

# Introduction à QCD

Gif 2007

Paris 24 - 28 septembre

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- 2) Le Lagrangien de QCD
- 3) Règles de Feynman  
ghosts, unitarité'
- 4) Renormalisation, liberté asymptotique
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# Théories de Jauges

H. Weyl (1918)

Relativité' générale et Electrodynamique

$$\vec{e}_i(x_i)$$

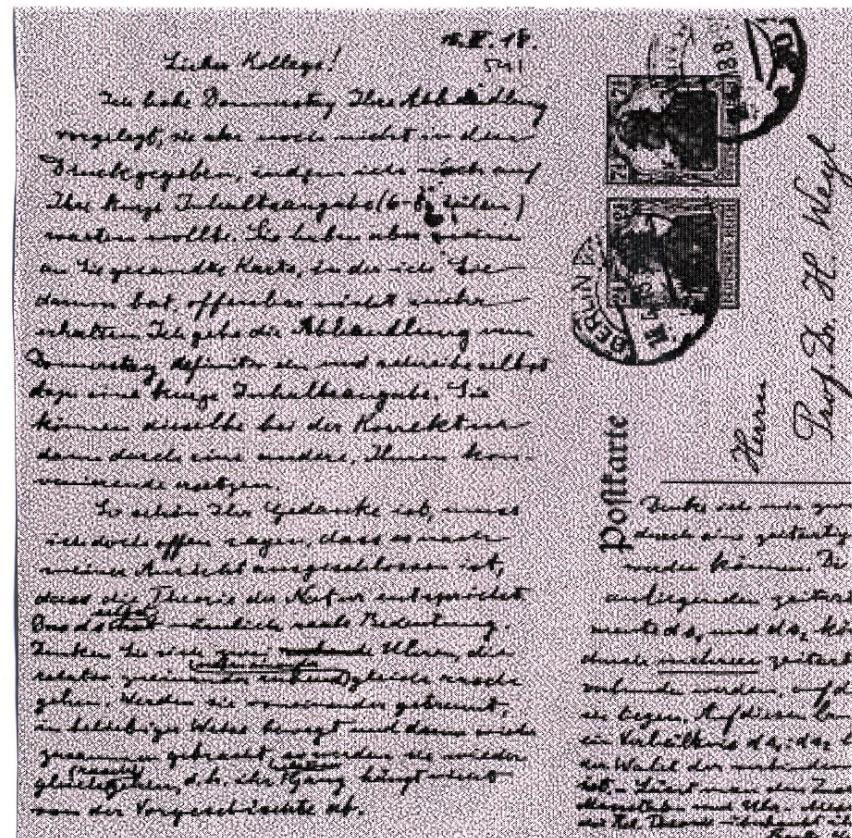
$$\vec{e}_i \cdot \vec{e}_j = g_{ij}(x)$$

$$\vec{v} = v^i(x) \vec{e}_i(x)$$

$$|\vec{v}|^2 = v^i v^j g_{ij}$$

$$\bar{g}_{ij}(x) = e^{2\lambda(x)} g_{ij}(x)$$

$$\bar{A}_\mu(x) = A_\mu - \partial_\mu \lambda(x)$$



N. Straumann, Photo 200

QED

$$\mathcal{L}_0 = \bar{\psi}(i\cancel{D} - m)\psi \quad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \quad \cancel{D} = \gamma^\mu \partial_\mu \quad \bar{\psi} = \psi^+ \gamma^0$$

$$(i\cancel{D} - m) \psi(x) = 0 \quad \rightarrow \quad (p - m) \psi(p) = 0$$

Invariance de  
fauge locale

$$\psi'(x) = e^{-ie\phi(x)} \psi(x) \quad i\cancel{D}\psi' = e\cancel{D}\phi e^{-ie\phi} \psi + ie e^{-ie\phi} \cancel{D}\psi$$

$$\Rightarrow A_\mu(x) : \quad \mathcal{L} = \bar{\psi}(i\cancel{D} - eA - m)\psi$$

$$A'_\mu = A_\mu + \partial_\mu \phi$$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$\mathcal{L} = \bar{\psi}(i\cancel{D} - m)\psi$$

$$(D_\mu \psi)' = e^{-ie\phi} D_\mu \psi$$

$$([D_\mu, D_\nu] \psi)' = e^{-ie\phi} [D_\mu, D_\nu] \psi$$

$$[D_\mu, D_\nu] \psi = ie F_{\mu\nu} \psi$$

champs classiques  
↑  $\partial \cdot A = 0$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F'_{\mu\nu} = F_{\mu\nu}$$

$$\mathcal{L}_{QED} = \bar{\psi} (i \not{D} - m) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \left( -\frac{\lambda}{2} (\partial_\mu A^\mu)^2 \right)$$

