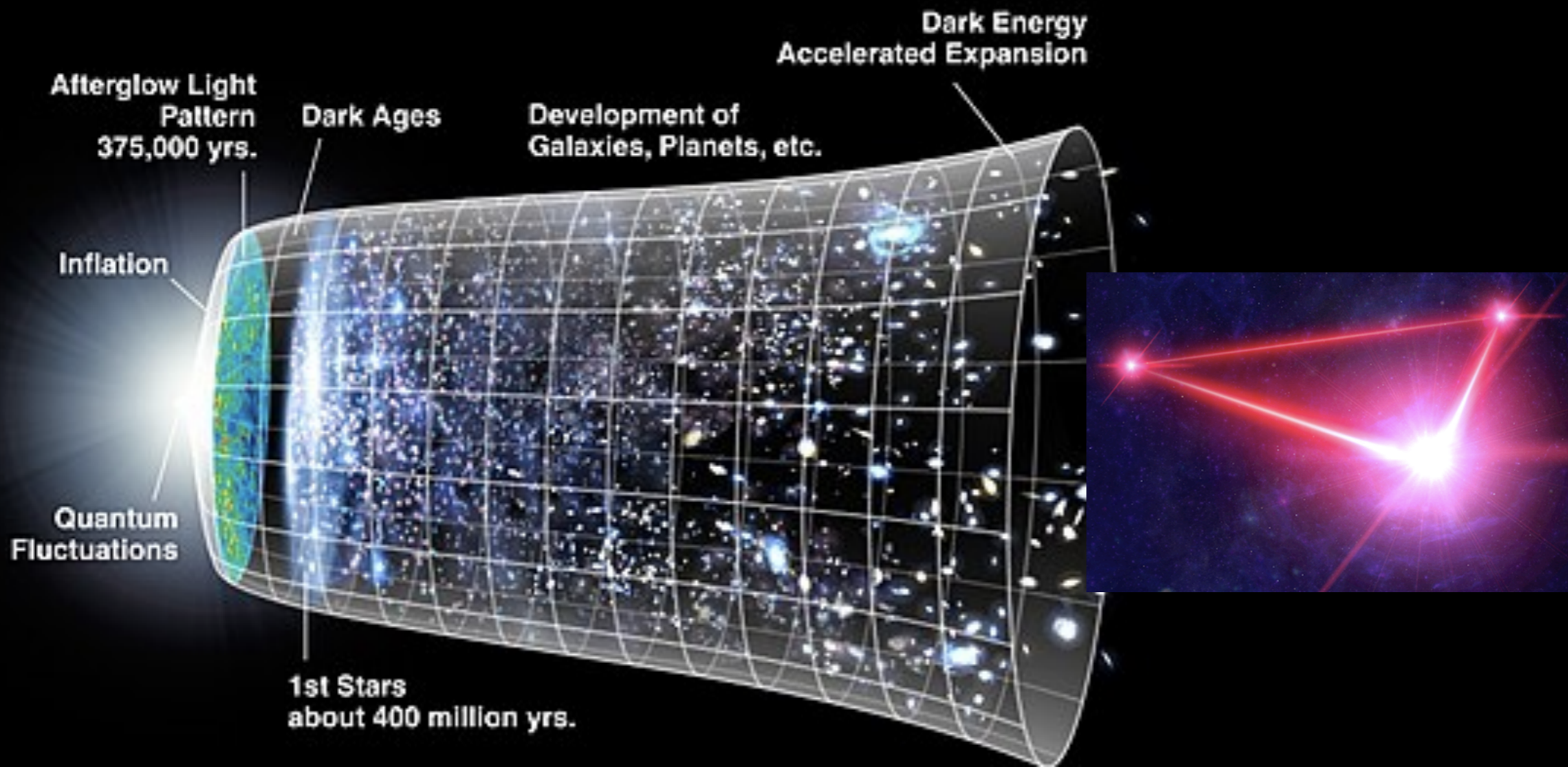
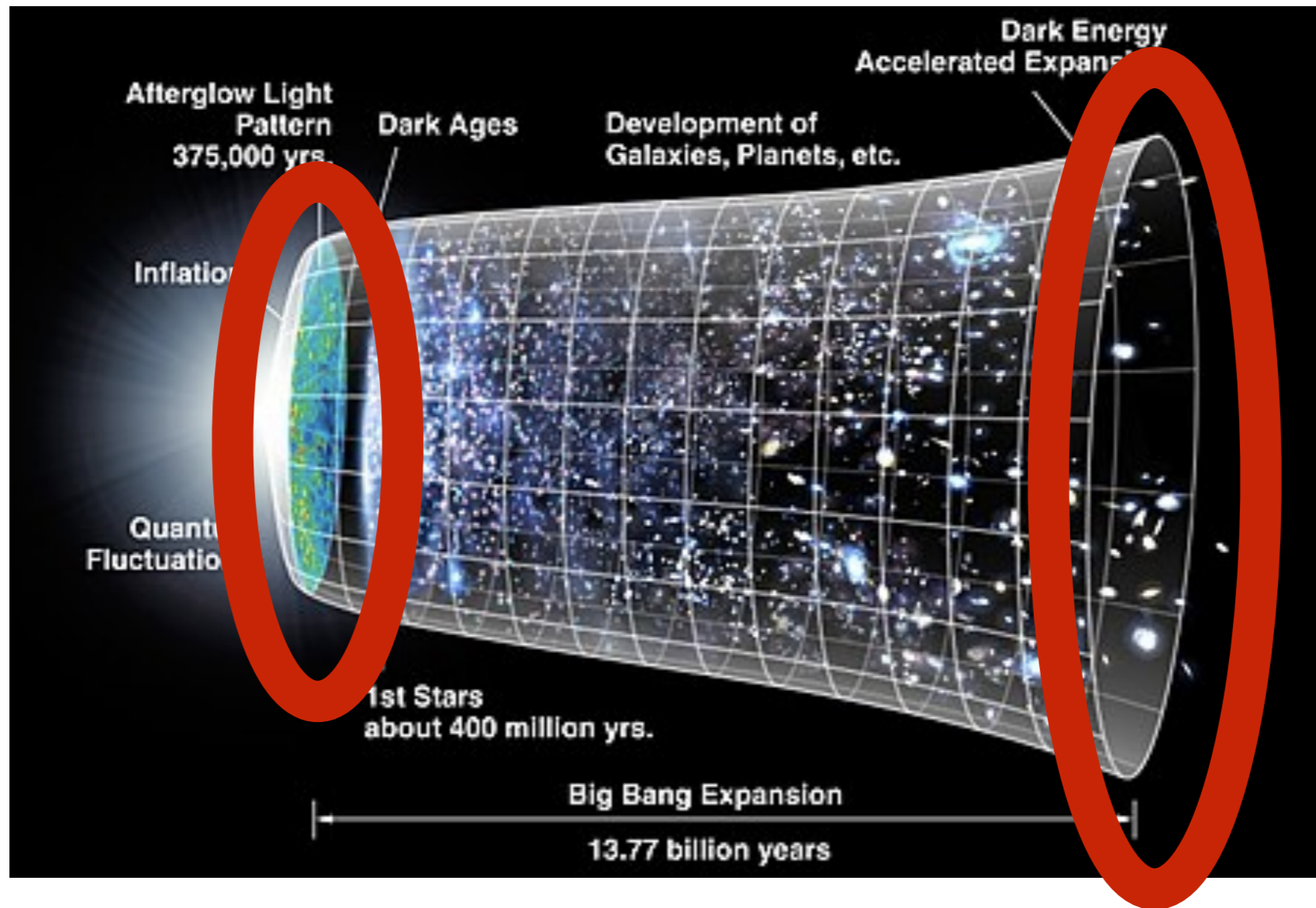


Cosmology with LISA

Chiara Caprini
CNRS (APC Paris)



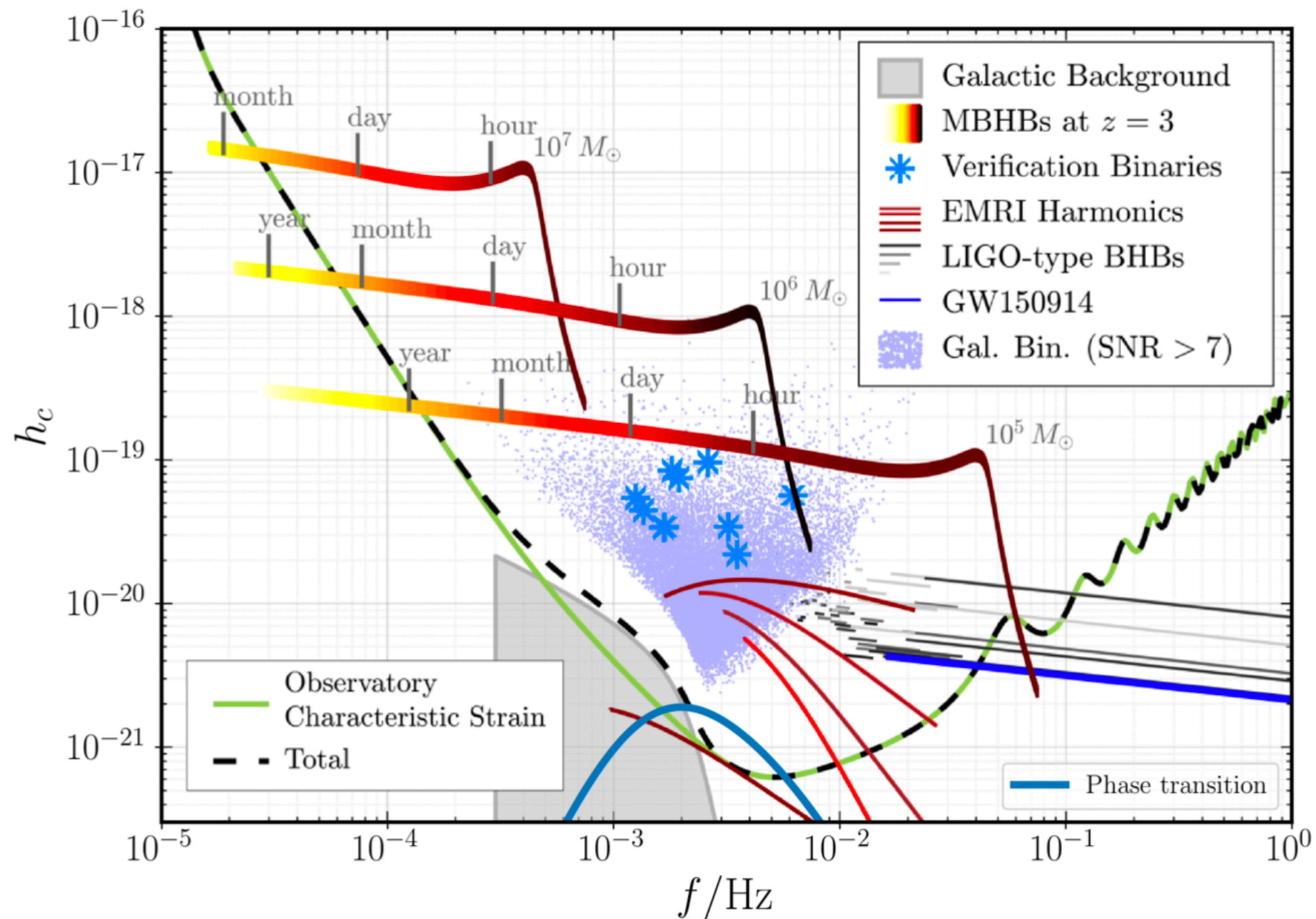
How can GW help to probe cosmology?



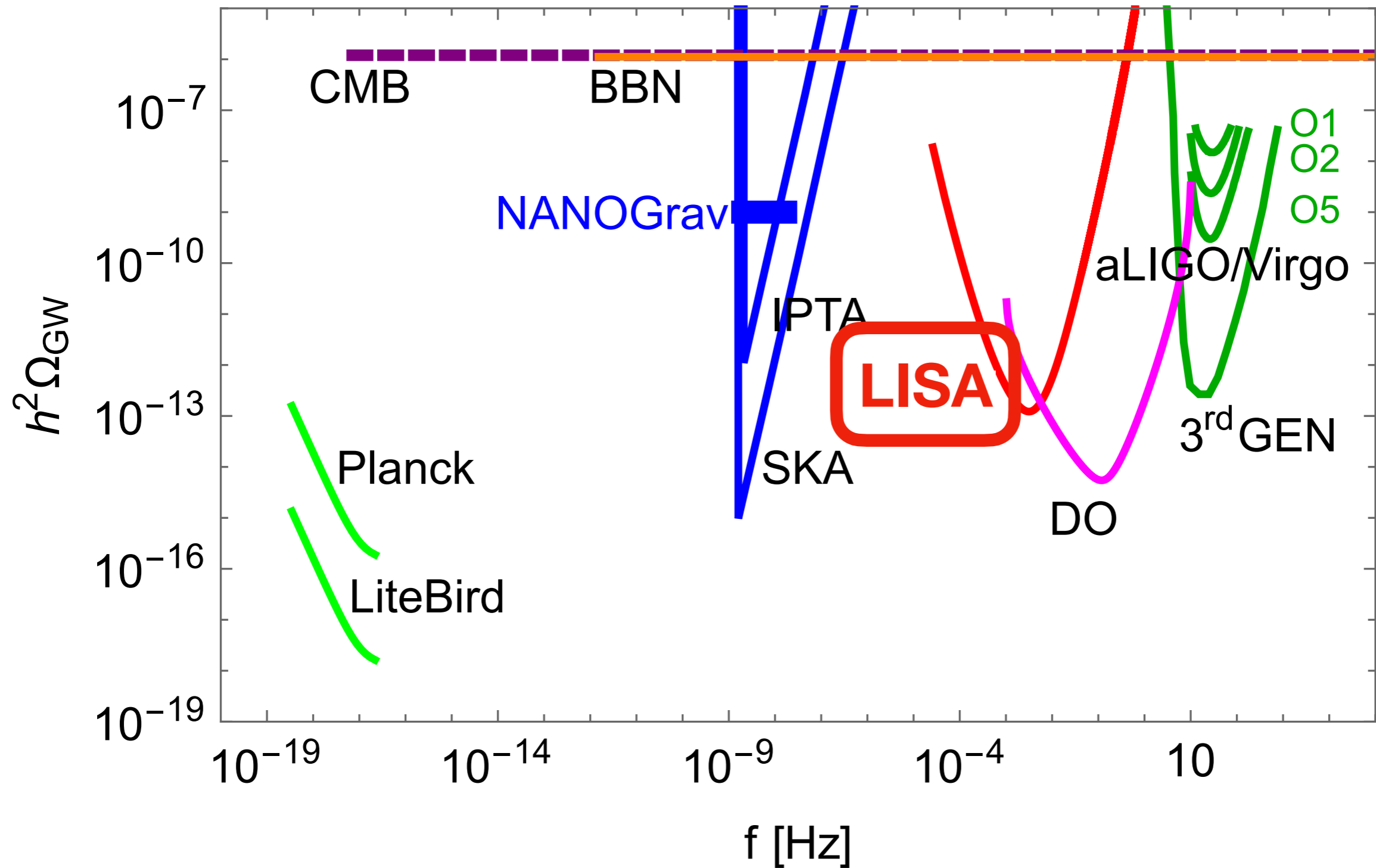
the stochastic GW background from primordial sources: test of early universe and high energy phenomena

use of GW emission from binaries to probe late-time dynamics and content of the universe

This talk: *overview* of LISA potential in probing both aspects,
SGWB and standard sirens



What is/will be known about the SGWB



GWs in the primordial universe

GWs are *tensor perturbations* of the FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$|h_{ij}| \ll 1$$

$$h^i_i = \partial_j h^j_i = 0$$

superimposed on the homogeneous and isotropic background

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G (\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$\ddot{h}_{ij} + \boxed{3H \dot{h}_{ij}} + k^2 h_{ij} = \boxed{16\pi G \Pi_{ij}^{TT}}$$

Hubble expansion

GW source:
tensor anisotropic stress

GWs in the primordial universe

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STANDARD INFLATION:

amplification of tensor metric vacuum fluctuations by the exponential expansion

GWs in the primordial universe

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$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

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superimposed on the homogeneous and isotropic background

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$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

ACTIVE GW SOURCE:
tensor anisotropic stress
can act an any time in the universe

SGWB from a generic stochastic source in the radiation era

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

Possible sources of tensor anisotropic stress in the early universe:

- Scalar field gradients $\Pi_{ij} \sim [\partial_i \phi \partial_j \phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2 (\rho + p) v_i v_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i \Psi, \partial_i \Phi$
- ...

Why sources in the early universe produce SGWBs?

A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\ell_* \leq H_*^{-1}$$

ℓ_* characteristic length-scale of the source
typical size of the tensor anisotropic stresses

Why sources in the early universe produce SGWBs?

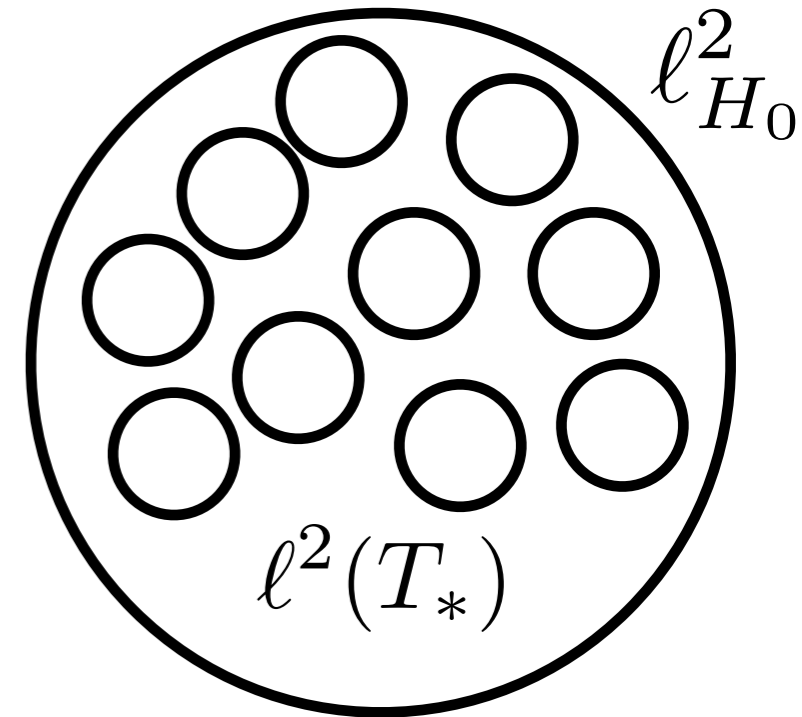
A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\ell_* \leq H_*^{-1}$$

Angular size on the sky today of a region in which the SGWB signal is correlated

$$\Theta_* = \frac{\ell_*}{d_A(z_*)}$$

Angular diameter distance



$$\Theta(z_* = 1090) \simeq 0.9 \text{ deg}$$

$$\Theta(T_* = 100 \text{ GeV}) \simeq 10^{-12} \text{ deg}$$

Suppose a GW detector angular resolution of 10 deg $\longrightarrow z_* \lesssim 17$

Only the statistical properties of the signal can be accessed

$h_{ij}(\mathbf{x}, t)$ must be treated as a random variable

Why sources in the early universe produce SGWBs?

A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\ell_* \leq H_*^{-1}$$

characteristic frequency of the GW signal

$$f_* = \frac{1}{\ell_*} \geq H_*$$

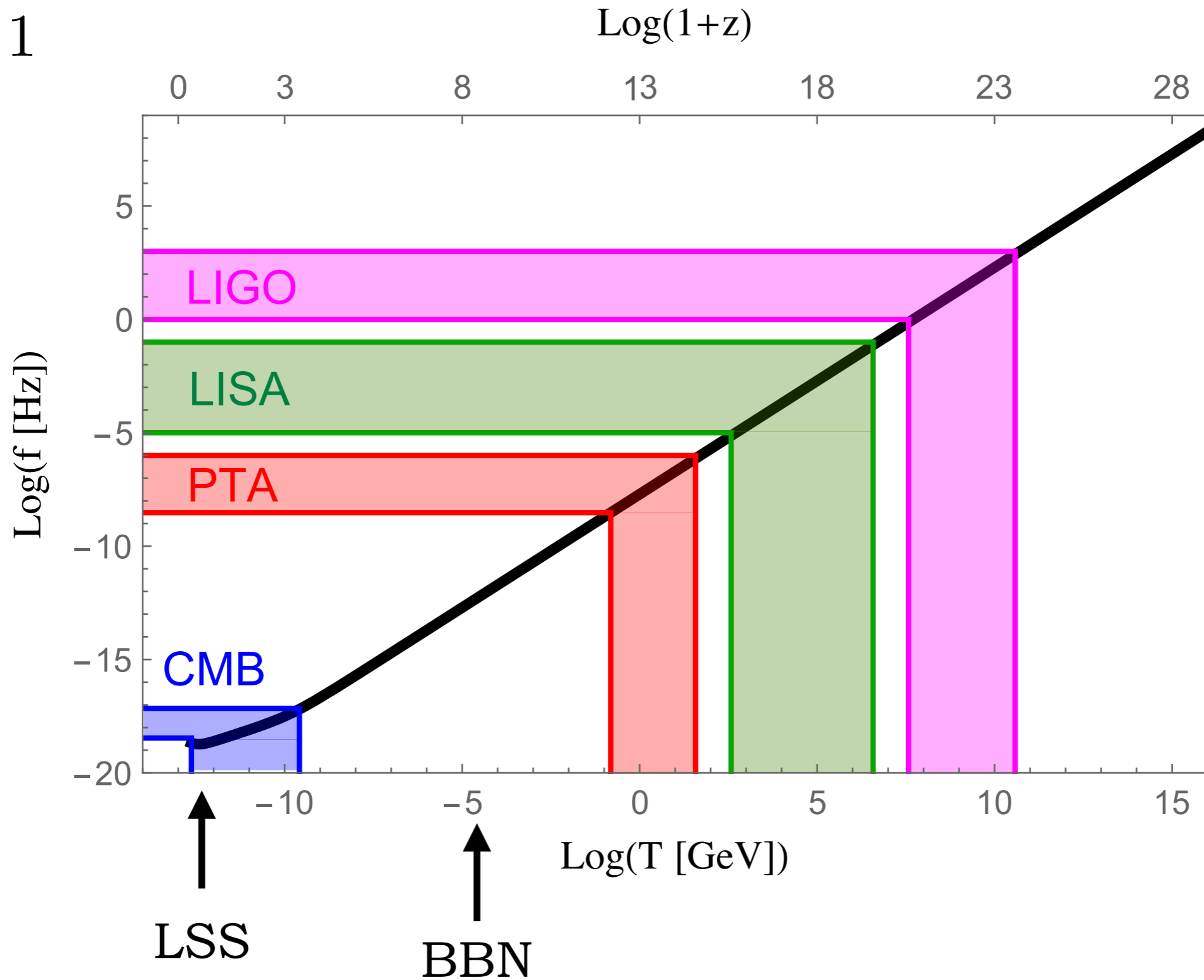
$$\epsilon_* = \ell_* H_*$$

Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

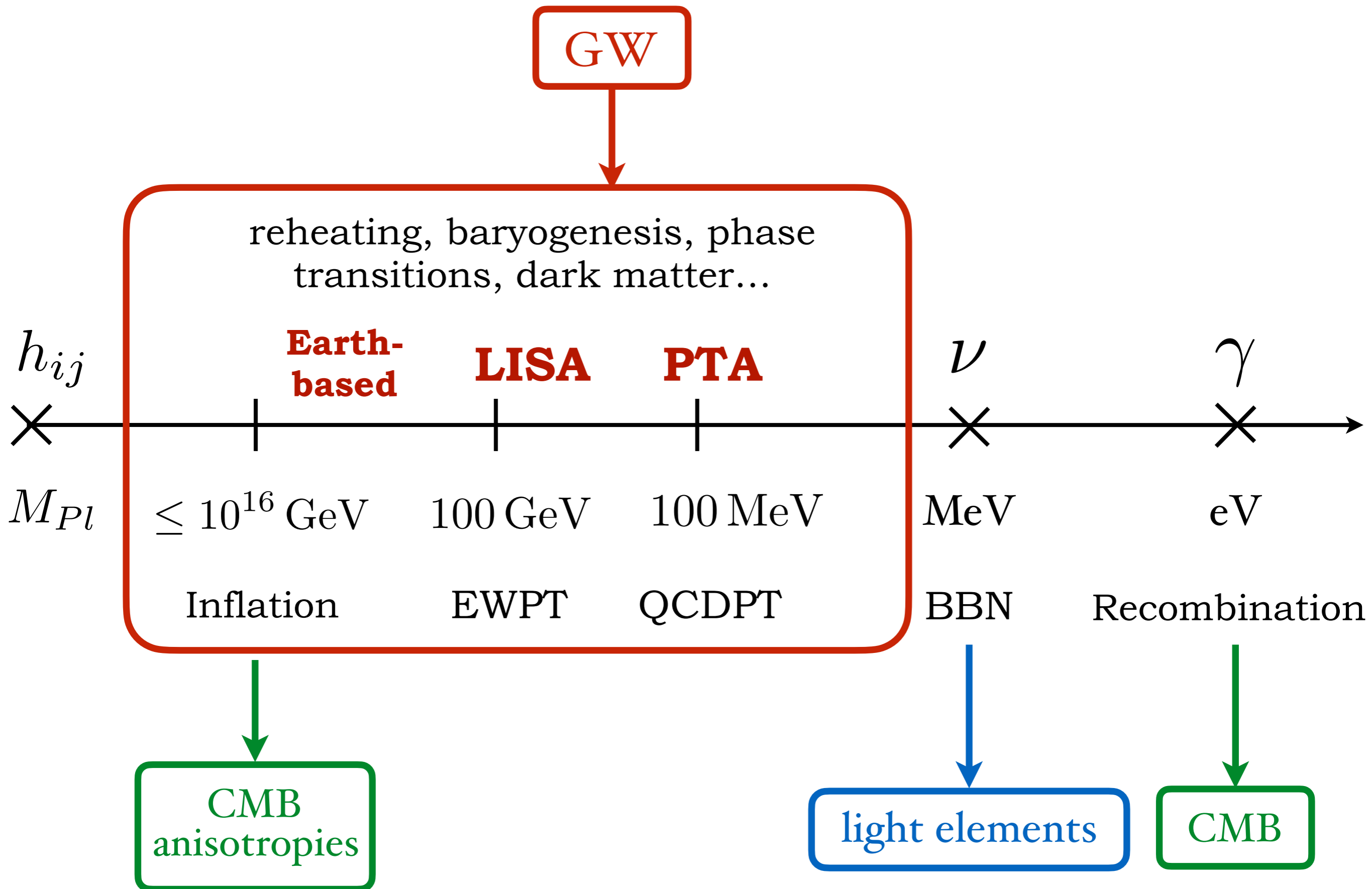
$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\text{GeV}} \text{ Hz}$$

