The muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

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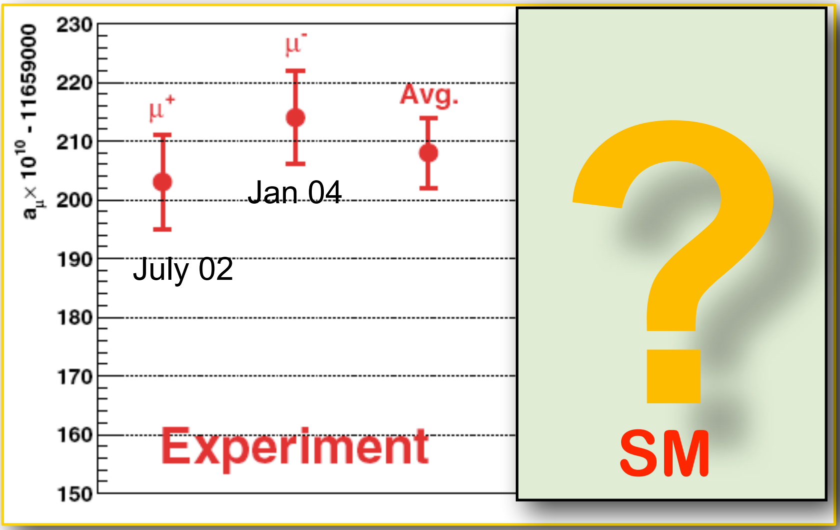
INFN Padova

Virtual Particle Physics in Paris (VP^3) Seminar
March 23rd 2021

- **Muon g-2: recent theory progress**
- **Muon g-2 $\Longleftrightarrow$ Δα connection**
- **The MUonE project**

Muon g-2: experimental status



- **BNL 821**: $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys})\times10^{-11}$ [0.5ppm].
- **Fermilab E989**: new muon g-2 experiment aims at $\pm16\times10^{-11}$ [0.14ppm]. First 3 data taking completed. First result expected on April 7th.
- **J-PARC**: Muon g-2 proposal. Phase-1 with ~ BNL precision.

Muon g-2: recent theory progress

White Paper of the Muon g-2 Theory Initiative:

arXiv:2006.04822

Muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

$+ \ 24.05050988\ (28)\ (\alpha/\pi)^3$

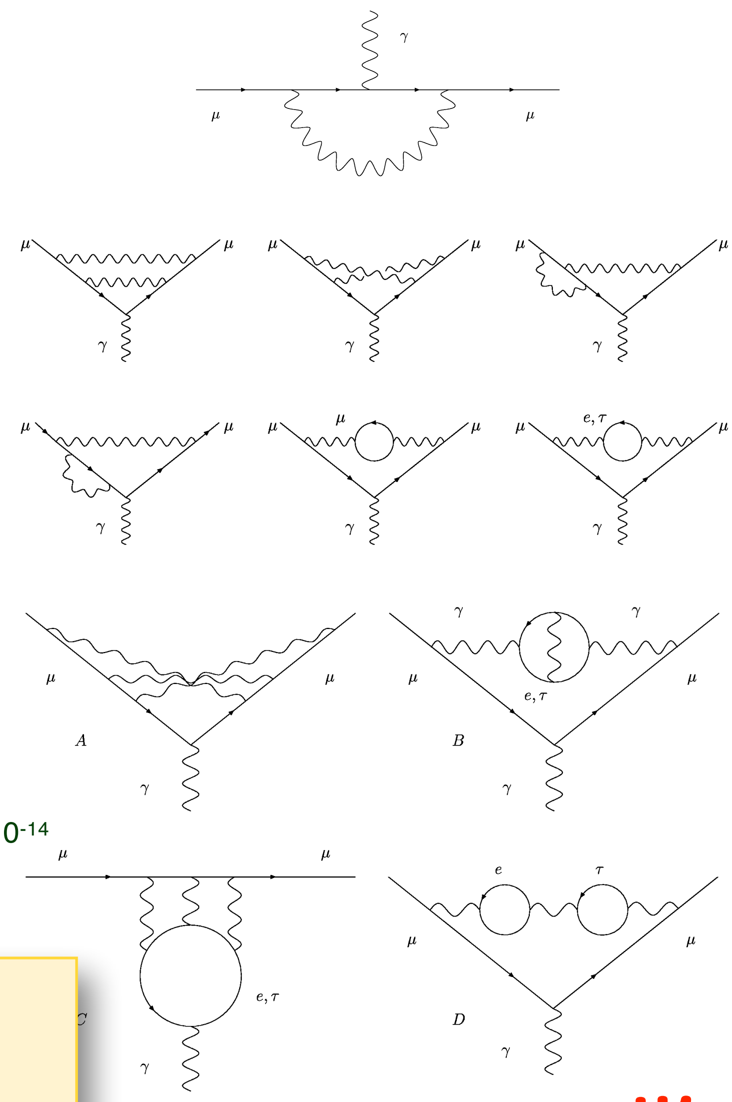
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP ’04;
Friot, Greynat & de Rafael ’05, Mohr, Taylor & Newell 2012

$+ \ 130.8780\ (60)\ (\alpha/\pi)^4$

Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term). COMPLETED²!

$+ \ 750.86\ (88)\ (\alpha/\pi)^5$ COMPLETED!

Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible $\delta a_\mu \sim 6\times10^{-14}$



Adding up, we get:

$a_\mu^{QED} = 116584718.931\ (19)(100)(23) \times 10^{-11}$

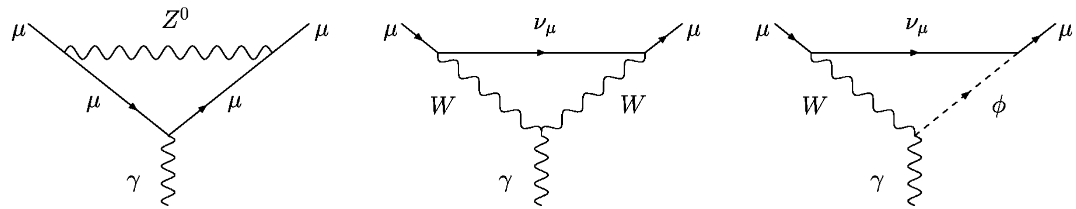
from 4-loop & 5-loop coeffs unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018

Muon g-2 TI WP value

Shift down in $a_\mu^{QED} \sim 1.4 \times 10^{-12}$ with new LKB Paris α(Rb) value (Morel et al 2020)