### Quantification covariante

$$\frac{\delta \mathcal{L}_{QED}}{\delta \partial_\nu A_\nu} = F^{\nu 0} (-g^{\nu 0} \lambda \partial \cdot A) = \Pi^\nu$$

$$\Pi^\nu = -\lambda \partial \cdot A$$

$$D^{\mu\nu}(k) = -\frac{i}{k^2 + i\varepsilon} \left( g^{\mu\nu} + \frac{1-\lambda}{\lambda} \frac{k^\mu k^\nu}{k^2 + i\varepsilon} \right)$$

$$T_{\mu\nu\rho\dots}(k_1, k_2, k_3, \dots) = \text{Diagram with loop and momentum } p_i^2 = m^2$$

$$k_1^\mu T_{\mu\nu\rho\dots} = 0 \quad \left( \begin{array}{l} \text{même si} \\ k_1^2 \neq 0 \end{array} \right)$$

# QCD

$$\text{Spin Isotopique : } P = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \psi = \begin{pmatrix} P \\ n \end{pmatrix} \quad \psi^+ \psi = 1 = |P|^2 + \sum_{a=1}^3 |\theta_a|$$

$$\psi' = U \psi \quad U^\dagger U = 1 \quad [U, H] = 0 \quad \tau_a^+ = \tau_a \quad \text{Tr}(\tau_a) = 0$$

$$U = e^{i \frac{\theta_0}{2} I} e^{-i \sum_{a=1}^3 \theta_a \frac{\gamma^a}{2}} \quad \theta_a \neq \theta_a(x) \quad \left[ \frac{\gamma^a}{2}, \frac{\gamma^b}{2} \right] = i \epsilon^{abc} \frac{\gamma^c}{2}$$

$U(1) \quad SU(2) \rightarrow \det(U) = 1 \quad \sum_{a=1}^3 \theta_a \frac{\gamma^a}{2} = \vec{\theta} \cdot \vec{\gamma} = \Theta$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \psi_i^* \psi_j \quad \left\{ \begin{array}{l} \sum_i \psi_i^* \psi_i = \psi^+ \psi \in \mathbb{1} \\ j_a = \psi^+ \frac{\gamma_a}{2} \psi \in \mathbb{3} \\ j'_a = j_a - i (\vec{\theta} \cdot \vec{T})_{ac} f_c \end{array} \right. \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad \vec{\phi} \cdot \vec{j} \in \mathbb{1}$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \psi_i^* \psi_j \quad \left\{ \begin{array}{l} \sum_i \psi_i^* \psi_i = \psi^+ \psi \in \mathbb{1} \\ j_a = \psi^+ \frac{\gamma_a}{2} \psi \in \mathbb{3} \\ j'_a = j_a - i (\vec{\theta} \cdot \vec{T})_{ac} f_c \end{array} \right. \quad (T^b)_{ac} = -i \epsilon^{bac}$$

$$\pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \bar{\pi}_3 \end{pmatrix} = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ (\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix} \quad \mathcal{L} = \bar{\psi} (i \not{D} - m) \psi + \frac{1}{2} (\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \mu \vec{\pi} \cdot \vec{\pi}) + g \vec{j}_5 \cdot \vec{\pi}$$



$$\frac{g^2}{\pi} \sim 1/4$$

$$SU_L(2) \rightarrow SU_F(3)$$

$$\vec{j}_5 = \bar{\psi} \gamma_5 \frac{\vec{\gamma}}{2} \psi$$

Yang et Mills (1954)

$$\partial_a(x)$$

$$g \vec{f}_\mu \cdot \vec{A}^\mu$$

$$\vec{f}_\mu = \bar{\psi} f_\mu \frac{\vec{\tau}}{2} \psi$$
$$\vec{A}_\mu = \begin{pmatrix} A'_\mu \\ A''_\mu \\ A'''_\mu \end{pmatrix}$$

$$D_\mu = \partial_\mu - ig \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu = \partial_\mu - ig A_\mu \quad (D_\mu \psi)' = U(x) D_\mu \psi$$

$$A'_\mu = U(x) A_\mu U^{-1}(x) - \frac{i}{g} (\partial_\mu U) U^{-1}$$

$$\text{Tr}(A_\mu A^\mu) \neq \text{Tr}(A'_\mu A'^\mu)$$

$$A''_\mu = A'_\mu + \epsilon^{abc} \Theta_b(x) A_c^\mu - \frac{1}{g} \partial_\mu \Theta_a$$

$$[D^\mu, D^\nu] \psi = -ig \left( \frac{\vec{\tau}^a}{2} F_a^{\mu\nu} \right) \psi$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig [A^\mu, A^\nu]$$

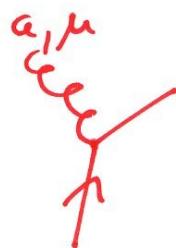
$$F'_{\mu\nu} =$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g \epsilon^{abc} A_b^\mu A_c^\nu$$