Characteristic frequency of the GW signal

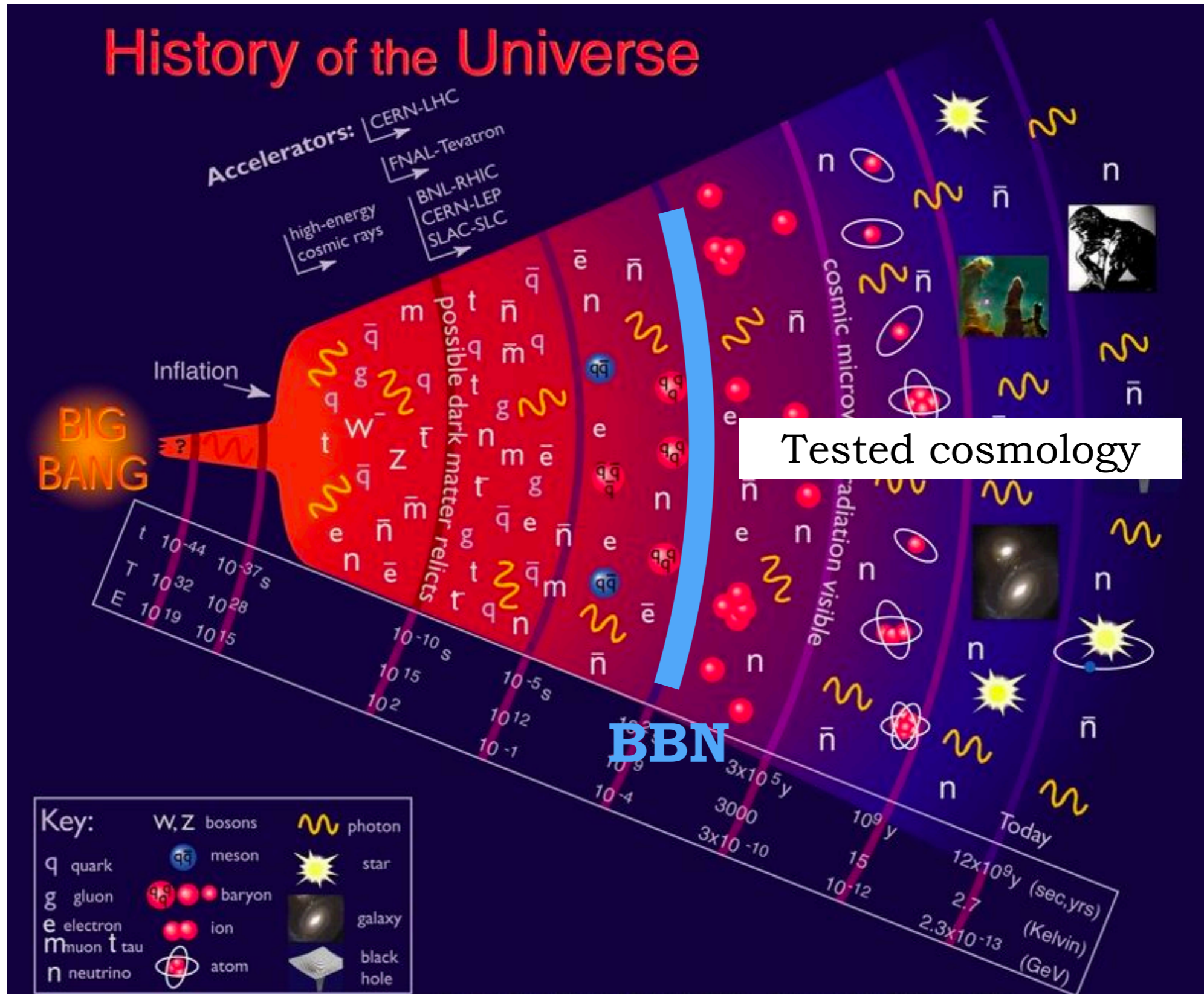
$$\epsilon_* = 1$$



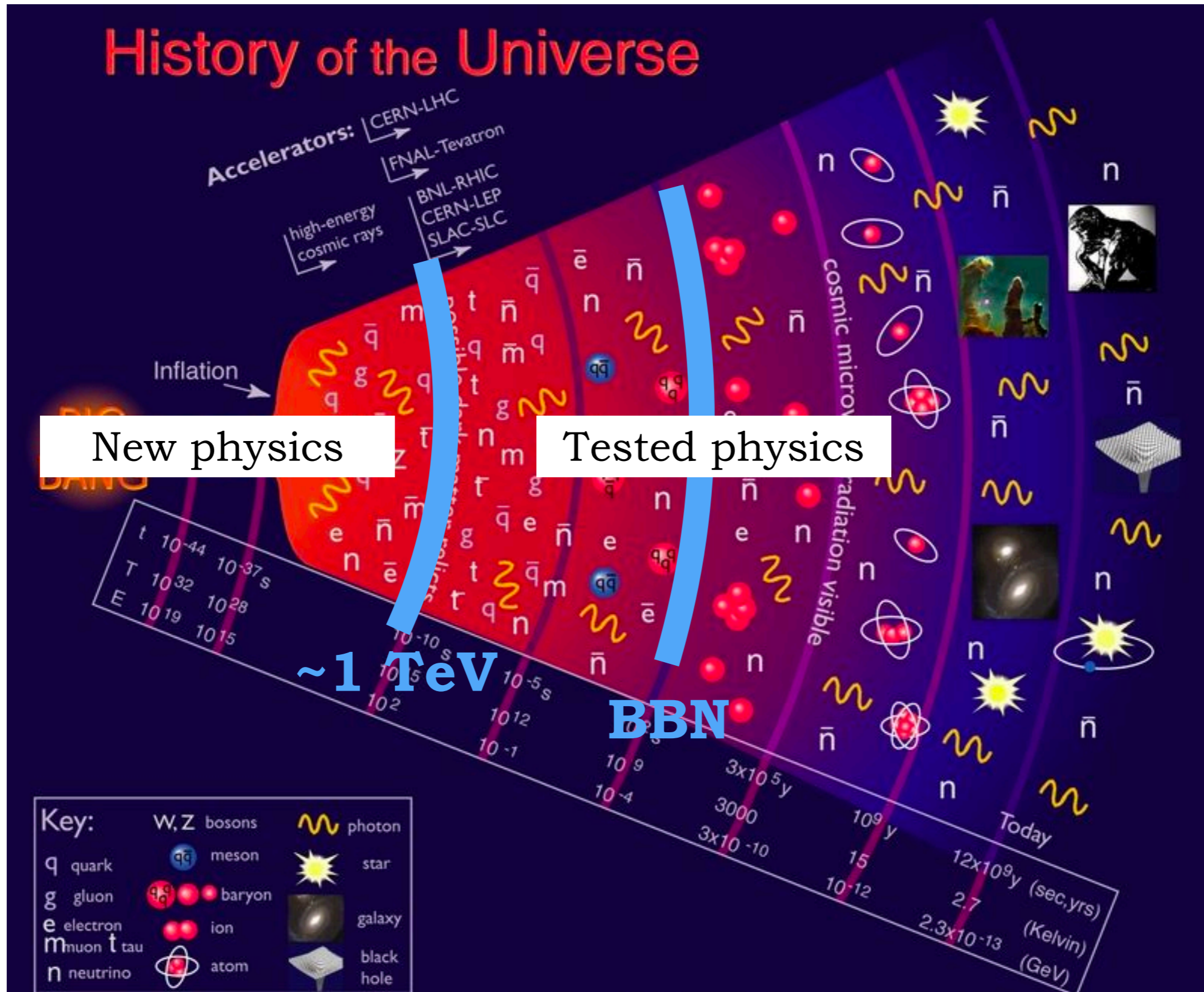
Discovery potential of primordial SGWB detection



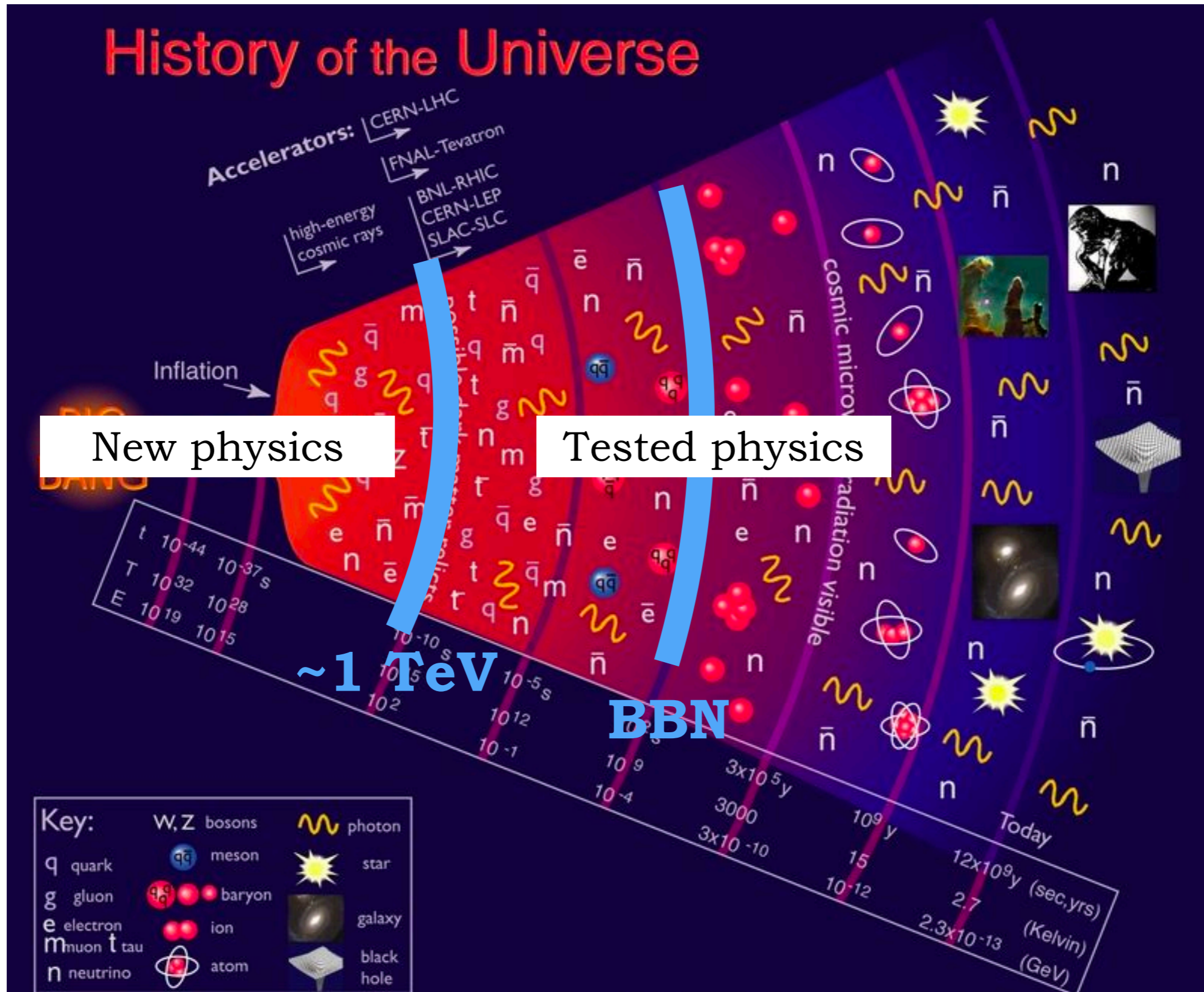
GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation —> **amazing discovery potential**



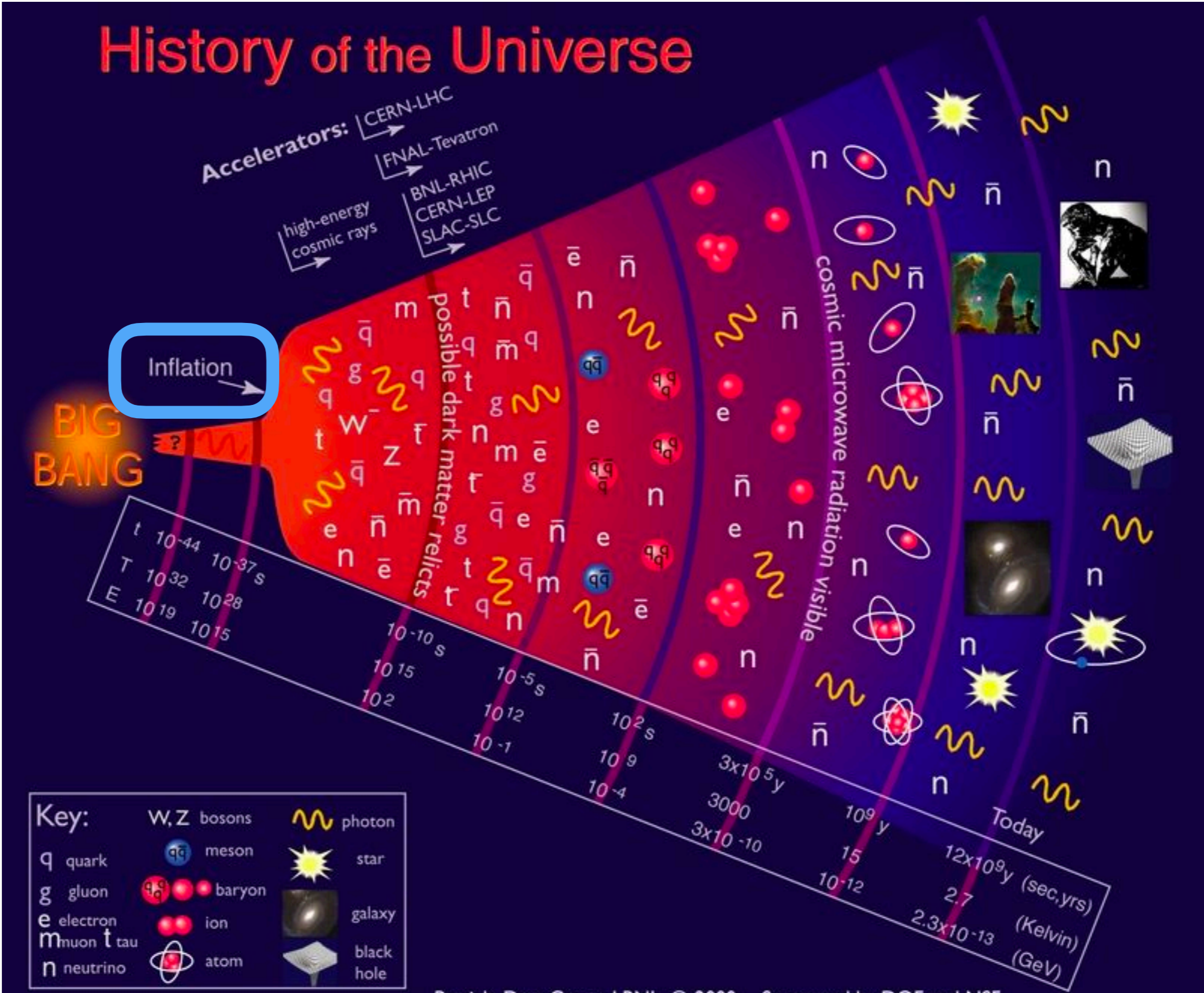
No guaranteed GW signal: predictions rely on untested phenomena, and are often difficult to estimate (non-linear dynamics, strongly coupled theories...)



Many GW generation processes are related to **PHASE TRANSITIONS**



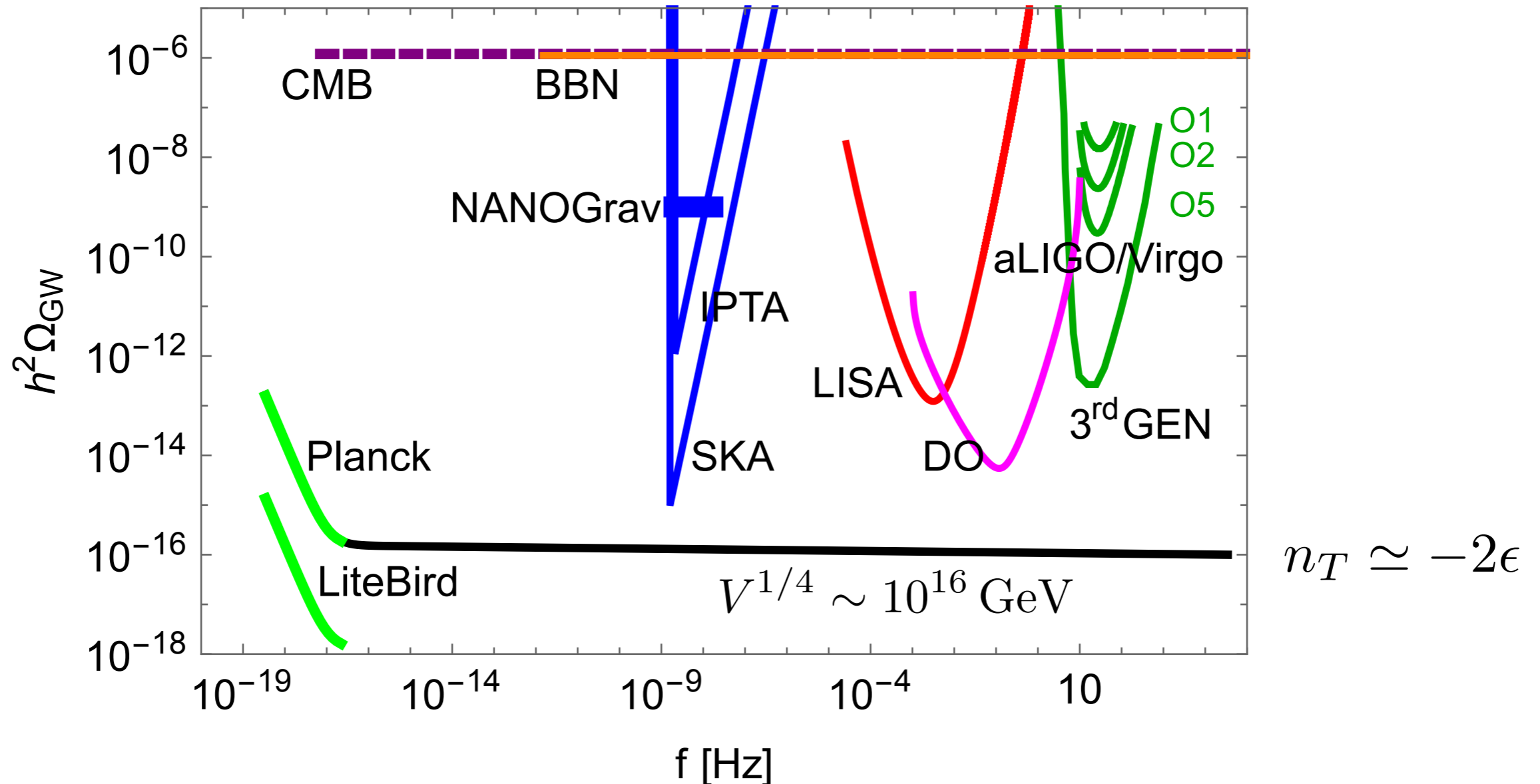
Inflation: phase transition of the Inflaton field



GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

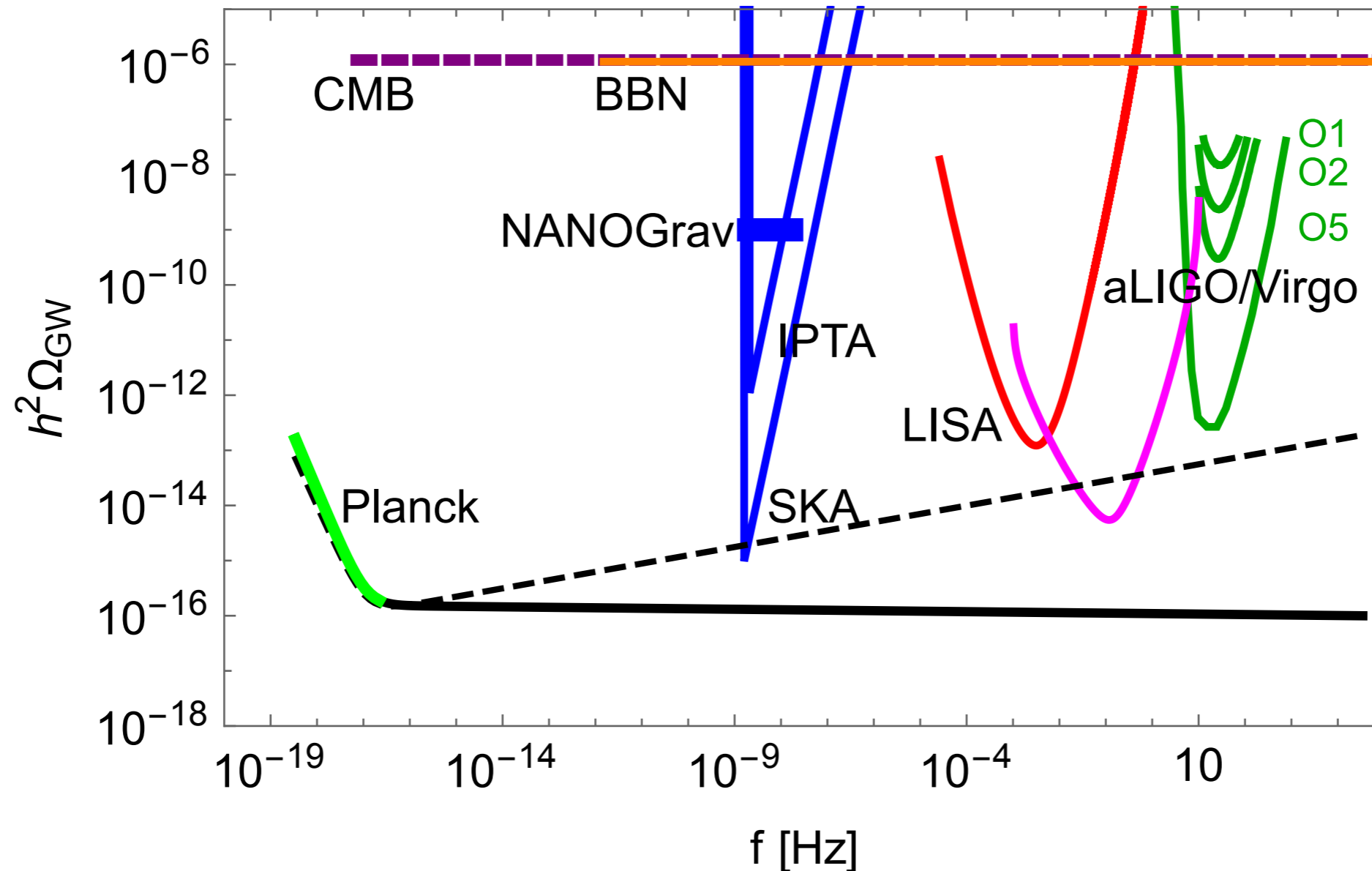
BUT! The signal in the standard slow roll scenario is too low and **(P)reheating** generates a signal with high amplitude, but at high frequency



GW signal from (non-standard) inflation

There is the possibility to enhance the signal going beyond the standard inflationary scenario: **adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state...**

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$



GW signal from (slow roll) inflation

- tensor spectrum $\mathcal{P}_h = \frac{2}{\pi} \frac{H^2}{m_{Pl}^2} \left(\frac{k}{aH} \right)^{-2\epsilon}$ $\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1$

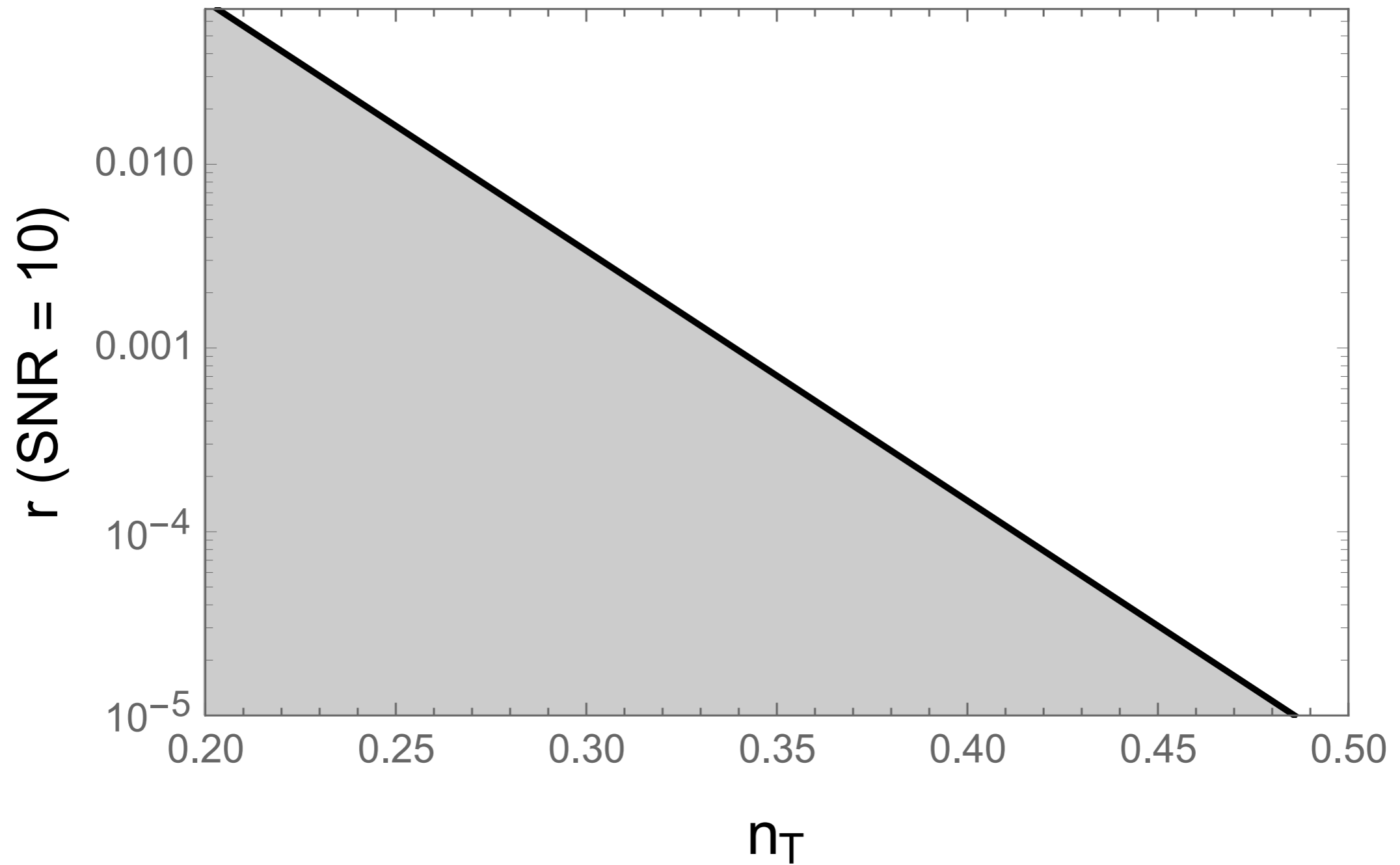
$$\Omega_{\text{GW}}(f) = \frac{3}{128} \Omega_{\text{rad}} r \mathcal{P}_{\mathcal{R}}^* \left(\frac{f}{f_*} \right)^{n_T} \left[\frac{1}{2} \left(\frac{f_{\text{eq}}}{f} \right)^2 + \frac{16}{9} \right]$$

- tensor to scalar ratio $r = \mathcal{P}_h / \mathcal{P}_{\mathcal{R}}$ $r_* \leq 0.07$ Planck+BICEP limit
- scalar amplitude at CMB pivot scale $\mathcal{P}_{\mathcal{R}}^* \simeq 2 \cdot 10^{-9}$ $k_* = \frac{0.05}{\text{Mpc}}$
- GW signal extended in frequency: $H_0 \leq f \leq H_{\text{inf}}$

continuous sourcing of GW as modes re-enter the Hubble horizon

general constraints on (r, n_T) from LISA

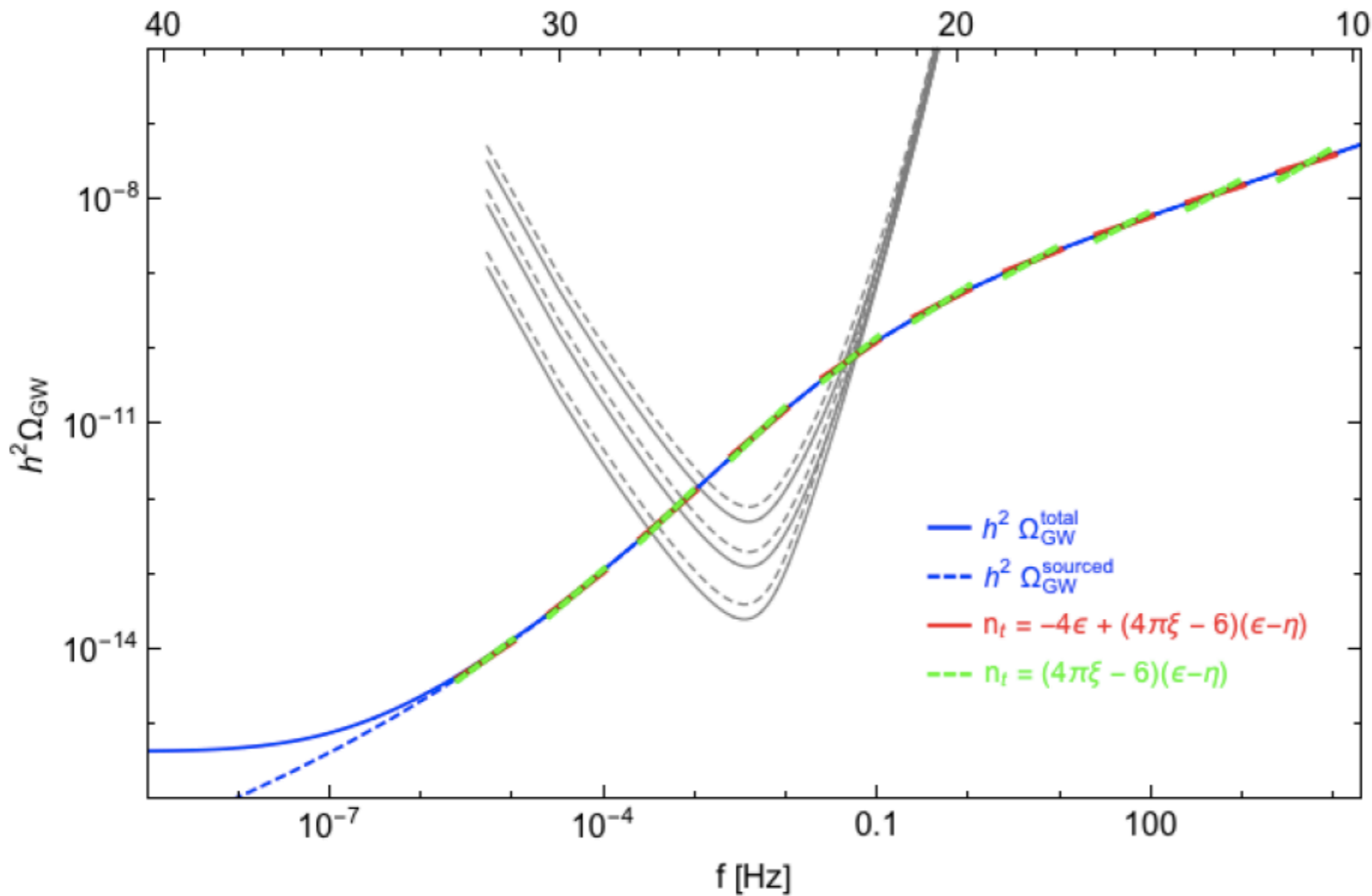
$k_* = 0.05/\text{Mpc}$



just one example, quite constrained by now:
inflaton-gauge field coupling

$$\Delta\mathcal{L} = -\frac{1}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$

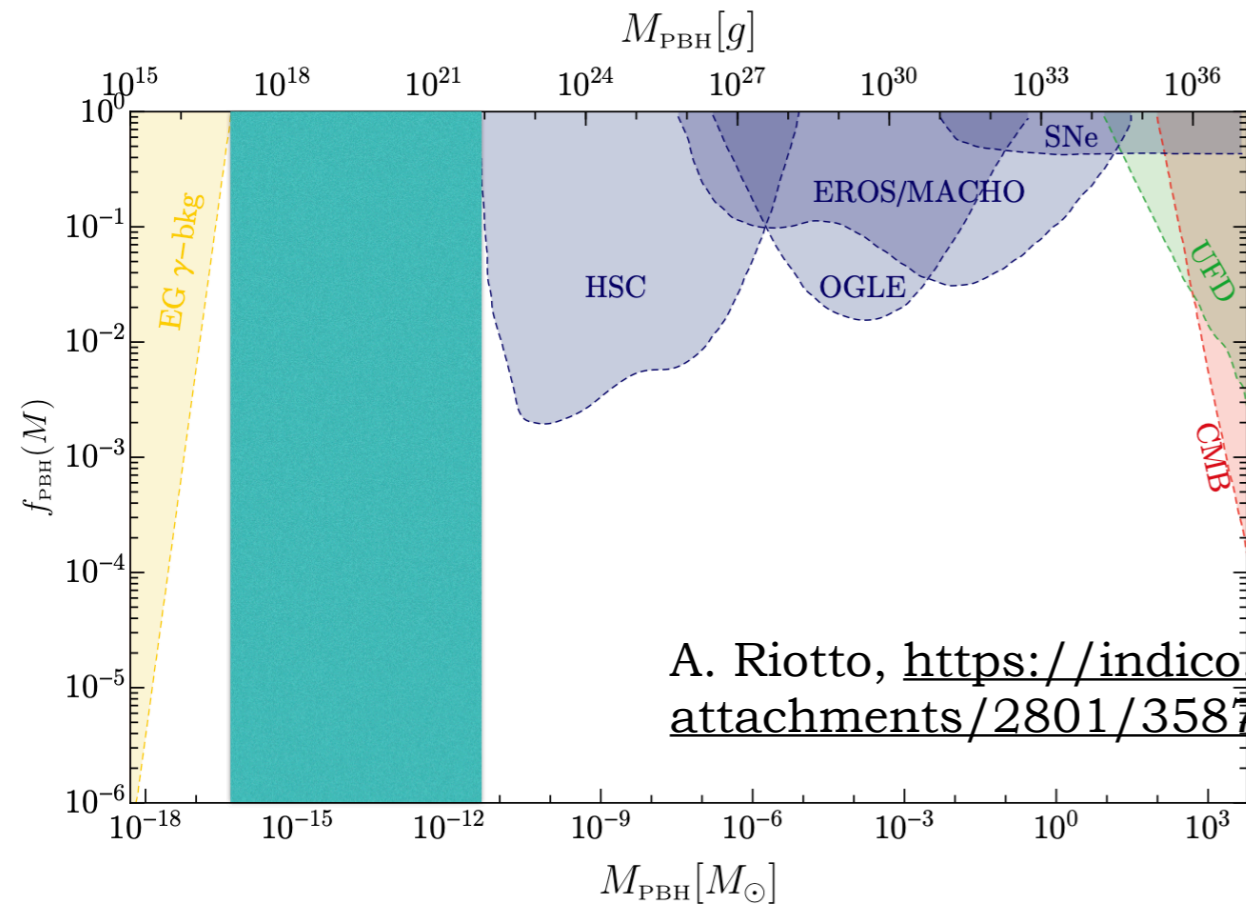


$$\Lambda = \frac{M_{Pl}}{35}$$

quadratic
inflaton
potential

OTHER SIGNATURES:
non-gaussianity, chirality

GW signal from second order scalar perturbations: PBH and LISA

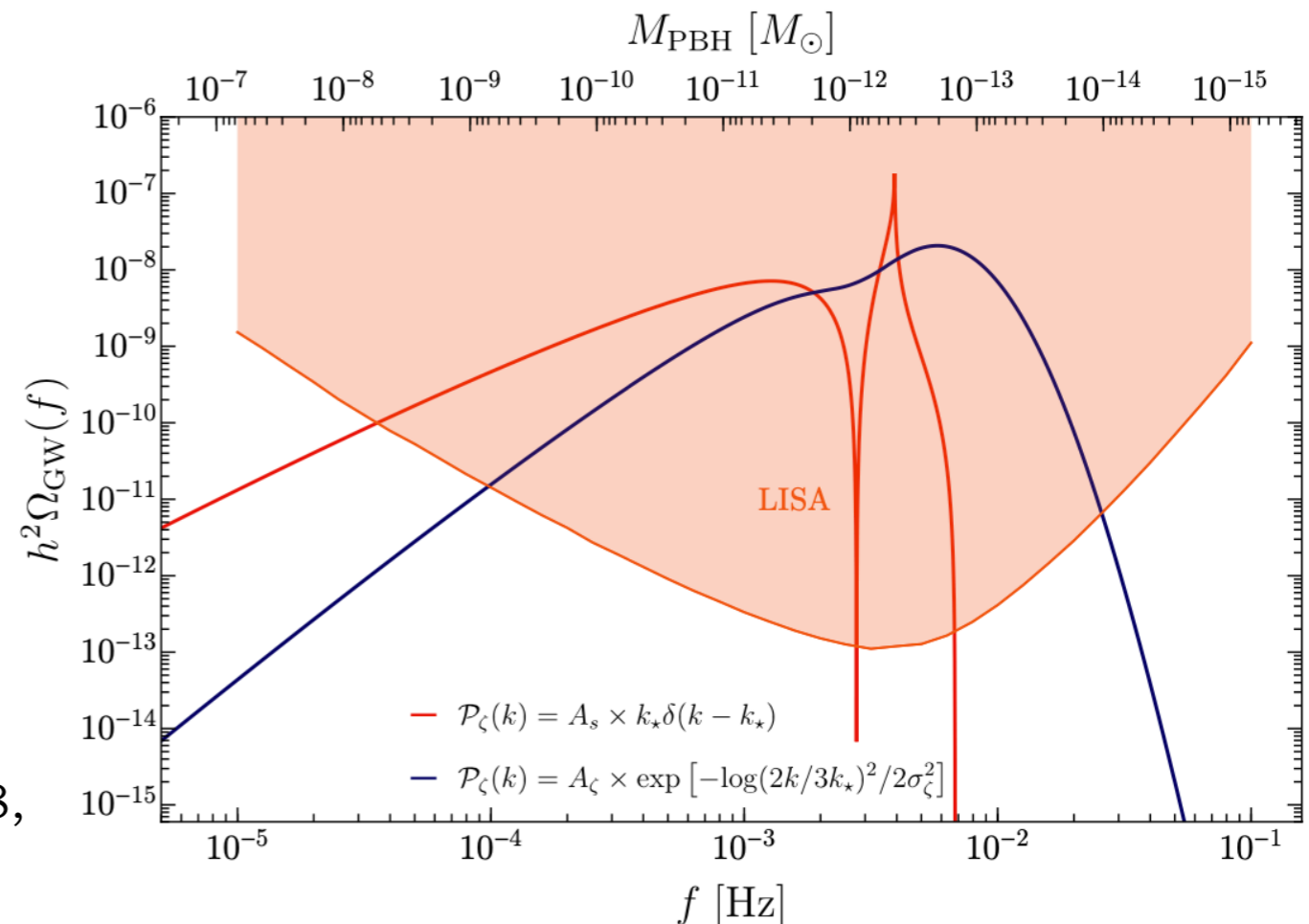


A. Riotto, <https://indico.math.cnrs.fr/event/5766/contributions/5153/attachments/2801/3587/Paris2021.pdf>

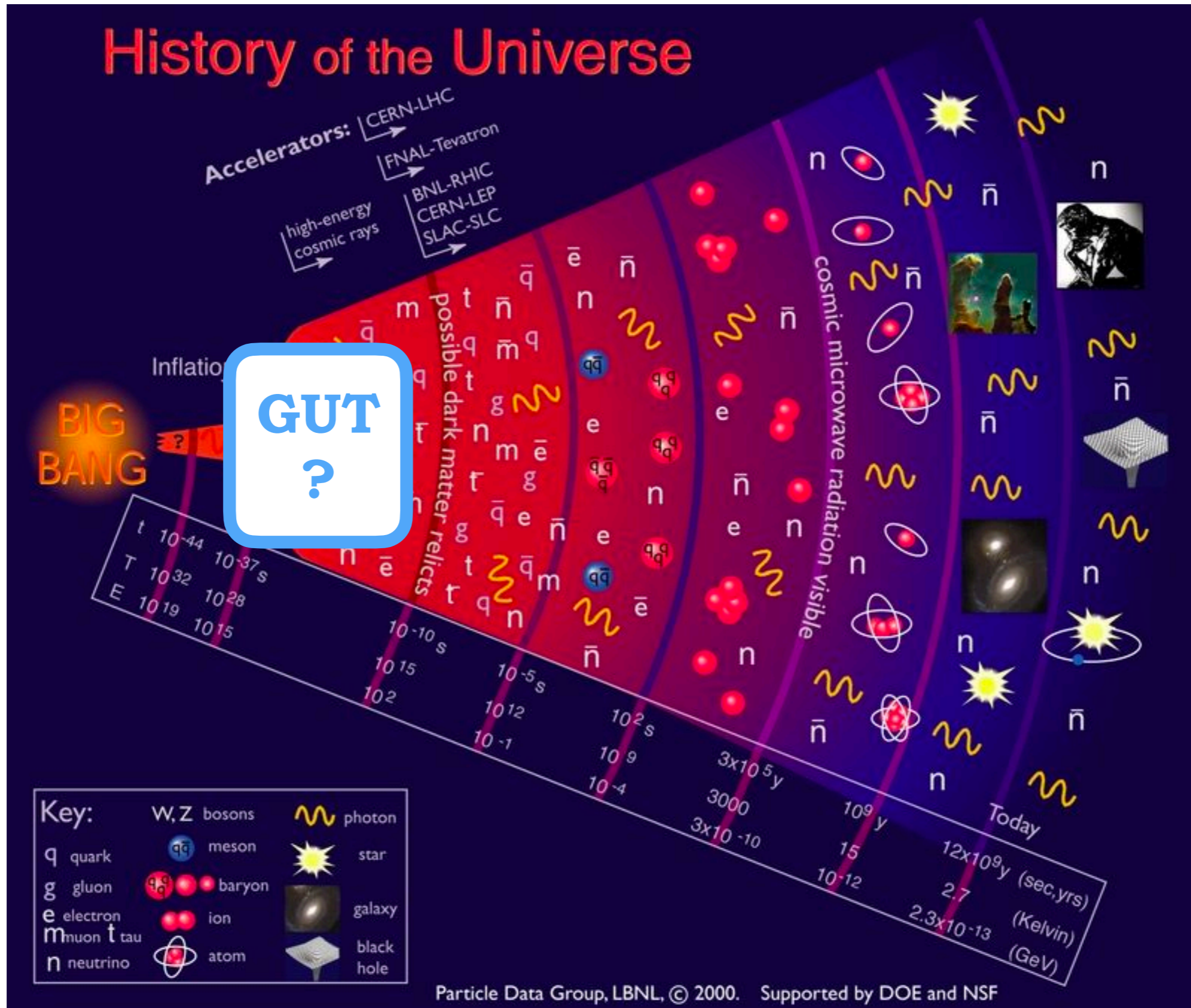
There is a mass window for which PBH can still constitute the whole of the dark matter

If one wants to produce PBH in this mass range, one also has an observable SGWB in LISA by second order scalar perturbations

