IPhU seminar (Marseille March 2021)

Institut
Physique de
l'Univers
Aix*Marseille Université

Estimating the covariance of cosmological observables : prospects and challenges for future redshift surveys.

Julien BEL
Centre de Physique Théorique (CPT)

Aix*Marseille
université

CPT
Centre de Physique Théorique

Outline

1. Motivation
2. Monte Carlo method
3. Power spectrum covariance

Motivation

The example of the cosmic microwave background (CMB):

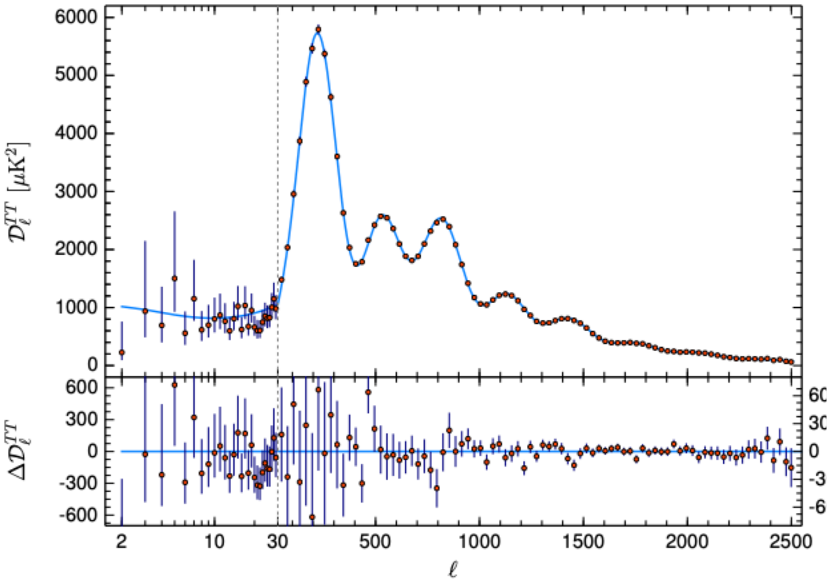
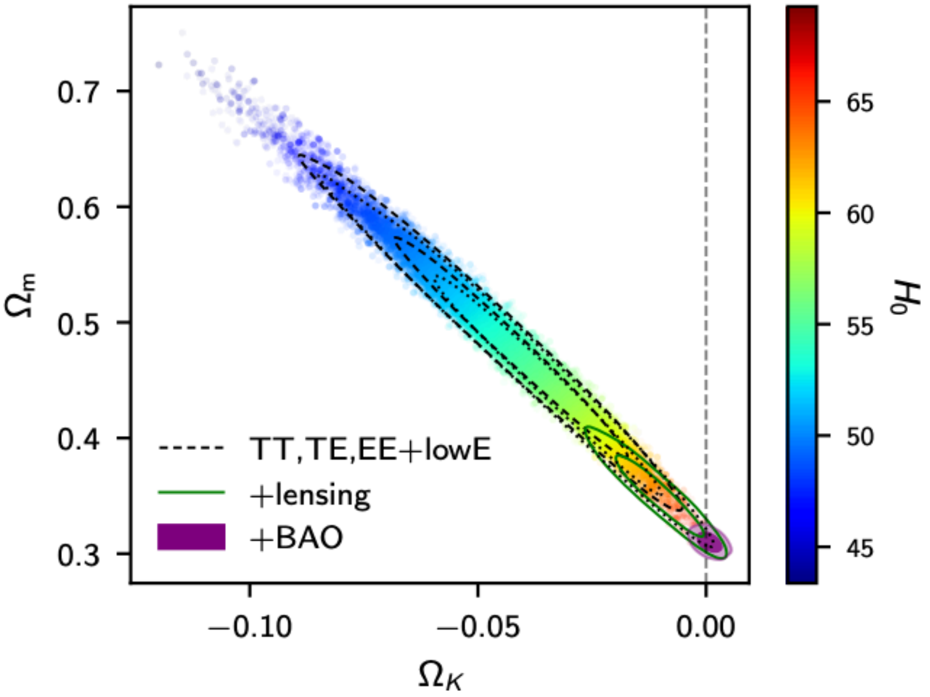


Fig. 1. *Planck* 2018 temperature power spectrum. At multipoles $\ell \geq 30$ we show the frequency-coadded temperature computed from the `Plik` cross-half-mission likelihood, with foreground and other nuisance parameters fixed to a best fit the base-ΛCDM cosmology. In the multipole range $2 \leq \ell \leq 29$, we plot the power spectrum estimates from the Co component-separation algorithm, computed over 86 % of the sky. The base-ΛCDM theoretical spectrum best fit to th TT,TE,EE+lowE+lensing likelihoods is plotted in light blue in the upper panel. Residuals with respect to this model are the lower panel. The error bars show $\pm 1\,\sigma$ diagonal uncertainties, including cosmic variance (approximated as Gaussian, including uncertainties in the foreground model at $\ell \geq 30$. Note that the vertical scale changes at $\ell = 30$, where the horizontal axis switches from logarithmic to linear.



Planck (2018)

Motivation

Problem: Combination of LSS probes

e.g. galaxy-galaxy lensing and redshift space galaxy clustering

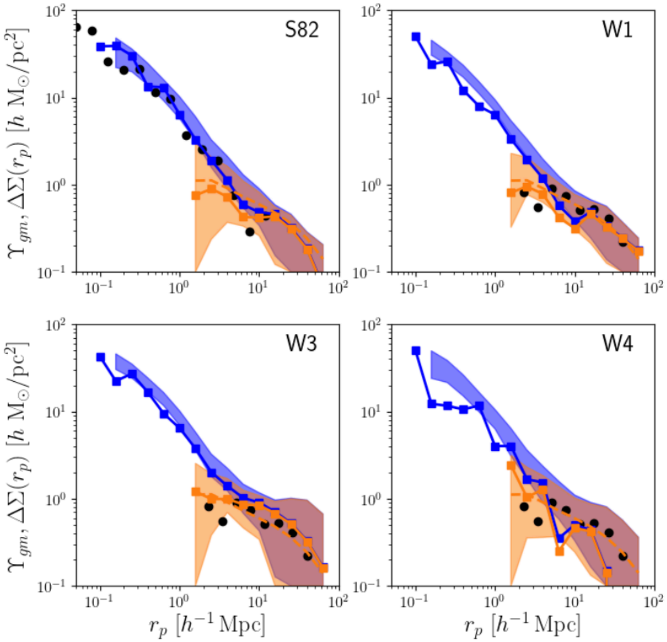


Fig. 9. Filtered Υ_{gm} and non-filtered $\Delta\Sigma$ GGL measurements with mocks (shaded regions), $\Delta\Sigma$ and Υ data (blue and cyan points respectively), and theory with a linear bias parameter $b_1 = 1.8$ (dashed line). Black dots in S82 panel represent $\Delta\Sigma$ measurements from L16, and Υ_{gm} measurements from Alam et al. (2016) in CFHTLS panels.

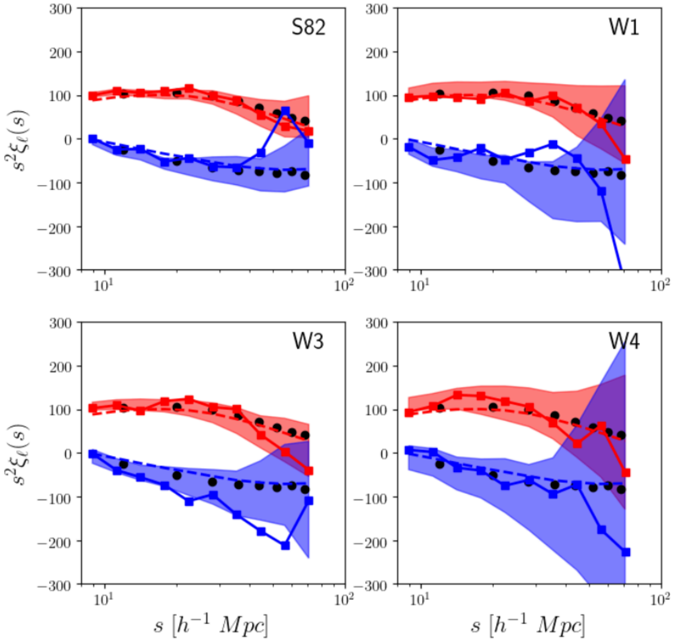


Fig. 8. Monopole (red) and quadrupole (blue) measurements with mock catalogs (shaded region), real data (solid lines) and theoretical predictions with a linear bias parameter $b_1 = 1.8$ (dashed lines). Black dots represent pre-reconstruction measurements with the full DR12v5 CMASS sample from Cuesta et al. (2016).