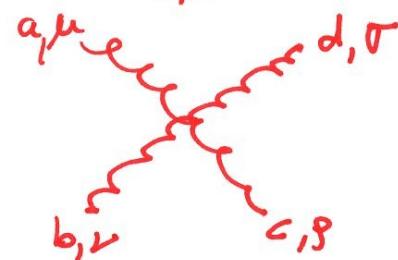
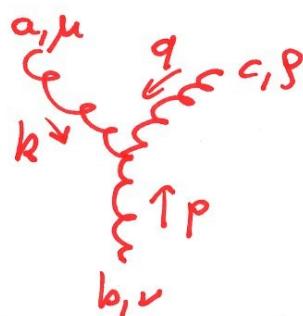
$$U F_{\mu\nu} U^{-1}$$

$$\text{Tr}(F'_{\mu\nu} F'^{\mu\nu}) = \text{Tr}(F_{\mu\nu} F^{\mu\nu}) = \frac{1}{2} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$\mathcal{L}_{\text{YM}} = \bar{\psi}(iD - m)\psi - \frac{1}{2} \text{Tr}(F^{\mu\nu}F_{\mu\nu}) - \left(-\frac{\lambda}{2} (\bar{Q}_a A_a^\mu)^2\right)$$



$$ig \Gamma^{\mu} \frac{q^a}{2}$$



$$g \epsilon^{abc} (g^{\mu\nu} (k-p)^\delta + \text{leftrightarrow}) = \Gamma^{\mu\nu\delta}$$

$$-ig^2 [\epsilon^{abe} \epsilon^{ade} (g^{\mu\delta} g^{\nu\tau} - g^{\mu\tau} g^{\nu\delta}) + \epsilon^{ace} \epsilon^{bde} (g^{\mu\nu} g^{\delta\tau} - g^{\mu\delta} g^{\nu\tau}) + \epsilon^{ade} \epsilon^{bce} (g^{\mu\nu} g^{\delta\tau} - g^{\mu\delta} g^{\nu\tau})]$$

$$k_\mu \Gamma^{\mu\nu\delta} \neq 0$$

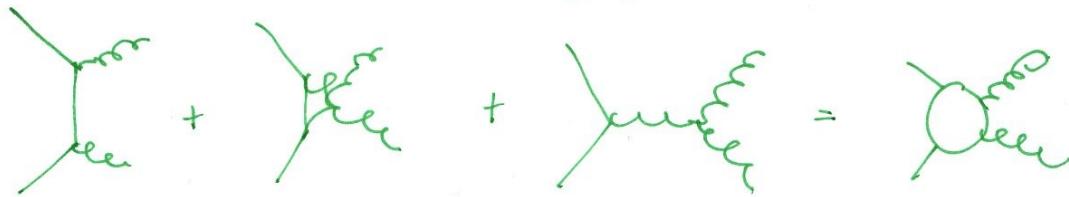
$$k_\mu \Gamma^{\mu\nu\delta} \epsilon_\nu(p) \epsilon_\delta(q) = 0 \quad \left\{ \begin{array}{l} k^2 \neq 0 \\ \epsilon(p) \cdot p = 0 \\ p^2 = 0 \end{array} \right.$$

Unitarité'

$$S^+ S = 1$$

$$S = 1 + iT$$

$$\text{Im} \langle \alpha | T | \alpha \rangle = \frac{1}{2} \sum_n \int d\mu s_n |\langle \alpha | T | n \rangle|^2$$

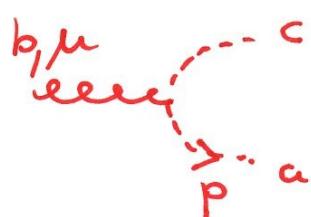


$$\text{Im} \quad \text{loop diagram} = \frac{1}{2} \int d\mu s \left| \text{loop diagram} \right|^2$$

$$\text{Im} \int d^4 k \frac{g^{\mu\nu}}{k^2 + i\varepsilon} \rightarrow -2\pi g^{\mu\nu} \int d^4 k \delta(k^2) \stackrel{?}{=} \sum_{\lambda=1,2} \int d^4 k \delta(k^2) \left| M^\mu \epsilon_\nu(k, \lambda) \right|^2$$

Feynman (1963)

Faddeev et Popov (1967)



$$-g \epsilon^{abc} p^\mu u \quad (\text{gauche covariante})$$

$$\gamma = (1, 0, 0, 0)$$

$$\begin{aligned} & - \frac{g^{\mu\nu}}{(k \cdot y)^2} \\ & + \frac{k^\mu k^\nu}{(k \cdot y)} \end{aligned}$$

$$\frac{i \delta^{ab}}{p^2 + i\varepsilon}$$

facteur -1 pour une boucle

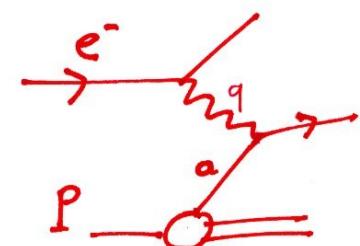
# Quarks, Gluons, Couleur

- $B = q\bar{q}q$

- $M = q\bar{q}$

	$I_3$	$B$	$S$	$Q$
u	$1/2$	$1/3$	$0$	$2/3$
d	$-1/2$	$1/3$	$0$	$-1/3$
s	$0$	$1/3$	$-1$	$-1/3$