N. Bartolo et al, arXiv:1810.12218, arXiv:1810.12224



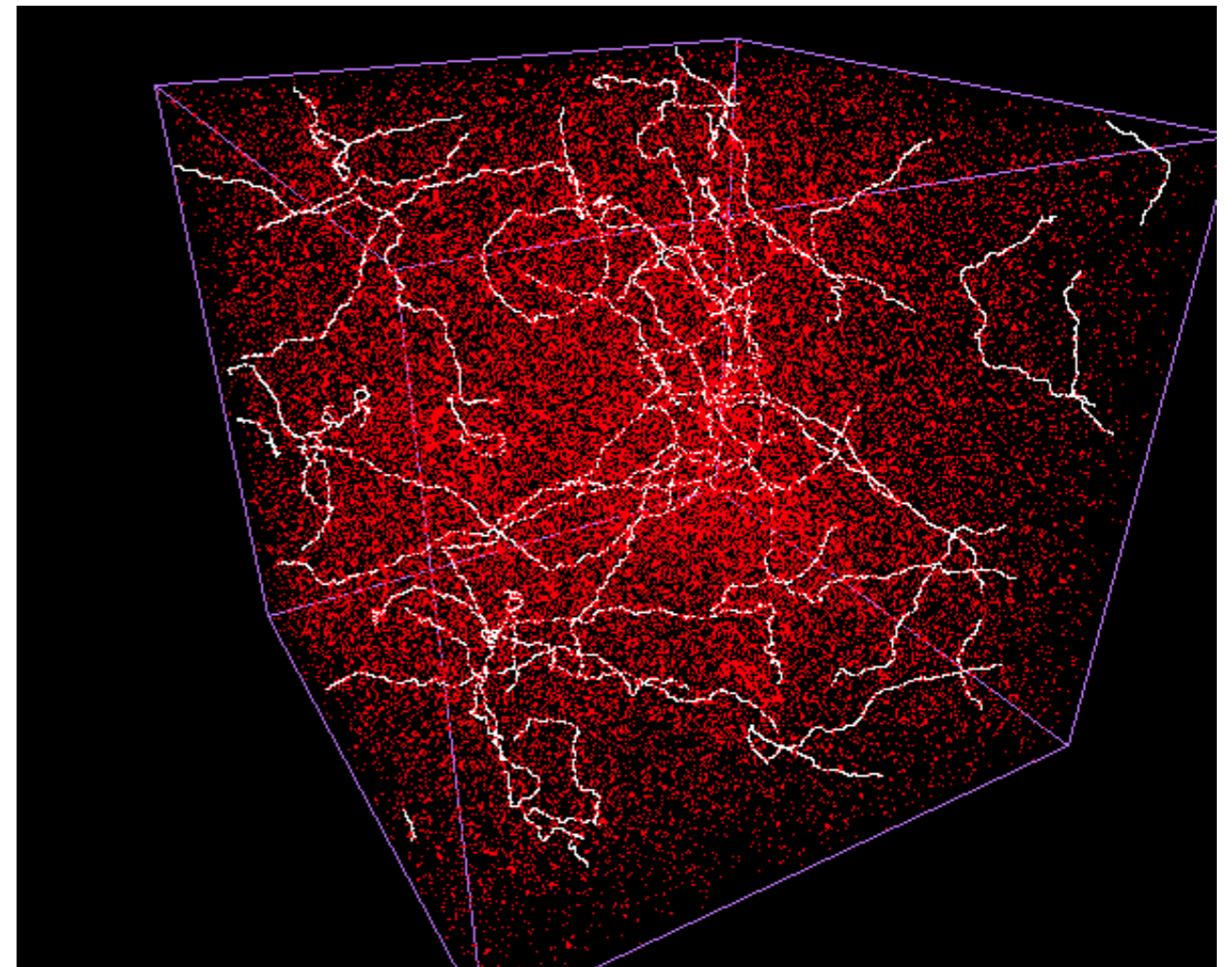
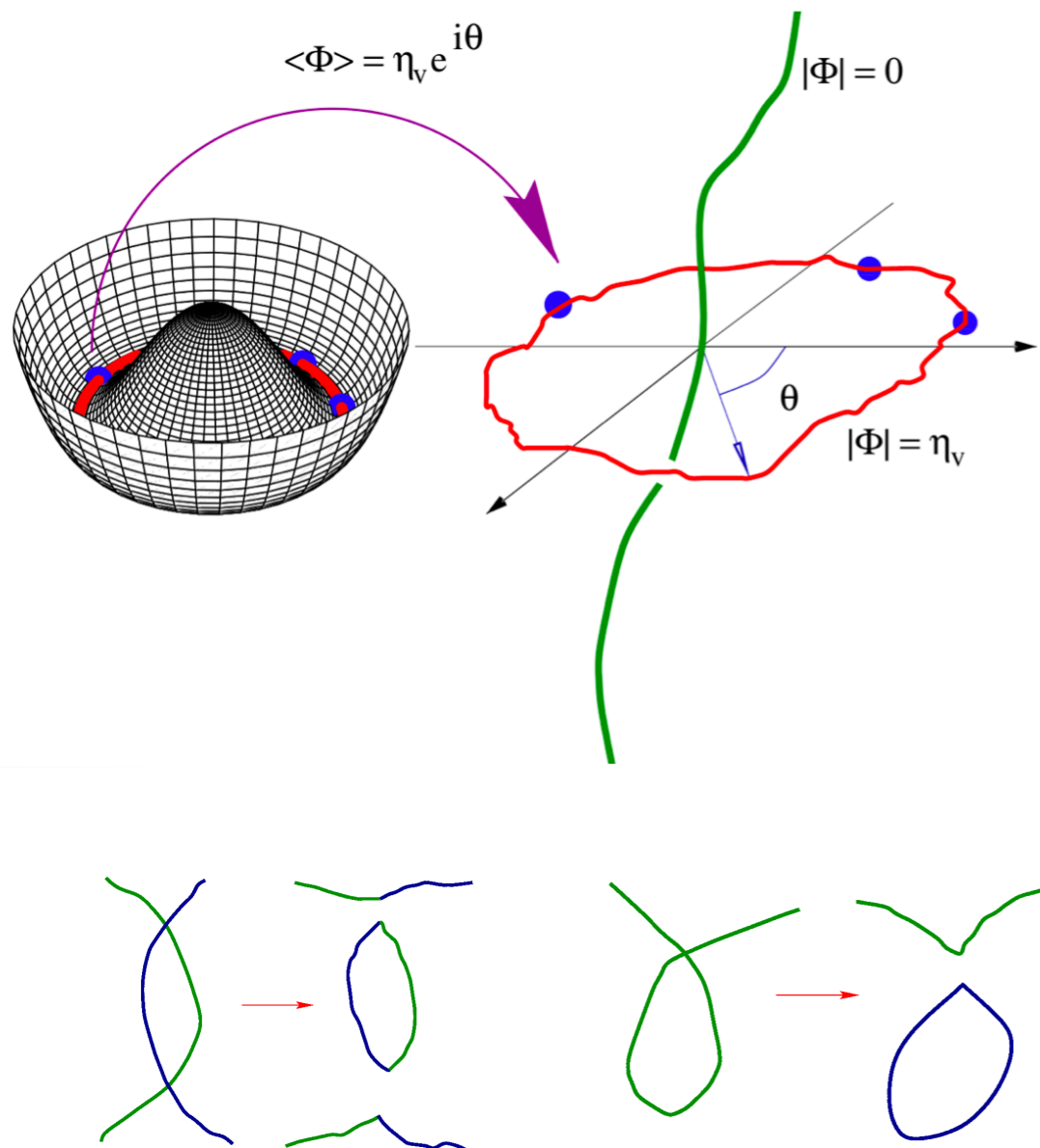
GUT phase transition or similar: related to the breaking of the symmetries of the high-energy theory describing the universe



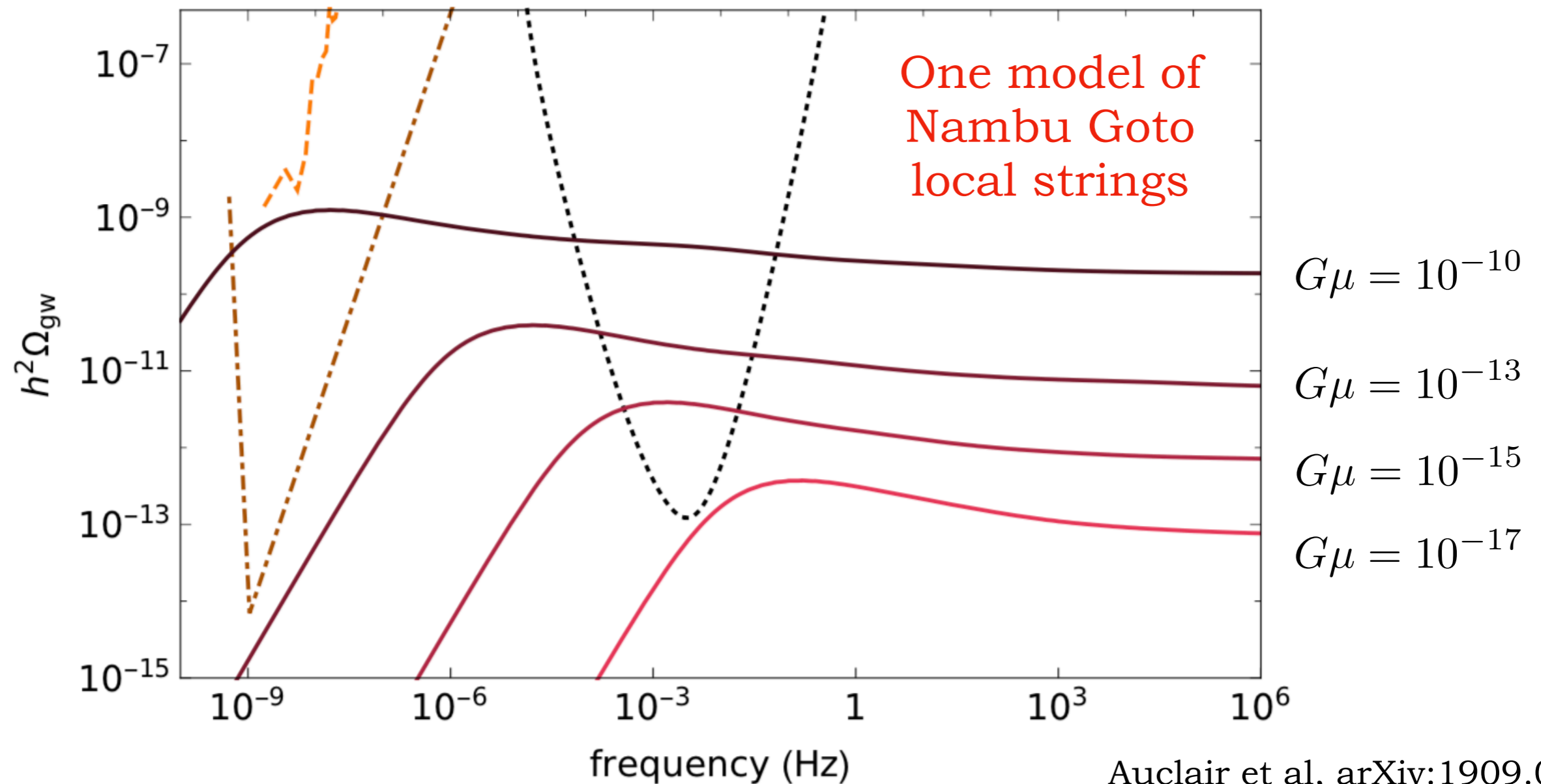
GW signal from cosmic strings

Cosmic strings (or other kind of topological defects) are non-trivial field configurations left-over after the phase transition has completed

A network of cosmic strings emits GWs
(though the results are very model dependent)

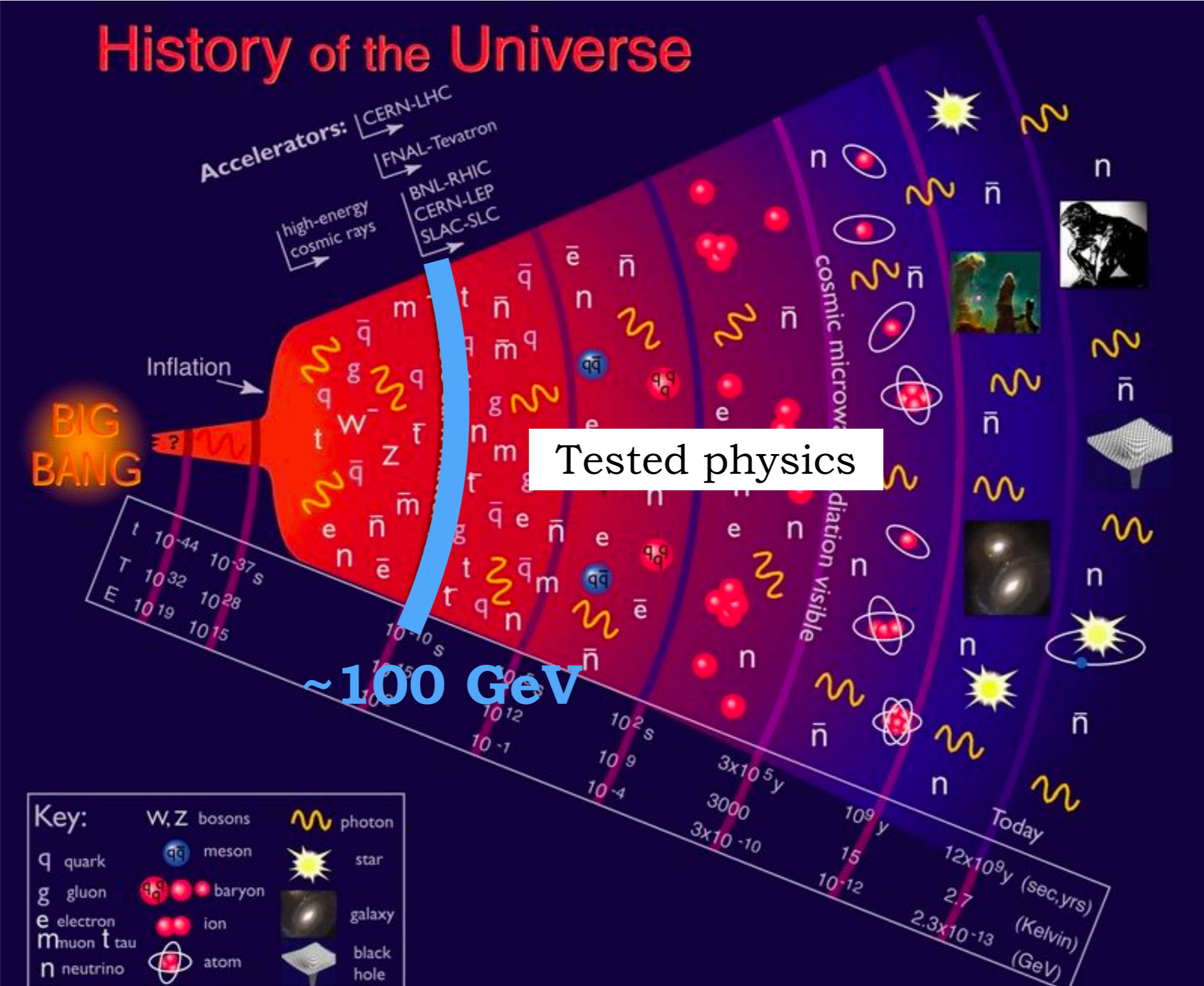


GW signal from cosmic strings



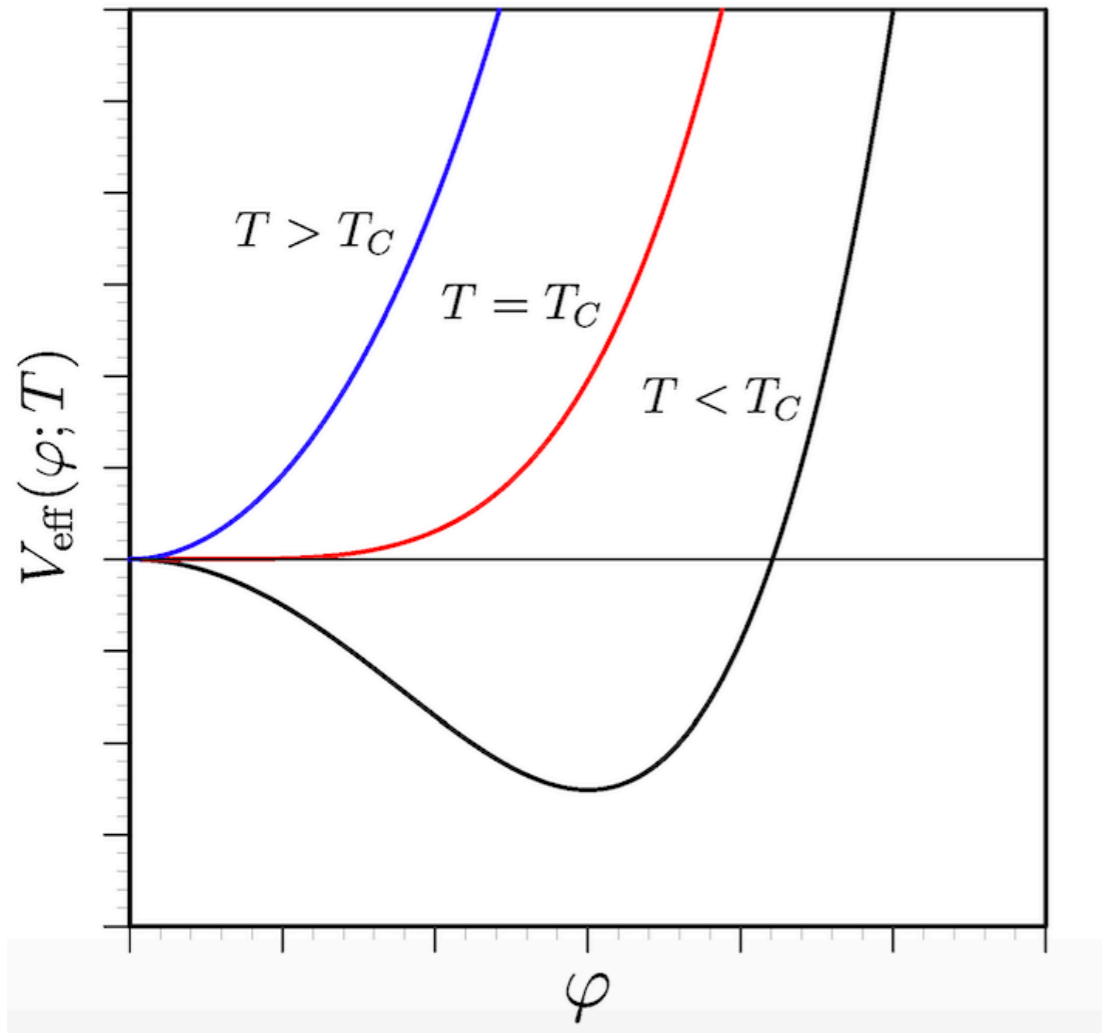
- The signal extends over many frequencies since the GW production is continuous throughout the universe evolution
- The energy density of the cosmic string network is a constant fraction of the universe's one

Electroweak phase transition

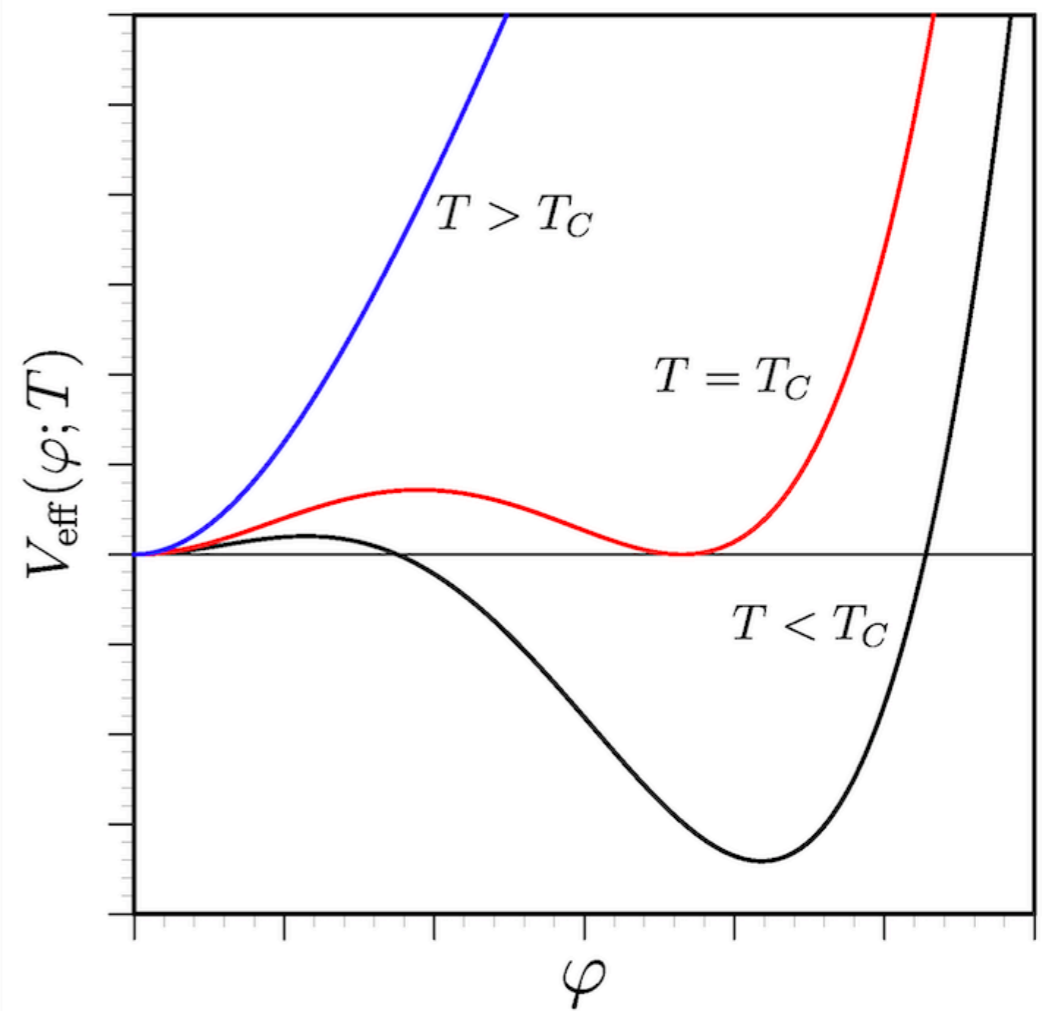


Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Second order phase transition