Jullo et al. (2019)

Motivation

From measurements to cosmological constraints :

Be $\vec{\theta}$ a vector containing parameters of a model

Be $\vec{x}$ an observable

$L(\vec{x} \mid \vec{\theta})$ likelihood $P(\vec{\theta} \mid \vec{x})$? posterior

$$P(\vec{\theta} \mid \vec{x}) = \frac{L(\vec{x} \mid \vec{\theta})\, P(\vec{\theta})}{P(\vec{x})}$$

$P(\vec{\theta}) \rightarrow$ prior

Motivation

Usual hypothesis: The likelihood is Gaussian

$$L(\vec{x}\,|\,\vec{\theta}) = \frac{1}{(2\pi)^{\frac{k}{2}}\sqrt{|G|}}\, \exp\left\{-\frac{1}{2}(\vec{x}-\vec{\mu})^T G^{-1}(\vec{x}-\vec{\mu})\right\}$$

where $\vec{\mu}=\vec{\mu}(\vec{\theta})$ is the expectation value given the model

G is the covariance matrix $(k \times k)$

minimisation of

$$\boxed{-2\ln\left[P(\vec{\theta}\,|\,\vec{x})\right] = \underbrace{\left[\vec{X}-\vec{\mu}(\vec{\theta})\right]^T G^{-1}\left[\vec{X}-\vec{\mu}(\vec{\theta})\right]}_{\equiv\ \chi^2} + \ln|G| - 2\ln P(\theta) + A}$$

precision matrix

Motivation

The key ingredient is the precision matrix :

ONE SHOULD

predict the precision matrix in any cosmological model

INSTEAD

estimate the covariance matrix from sample realisations in a given cosmological model

1) N-body simulations

2) Approximate methods

3) Monte Carlo realisations

$\gtrsim 1000$ realisations

Monte Carlo method

Problem:

The galaxy/matter density field is a non-gaussian field

i.e. $$P(\delta_1, \delta_2) \neq \frac{1}{2\pi |C|^{1/2}} \exp\left\{ -\frac{1}{2} (\delta_1, \delta_2)\, C^{-1} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \right\}$$

where $\delta_i \equiv \frac{\rho_i}{\bar{\rho}} - 1$ (density contrast)

the 1-point PDF is also non-gaussian

use a local transform to generate non-Gaussian PDFs

Monte Carlo method

Local transform (probability conservation):

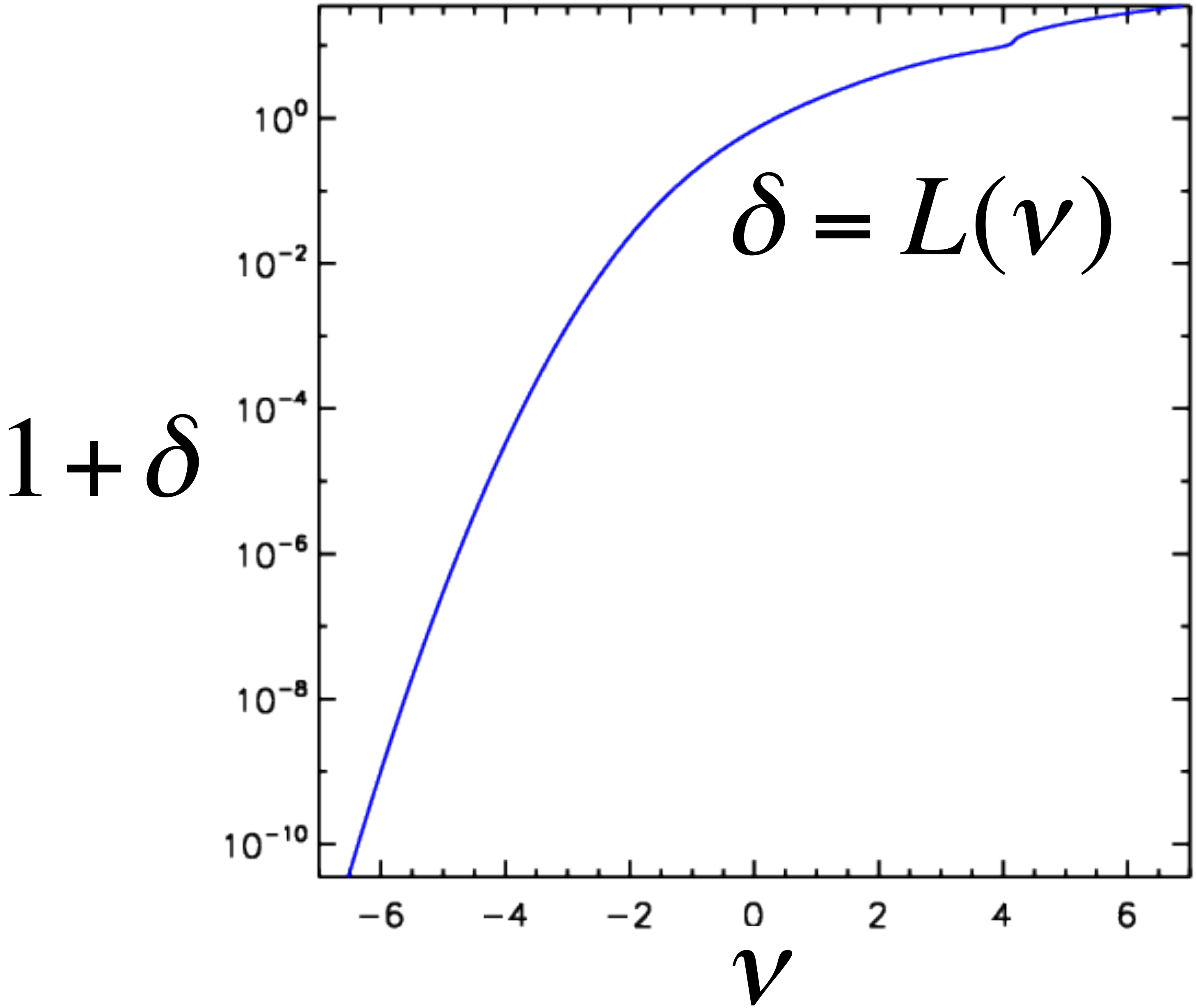
$$\int_{-1}^{\delta} P_{\Gamma}(\delta')\,d\delta' = \int_{-\infty}^{\nu} G(\nu')\,d\nu'$$

target PDF

Centred reduced Gaussian

$$\boxed{\delta = L(\nu)}$$

Monte Carlo method



Monte Carlo method

The Gaussian field must be specified in Fourier space through its power spectrum:

$$\nu_{\vec{k}} = \alpha_{\vec{k}} + i\,\beta_{\vec{k}}$$

where $V[\alpha_{\vec{k}}] = V[\beta_{\vec{k}}] = \frac{P_\nu(\vec{k})}{2} \longrightarrow$ Power spectrum

Minimum requirement:

match the power spectrum of the non-Gaussian density field

$P_\delta(\vec{k})$ or $\xi_\delta(\vec{r})$ (2-point correlation function)

