq 10^{-13}$)

SNR for Einstein Telescope & parity-odd dipole

Einstein Telescope=Proposed underground LISA
(10km arms)

Optimal SNR for ET
(assuming scale-invariant Ω_{GW})

$$SNR \simeq 2000 \left(\frac{v}{10^{-3}}\right) \left(\frac{|\sum_\lambda \lambda \Omega_{GW}|}{6 \times 10^{-8}}\right) \sqrt{\frac{T}{3 \text{ years}}}$$

(Same as LISA!)

But note sensitivity at *~100 Hz* instead of *~.01 Hz*

(2) use >1 non-coplanar detectors

Ground based interferometers

In small frequency approximation
(good for ground-based interferometers)

$$2\pi k\, L \ll 1$$

response functions simplify:
analytical expressions!

Flanagan 93,
Allen Romano 97,
Seto Taruya 06, 07

Ground based interferometers

Defining…

$$D_i^{ab} \equiv \frac{\hat{U}_i^a \hat{U}_i^b - \hat{V}_i^a \hat{V}_i^b}{2}$$

unit vectors
directed along
arms to i-th interferometer

$$\kappa \equiv 2\pi k |\vec{x}_i - \vec{x}_j| \quad , \quad \hat{s}_{ij} \equiv \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_i - \vec{x}_j|}$$

location of
i-th interferometer

$$f_A(\kappa) \equiv \frac{j_1(\kappa)}{2\kappa} + \frac{1-\kappa^2}{2\kappa^2} j_2(\kappa) \quad , \quad f_B(\kappa) \equiv \frac{j_1(\kappa)}{\kappa} - \frac{5-\kappa^2}{\kappa^2} j_2(\kappa)$$

$$f_C(\kappa) \equiv \frac{-7 j_1(\kappa)}{4\kappa} + \frac{35-\kappa^2}{4\kappa^2} j_2(\kappa) \quad ,$$

$$f_D(\kappa) \equiv \frac{j_1(\kappa)}{2} - \frac{j_2(\kappa)}{2\kappa} \quad , \quad f_E(\kappa) \equiv -\frac{j_1(\kappa)}{2} + 5\frac{j_2(\kappa)}{2\kappa} \quad ,$$