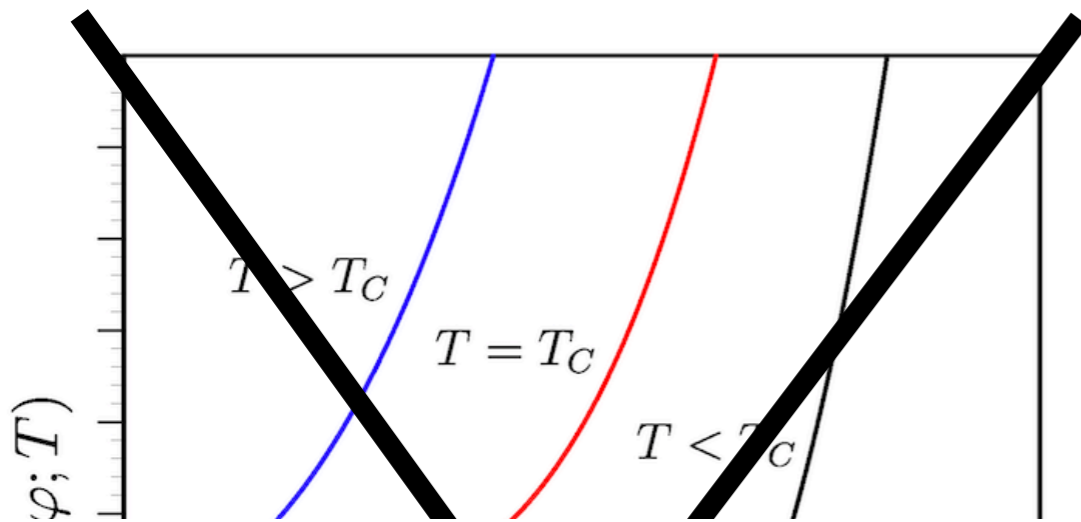


First order phase transition



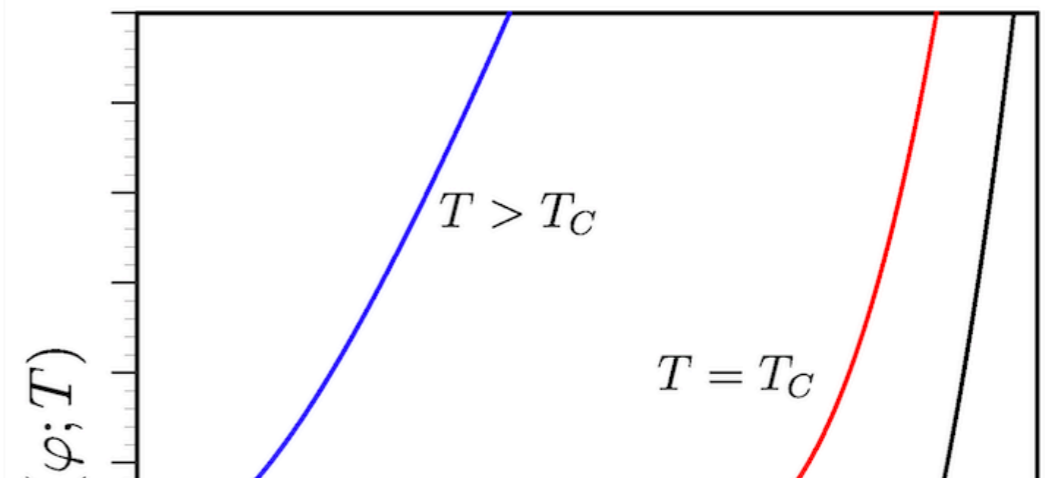
Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Second order phase transition

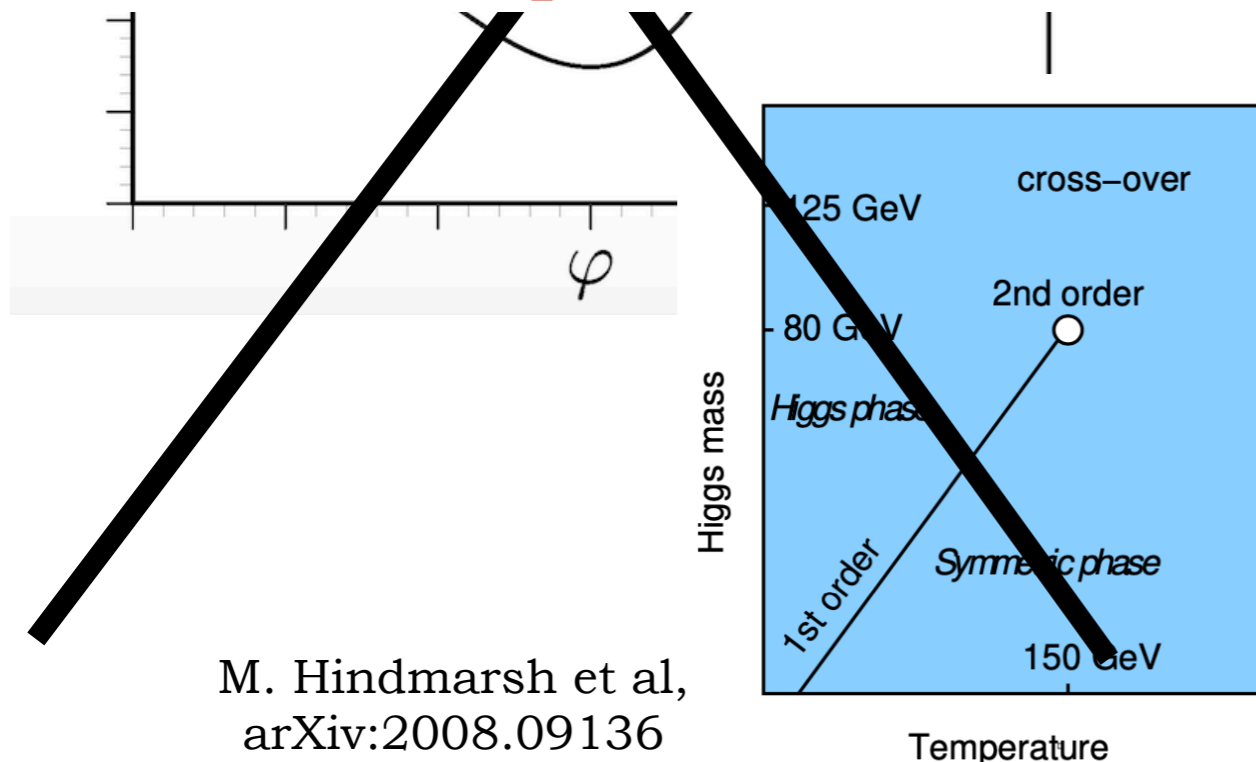


**Standard Model of particle physics:
no GW production**

First order phase transition



**Beyond Standard Model of particle physics:
GW production possible**

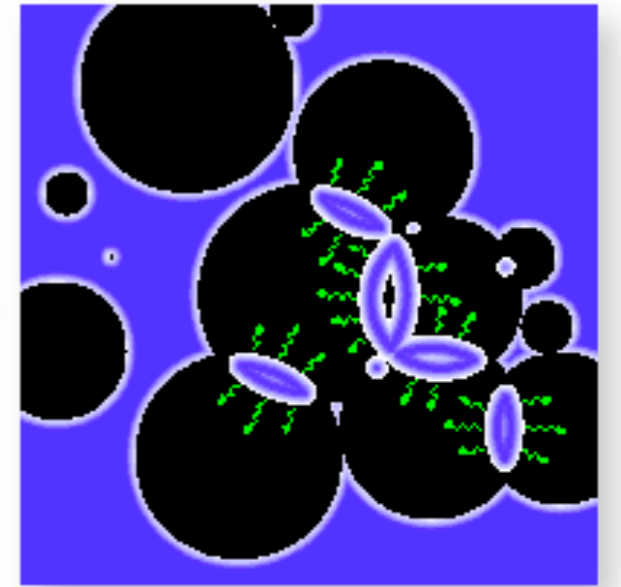
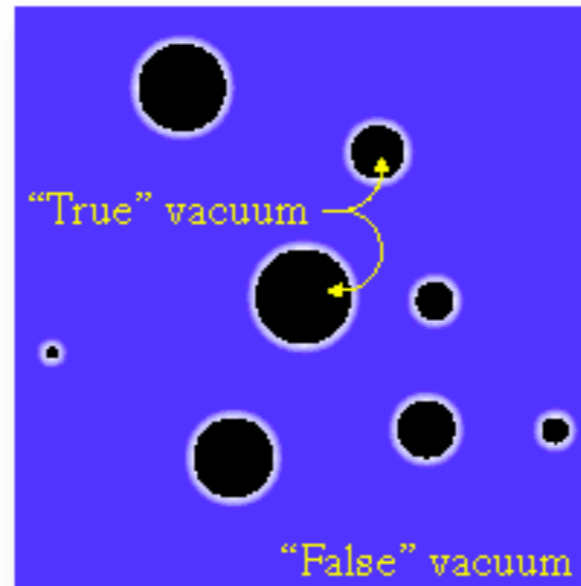
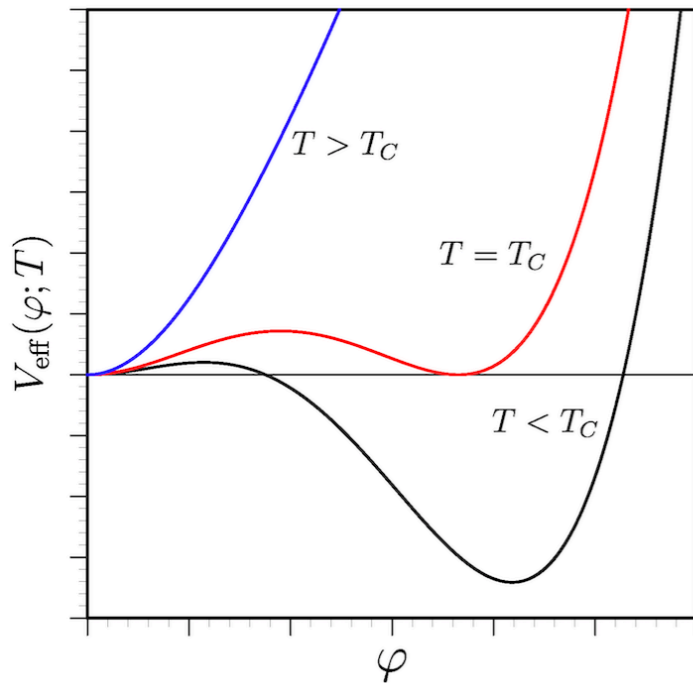


M. Hindmarsh et al,
arXiv:2008.09136

Very strong case for LISA

- singlet/multiplet extensions of MSSM (SUSY motivated or not)
- SM plus dimension six operator (EFT approach)
- Dark Matter sector uncoupled to the SM
- Warped extra dimensions

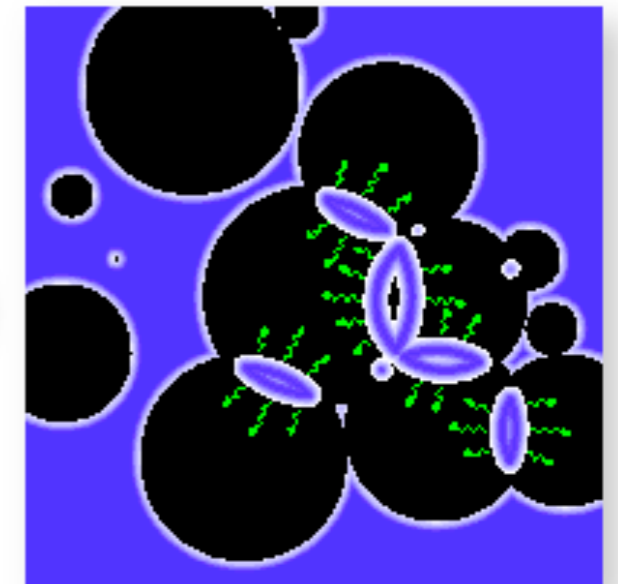
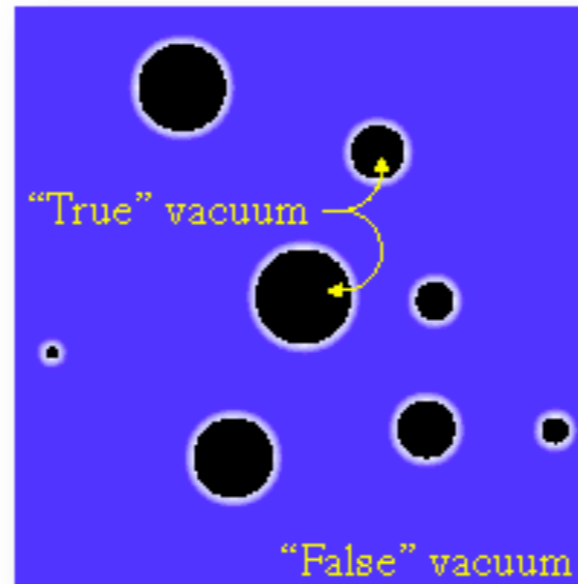
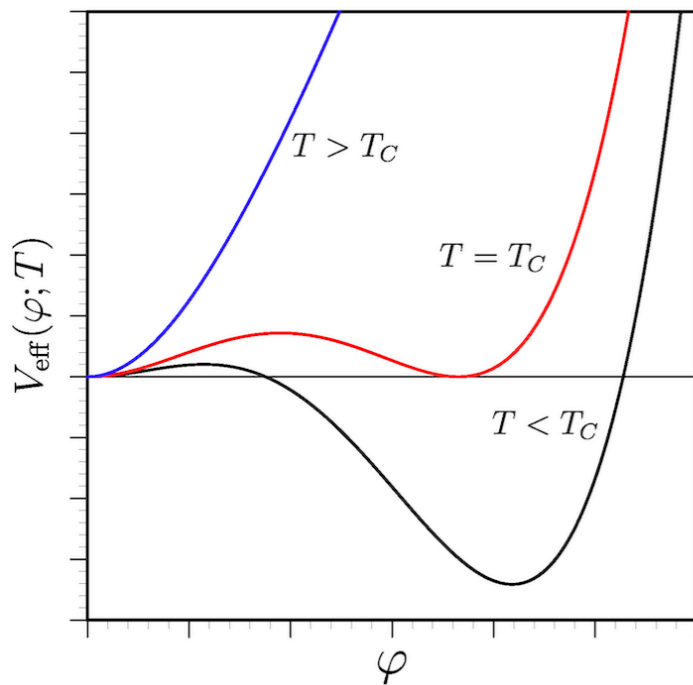
GW signal from the EW phase transition



$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

- collisions of bubble walls $\Pi_{ij} \sim [\partial\phi_i \partial\phi_j]^{TT}$
- sound waves and turbulence in the fluid $\Pi_{ij} \sim [\gamma^2 (\rho + p) v_i v_j]^{TT}$
- primordial magnetic fields (MHD turbulence) $\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$

GW signal from the EW phase transition



The characteristic scale of the tensor stresses determine the GW frequency:
connected to the bubble size

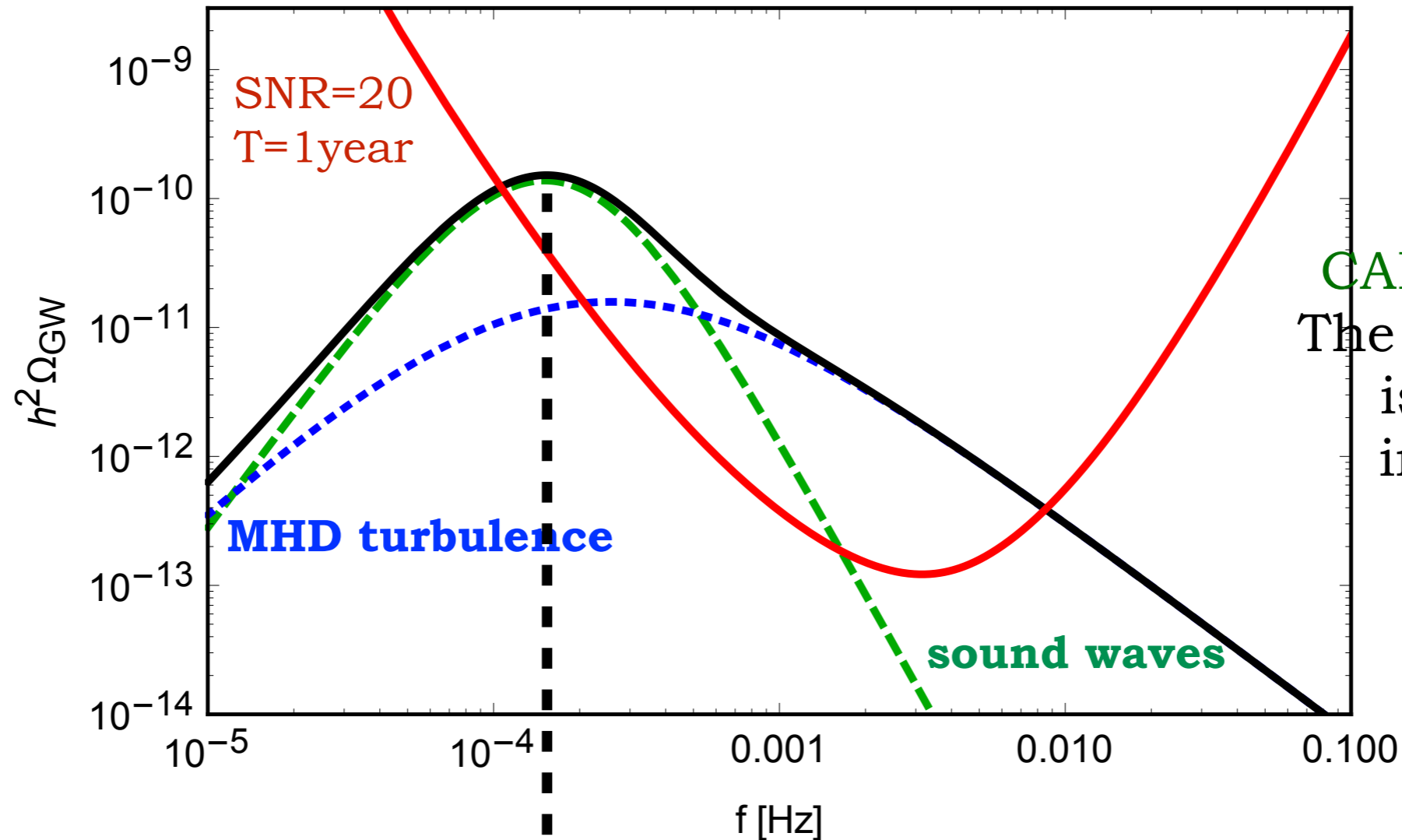
$$\epsilon_* = \ell_* H_*$$

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\text{GeV}} \text{ Hz}$$

$$T_{\text{EW}} \sim 100 \text{ GeV} \quad \ell_* H_* \simeq 0.01 \quad \longrightarrow \quad \boxed{f \sim \text{mHz}} \quad \text{LISA}$$

One example of GW signal from the EW phase transition “Higgs portal” scenario

$T_* = 59.6 \text{ GeV}$, $\alpha = 0.17$, $\beta/H_* = 12.5$



CAREFUL NOTE:
The spectral shape
is still under
investigation

$$f_* \sim \frac{1}{\ell_*}$$

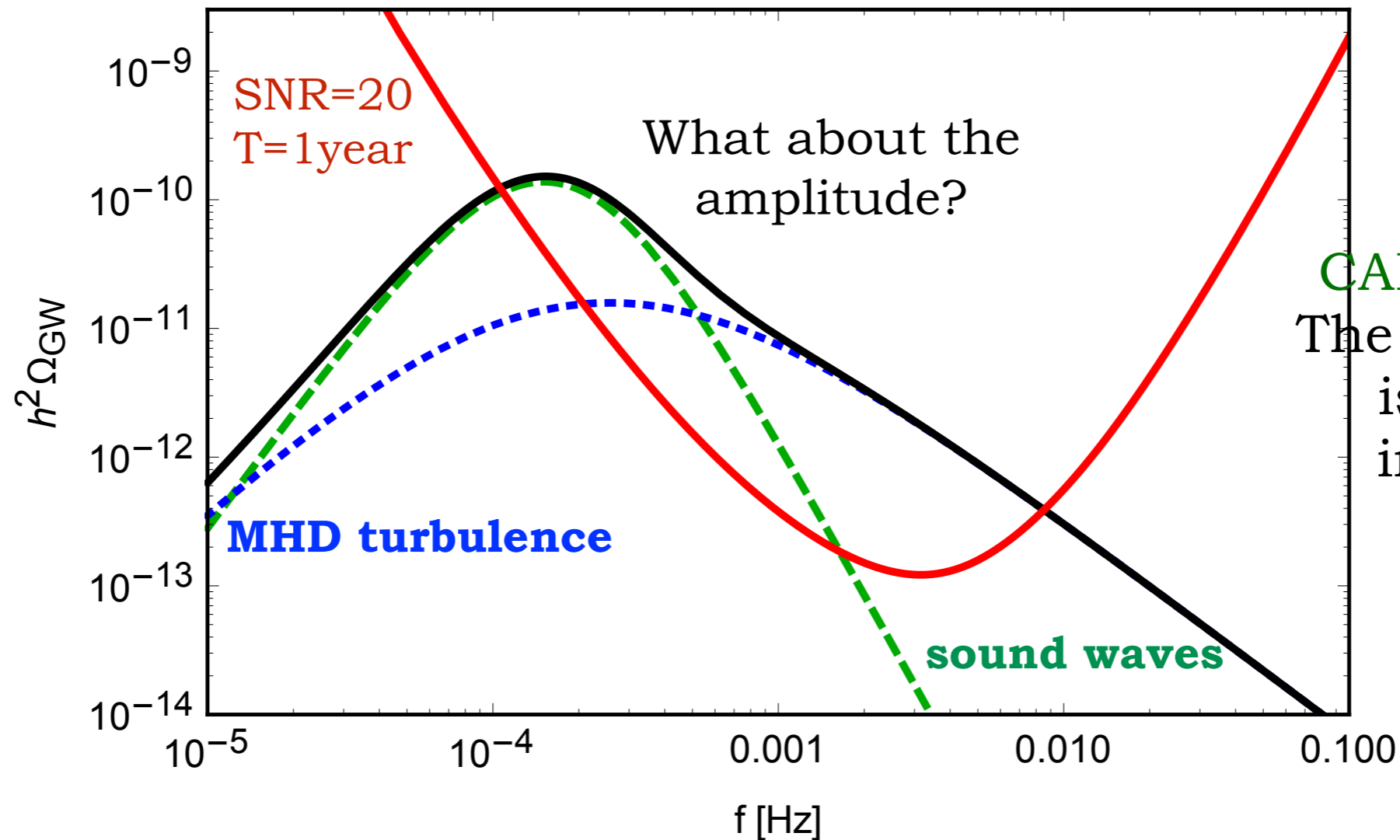
$$\beta = \frac{d}{dt} \ln \mathcal{P}(T_*)$$