- **One-loop term:**



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

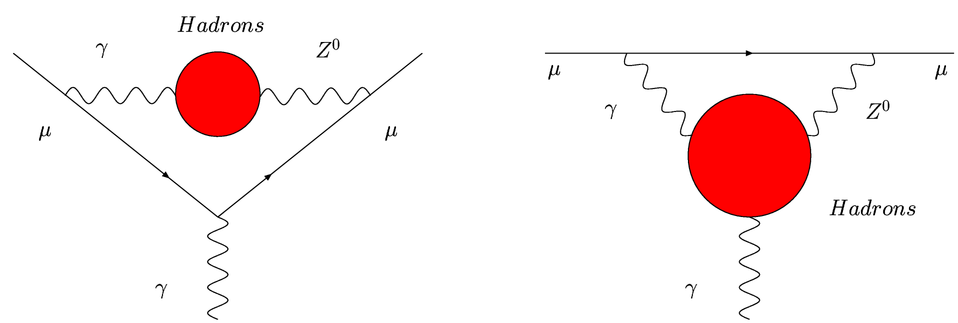
- **One-loop plus higher-order terms:**

$a_\mu^{\text{EW}} = 153.6\ (1.0) \times 10^{-11}$

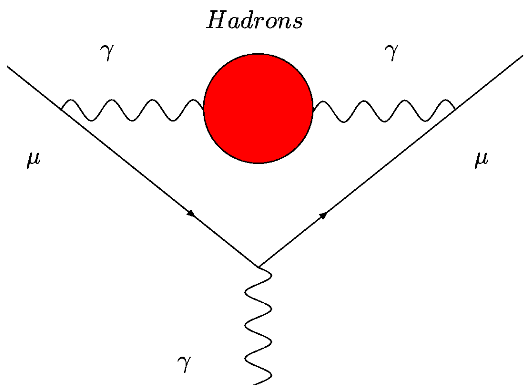
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

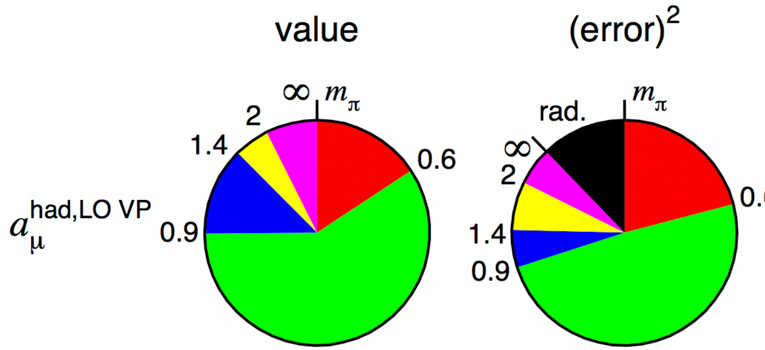
Hadronic loop uncertainties (and 3-loop nonleading logs).

Muon g-2 TI WP: arXiv:2006.04822



The hadronic LO contribution





Keshavarzi, Nomura, Teubner 2018

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds \, K(s) \, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

a_μ^{HLO} = 6895 (33) x 10^{-11} F. Jegerlehner, arXiv:1711.06089

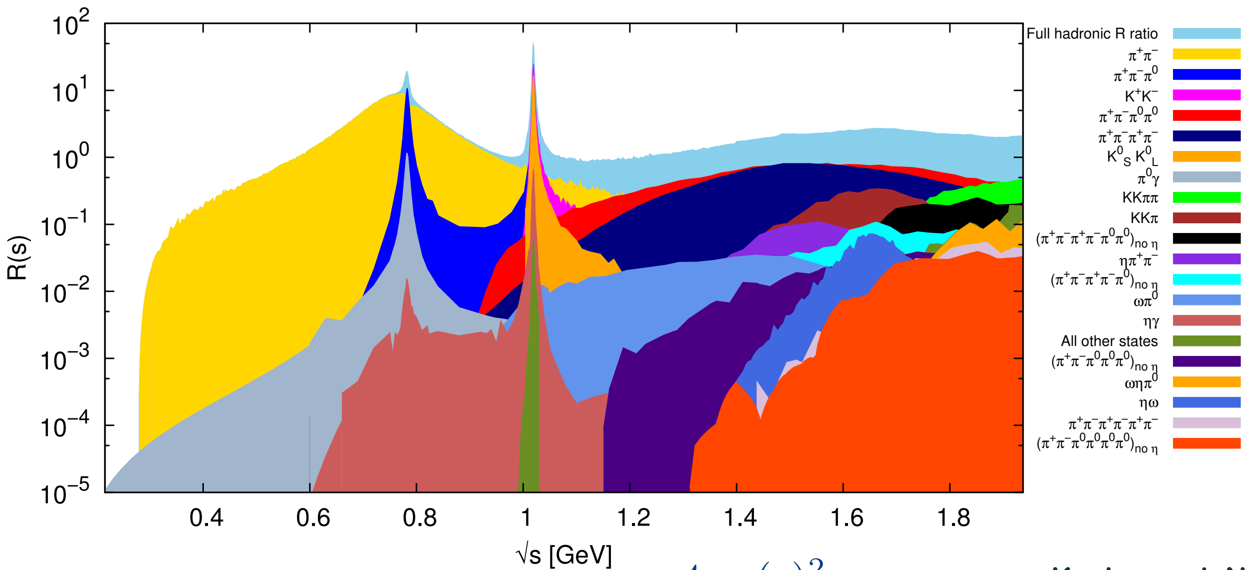
= 6939 (40) x 10^{-11} Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

= 6928 (24) x 10^{-11} Keshavarzi, Nomura, Teubner, arXiv:1911.00367

= 6931 (40) x 10^{-11} (0.6%) Muon g-2 TI WP: arXiv:2006.04822

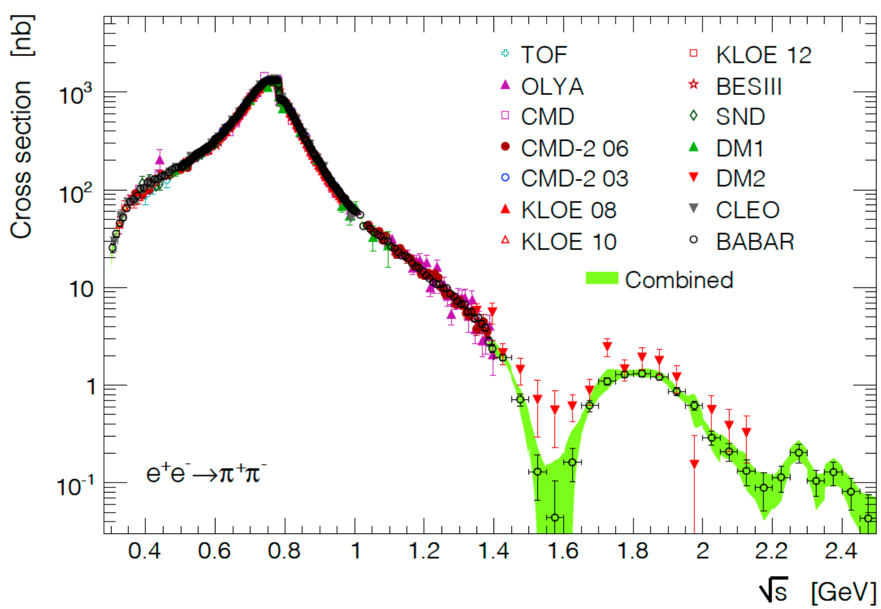
- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Great progress in lattice QCD results. BMW 2020 result with 0.7% precision: a_μ^{HLO} = 7087(53)x10^{-11}. Tension with dispersive evaluations.** S. Borsanyi et al. 2002.12347.