Monte Carlo method

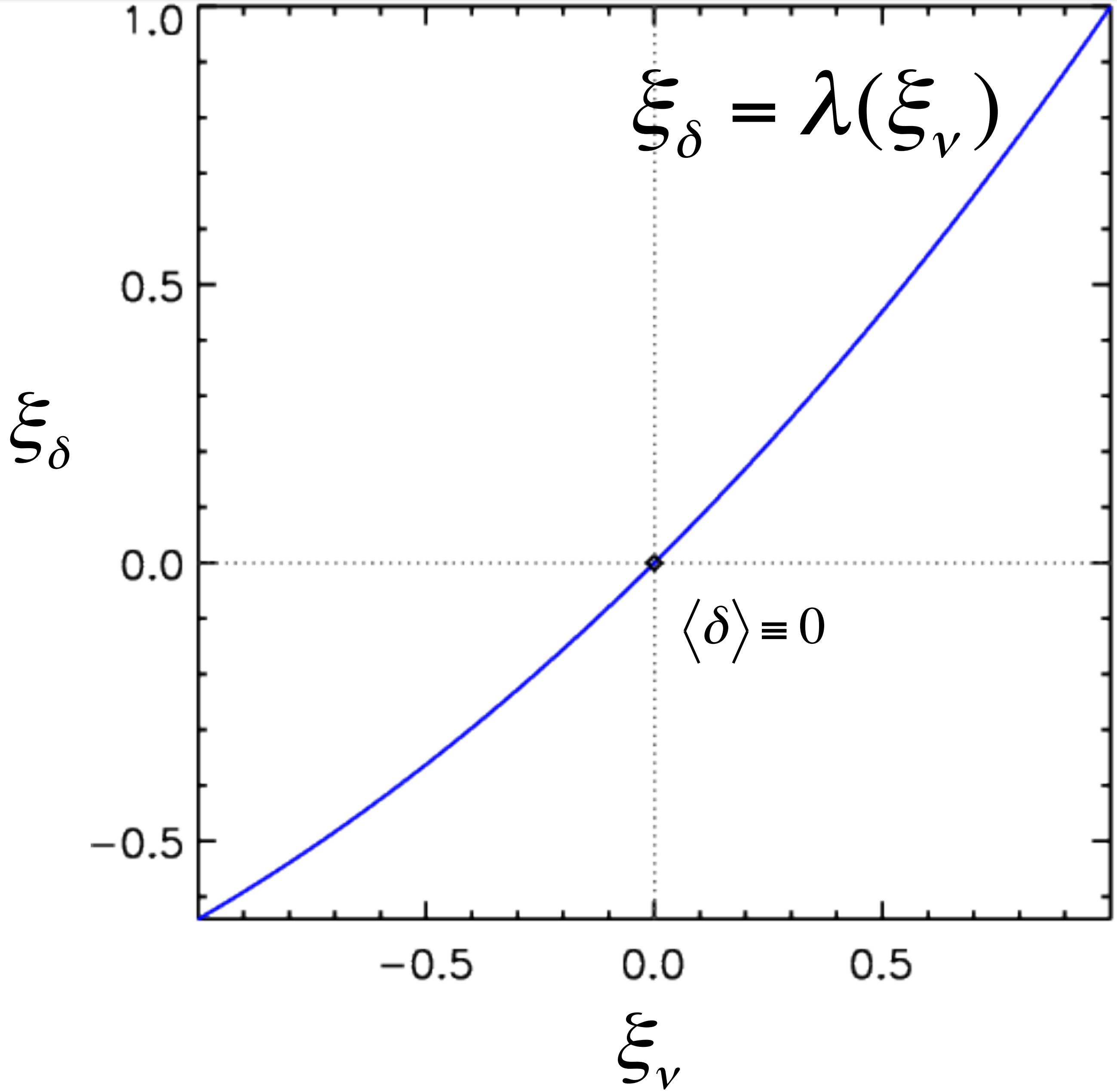
One can express the 2-point correlation as: $\xi_\delta \equiv \langle \delta_1 \, \delta_2 \rangle$

$$\xi_\delta = \int L(\nu_1) L(\nu_2) B(\nu_1, \nu_2, \xi_\nu) d^2\vec{\nu}$$

$B(\nu_1, \nu_2, \xi_\nu)$: Centred reduced bivariate Gaussian

$$\boxed{\xi_\delta = \lambda(\xi_\nu)}$$

Monte Carlo method



Monte Carlo method

Be $P(k)$ the power spectrum of the galaxy field

$$\xi_\delta(r) = 4\pi \int_0^\infty k^3 P(k) \frac{\sin(kr)}{kr} d\ln k$$

$$\xi_\nu = \lambda^{-1}(\xi_\delta)$$

$$P_\nu(k) = \frac{1}{2\pi^2} \int_0^\infty r^3 \xi_\nu(r) \frac{\sin(kr)}{kr} d\ln r$$

Monte Carlo method

Be $P(k)$ the power spectrum of the galaxy field

$$\xi_\delta(r) = 4\pi \int_0^\infty k^3 P(k) \frac{\sin(kr)}{kr} d\ln k$$

$$\xi_\nu = \lambda^{-1}(\xi_\delta)$$

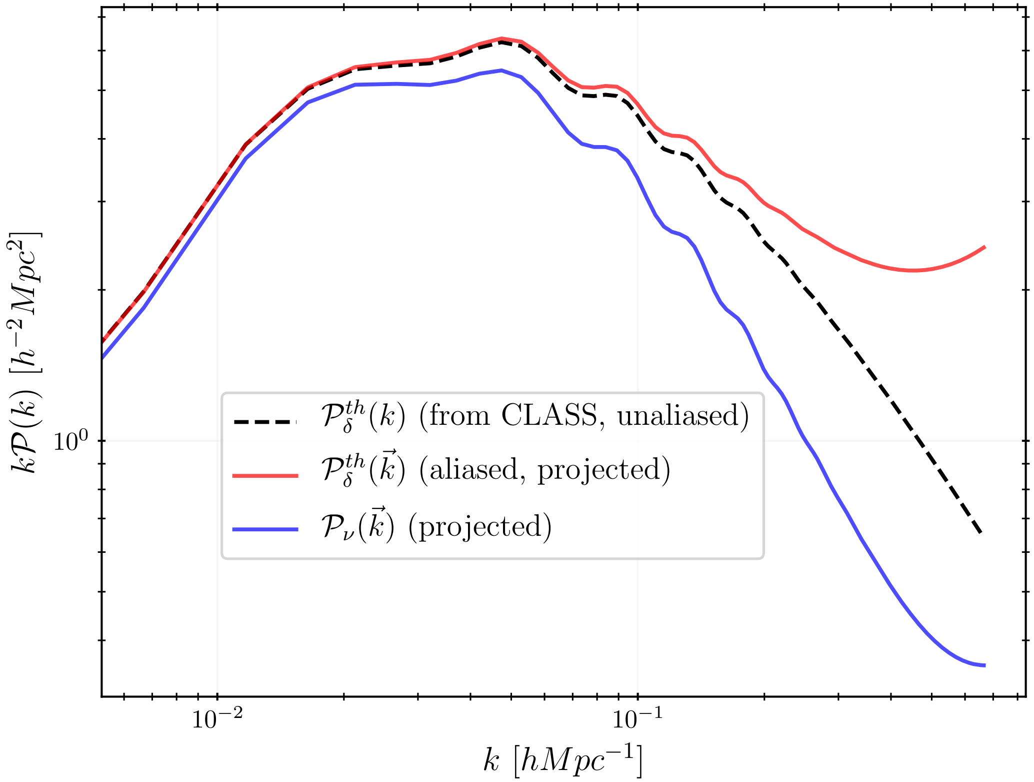
Key: 3D Fourier transform and aliasing

Monte Carlo method

Baratta et al. (2020)

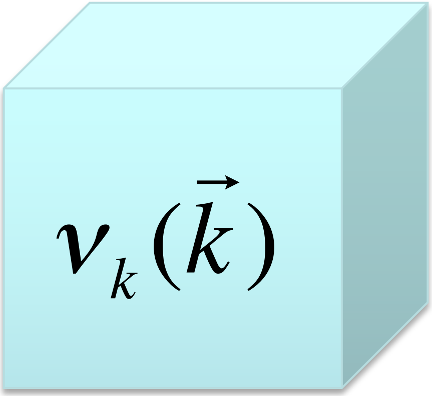
We adopted the following pipeline:

$$P_\delta^{th}(k) \xrightarrow[\text{eq.(11)}]{\text{3D aliasing}} P_\delta^{th}(\overrightarrow{k}) \xrightarrow{FFT^{-1}} \xi_\delta^{th}(\overrightarrow{x}) \xrightarrow[\text{eq.(B.12)}]{\lambda^{-1}} \xi_\nu(\overrightarrow{x}) \xrightarrow{FFT} P_\nu(\overrightarrow{k})$$