Ground based interferometers

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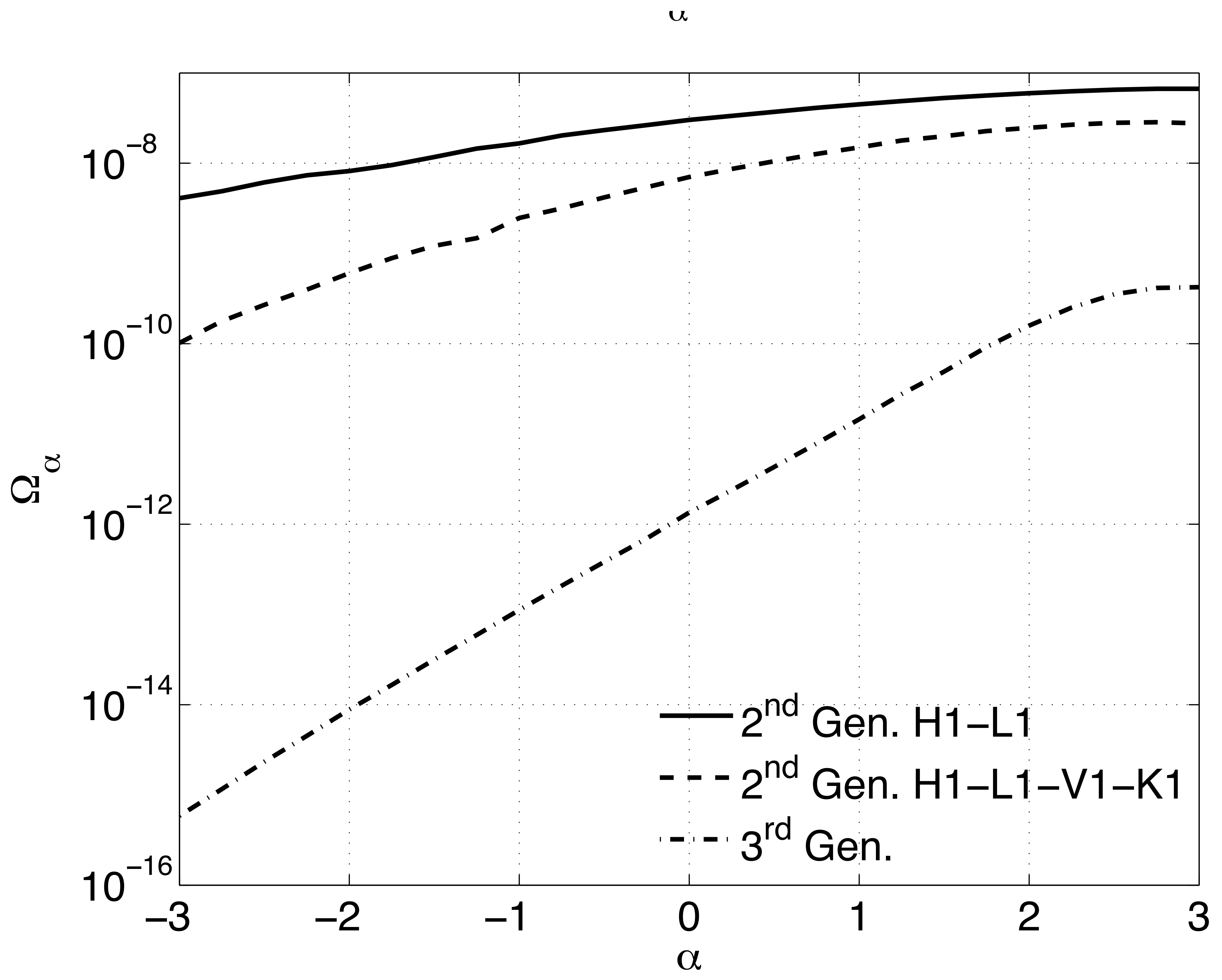
$$
\begin{aligned}
\mathcal{M}_{ij}^{\lambda}(k) \; = \;& f_A(\kappa)\;\mathrm{tr}\left[D_i D_j\right] + f_B(\kappa)\;\left(D_i \hat{s}_{ij}\right)^a \left(D_j \hat{s}_{ij}\right)^a + f_C(\kappa)\;\left(D_i \hat{s}_{ij}\hat{s}_{ij}\right)\left(D_j \hat{s}_{ij}\hat{s}_{ij}\right) \\
+ \;& \lambda\, f_D(\kappa)\;\left[D_i D_j\right]^{ab} \epsilon_{abc}\hat{s}_{ij}^{c} + \lambda\, f_E(\kappa)\left(D_i \hat{s}_{ij}\right)^a \left(D_j \hat{s}_{ij}\right)^b \epsilon_{abc}\hat{s}_{ij}^{c}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_{ij}^{\lambda}(k,\hat{v}) = \;& f_A'(\kappa)\;\hat{v}_e\hat{s}_e\left(D_i D_j\right)^{aa} \\
& + \left[f_B'(\kappa) - 2\,\frac{f_B(\kappa)}{\kappa}\right]\hat{v}_e\hat{s}_e\left(D_i\hat{s}_b\right)^a\left(D_j\hat{s}\right)^a + \frac{f_B(\kappa)}{\kappa}\left[\left(D_i\hat{v}\right)^a\;\left(D_j\hat{s}\right)^a + \left(D_i\hat{s}\right)^a\;\left(D_j\hat{v}\right)^a\right] \\
& + \left[f_C'(\kappa) - 4\,\frac{f_C(\kappa)}{\kappa}\right]\hat{v}_e\hat{s}_e\;\left(D_i\hat{s}\hat{s}\right)\;\left(D_j\hat{s}\hat{s}\right) + 2\frac{f_C(\kappa)}{\kappa}\left[\left(D_i\hat{s}\hat{v}\right)\;\left(D_j\hat{s}\hat{s}\right) + \left(D_i\hat{s}\hat{s}\right)\;\left(D_j\hat{s}\hat{v}\right)\right] \\
& + \lambda\left[f_D'(\kappa) - \frac{f_D(\kappa)}{\kappa}\right]\hat{v}_e\hat{s}_e\left(D_i D_j\right)^{ab}\epsilon_{abc}\hat{s}_c + \lambda\frac{f_D(\kappa)}{\kappa}\left(D_i D_j\right)^{ab}\epsilon_{abc}\hat{v}_c \\
& + \lambda\left[f_E'(\kappa) - 3\frac{f_E(\kappa)}{\kappa}\right]\hat{v}_e\hat{s}_e\;\left(D_i\hat{s}\right)^a\;\left(D_j\hat{s}\right)^b\;\epsilon_{abc}\hat{s}_c \\
& + \lambda\frac{f_E(\kappa)}{\kappa}\left\{\left[\left(D_i\hat{v}\right)^a\;\left(D_j\hat{s}\right)^b + \left(D_i\hat{s}\right)^a\;\left(D_j\hat{v}\right)^b\right]\epsilon_{abc}\hat{s}_c + \left(D_i\hat{s}\right)^a\;\left(D_j\hat{s}\right)^b\;\epsilon_{abc}\hat{v}_c\right\}
\end{aligned}
$$

helicity-dependent

Ground based interferometers

Crowder, Namba, Mandic, Mukohyama, Peloso 12



assuming a SGWB with maximal parity violation ($\Pi = +1$), the lines denote Ω_α needed for a given α to detect the SGWB and exclude $\Pi = 0$ at 95% confidence, using two second-generation detector networks and the example of a third-generation detector pair.