The low-energy hadronic cross section



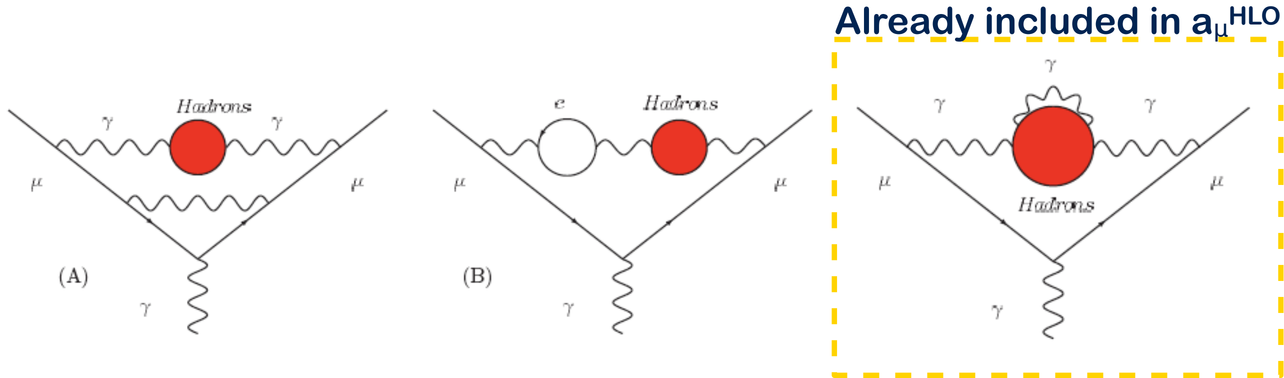
$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner
PRD 2018



Davier, Hoecker, Malaescu, Zhang
EPJC 2020

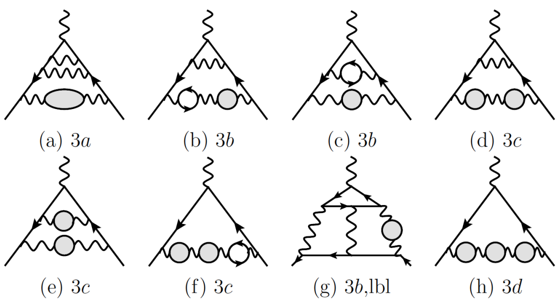
- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:



$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$

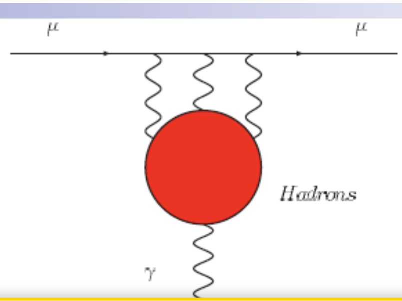
Krause '96; Keshavarzi, Nomura, Teubner 2019; Muon g-2 TI WP.

- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:



$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$

Kurz, Liu, Marquard, Steinhauser 2014



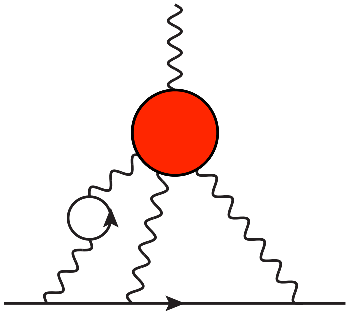
- **Hadronic light-by-light at O(α^3)**
  - This term had a troubled life! But nowadays:

$a_\mu^{HNLO}(lbl) =$	$+ 80\ (40) \times 10^{-11}$	Knecht & Nyffeler ’02
$=$	$+136\ (25) \times 10^{-11}$	Melnikov & Vainshtein ’03
$=$	$+105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein ’09
$=$	$+ 100\ (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263
$=$	$+ 92\ (19) \times 10^{-11}$	Muon g-2 TI WP, 2006.04822

  - Significant improvements due to data-driven dispersive approach.
  - Great progress on the lattice! RBC/UKQCD result: $79(35)\times10^{-11}$ arXiv:1911.08123

- **Hadronic light-by-light at O(α^4)**

$$a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}$$



Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; Muon g-2 TI WP, 2006.04822

Comparing the SM prediction with the measured muon g-2 value:

$$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$$

BNL E821

$$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$$

Muon g-2 TI

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 279\ (76) \times 10^{-11}$$

3.7 σ

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$ $\Delta\alpha_{had}^{(5)} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