Monte Carlo method

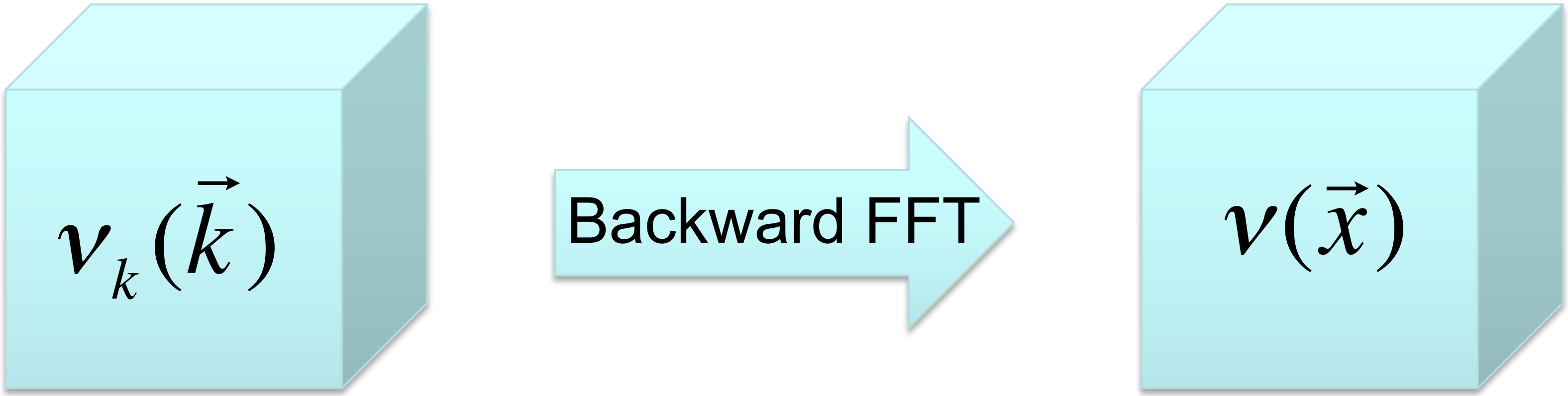
3-D mesh in Fourier space



$$v_k(\vec{k})$$

Box of size L

Monte Carlo method



3-D mesh in Fourier space

$v_k(\vec{k})$

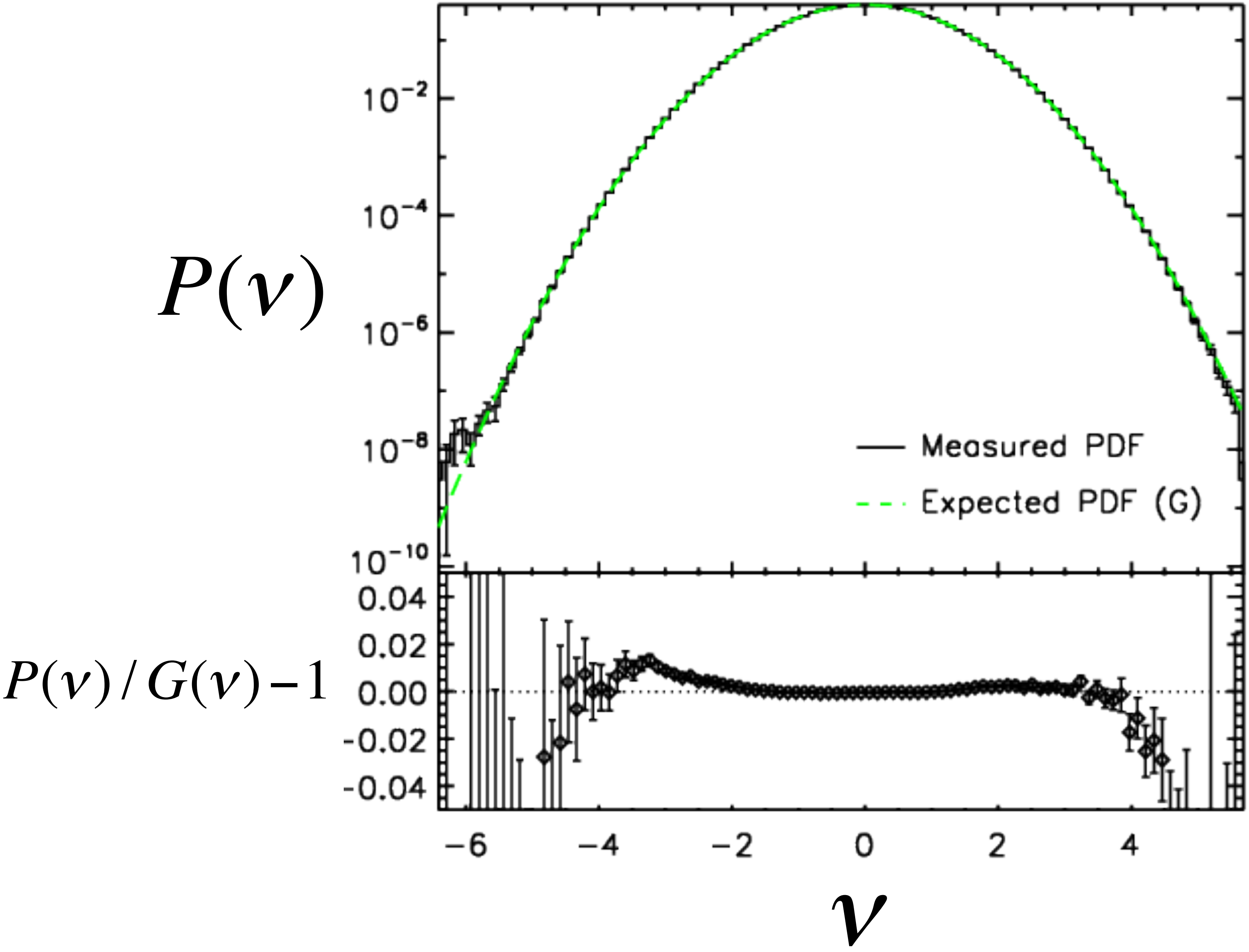
Backward FFT

3-D mesh in real space

$v(\vec{x})$

Box of size L

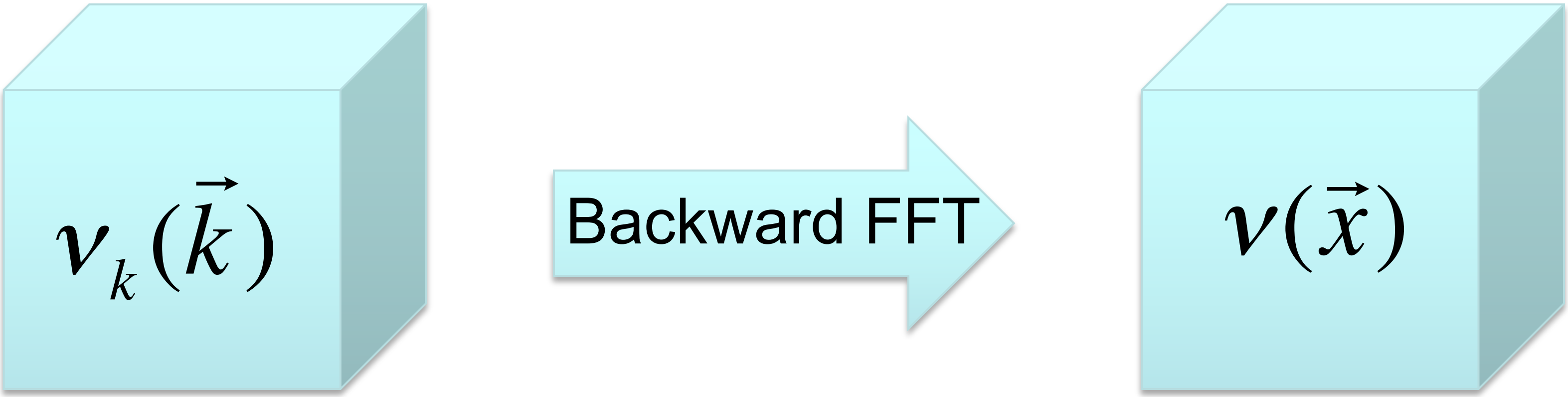
Monte Carlo method



Monte Carlo method

3-D mesh in Fourier space