nucleation rate

$$R_* = \frac{(8\pi)^{\frac{1}{3}}}{\beta} \text{Max}(v_w, c_s)$$

One example of GW signal from the EW phase transition “Higgs portal” scenario

$T_* = 59.6 \text{ GeV}$, $\alpha = 0.17$, $\beta/H_* = 12.5$



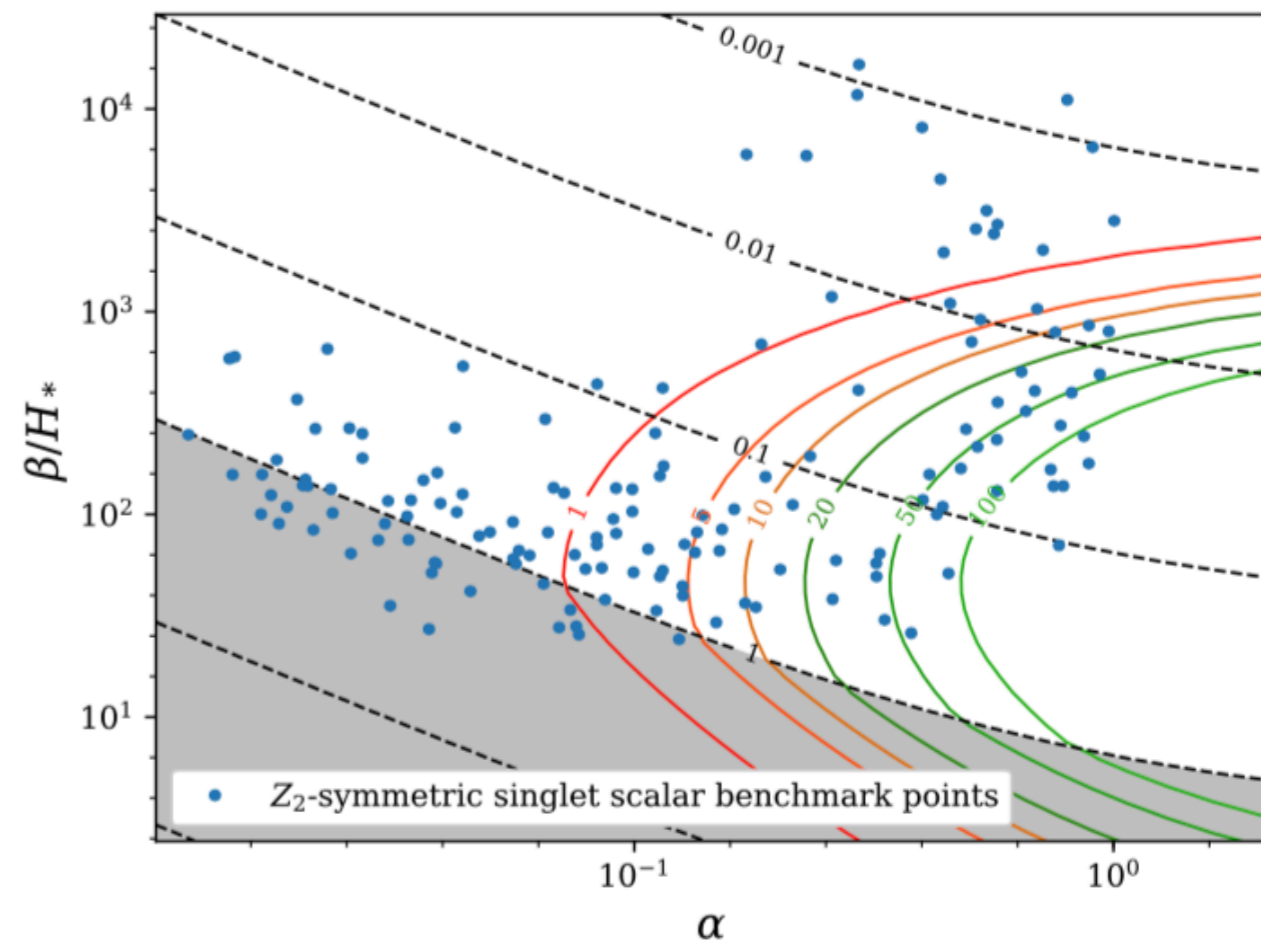
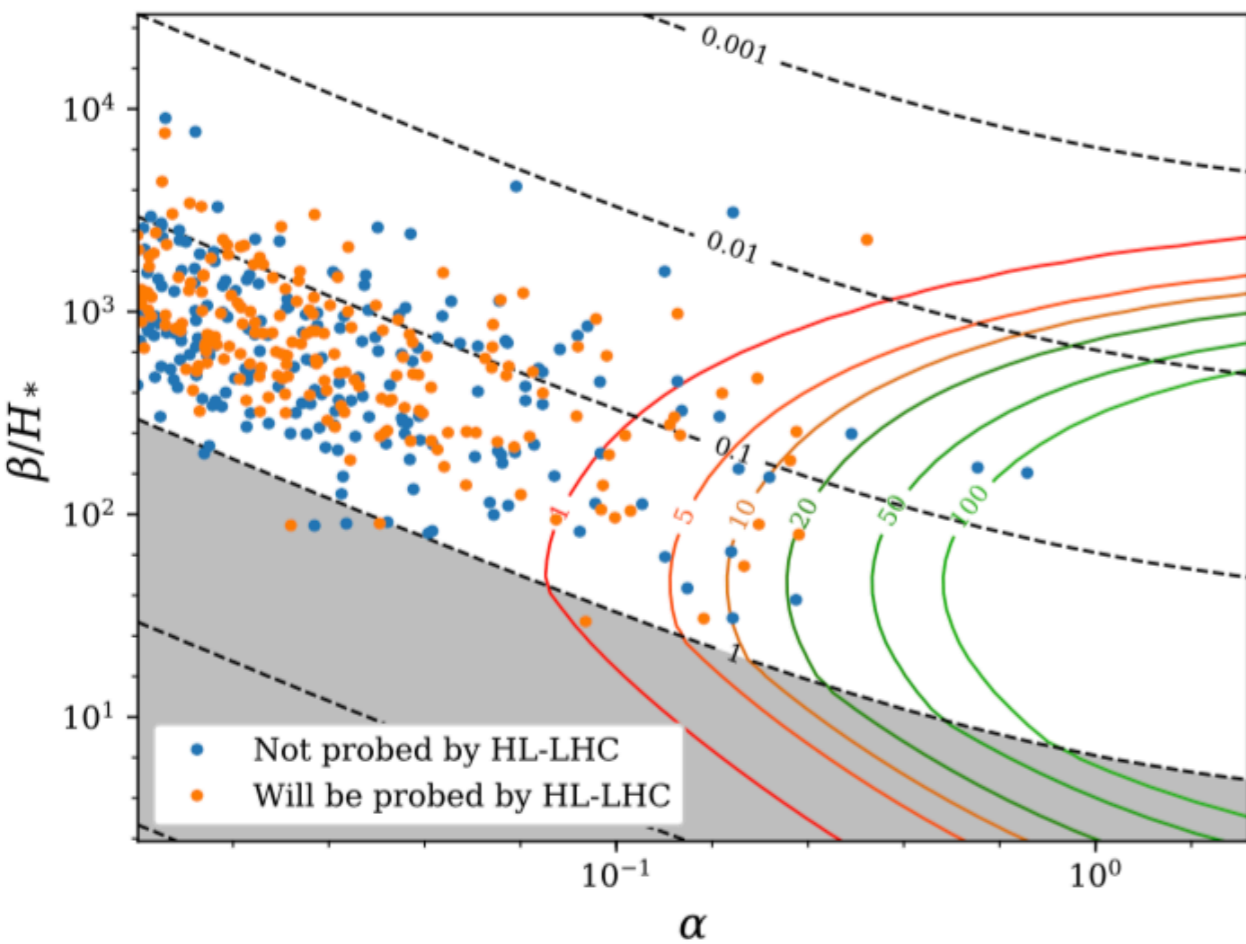
$$\frac{d\Omega_{\text{GW}}}{d \ln k} \sim \Omega_{\text{rad}}^0 \left(\frac{\Pi}{\rho_{\text{rad}}^*} \right)^2 \frac{(H_* R_*)^2}{H_* R_* + \sqrt{\Pi/\rho_{\text{rad}}^*}}$$

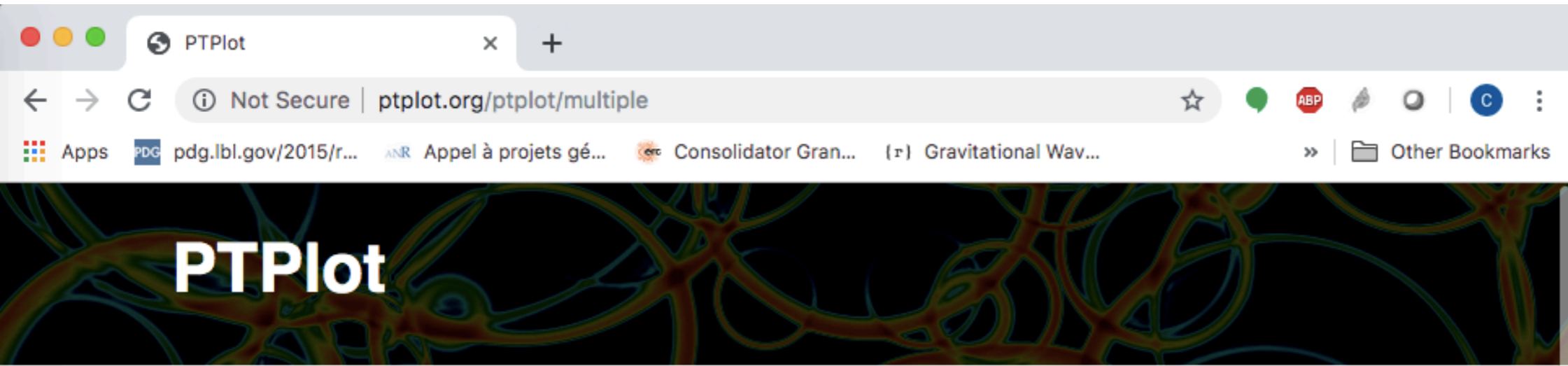
Fraction of the “tensor” energy density
available for GW production

Source dynamics

LISA (mHz) is sensitive to energy scales around the **TeV scale**, so it can probe the EWPT in BSM models and more exotic PTs beyond the EWPT

connections with baryon asymmetry, dark matter : LISA could act as a probe of BSM physics, complementary to colliders





PTPlot: Plot multiple parameter points

Note that the input table should be a comma-separated list of pairs of α_θ , β/H_* and (optionally) a label for each point (Math mode TeX is allowed, surrounded by \$ signs, in the label column is ignored).

NB: β/H_* against α plots require v_w to be fixed.

Wall velocity v_w :

Transition temperature T_* :

Degrees of freedom g_* :

Mission profile Science Requirements Document (3 years)
 Science Requirements Document (7 years)

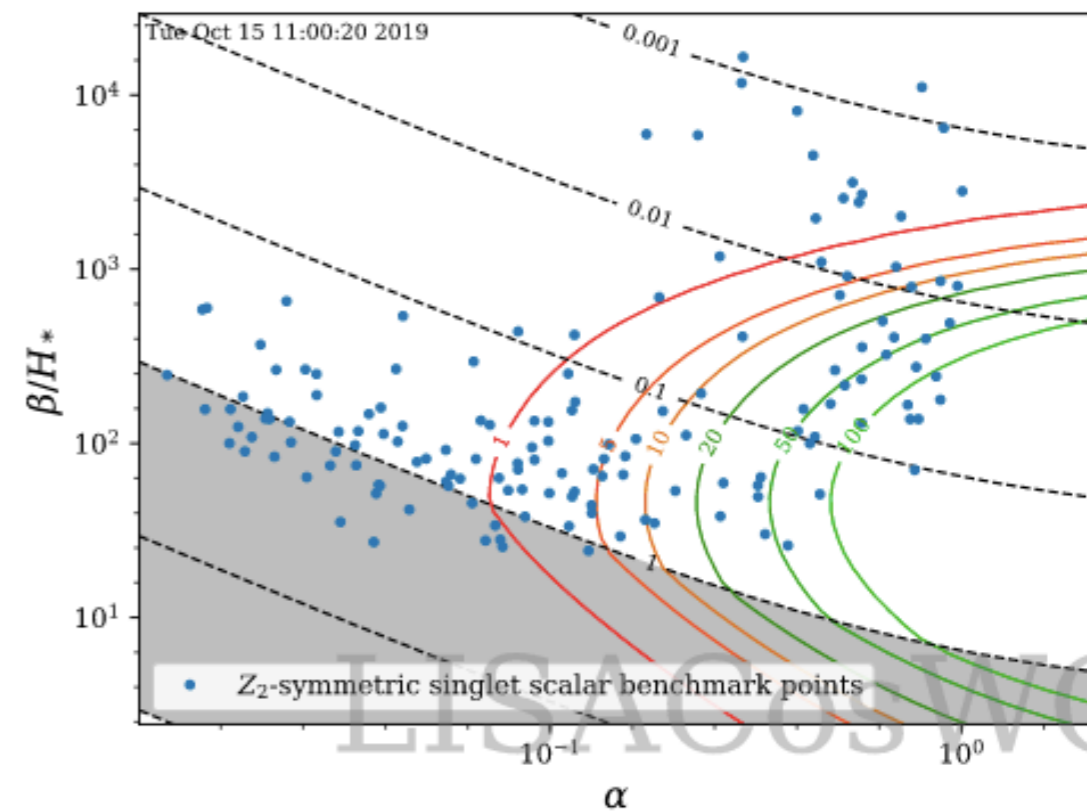
Input table:

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Render alpha/beta SVG plot

PTPlot

PTPlot: Z_2 -symmetric singlet scalar benchmark points



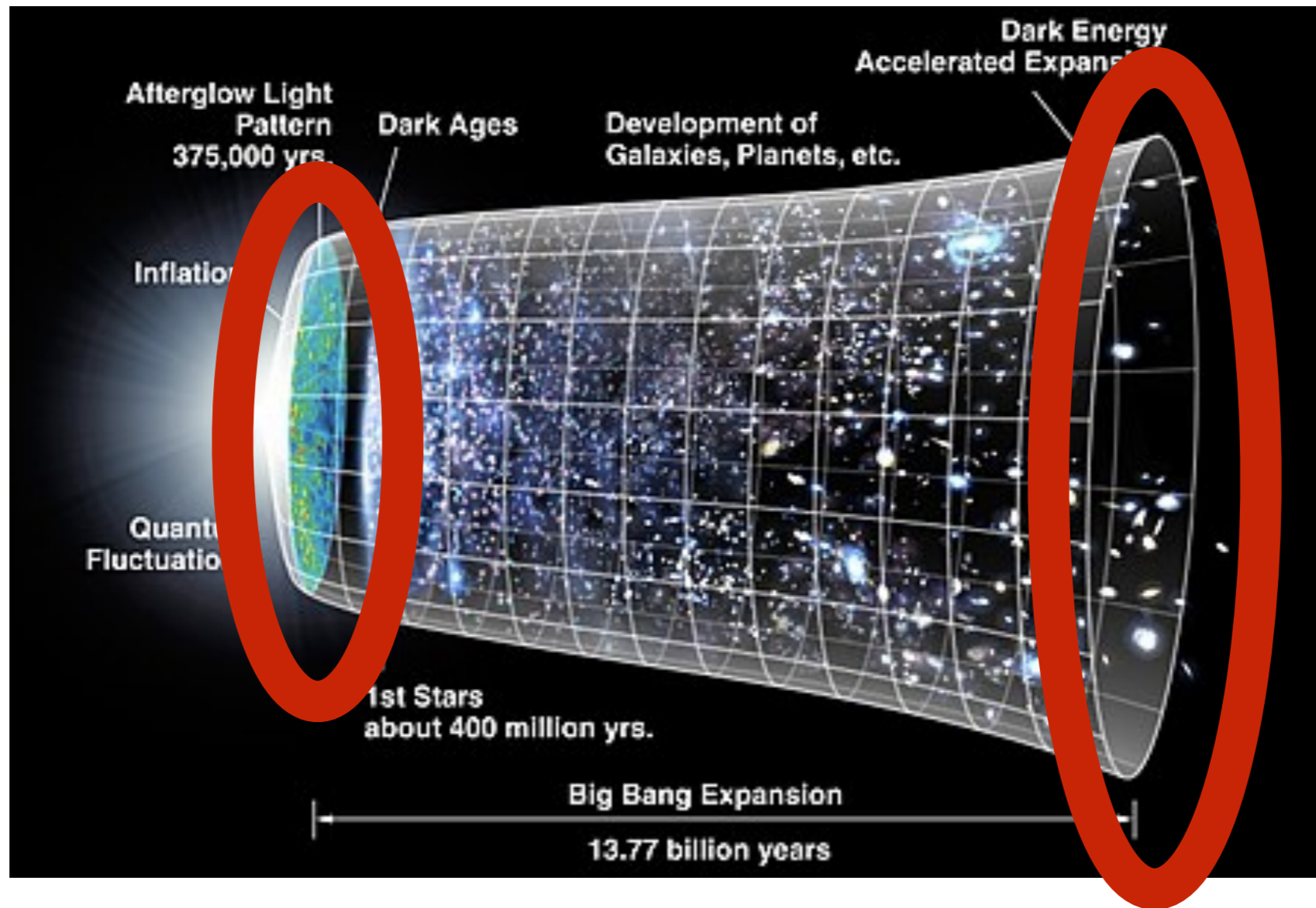
Future prospects concerning GW from PT

- near-linear regime well understood: acoustic waves are the dominant source of GW. Role of non-linearities to be better understood
- most favourable scenarios beyond SM lead to strong PTs that maximise the signal -> short characteristic time of formation of non-linearities
- the GW signal spectral shape must be predicted very accurately to make detection forecasts
- are we able to distinguish scenarios? improve the link spectrum vs. PT parameters (in particular, role of the wall velocity)
- data analysis techniques must be developed (type SGWBinner, to perform foreground subtraction...)
- ...

To summarise the part about SGWB from the primordial universe:

- Several phase transitions might have occurred in the early universe, leading to GW production
- **Inflation**: new physics but observationally motivated, extended GW signal in frequency, only accessible by CMB unless one goes beyond the standard slow roll scenario. **Particularly interesting for LISA IMO: PBH**
- **Cosmic strings**: new physics but theoretically motivated, GW signal strongly model dependent, extended in frequency, can be accessed/constrained at PTA, LISA, LIGO/Virgo. **LISA can provide good constraints**
- **Electroweak PT**: at the limit of tested physics, GW signal can be accessed/constrained only for models beyond the standard model of particle physics —> tests of models, complementary to particle colliders: **Particularly interesting for LISA**
- **SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner -> must improve data analysis techniques!**

How can GW help to probe cosmology?



the stochastic GW background from primordial sources: test of early universe and high energy phenomena

use of GW emission from binaries to probe late-time dynamics and content of the universe

Using GWs to measure the background expansion of the universe

$$H(z) = H_0 \sqrt{\Omega_M (z+1)^3 + (1 - \Omega_\Lambda - \Omega_M) (z+1)^2 + \Omega_\Lambda \exp\left[-\frac{3w_a z}{z+1}\right] (z+1)^{3(1+w_0+w_a)}}$$

- Hubble factor, written for one specific model of dark energy
- No contribution from radiation, negligible in the late universe

$$d_L(z) = (1+z) \mathcal{G} \left(\int_0^z \frac{dz'}{H(z')} \right) \quad \mathcal{G} = 1$$

Flat space hyper surfaces

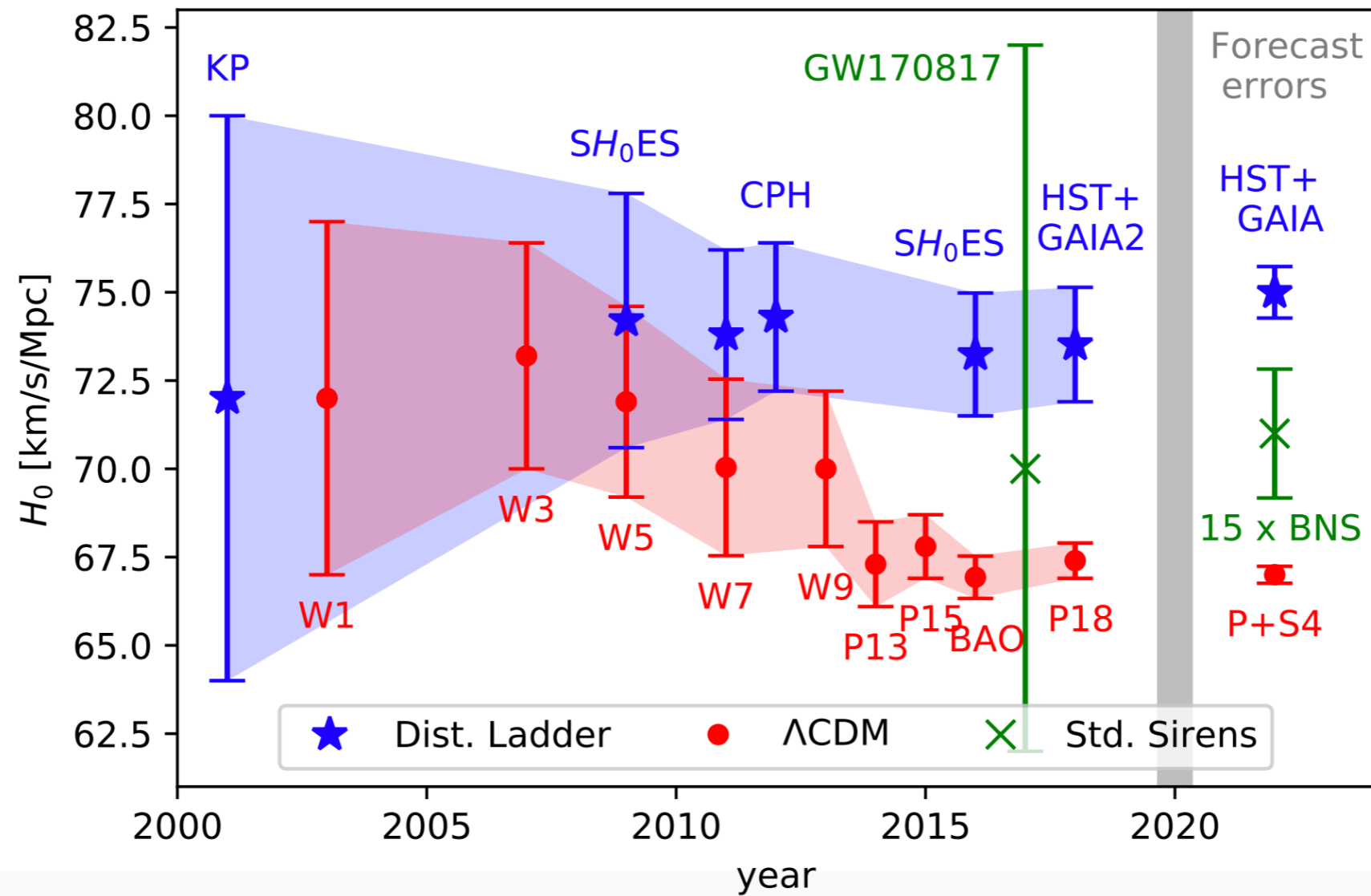
Measuring the luminosity distance as a function of redshift provides access to the cosmological parameters, in particular H_0 at low redshift

$$z \ll 1$$

$$cz = H_0 d_L(z)$$

Hubble law

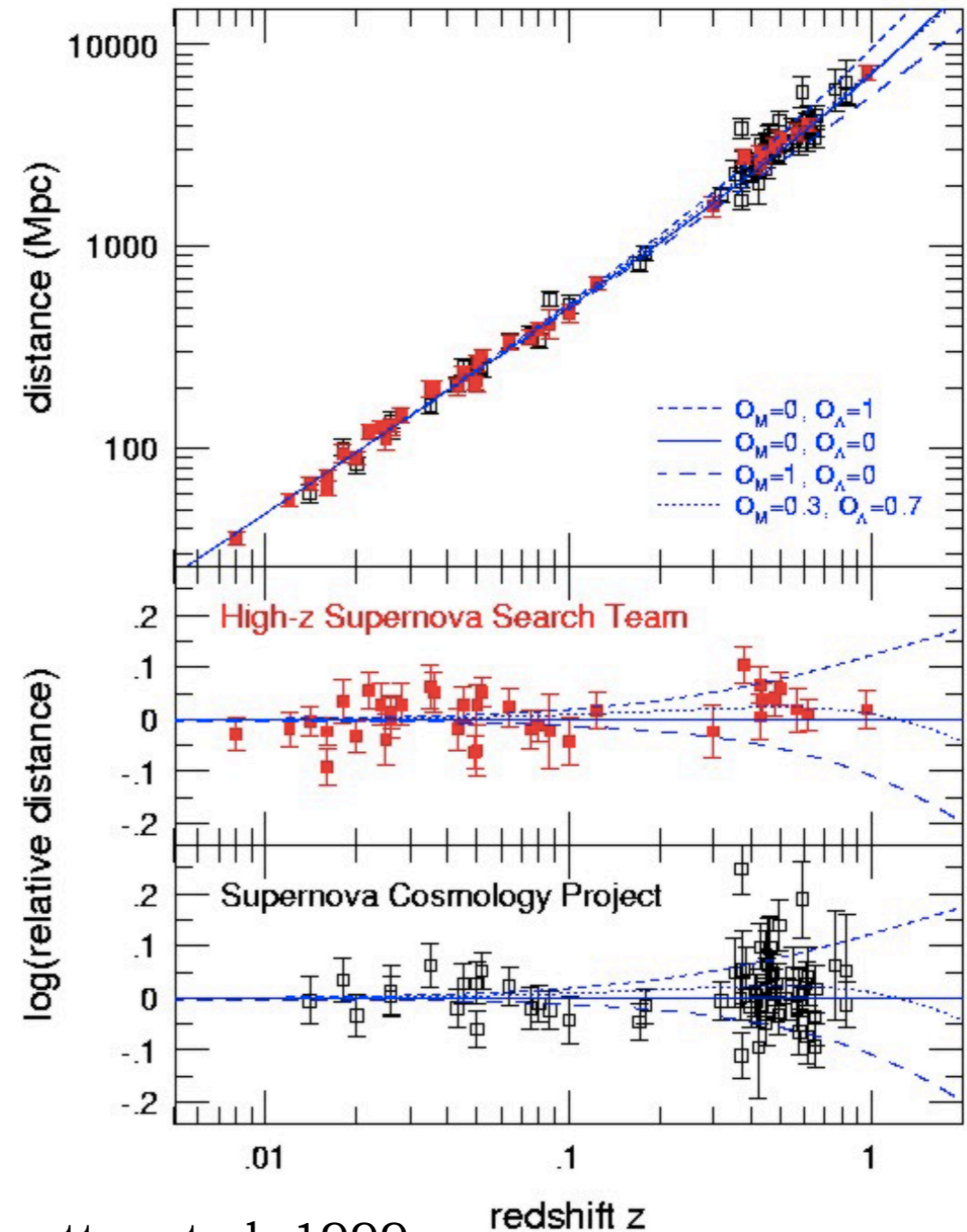
Using GWs to measure the background expansion of the universe



- Remarkable agreement if ones thinks that the two methods measure physical phenomena that are 13 billion years apart
- Not enough in the era of precision cosmology
- Does this require new physics?