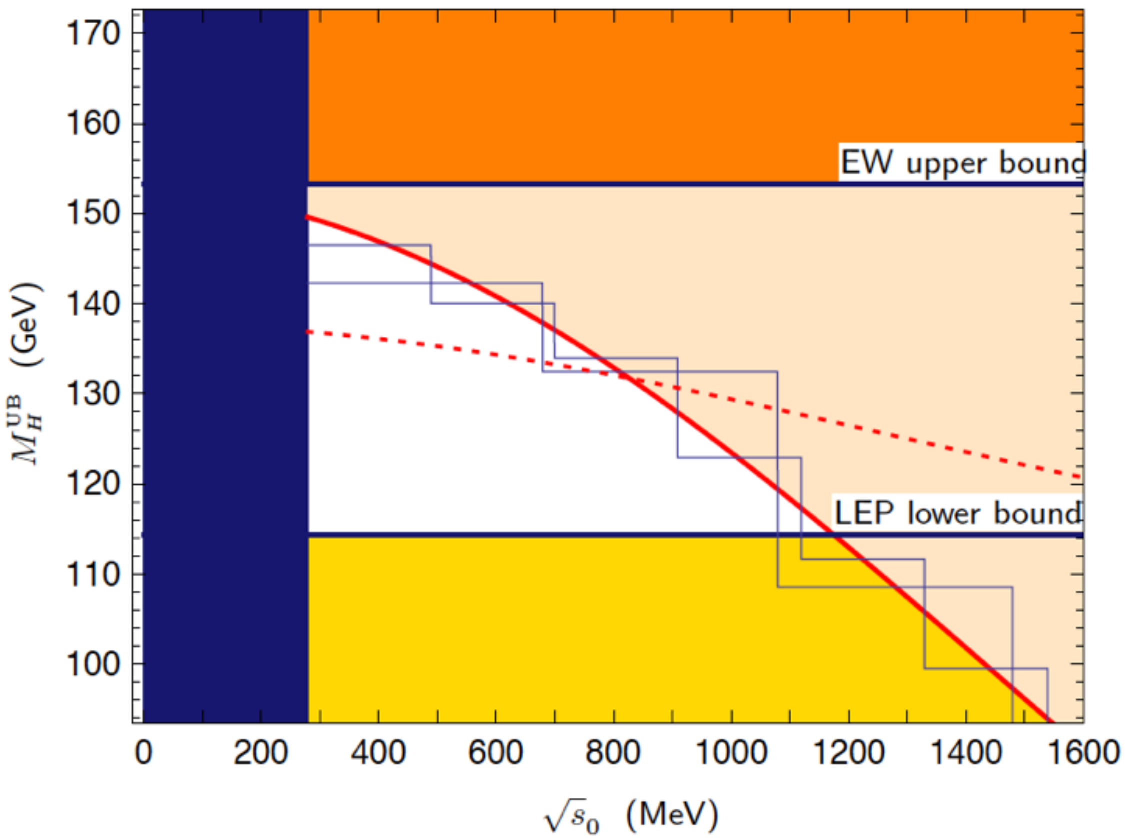
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ε>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \longrightarrow$$

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



Marciano, MP, Sirlin, PRD 2008

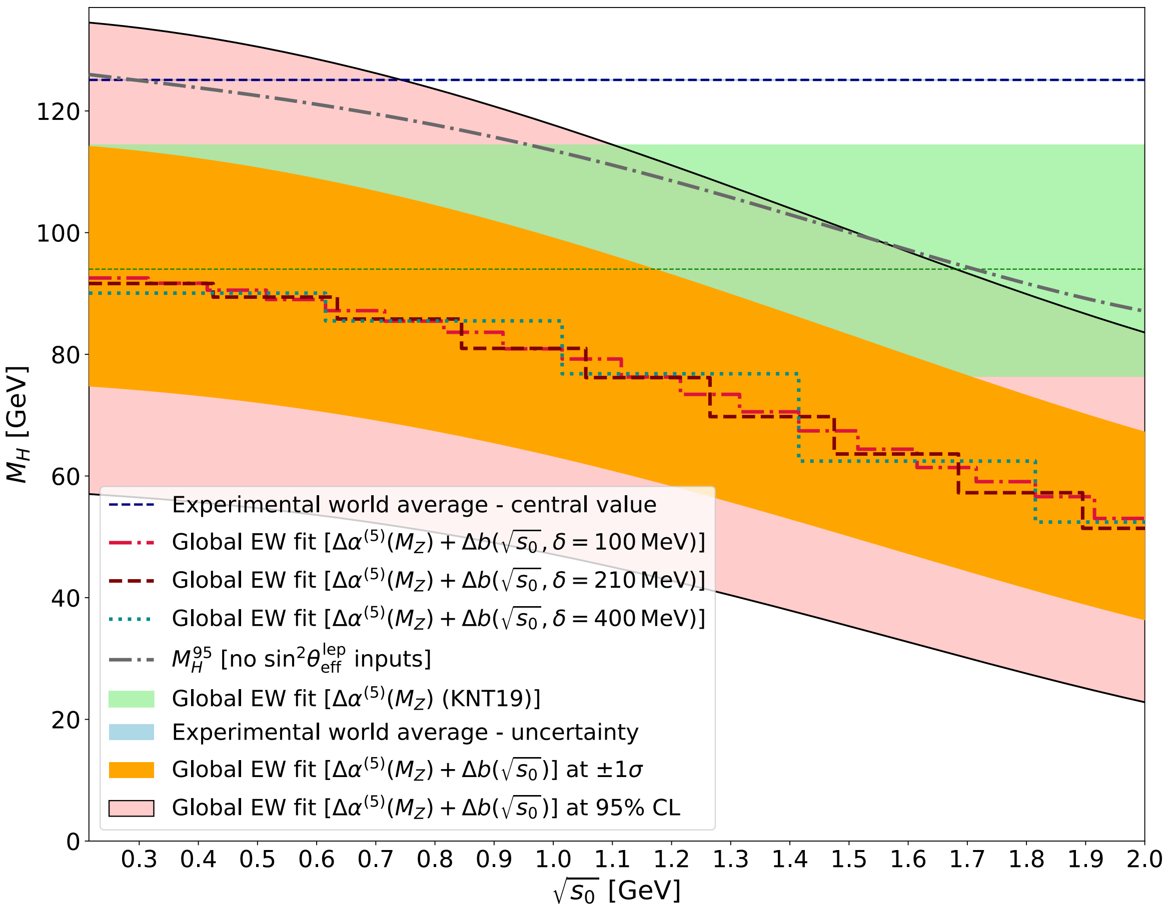
Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to $\sigma(s)$, theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	…
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	…
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	…
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	…
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	…
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	…
R^0_l	20.767(25)	[108]	20.732(4)	…
R^0_c	0.1721(30)	[108]	0.17222(8)	…
R^0_b	0.21629(66)	[108]	0.21581(8)	…
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	…
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	…
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	…
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	…
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	…
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	…
A_c	0.670(27)	[108]	0.6679(2)	…
A_b	0.923(20)	[108]	0.93462(7)	…
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

Muon g-2: connection with the SM Higgs mass (2020)