$\rightarrow SU_3$



$$G_{a/P}(z)$$

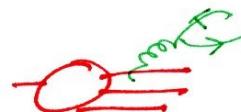
$$z = \frac{p_a^2}{P^2}$$

$$x_{Bj} = \frac{Q^2}{2P \cdot q} \quad Q^2 = -q^2$$

$$F_2 = \sum_a e_a^2 x_{Bj} G_{a/P}(x_{Bj})$$

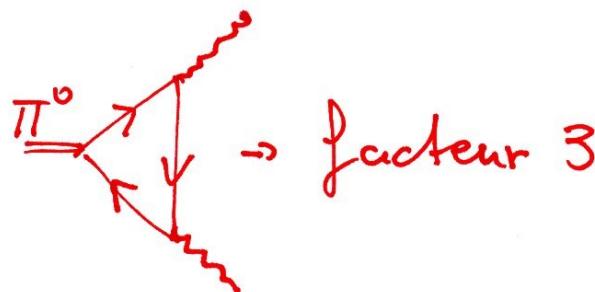
$$\sum_a \int_0^1 dx \times G_{a/P}(x) \sim \frac{1}{2}$$

Scaling  
de  
 $B_j$



$$\int_0^1 dx (G_u(x) - G_{\bar{u}}(x)) = 2$$

- $R = \frac{\sigma(e^+e^- \rightarrow \text{Hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{\sum_q e_q^2 |\Delta C^q|^2}{|\Delta C^{\mu^+ \mu^-}|^2} = 3 \sum_q e_q^2$



$$\psi_B = E(x_i) S(s_i) U(g_i) A_c$$

$$B = \Delta^{++}$$

# QCD (1972-73)

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$\rightarrow SU_c(3)$$

$$\left[ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f^{abc} \frac{\lambda^c}{2}$$

$$U(x) = e^{-i \sum_{a=1}^8 \theta_a(x) \frac{\lambda^a}{2}}$$

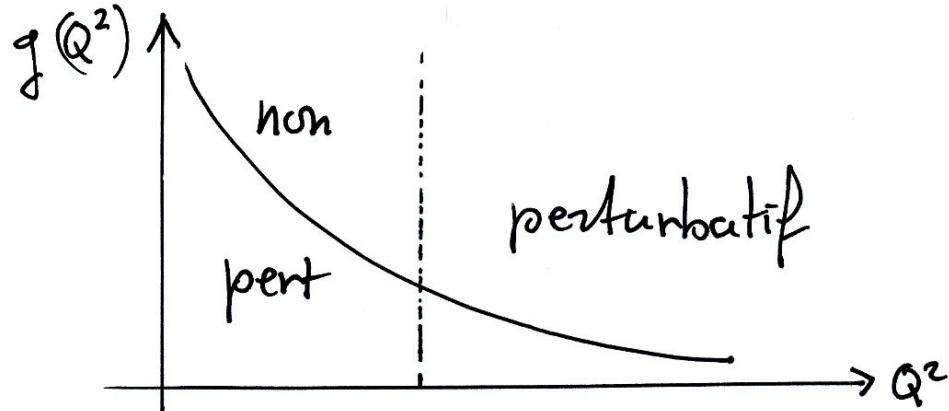
$$\lambda_a^+ = \lambda_a^-$$

$$\text{Tr}(\lambda_a) = 0$$

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A_{a\mu} \quad \rightarrow \quad F_{a\mu\nu}^{ab} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_{b\mu}^\mu A_c^\nu$$

$$\mathcal{L}_{QCD} = \sum_q \bar{q} (i D - m_q) q - \frac{1}{4} F_{a\mu\nu}^{ab} F_{b\mu\nu}^a$$

Liberté asymptotique



non perturbatif

$\left\{ \begin{array}{l} q, g \rightarrow \text{hadrons} \\ \text{confinement} \\ \text{BSSE } \chi_{\text{sym}} \\ \text{Plasma de } q \text{ et } g \\ \text{vide} \end{array} \right.$

perturbatif

$\left\{ \begin{array}{l} \text{tests de la théorie} \\ \text{NLO, NNLO} \\ \text{fond pour la nouvelle physique} \end{array} \right.$

Gross  
Politzer  
Wilczek  
Fritsch  
Gell-Mann  
Weinberg  
:

## Renormalisation, GR, liberté asymptotique

$$\mathcal{L}_{QCD} = \bar{q} i \not{\partial} q + g \bar{q} \not{A} q + \dots \xrightarrow{k_\mu \rightarrow \infty} \int \frac{d^d p}{(2\pi)^d} (ig)^2 \frac{\gamma^\mu (p+k) \gamma_\mu}{(p+k)^2} \left(\frac{-i}{p^2}\right) \frac{\lambda^a \lambda^a}{2} \sim k \int \frac{dx}{x}$$

rég.  
dimens.