3-D mesh in real space



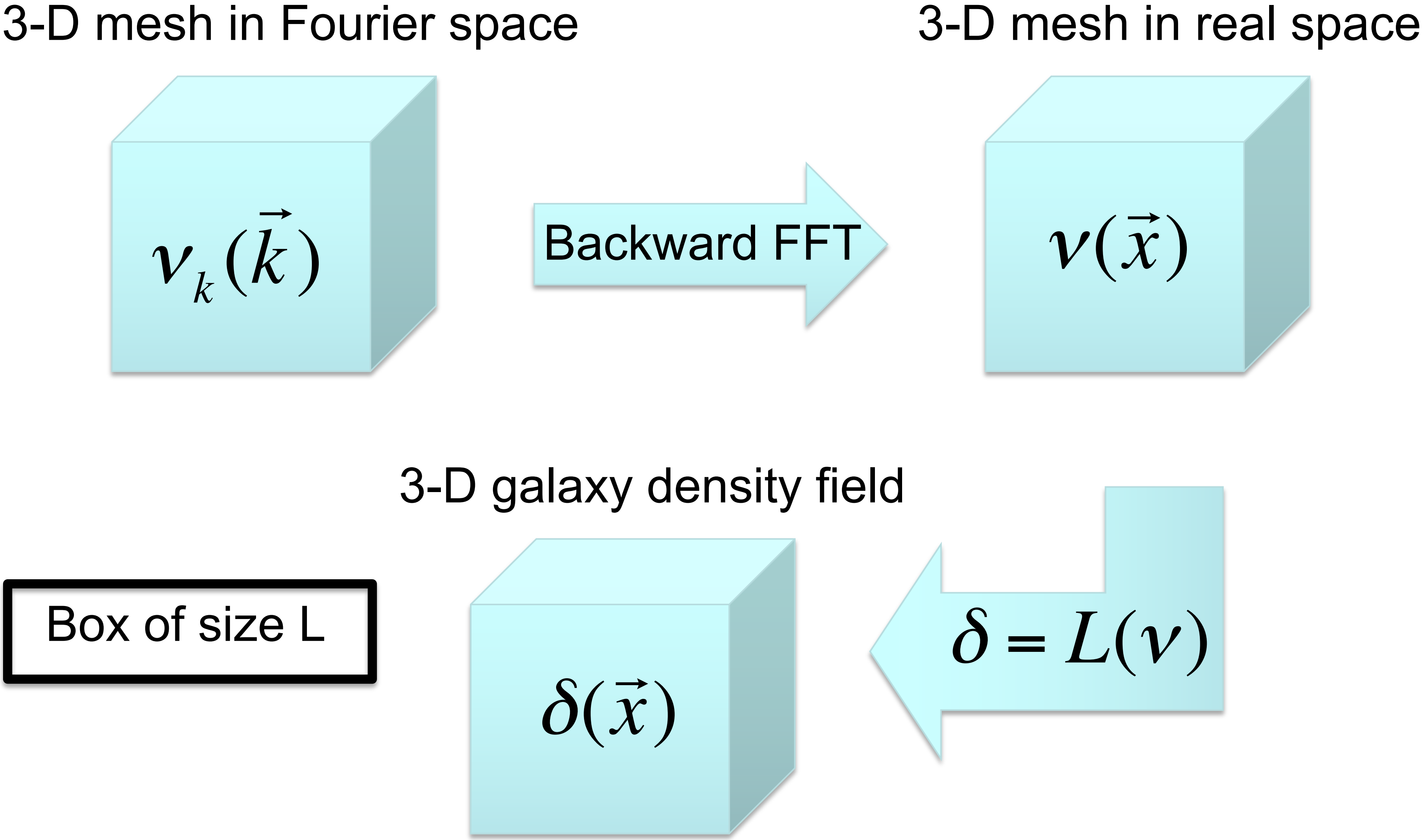
$v_k(\vec{k})$

Backward FFT

$v(\vec{x})$

Box of size L

Monte Carlo method



3-D mesh in Fourier space

$\nu_k(\vec{k})$

Backward FFT

3-D mesh in real space

$\nu(\vec{x})$

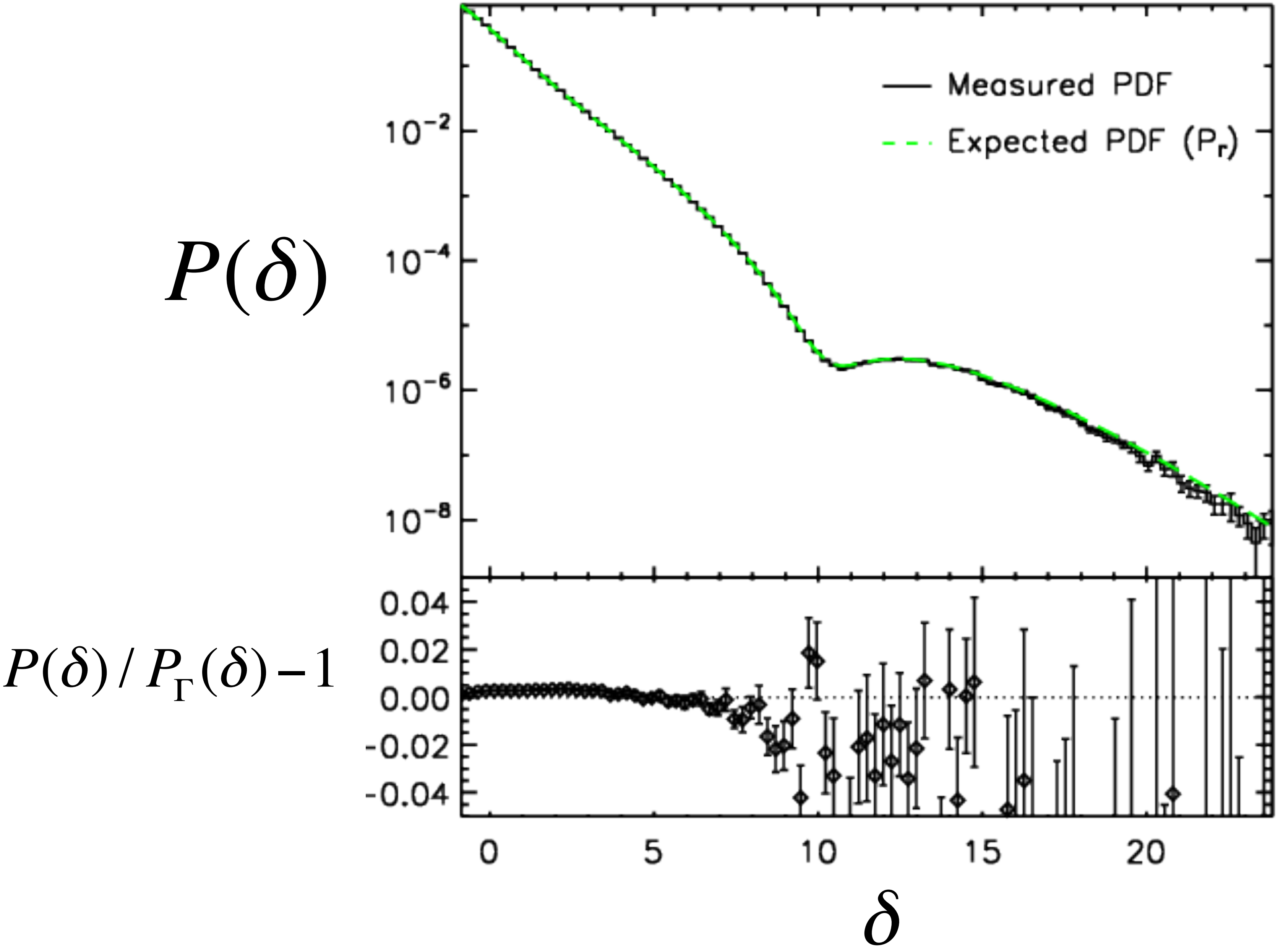
$\delta = L(\nu)$

3-D galaxy density field

$\delta(\vec{x})$

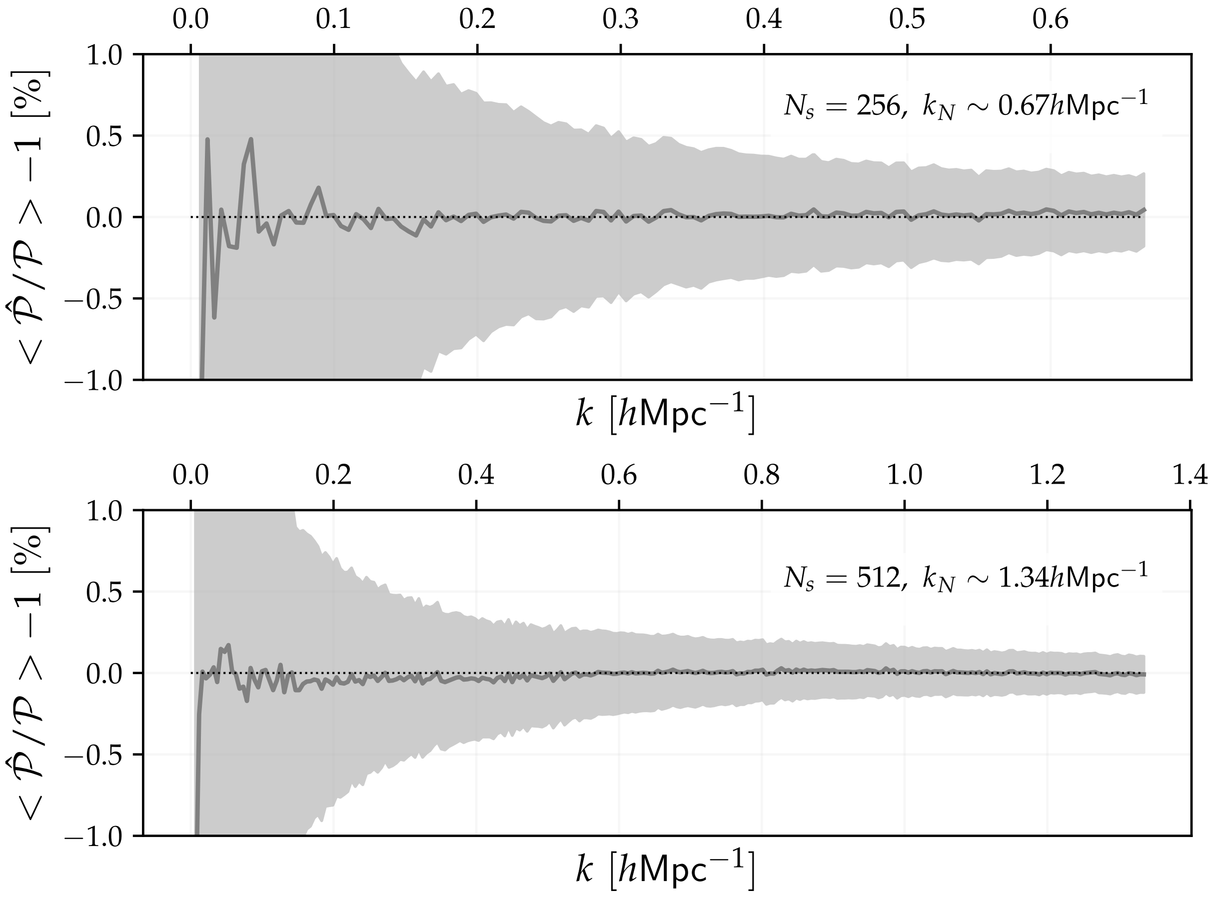
Box of size L