Measurement of $d_L(z)$: standard candles

Nobel prize in physics 2011:
discovery of the late-time acceleration of the universe



Riess et al. 1998

Perlmutter et al. 1999

redshift z

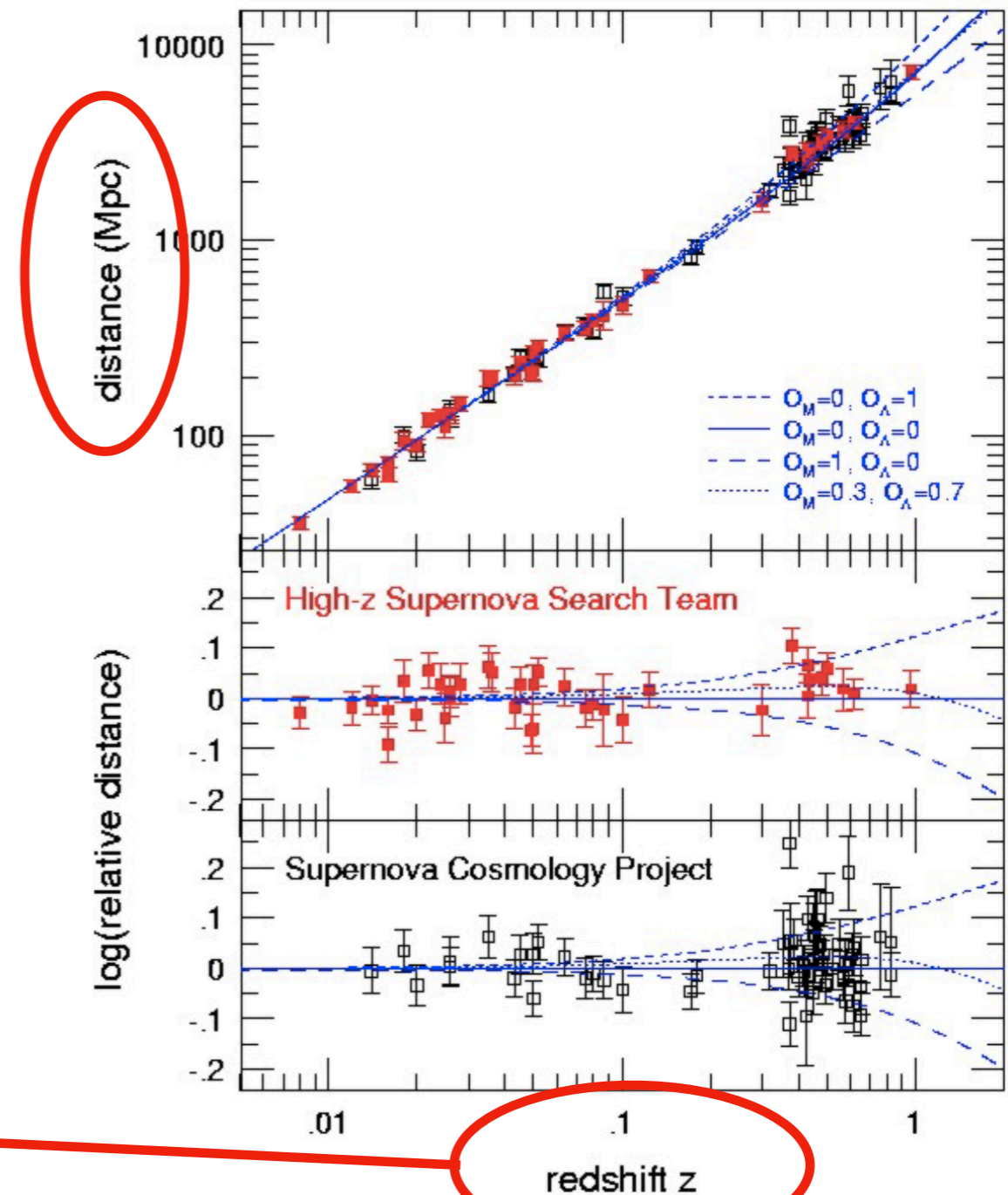
Measurement of $d_L(z)$: standard candles

Nobel prize in physics 2011:
discovery of the late-time acceleration of the universe

$$d_L(z) = \sqrt{\frac{L}{4\pi\mathcal{F}}}$$

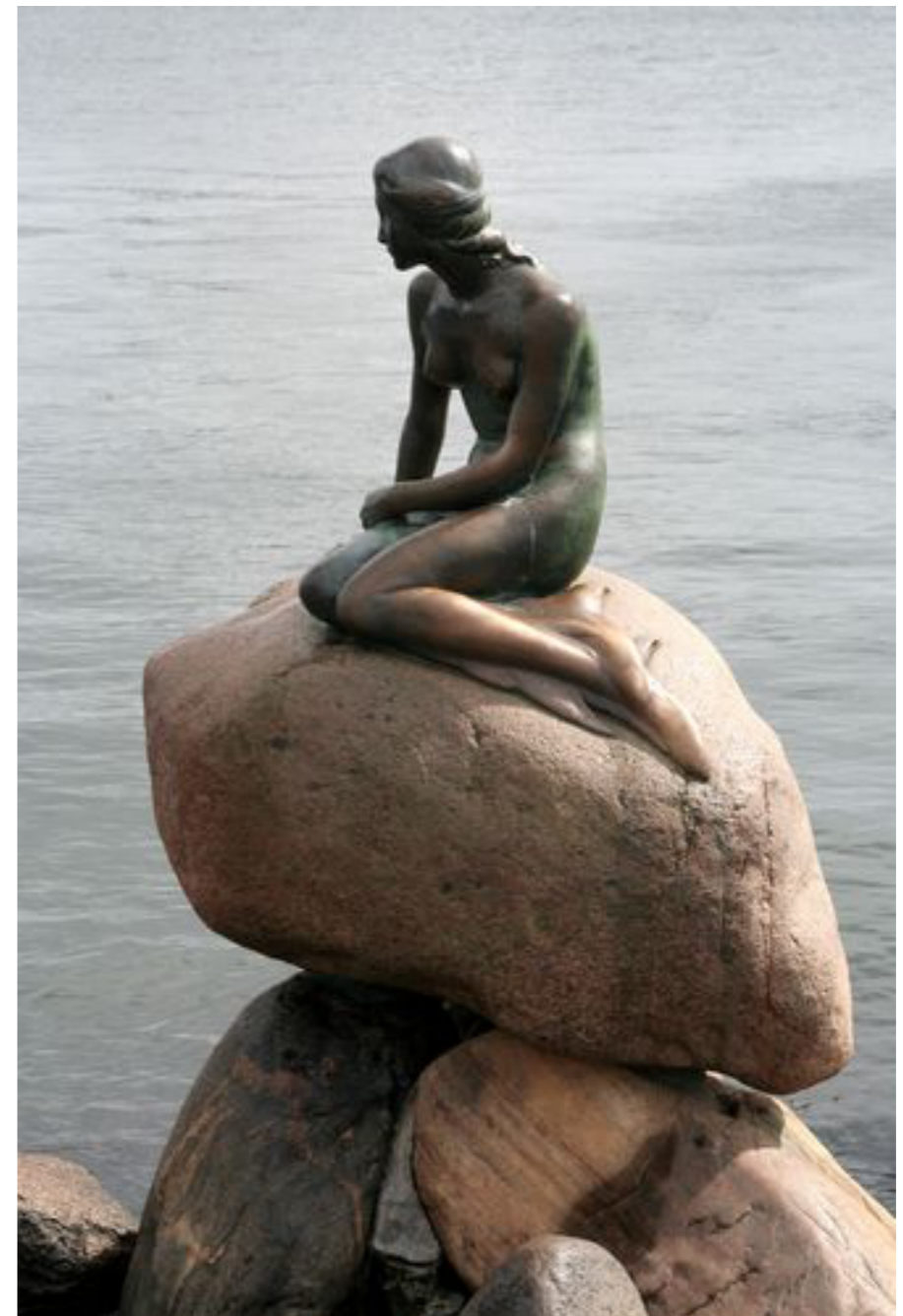
- Flux measured directly
- Intrinsic luminosity known from calibration
- Measurement of the luminosity distance is **NOT SO EASY**

Redshift measured directly
from the optical emission
EASY!



Measurement of $d_L(z)$: standard sirens

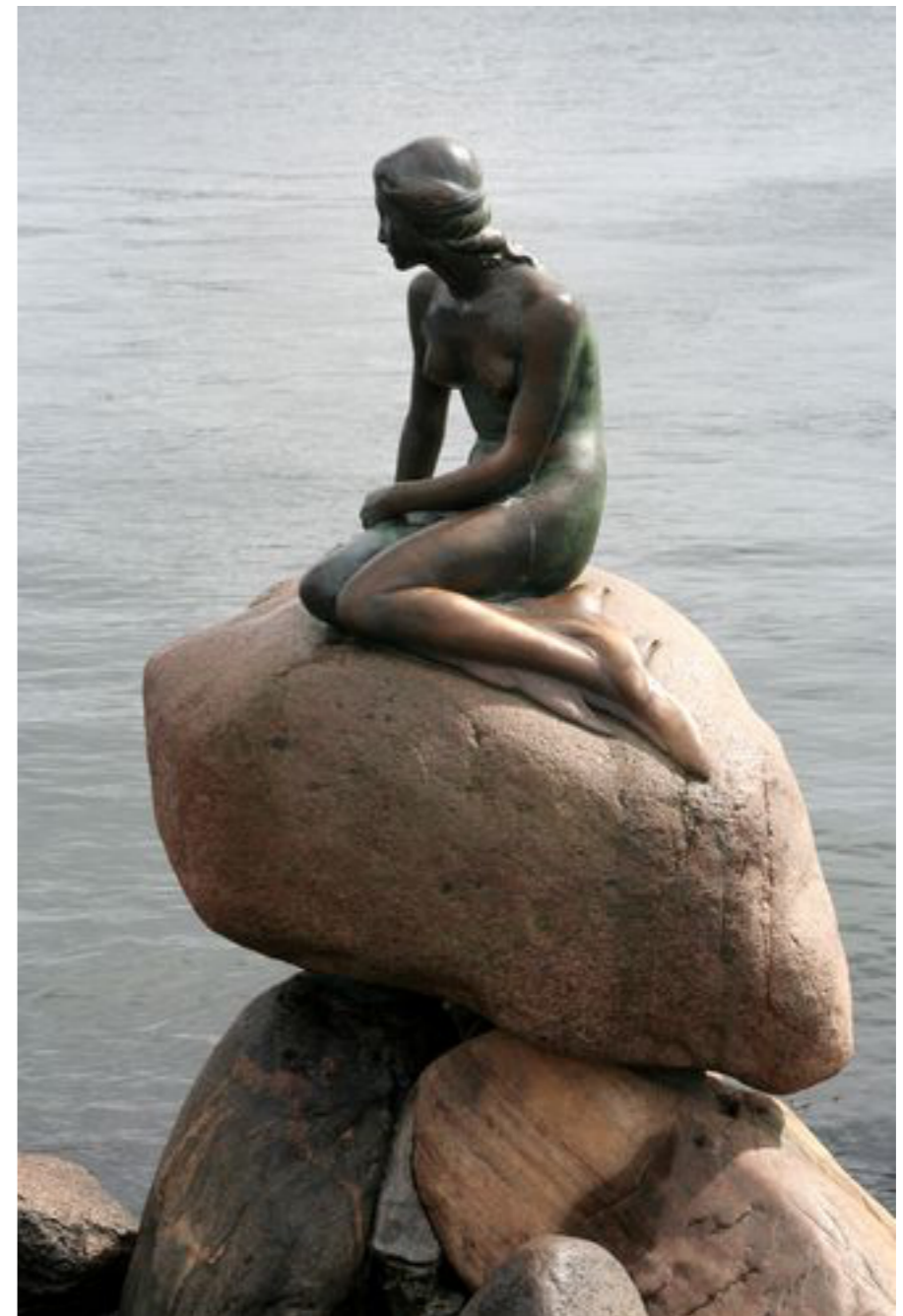
GW emission by compact binaries
can also be used to test the expansion of the universe



Measurement of $d_L(z)$: standard sirens

GW emission by compact binaries
can also be used to test the expansion of the universe

- Measurement of the luminosity distance: no calibration needed, **EASY AND DIRECT**
- Measurement of the redshift: **IMPOSSIBLE!**



Inspiral of compact binaries at cosmological distance

$$h_+(t_S, \theta, \varphi) = \frac{4}{a(t_O)r} (G M_c)^{5/3} [\pi f(t_S^{\text{ret}})]^{2/3} \left(\frac{1 + \cos^2 \theta}{2} \right) \cos(2\Phi(t_S^{\text{ret}}))$$

$$h_\times(t_S, \theta, \varphi) = \frac{4}{a(t_O)r} (G M_c)^{5/3} [\pi f(t_S^{\text{ret}})]^{2/3} \cos \theta \sin(2\Phi(t_S^{\text{ret}}))$$



Propagation effect at
the observer



Still measured by the source clock,
we want it in the observer's clock

Inspiral of compact binaries at cosmological distance

$$h_+(\tau, \theta, \varphi) = \frac{4}{d_L(z)} (G \mathcal{M}_c)^{5/3} [\pi f(\tau)]^{2/3} \left(\frac{1 + \cos^2 \theta}{2} \right) \cos(2\Phi(\tau))$$

$$h_\times(\tau, \theta, \varphi) = \frac{4}{d_L(z)} (G \mathcal{M}_c)^{5/3} [\pi f(\tau)]^{2/3} \cos \theta \sin(2\Phi(\tau))$$

$$\mathcal{M}_c = (1 + z) M_c$$

Redshifted chirp mass

degeneracy among the redshift and the true chirp mass

Assuming that the redshift is constant

$$\dot{f}_O = \frac{96\pi^{8/3}}{5} (G \mathcal{M}_c)^{5/3} f_O^{11/3}$$

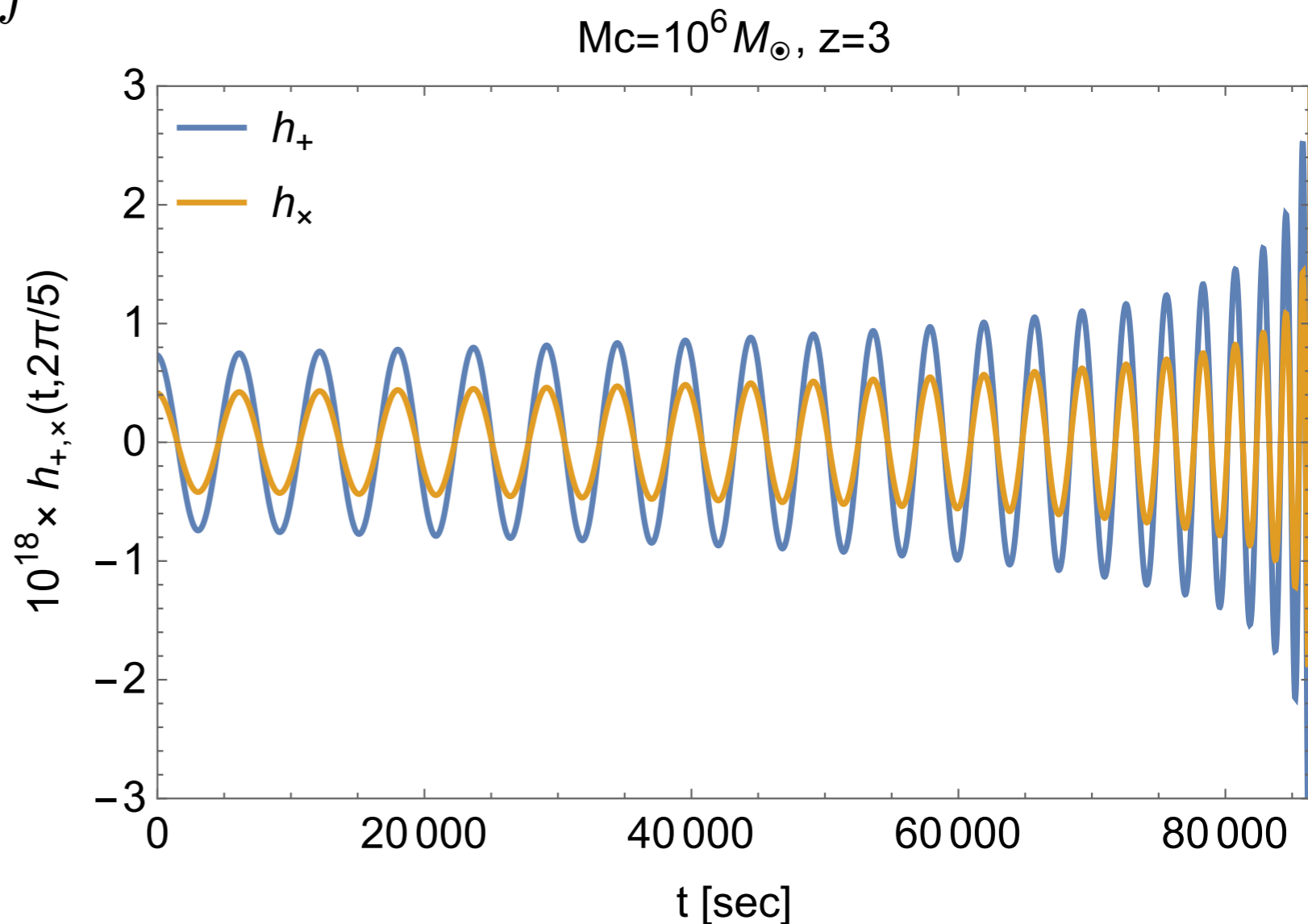
Measurement of $d_L(z)$

$$h_+(\tau, \theta, \varphi) = \frac{4}{d_L(z)} (G \mathcal{M}_c)^{5/3} [\pi f(\tau)]^{2/3} \left(\frac{1 + \cos^2 \theta}{2} \right) \cos(2\Phi(\tau))$$

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$$\dot{f} = \frac{96\pi^{8/3}}{5} (G \mathcal{M}_c)^{5/3} f^{11/3}$$

- From f and \dot{f} measure the redshifted chirp mass
- From the ratio among h_+ and h_\times measure the inclination of the orbit
- **Get a direct measurement of d_L**



Measurement of $d_L(z)$

$$\mathcal{M}_c = (1 + z)M_c$$

How can we break this degeneracy and use GW emission to build the Hubble diagram $d_L(z)$?

There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- **Direct method:** directly identify the galaxy hosting the event, via the measurement of a (transient) electromagnetic counterpart
- **Statistical method:** cross-correlate the sky position given by the GW measurement with galaxy catalogues
- Assume that one knows or constraints the intrinsic mass of the object

Measurement of $d_L(z)$

$$\mathcal{M}_c = (1 + z)M_c$$

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There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- **Direct method:** directly identify the galaxy hosting the event, via the measurement of a (transient) electromagnetic counterpart
 - **Earth-based interferometers:** sources with counterparts are NS-NS binaries and perhaps NS-BH binaries
 - **LISA:** sources with expected counterparts are massive BH-BH binaries at the centre of galaxies