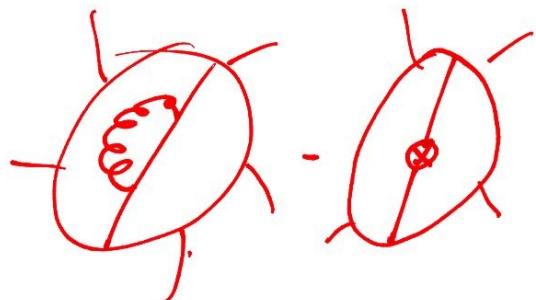
$$\left[ \frac{d^n p}{(2\pi)^n} \quad \{ \gamma^\mu, \gamma^\nu \} = g^{\mu\nu} \quad \text{tr}(II) = 4 \quad \gamma^\mu \gamma^\nu \gamma_\mu = (2-d) \gamma^\nu \quad d = 4 - 2\varepsilon \right]$$

(PS:  $d = 4 - \varepsilon$ )

$$= \frac{i g^2}{(4\pi)^{d/2}} k C_2(r) \Gamma\left(2 - \frac{d}{2}\right) \int_0^1 \frac{dx (1-x)}{(-x(1-x)k^2)^{2-d/2}} (d-2)$$

$$= \frac{i g^2}{(4\pi)^2} k C_2(r) \Gamma(\varepsilon) \frac{(4\pi)}{(-k^2)} \frac{1}{(-k^\varepsilon)}^\varepsilon \underbrace{\int_0^1 dx (1-x)^{1-\varepsilon} x^{-\varepsilon} z(1-\varepsilon)}_{1 + C \cdot \varepsilon}$$

$$g^2 = g^2(\mu) (\mu^2)^\varepsilon \rightarrow \frac{i g^2(\mu)}{(4\pi)^2} k C_2(r) \left[ \frac{1}{\varepsilon} - \gamma + \ln 4\pi + \ln \left( \frac{\mu^2}{-k^2} \right) + C \right]$$



$$\mathcal{L}_{QCD}^* = \mathcal{L}_{QCD} + \bar{q} i \not{\partial} q \Delta_{2F} \left\{ \begin{array}{l} \frac{g^2(\mu)}{(4\pi)^2} C_2(r) \frac{1}{\varepsilon} \\ \frac{g^2(\mu)}{(4\pi)^2} C_2(r) \left( \frac{1}{\varepsilon} - \gamma + \ln 4\pi \right) \end{array} \right. \begin{array}{l} \text{MS} \\ \overline{\text{MS}} \end{array}$$

Schéma de renormalisation

$$L_{\text{actD}}^{\circ} = (1 + \Delta_{2F}) \bar{q} i \not{D} q = \bar{q}^{\circ} i \not{D} q^{\circ} \quad \begin{aligned} q^{\circ} &= \sqrt{Z_{2F}} q \\ Z_{2F} &= 1 + \Delta_{2F} \end{aligned}$$

$$L_{\text{actD}}^{\circ} = L_{\text{actD}} - L_{\text{CT}}$$

$$\frac{1}{2} \sum \text{loop diagrams} - \sum_q \text{loop diagrams} + \frac{1}{2} \text{loop diagrams} - \text{loop diagram} \rightarrow \Delta_{3YM} (-g^{\mu\nu} p^2 + p^\mu p^\nu)$$

$$A_\mu^{\circ} = \sqrt{Z_{3YM}} A_\mu$$

$$\text{loop diagrams} + \text{loop diagrams} + \dots - \text{loop diagram} - \text{loop diagram} - \text{loop diagram} - \text{loop diagram} \rightarrow \Delta_{1YM}$$

$$\text{loop diagram} + \dots + \text{loop diagram} + \dots \rightarrow \Delta_5$$

$$\text{loop diagram} + \text{loop diagram} \rightarrow -\Delta_{1F} g \bar{q} A q$$

$$\text{loop diagram} \rightarrow \tilde{\Delta}_3 \text{ et } \tilde{\Delta}_1$$

$$g \bar{q} A q + \Delta_{1F} g \bar{q} A q = Z_{1F} g \bar{q} A q = \frac{Z_{1F} g}{Z_{2F} (Z_{3YM})^{1/2}} \bar{q}^{\circ} A^{\circ} q^{\circ}$$