Monte Carlo method



Monte Carlo method

with 10 000 realisations



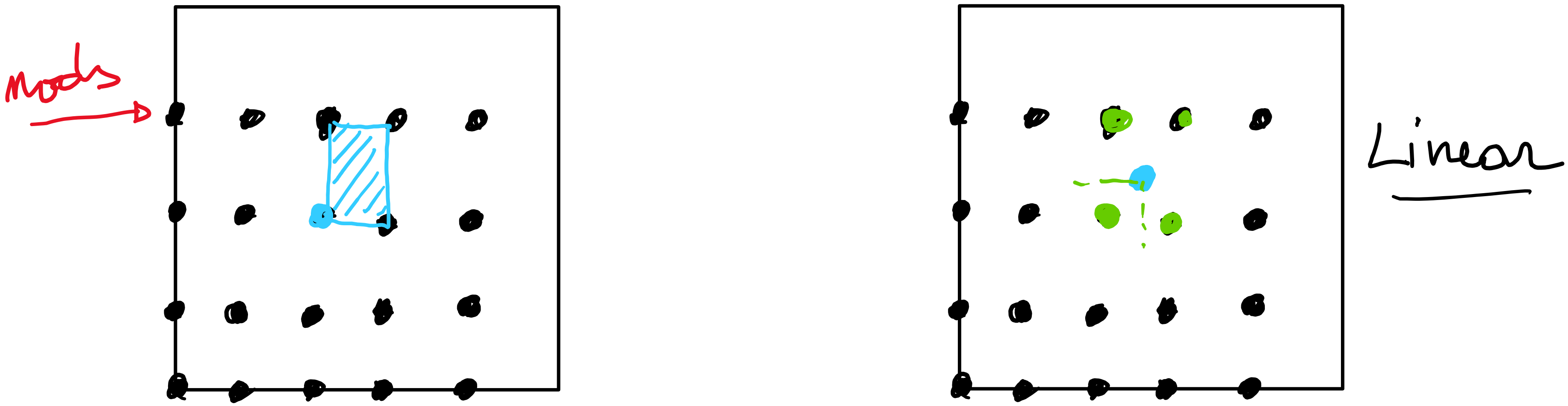
Baratta et al. (2020)

Monte Carlo method

Point process:
populate the density field with objects

Local Poisson process approximation: $P\left[N|\Lambda\right]=\frac{\Lambda^N}{N!}e^{-\Lambda}$ *(Layser 1956)*

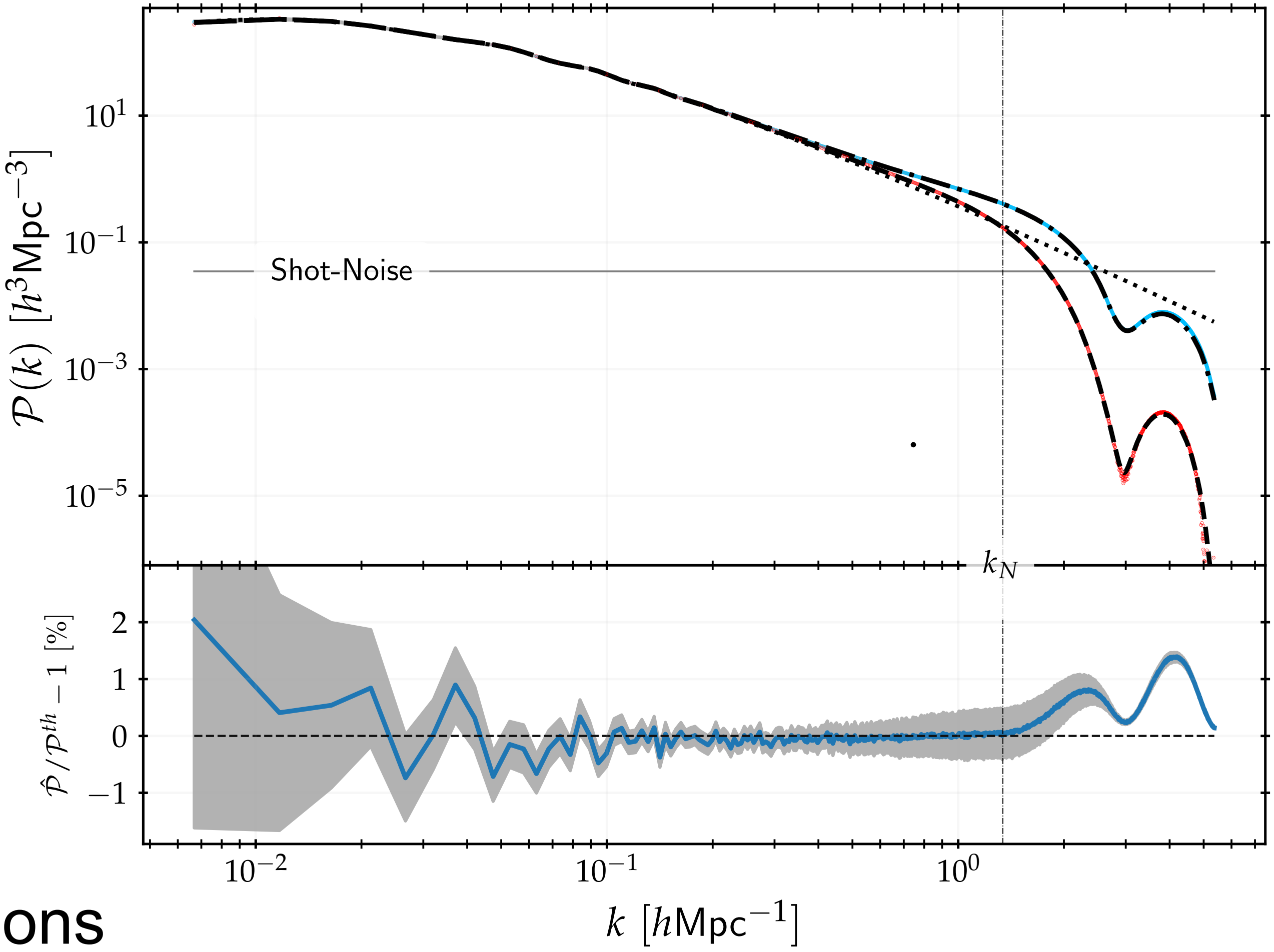
$$\Lambda=\int_{\text{cell}}\rho(\vec{x})\,d^3\vec{x}$$