Δα

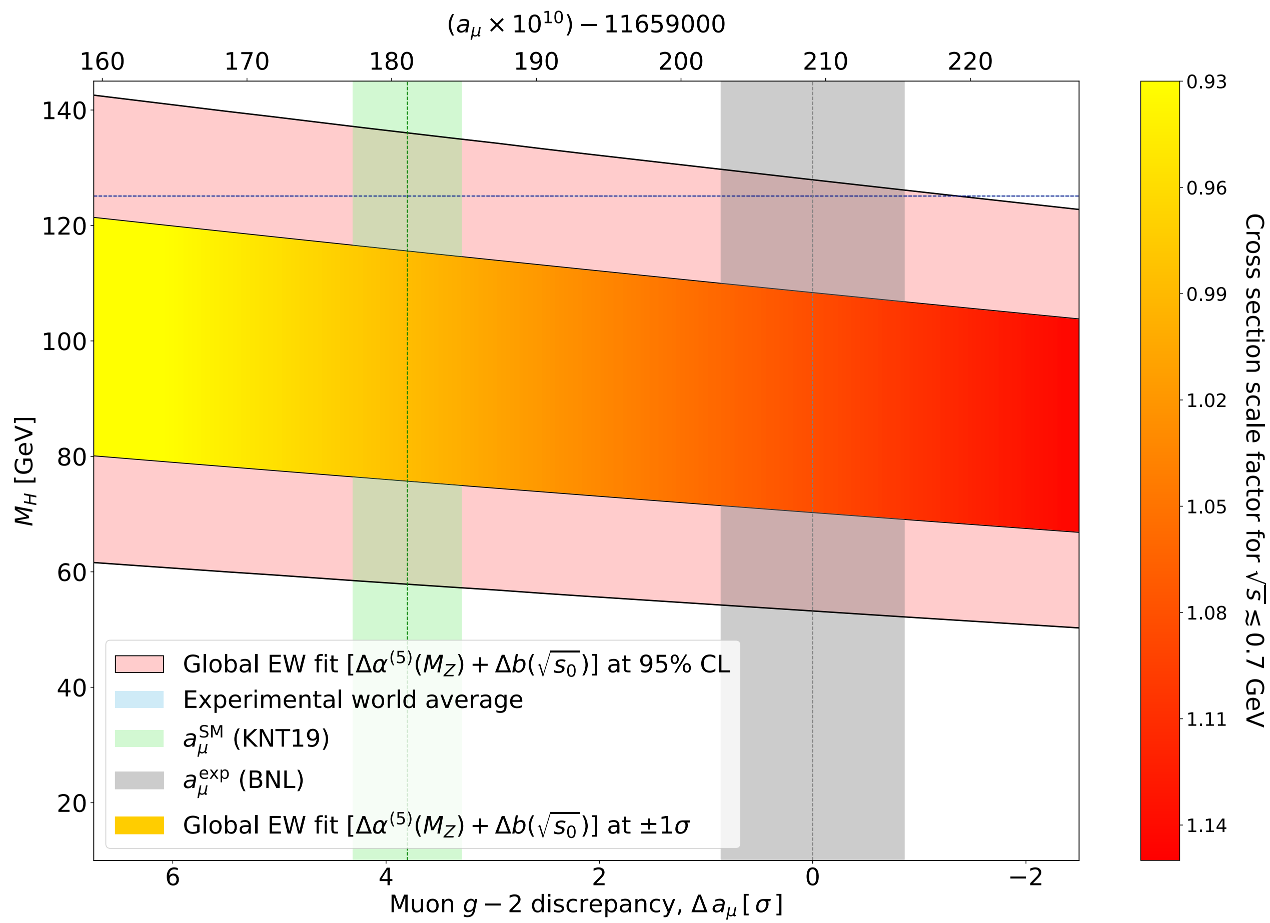


Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

The muon g-2: connection with the SM Higgs mass (2020) - II

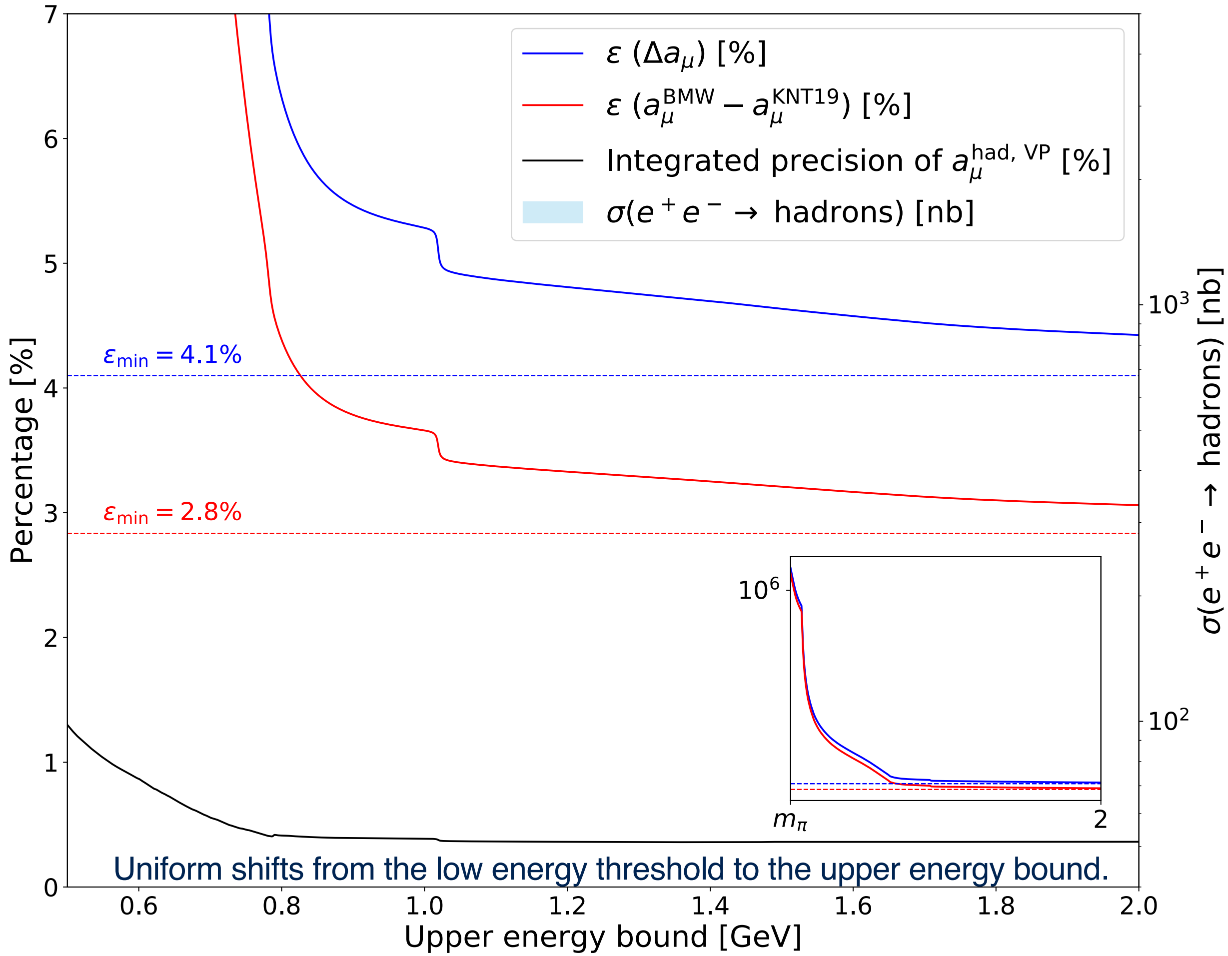
Δα



Uniform scaling of σ(s) below ~0.7 GeV? +9% required!