$$g_0^F = \frac{Z_{IF}}{Z_{2F}(Z_{3YM})^{1/2}} g$$

$$g_0^{YM} = \frac{Z_{1YM}}{Z_{3YM}^{1/2}} g$$

$$z_i = 1 + \Delta_i$$

$$\frac{Z_{3YM}}{Z_{1YM}} = \frac{Z_{2F}}{Z_{IF}} = \frac{\tilde{Z}_3}{\tilde{Z}_1} = \frac{Z_{1YM}}{Z_5} \quad (\text{Slaunov-Taylor})$$

$$g^0 = Z_g g = Z_g g(\mu) \mu^\epsilon$$

$$\mu \frac{\partial g^0}{\partial \mu} = 0$$

$$\mu \frac{\partial g(\mu)}{\partial \mu} = -\epsilon g(\mu) - g(\mu) \mu \frac{\partial \ln Z_g}{\partial \mu} = \beta(g(\mu), \epsilon)$$

$$Z_g = 1 - \frac{g^2(\mu)}{\epsilon} \beta_0$$

$$\mu \frac{\partial g(\mu)}{\partial \mu} = -\epsilon g(\mu) - g^3(\mu) \beta_0 - (g^5(\mu) \beta_1 + g^7(\mu) \beta_2 + g^9(\mu) \beta_3)$$

$$(\epsilon=0) \quad g^2(\mu) = \frac{g^2(\mu_0)}{1 + g^2(\mu_0) \beta_0 \ln \frac{\mu}{\mu_0}} = \frac{1}{\beta_0 \ln \frac{\mu}{\mu_0}}$$

$$\beta_0 = \frac{1}{(4\pi)^2} \frac{33 - 2N_F}{3} = \frac{1}{(4\pi)^2} \frac{11 \cdot C(G) - 4 \cdot C(N)}{3}$$

$$T_{phys}(\underbrace{Q^2}_{\ln Q^2/\mu^2}, \underbrace{\mu^2}_{\ln Q^2/\mu^2}, g(\mu)) = T_{phys}(Q^2, Q^2, g(Q^2))$$

$SU(N)$

- représentation fondamentale à  $N$  dimensions  
 $N^2 - 1$  matrices hermitiennes  $\text{Tr}(T^a) = 0$   
 $T^a = \frac{\lambda^a}{2}$
- représentation adjointe à  $N^2 - 1$  dimensions  
 $(T^a)_{bc} = i f^{abc}$
- $T_r^a T_r^a = C_2(r) \mathbb{1}$       represent.  $r$   
 $\text{Tr}(T_r^a T_r^b) = C(r) \delta^{ab}$