Monte Carlo method

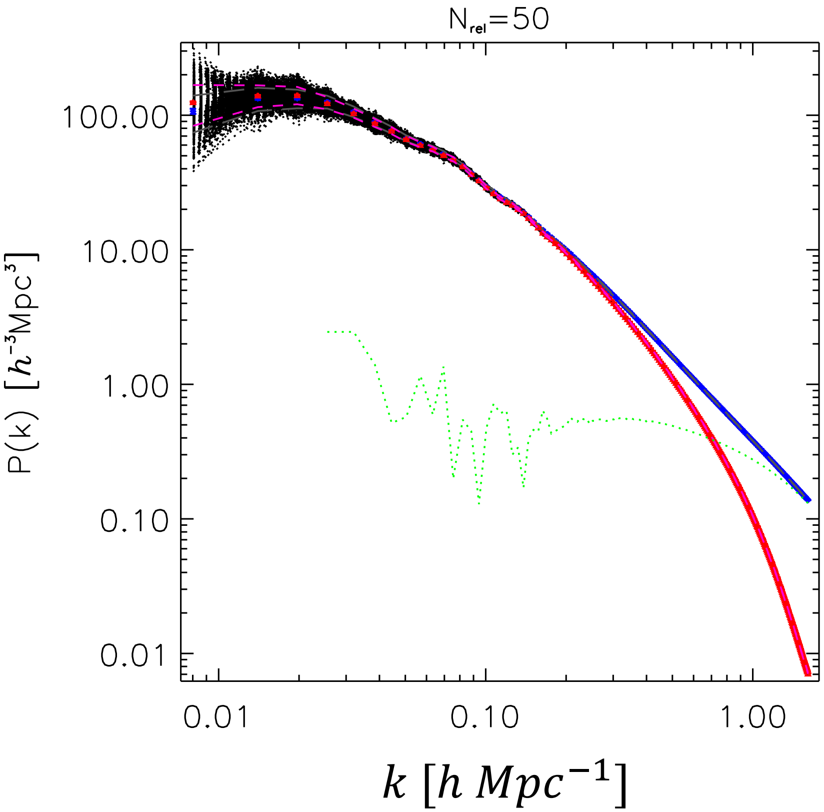
Point process:
populate the density field with objects

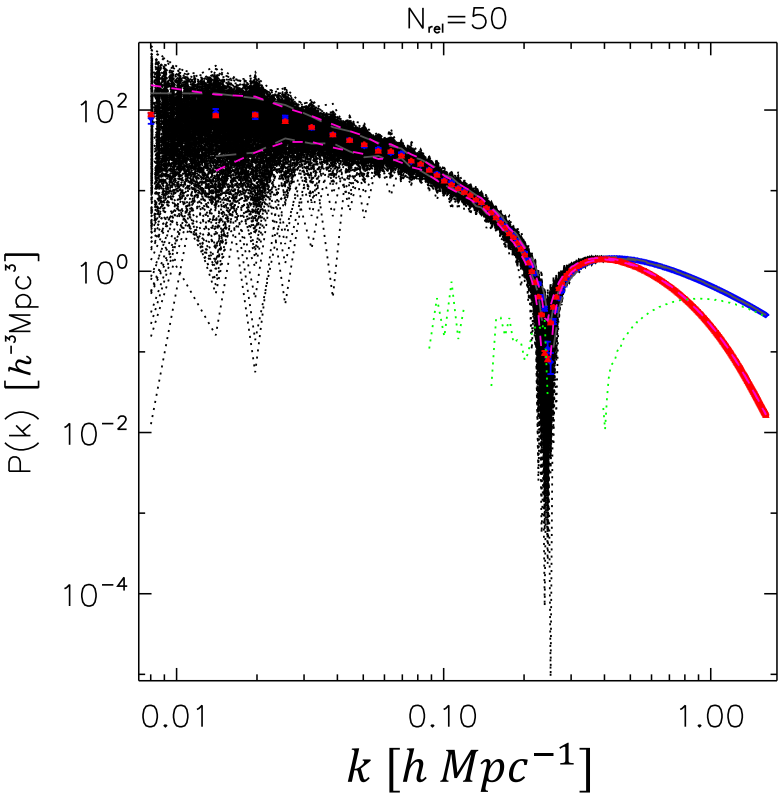
Baratta et al. (2020)