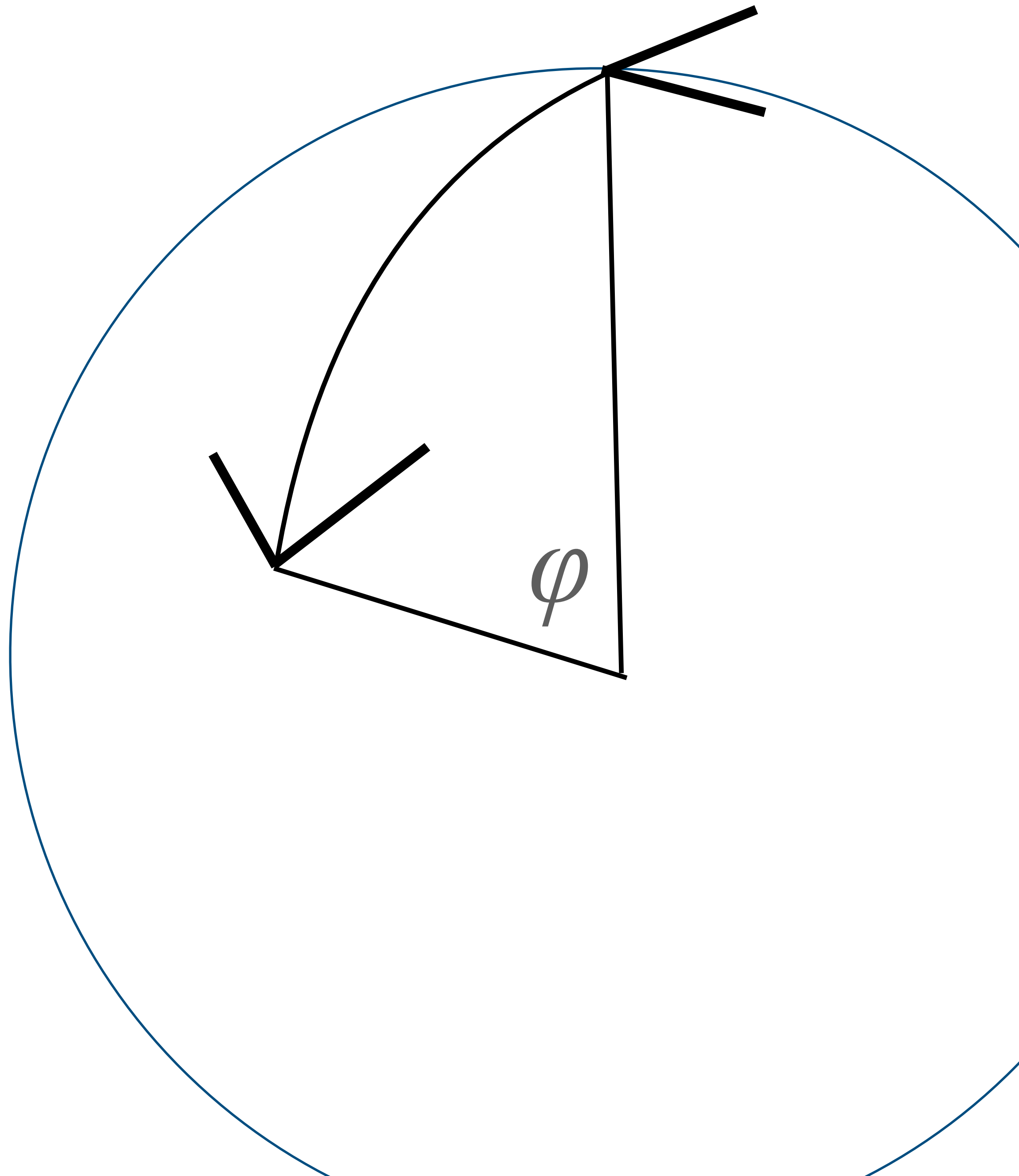
Ground based interferometers

Monopole sensitivity to chiral GW

~.76 for LIGO-L/Kagra

$$\mathcal{M}_{ij}^{+} - \mathcal{M}_{ij}^{-} = \frac{\kappa^2 \left(-3 + \cos\phi\right) j_0\left(\kappa\right) + \left[3\left(7 - \kappa^2\right) + \left(9 + \kappa^2\right)\cos\phi\right] j_2\left(\kappa\right)}{24\kappa} \sin\left(\frac{\phi}{2}\right) \sin\left[2\left(\alpha + \beta\right)\right]$$