fondamentale

$$C(N) = \frac{1}{2}$$



$$\rightarrow \text{Tr}\left(\frac{\lambda^a}{2} \frac{\lambda^b}{2}\right)$$

$$C_2(N) = \frac{N^2 - 1}{2N}$$

mass

$$\frac{\lambda^a}{2} \frac{\lambda^a}{2}$$

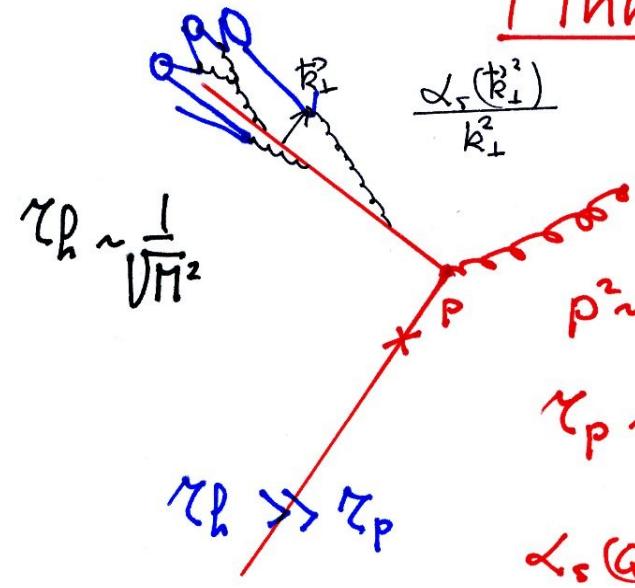
adjointe

$$C(G) = N$$



$$C_2(G) = N$$

# Annihilation $e^+e^-$



$$\frac{\alpha_s(\vec{k}_\perp^2)}{k_\perp^2}$$

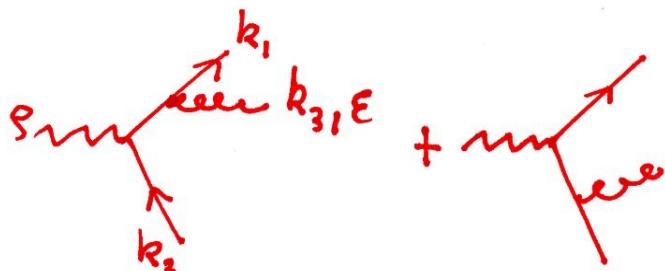
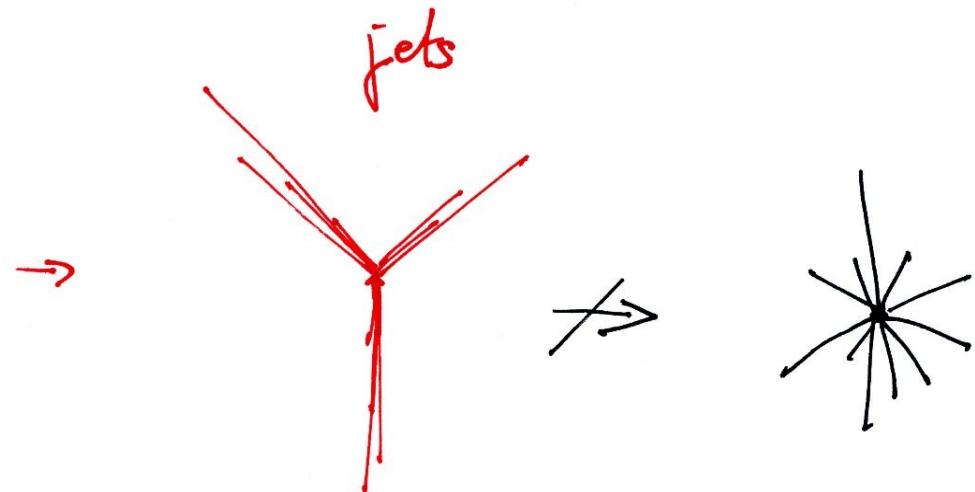
$$\gamma_h \sim \frac{1}{M^2}$$

$$\gamma_h \gg \gamma_p$$

$$p^2 \sim Q^2$$

$$\gamma_p \sim \frac{1}{Q^2}$$

$$\alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi}$$



$$\frac{\lambda}{2} \bar{u}(k_1) \gamma^\mu \epsilon_\mu(\lambda, k_3) \frac{(k_1 + k_3)}{(k_1 + k_3)^2} \gamma^\nu u(k_2) \stackrel{|\vec{k}_3| \rightarrow 0}{\simeq} \frac{\lambda}{2} \frac{2 \epsilon \cdot k_1}{2 k_1 \cdot k_3} \bar{u}(k_1) \gamma^\nu u(k_2)$$

$$\sum_{\lambda=1,2} \epsilon_\mu(k_3, \lambda) \epsilon_\nu(k_3, \lambda) = -g_{\mu\nu} + \dots$$

$$\rightarrow k_1^2 = 0$$

$$\rightarrow k_1 \cdot k_2$$

$$d\Gamma \propto \alpha_s C_2(3) \frac{k_1 \cdot k_2}{(k_1 \cdot k_3)(k_2 \cdot k_3)} \frac{d\vec{k}_3}{2 k_3}$$

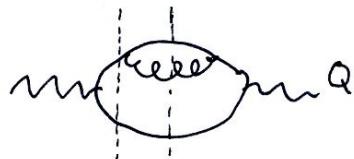
$$k_1 \cdot k_3 = |\vec{k}_1| |\vec{k}_3| (1 - \cos \theta_{13})$$

$$\left\{ \begin{array}{l} |\vec{k}_3| \rightarrow 0 \quad \frac{d|\vec{k}_3|}{|\vec{k}_3|} \sim \log \frac{1}{\lambda} \\ \frac{d\theta_{13}}{\theta_{13}} \sim \log \frac{1}{m} \end{array} \right. \begin{array}{l} \text{-Colinéaire} \\ \text{-de masse} \end{array}$$

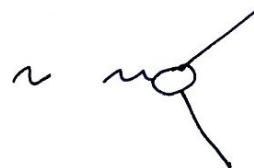
# Théorème de Lee-Nauenberg

## Kinoshita

$$\begin{array}{c} \langle ee \nu e \rangle + \langle \bar{\omega} \rangle \\ \langle ee \nu e \rangle + \langle \bar{B} \rangle \\ \langle ee \nu e \rangle + \langle \bar{D} \rangle \\ \langle ee \nu e \rangle + \langle \bar{\Xi} \rangle \end{array}$$



$$\text{outgoing } \vec{k}_1, \vec{k}_2, \vec{k}_3 \quad \left\{ \begin{array}{l} |\vec{k}_3| \rightarrow 0 \\ \vec{k}_3 \parallel \vec{k}_1, \vec{k}_2 \end{array} \right.$$