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?



Shifts below ~1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

What happens to the electron g-2?

The electron g-2 no longer provides the best value of α

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → "g_e-2" determination of alpha:

  $\alpha^{-1} = 137.035\,999\,151\ (33)$ [0.24 ppb]

- The best determination of α is obtained via atomic interferometry:

  $\alpha^{-1} = 137.035\,999\,046\ (27)$ [0.20 ppb] Parker et al, Science 360 (2018) 192 (Cs)

  $\alpha^{-1} = 137.035\,999\,206\ (11)$ [0.08 ppb] Morel et al, Nature 588 (2020) 61 (Rb)

Great improvement in precision, but 5.4σ difference!

Electron g-2: SM vs experiment

- Using

  $\alpha = 1/137.035\,999\,046\,(27)$ [Cs 2018]

  $\alpha = 1/137.035\,999\,206\,(11)$ [Rb 2020]

  the SM prediction for the electron g-2 is, respectively:

$$a_e^{SM} = 115\,965\,218\,16.16\,(0.11)\,(0.08)\,(2.28) \times 10^{-13} \text{ [Cs18]}$$
$$= 115\,965\,218\,02.64\,(0.11)\,(0.08)\,(0.93) \times 10^{-13} \text{ [Rb20]}$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The (EXP – SM) difference is:

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -8.9\,(3.6) \times 10^{-13} \text{ [2.5}\sigma\text{] [Cs18]}$$
$$= +4.7\,(3.0) \times 10^{-13} \text{ [1.6}\sigma\text{] [Rb20]}$$

[QED 5-loop $a_e^{QED5} = 4.6 \times 10^{-13}$]

- NP sensitivity limited only by the experimental errors in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector.

Giudice, Paradisi, MP 2012

- Using α(Rb2020), the sensitivity is $\delta\Delta a_e = 3.0 \times 10^{-13}$, ie ($\times 10^{-13}$):

$$\underbrace{(0.1)_{\rm QED5}, \quad (0.1)_{\rm HAD},}_{(0.2)_{\rm TH}} \quad (0.9)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\rm EXP}}$$

- The (g-2)$_e$ experimental error may soon drop below 10^{-13} → a_e sensitivity below 10^{-13} may soon be reached!

- In a broad class of BSM theories, contributions to a_l scale as

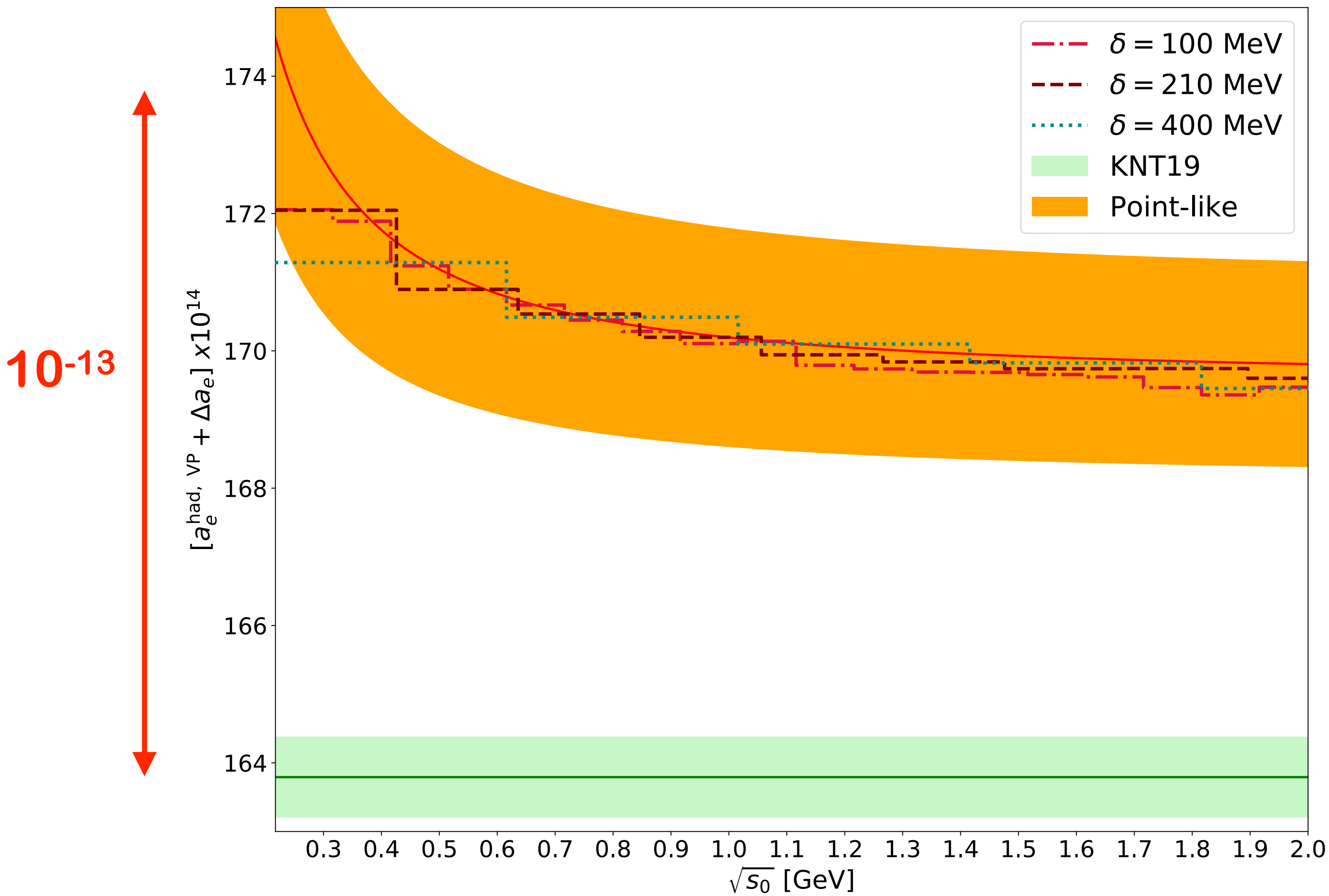
$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