Measurement of $d_L(z)$

$$\mathcal{M}_c = (1 + z)M_c$$

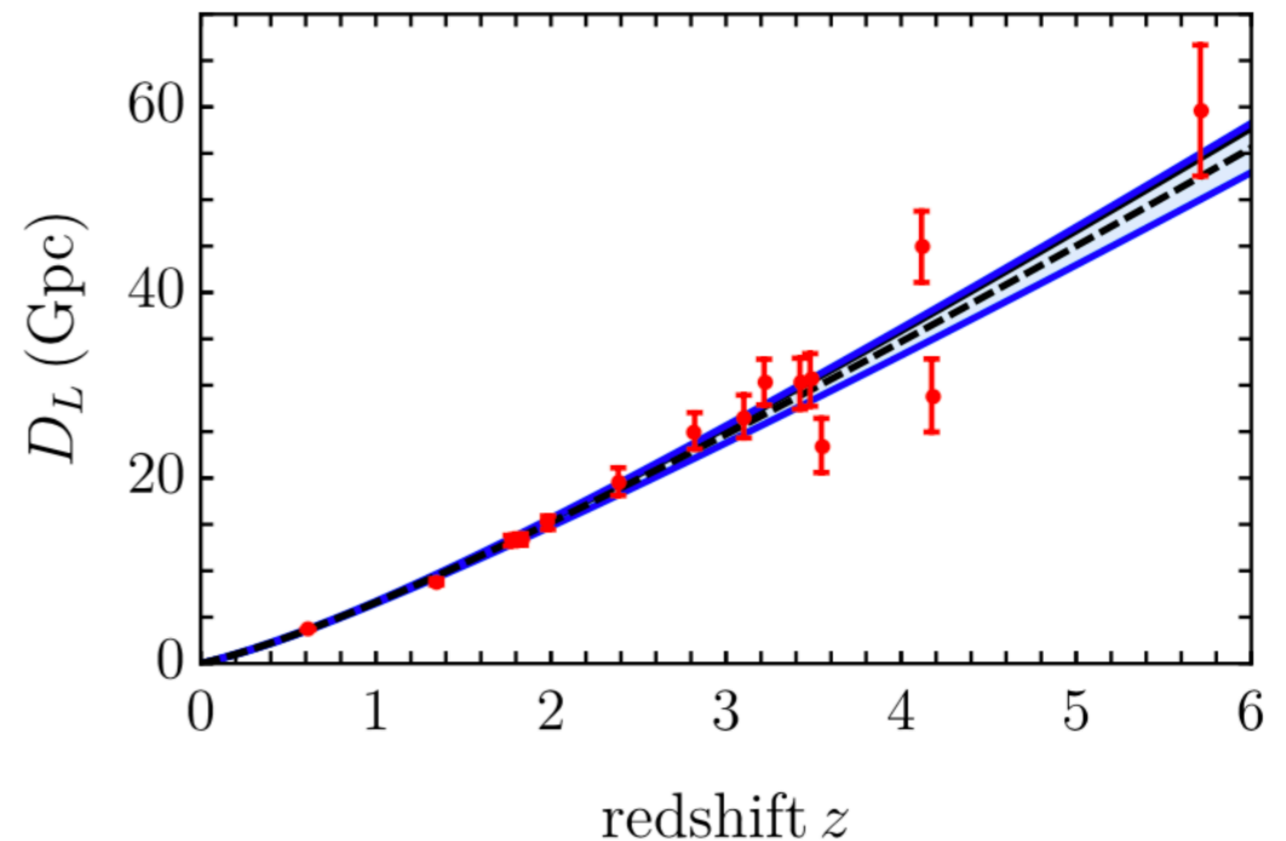
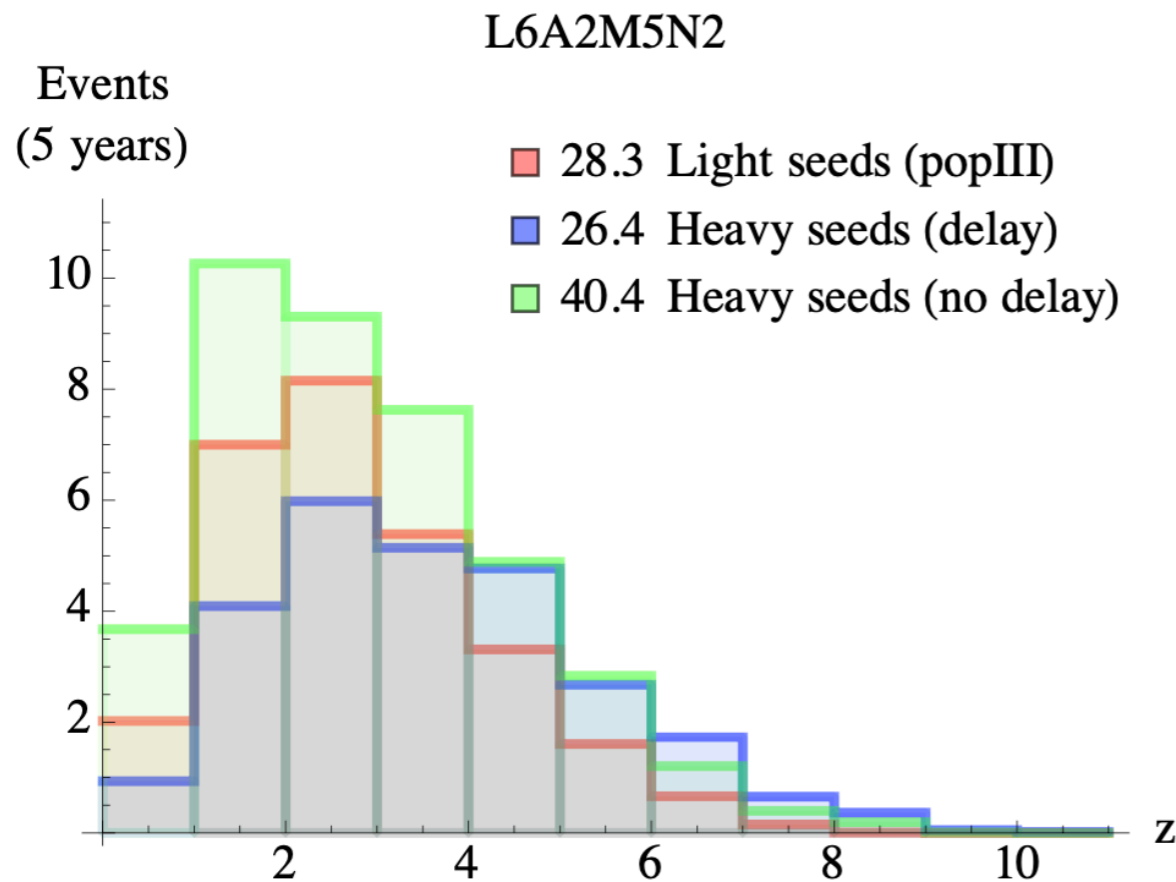
How can we break this degeneracy and use GW emission to build the Hubble diagram $d_L(z)$?

There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- **Statistical method:** in the absence of a counterpart, one can cross-correlate with a galaxy catalogue to associate to the GW event a group of galaxies with redshift compatible with the redshift range inferred from the GW measurement with priors on the cosmological parameters. One then searches for the unique set of cosmological parameters aligning each event on the same $d_L(z)$ relation
 - **Earth-based interferometers:** this method can be used with stellar mass BH-BH binaries (the most numerous sources)
 - **LISA:** this method can be used with Extreme Mass Ratio Inspirals (EMRIs)

Direct method with LISA

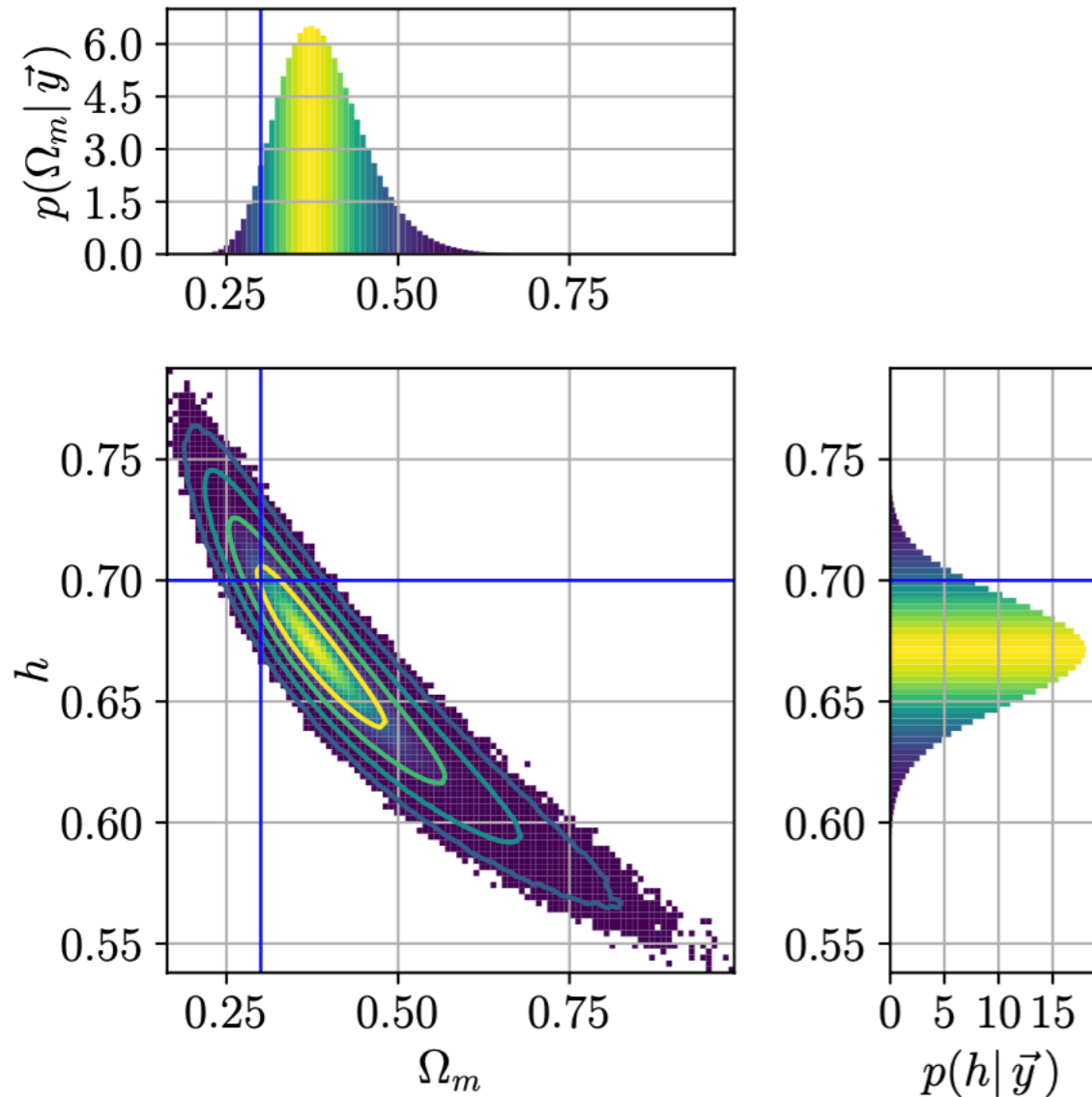
- **Massive BH binaries mergers** are expected to have counterparts if they occur in gaseous disks at the centre of galaxies (the rate of these events is uncertain though)
- The counterpart is expected in the **radio emission**, followed by **optical identification of the host galaxy**
- One must select events with high SNR and **good sky localisation** (few!)
- **Weak lensing** (and peculiar velocity errors) affect the measurement of d_L



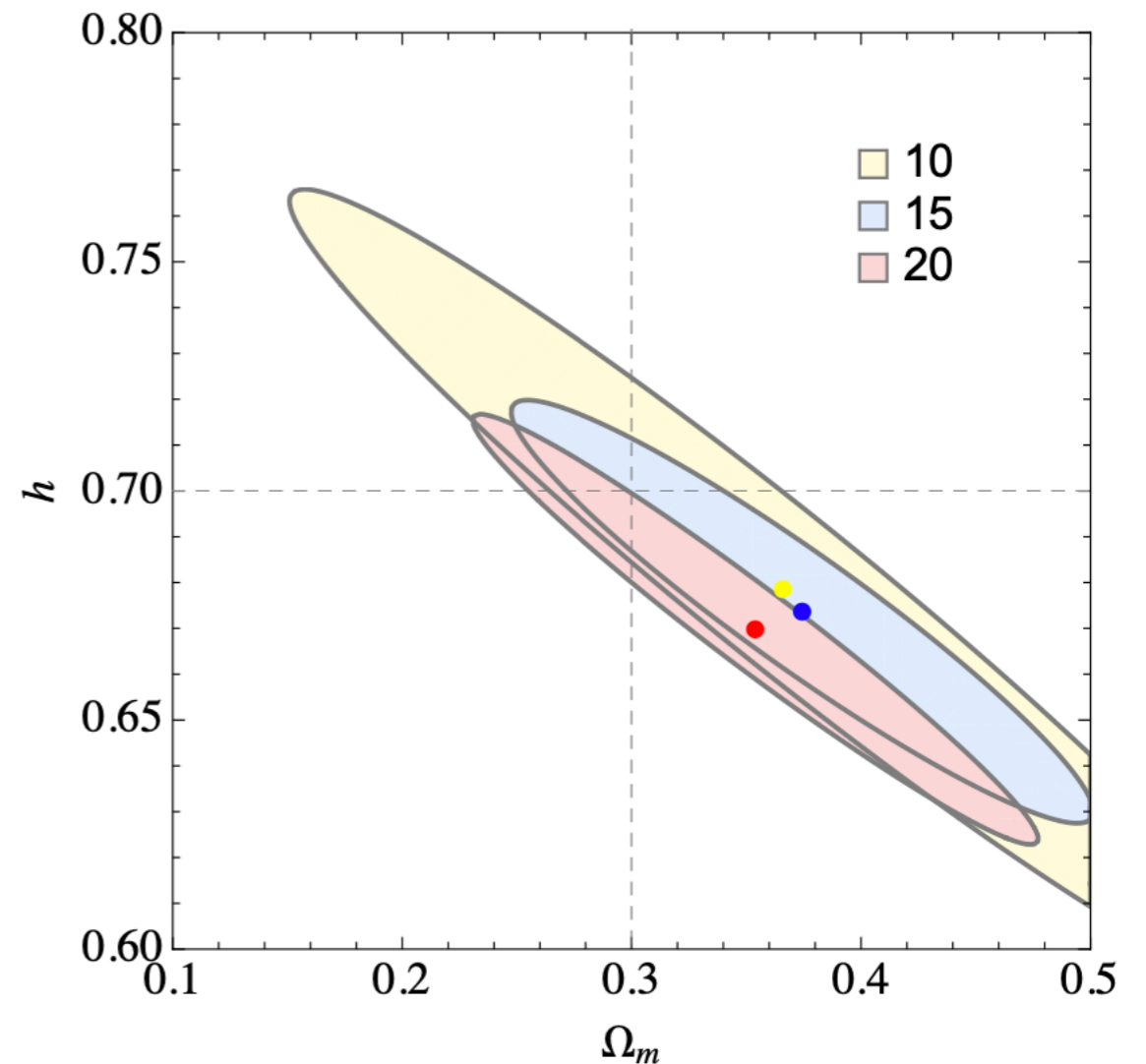
Direct method with LISA

Speri et al, arXiv:2010.09049

5% constraints on H_0 with
15 standard sirens



Reducing to 2% with 40
standard sirens



Direct method with LISA

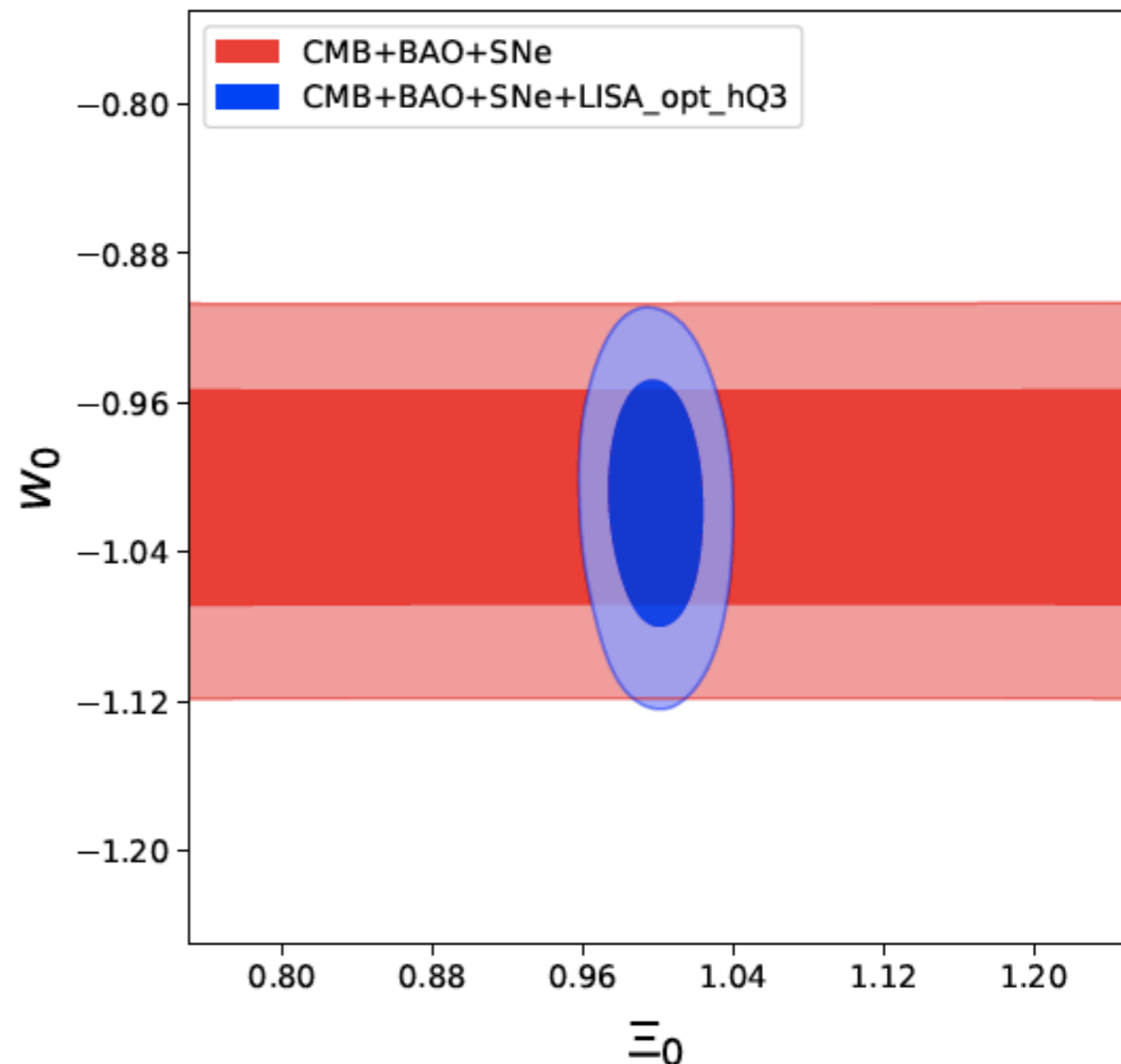
Belgacem et al, arXiv:1906.01593

LISA MBHB standard sirens can be used to test modified gravity theories

$$\tilde{h}_A'' + 2\mathcal{H}[1 - \delta(\eta)]\tilde{h}_A' + k^2\tilde{h}_A = 0$$

general parametrisation:

$$\frac{d_L^{\text{gw}}(z)}{d_L^{\text{em}}(z)} = \Xi_0 + \frac{1 - \Xi_0}{(1+z)^n}$$



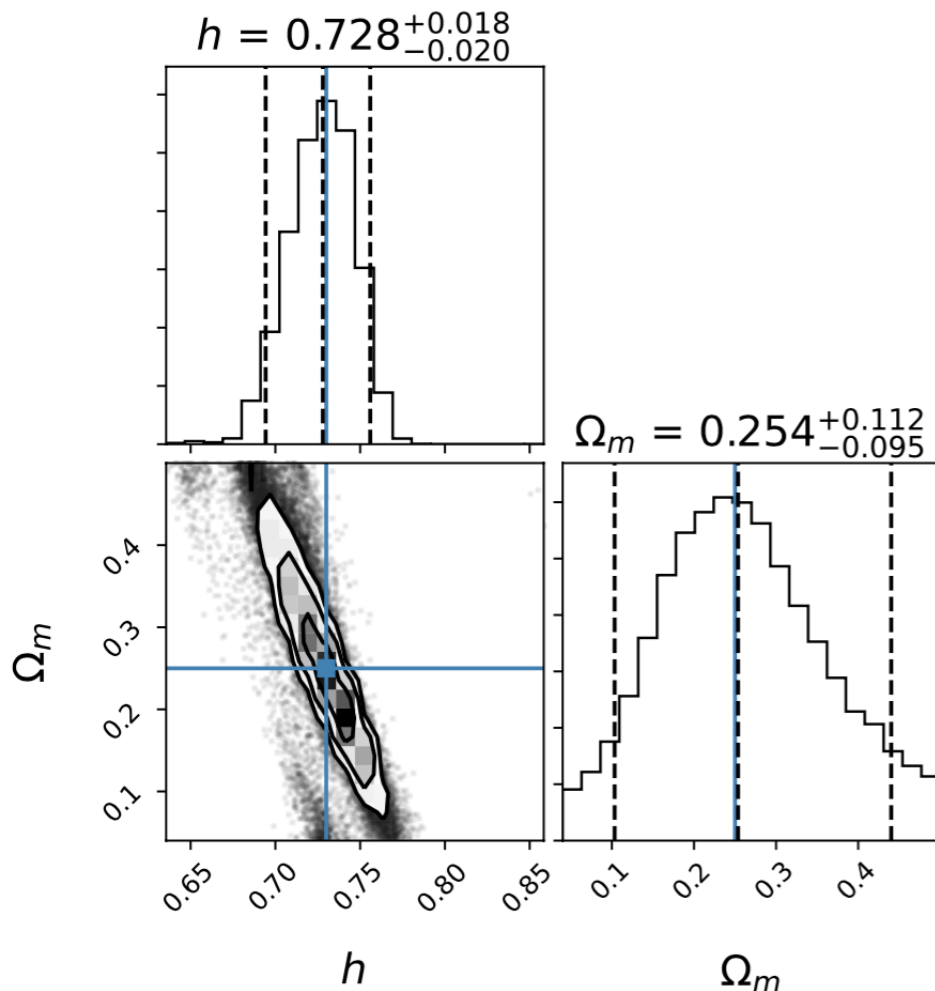
Statistical method with LISA

It can be applied to sources for which no counterpart is expected, present at sufficiently high redshift

- **SOBHB** sources with high enough SNR are too rare to be used as dark sirens
- **EMRIs** with $\text{SNR} > 100$ can be used for cosmology
Results depend on the rates, which are uncertain

Laghi et al, arXiv:2102.01708

Model M1 (*fiducial*)



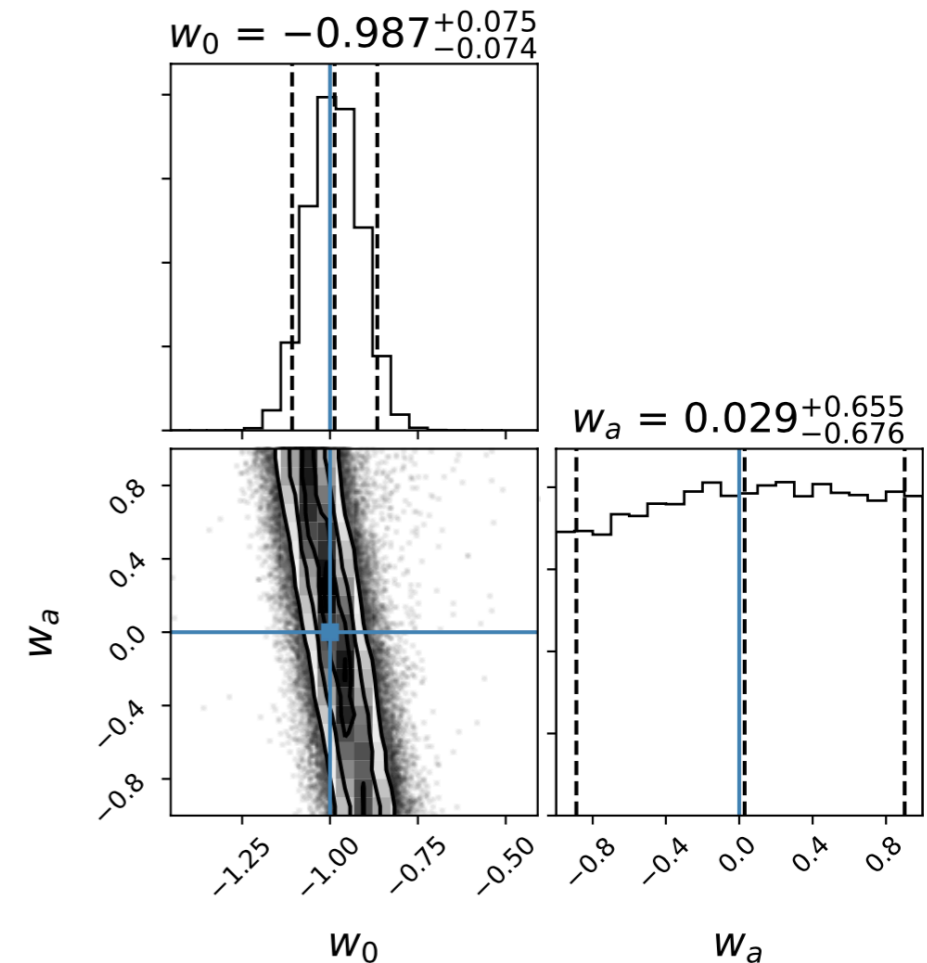
A 4-year LISA mission can provide:

3% constraints on H_0

7% constraints on w_0

DE, 4yr

Model M1 (*fiducial*)



To summarise the part about testing the expansion of the universe with standard sirens:

- GWs emission from compact binaries provide clean measurements of the luminosity distance which does not require calibration
- The construction of the Hubble diagram $d_L(z)$ can be performed with and without electromagnetic counterpart
- One can then determine H_0 in a way which is fully independent of both CMB and SNIa
- The network of Earth-based interferometers has already started to provide results, especially thanks to GW170817
- **LISA** can provide a **percent measurement of H_0** by combining the results of bright and dark standard sirens, as well as constraint the **expansion of the universe at high redshift** with bright sirens
- **The future is bright in what concerns GWs as a cosmological probe of the late expansion of the universe, and the construction of the Hubble diagram with GWs will help pinning down the present tension on the measurement of the Hubble constant**