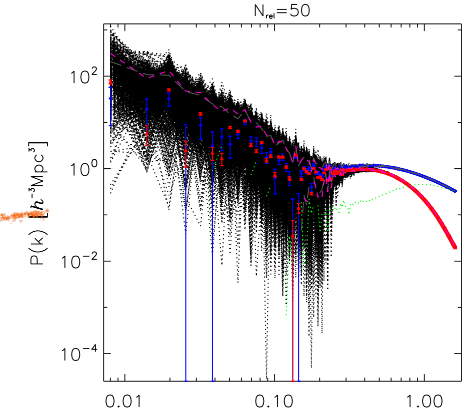
100 realisations

Redshift space power spectrum





Preliminary

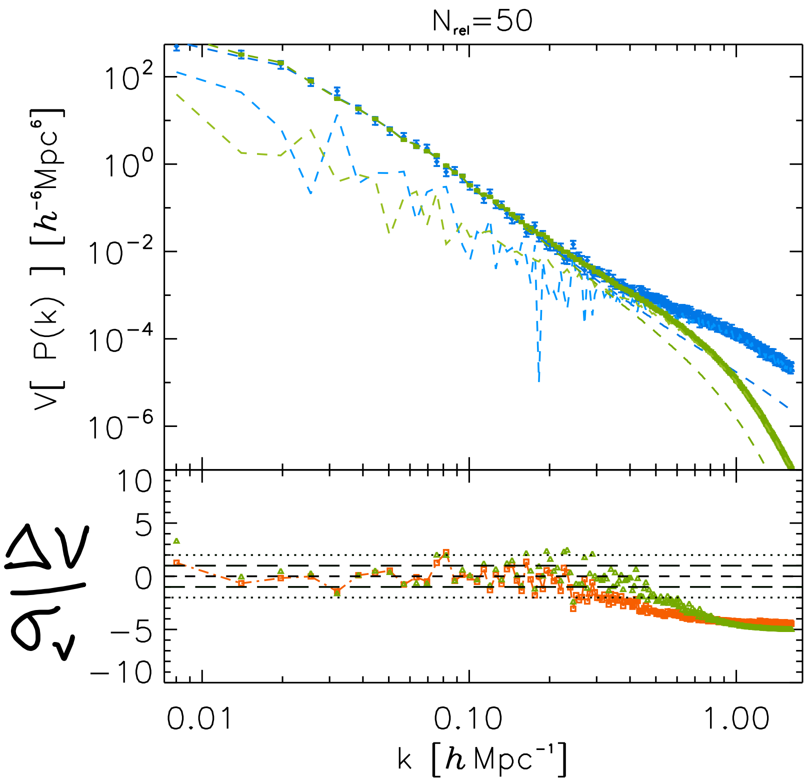


1000 realisations

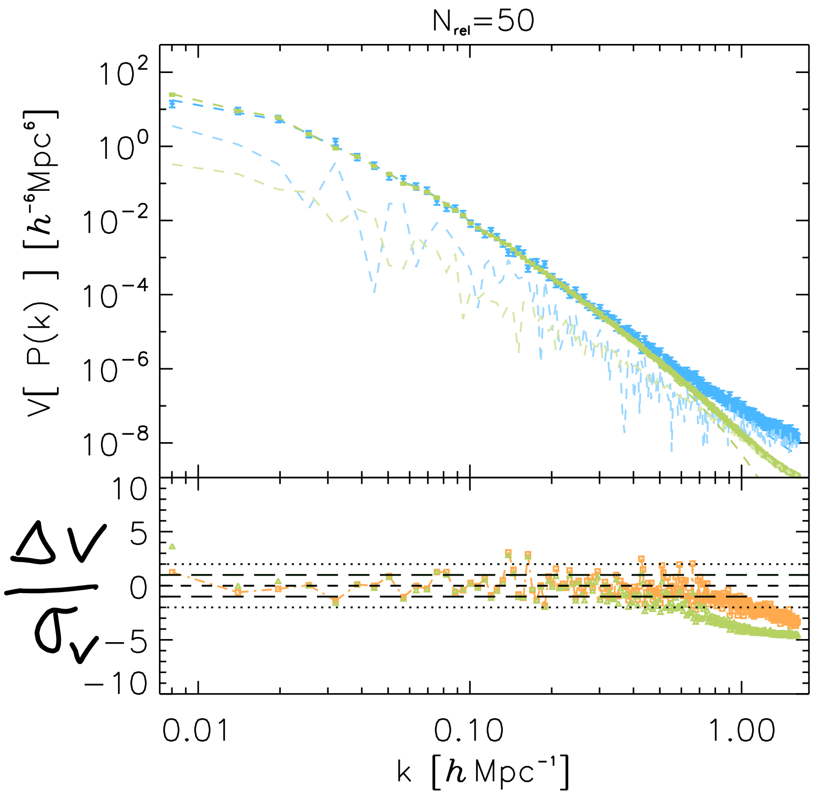
Power spectrum covariance matrix

Comparison with the DEMNUni series of 50 N-body simulations (Carbone et al. 2016).

Preliminary

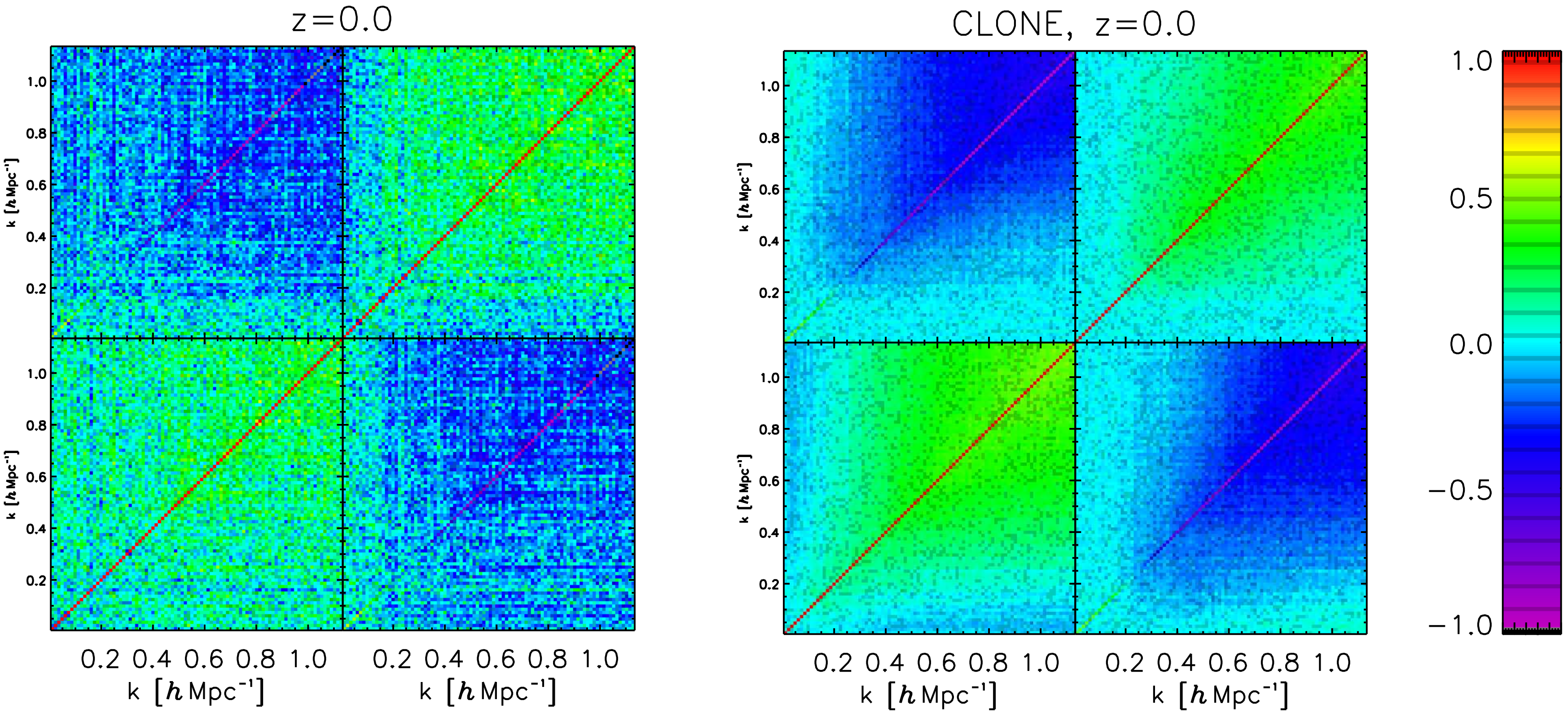


$z = 0$



$z = 2$

Redshift space covariance matrix



$P^{(0)}_{(k)}$ $P^{(2)}_{(k)}$ $P^{(0)}_{(k)}$ $P^{(2)}_{(k)}$

Preliminary

50 DEMNUni

1000 CLONES

In a nutshell:

1) The covariance matrix is a key ingredient to constrain cosmological parameters
2) We developed a Monte Carlo method to estimate the covariance matrix of clustering observables *(Baratta et al. 2020)* $P(k)$; $\xi(r)$; $C_\ell(z,z')$
3) We have shown that we are able to predict the expected power spectrum even on scales below Nyquist *(Baratta et al. 2020)*
4) We are comparing the method to N-body simulations in real and in redshift space
5) We are testing how the Gaussian hypothesis for the likelihood could affect cosmological inferences *(Euclid Work Package)*
6) Extend the Monte Carlo to spatially curved universes
7) Study the cosmological dependence of the covariance

DEMNUni simulations (phase II)

- 8×10^6 cpu-hours on BGQ/FERMI at CINECA (PI: C. Carbone)
- 10 mixed dark matter cosmological simulations for CMB and LSS analysis in the presence of evolving dark-energy (w_0, w_a) and massive neutrinos
- Baseline Planck cosmology
- Gadget-3 with ν-particle component (Viel et al. 2010)
- box-side size: 2 Gpc/h
- particle number: 2×2048^3 (CDM+ν)
- CDM mass: $8 \times 10^{10}\ M_\odot/h$ (neutrino particle mass depends on M_ν)
- softening length: 20 kpc/h
- starting redshift: $z_{in}=99$

$$k_{\rm nr} \;=\; 0.018(m_\nu/1{\rm eV})^{1/2}\Omega_m^{1/2}h/{\rm Mpc}$$

Interest:

the full hierarchy of N-point correlation functions can be predicted

$$\langle\delta_1...\delta_N\rangle = \int \mathrm{L}(\nu_1)...\mathrm{L}(\nu_N)\mathcal{B}^{(N)}(\vec{\nu},C_\nu)\mathrm{d}\nu_1...\mathrm{d}\nu_N,$$

where $B^{(N)}\,(\vec{v},C) = \frac{1}{(2\pi)^{N/2}\left|C\right|^{1/2}}\exp\left\{-\frac{1}{2}\vec{v}^{\mathrm{T}}C\vec{v}\right\}$

and $C_{ij} = \xi_\nu(x_i,x_j)$

$$C_{ij} = \frac{P(k_i)^2}{M_{k_i}}\delta^K_{ij} + k_F^3\bar{T}(k_i,k_j),$$

$$\bar{T}(k_i,k_j) = \int_{k_i}\int_{k_j} T(\vec{k}_1,-\vec{k}_1,\vec{k}_2,-\vec{k}_2)\frac{\mathrm{d}^3\vec{k}_1}{V_{k_i}}\frac{\mathrm{d}^3\vec{k}_2}{V_{k_j}},$$