Giudice, Paradisi & MP, JHEP 2012

Shift of the electron g-2

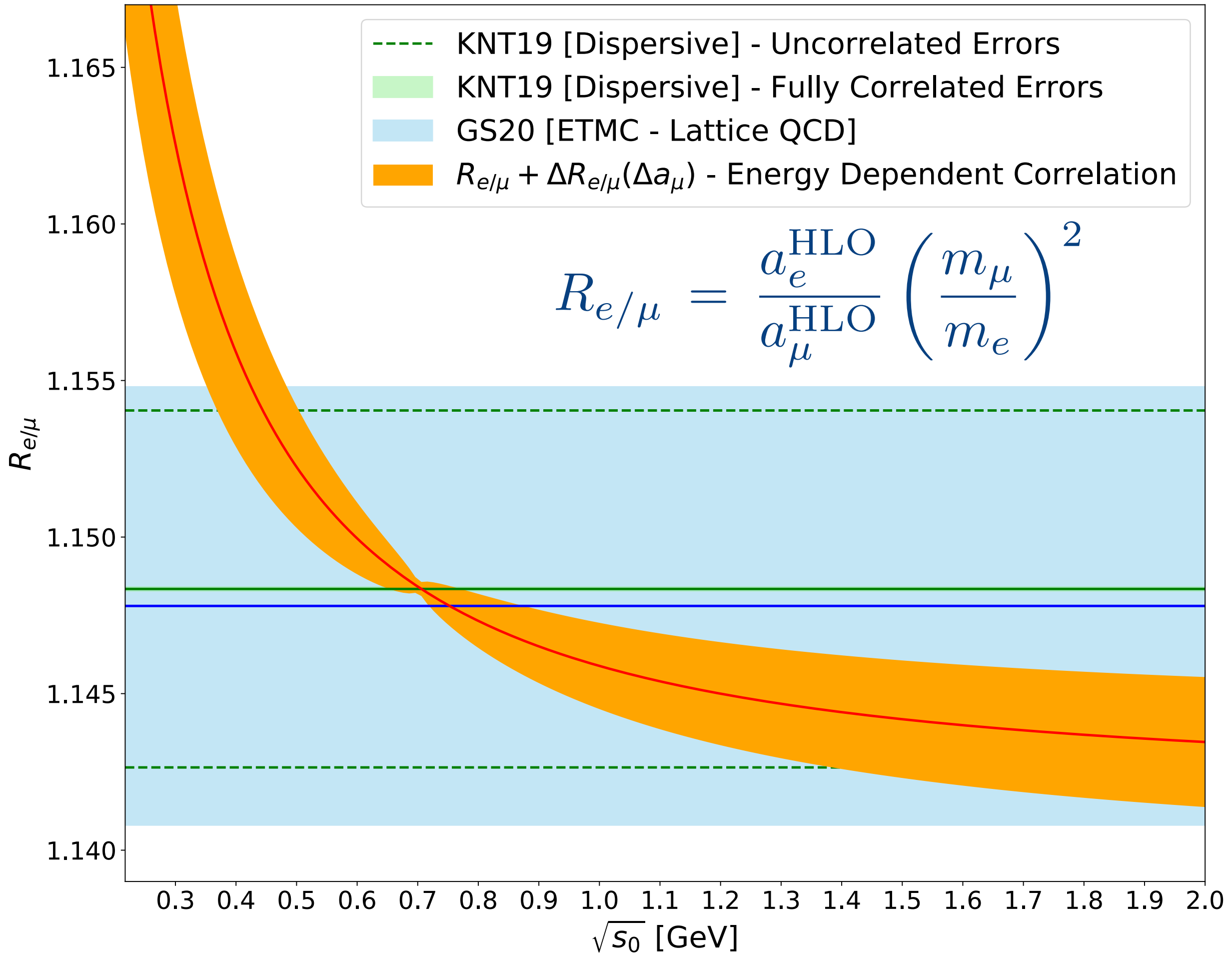


Shifts $\Delta\sigma(s)$ to fix Δa_μ only slightly change Δa_e

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19. Possible future bounds on very low energy shifts Δσ(s)?

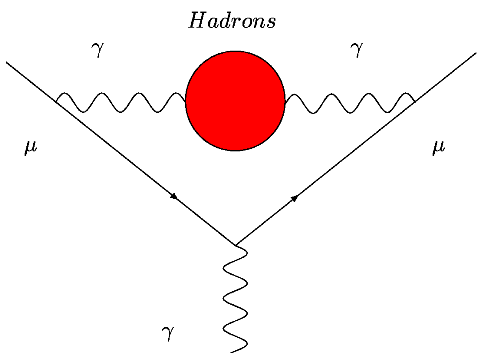
Keshavarzi, Marciano, MP, Sirlin, PRD 2020

- Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits," arXiv:2003.04886.
- Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.
- Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit," arXiv:2008.08107.
- Colangelo, Hoferichter and Stoffer, "Constraints on the two-pion contribution to hadronic vacuum polarization," arXiv:2010.07943.

The MUonE project

Spacelike proposal for a_μ^{HLO}

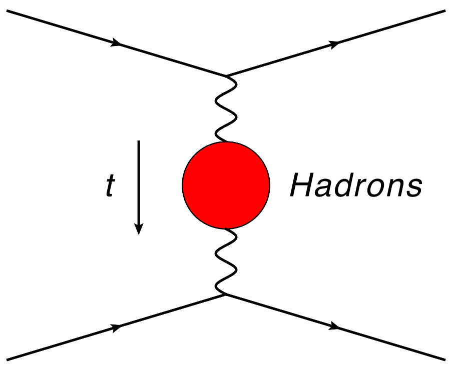
- **The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

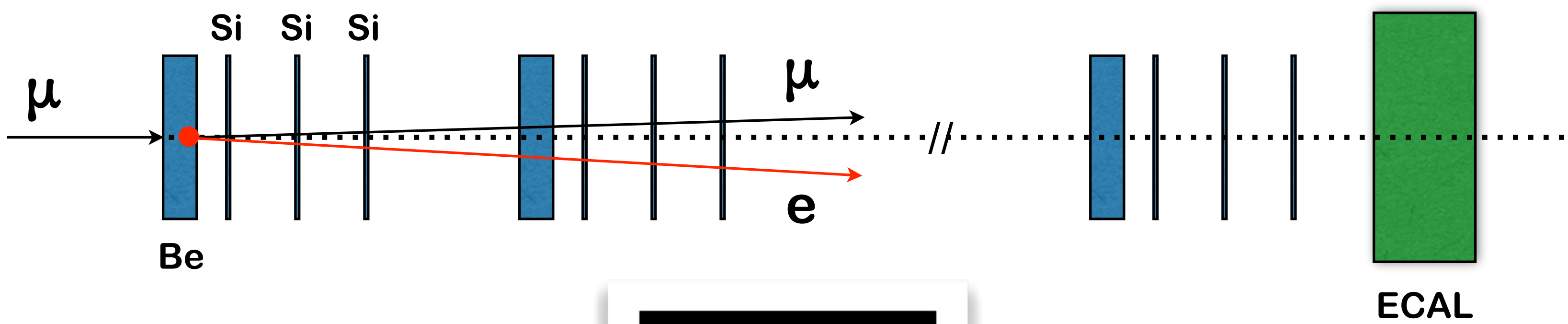
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