Maximized with detectors at antipodes rotated by 45°

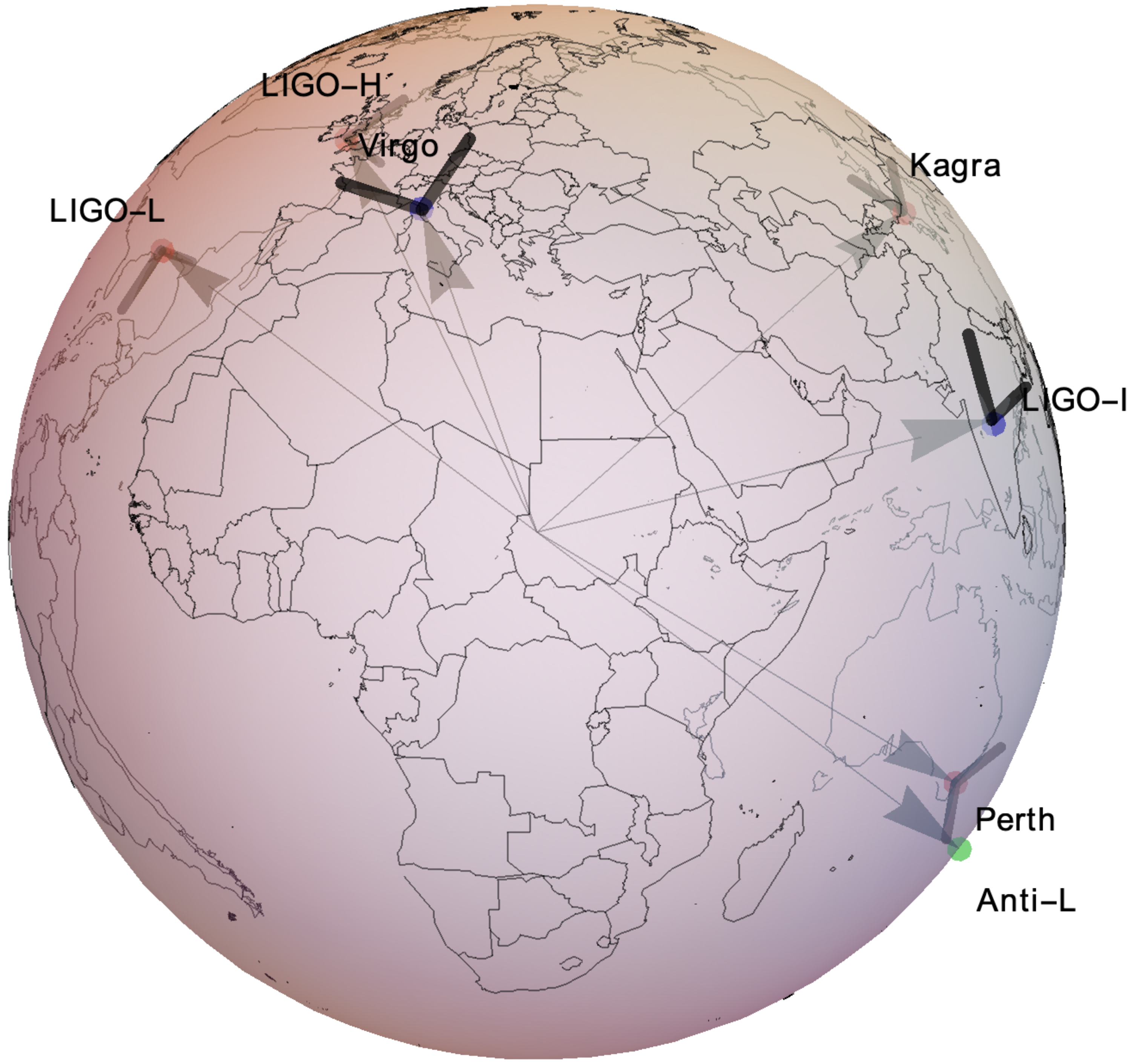


Ground based interferometers

Antipodes to LIGO-L, LIGO-H, Virgo and KAGRA are all in Ocean

Antipode to LIGO-I, “reasonably close” to Perth, Australia

Maximized with detectors at antipod
rotated by 45°



To sum up…

- Theoretical motivation for chiral SGWB, smoking gun of primordial origin?

- Parity violating SGWB detectable by existing network of ground-based detectors if $\Omega_{GW} \sim 10^{-8}$

- Detectable by LISA (@$.01\ Hz$) or ET (@$100\ Hz$) if $\Omega_{GW} \sim 10^{-11}$