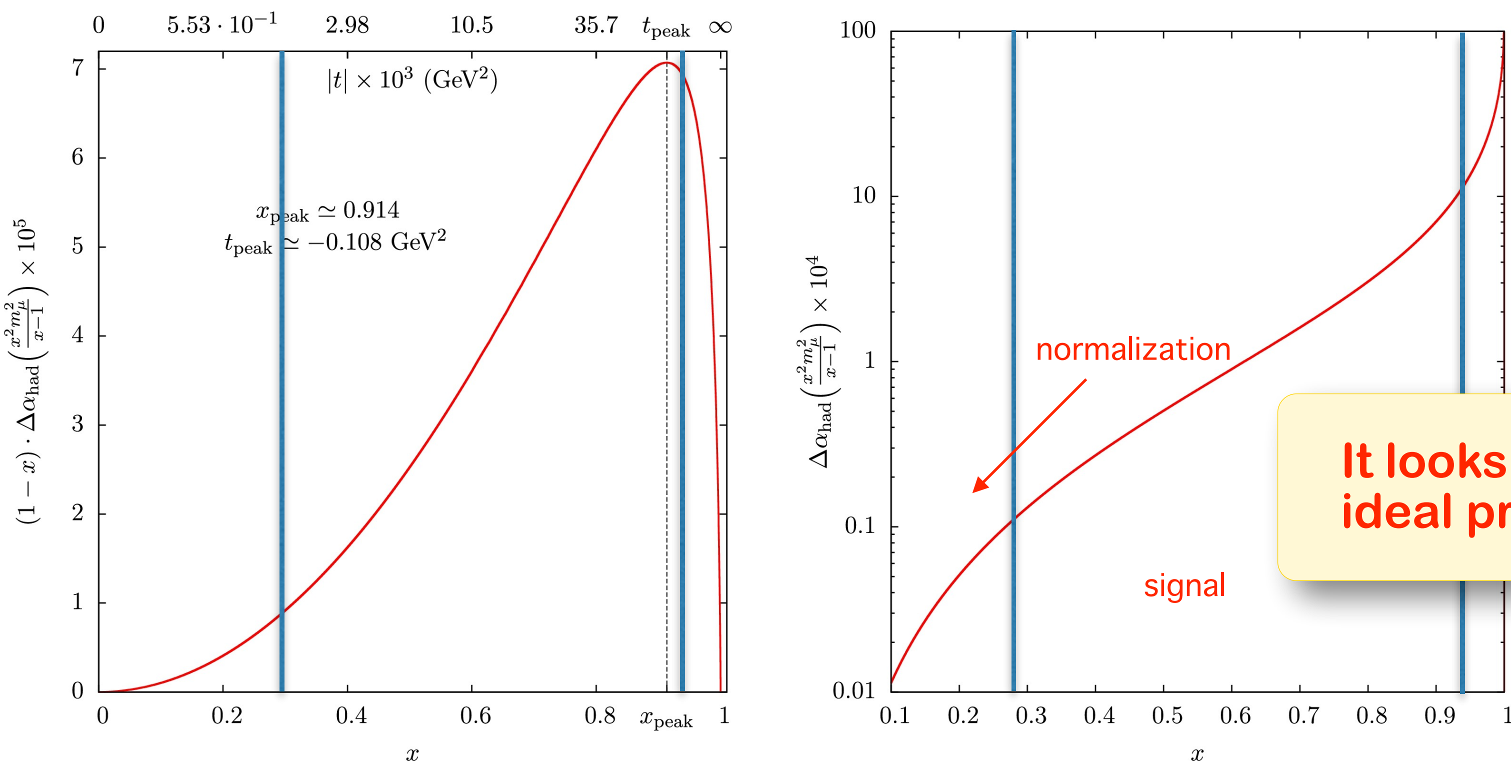
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

- For a 150 GeV muon beam ($\sqrt{s}$~400 MeV), MUonE's scan region extends up to x=0.932, ie beyond the peak! (the peak is at x=0.914)

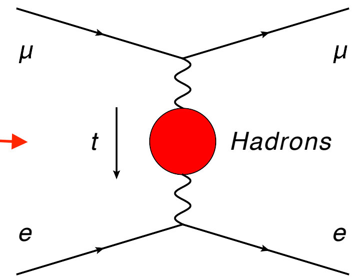
- The high-energy region inaccessible to MUonE contributes only 13% of a_μ^{HLO} integral. It can be determined with timelike data and/or lattice QCD results. Already obtained via lattice QCD! Giusti&Simula and Marinkovic´&Cardoso 2019



It looks like an ideal process!

- Statistics: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2-3 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7\ nb^{-1}$.
- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.
- Systematic effects must be known at ≲ 10ppm!
- Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111
- LoI submitted to CERN SPSC in 2019: Test run approved for 2021.
- Full-statistics run hopefully in 2022–24.

- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!



- **Fully differential fixed-order MC @ NLO ready** Pavia and PSI 2018-19
- **NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude close to completion.** Padova 2017 - present
- **Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation** Pavia and PSI 2020
- **NNLO hadronic effects computed** Padova and KIT 2019
- **Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes** PSI 2019-present
- **New Physics at MUonE?** Padova and Heidelberg 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

Padova
Europe/Rome timezone

Overview
Venue
Timetable
Logistic
Map

Support

MUonE theory workshops: Padova 2017, Mainz 2018, Zurich 2019
Next MUonE theory workshop: MITP Mainz 1-5.03 2021 (postponed)

Conclusions

- Is the present Δa_μ discrepancy due to missed contributions in the hadronic $\sigma(s)$? Unlikely:

  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- The fate of the Δa_μ discrepancy should soon be decided by the new Muon g-2 experiment at Fermilab and the follow-up exp. at J-PARC:

  Should a_μ^{exp} agree with the SM prediction calculated from DRs, it will mark the end of an era that has strongly challenged theoretical creativity and computational innovation.

  Alternatively, confirmation of the discrepancy will strengthen the NP interpretation and the quest for its underlying origin.

- MUonE at CERN will provide a new independent spacelike determination of a_μ^{HLO} alternative to the DR and lattice ones.