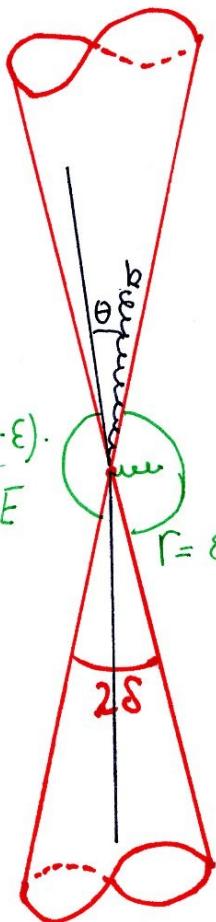
Dans une théorie des champs de masse nulle, les sections efficaces n'auront pas de divergence soft si l'on somme sur les états (initiaux et finaux) dégénérés.

$$R = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum e_q^2 \left( 1 + \frac{\alpha_s(\mu)}{\pi} + \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 r_2(\mu^2, Q^2) \right)$$

$$r_2 = \frac{8\pi^2}{\beta_0^2} \log \frac{Q^2}{\mu^2} + g_2$$

$$\frac{R}{3 \sum e_q^2} = 1 + \underbrace{\frac{\alpha_s(\mu)}{\pi} \left( 1 + 4\pi\beta_0 \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2} \right)}_{\text{NLO}} + \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \overset{\uparrow \text{NLO}}{r_2}$$

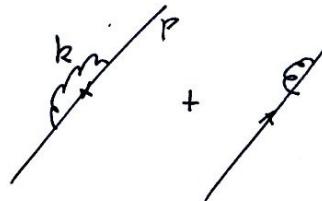
$$\frac{1}{\pi} \frac{\alpha_s(\mu)}{1 + \alpha_s(\mu) 4\pi\beta_0 \log \frac{Q^2}{\mu^2}} = \frac{1}{\pi} \frac{1}{4\pi\beta_0 \log \frac{Q^2}{\mu^2}} \underset{\text{NLO}}{=}$$



$$E_1 + E_2 > (1-\epsilon) \cdot 2E$$

## Jets de Sterman-Weinberg

$$E = \frac{\sqrt{Q^2}}{2}$$



$$\left( -\frac{1}{\epsilon} + \ln \frac{\mu^2}{p^2} \right) - \left( -\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2} \right) = \ln \left( \frac{Q^2}{p^2} \right)$$

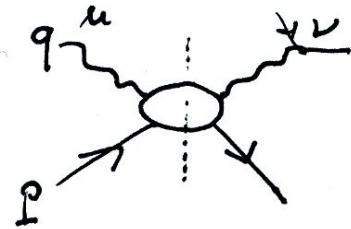
$\alpha_s(Q^2)$

$$\int_0^E \frac{dk}{k} \left( \frac{d\sigma(\delta)}{d\theta} \right) - \int_0^E \frac{dk}{k} \left( \frac{\pi/2}{\theta} \right) = - \int_0^E \frac{dk}{k} \left( \frac{\pi/2}{\theta} \right) \sigma(\delta)$$

$$+ \int_0^{2E\epsilon} \frac{dk}{k} \left( \frac{\pi/2}{\theta} \right) \sigma(\delta) \rightarrow - \int_{2E\epsilon}^E \frac{dk}{k} \left( \frac{d\theta}{\theta} \right) \Rightarrow \left( - \ln 2\epsilon \ln \delta + \dots \right) \cdot \frac{\alpha_s}{\pi} C_F$$

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_B \left( 1 - \frac{\alpha_s}{\pi} C_F 4 \ln 2 \epsilon \ln \delta + \dots \right)$$

# DIS



$$W^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} \right) W_1 + \left( P^\mu - q^\mu \frac{P \cdot q}{Q^2} \right) (\mu \leftrightarrow \nu) W_2$$

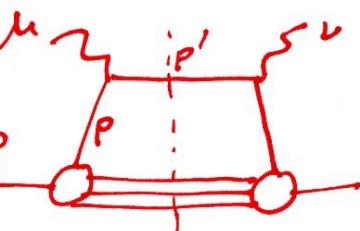
$$Q^2 = -q^2 \gg M_P^2$$

$$P \cdot q$$

$$F_1(x_{Bj}, Q^2) = W_1$$

$$F_2(x_{Bj}, Q^2) = P \cdot q W_2 / 2$$

$$x_{Bj} = \frac{Q^2}{2 P \cdot q}$$



$$W^{\mu\nu} = \int G(p^2, \vec{p}_\perp^2, z) dp^2 d\vec{p}_\perp^2 \frac{dz}{z^2} M_{(c)}^{\mu\nu}(p^2, p \cdot q, q^2)$$

$$z = \frac{P_0 + P_2}{2 P_2}$$

$$(1-z) \langle p^2 \rangle > |\vec{p}_\perp^2|$$

$$M_{(c)}^{\mu\nu} = e_q^2 \pi \text{Tr} (p^\mu \gamma^\nu (p+q) \gamma^\alpha) \underbrace{dp' \delta(p'^2) \delta^4(p+q-p')}_{\delta((p+q)^2) = \frac{1}{2p \cdot q} \delta(1 - \frac{Q^2}{2p \cdot q})}$$

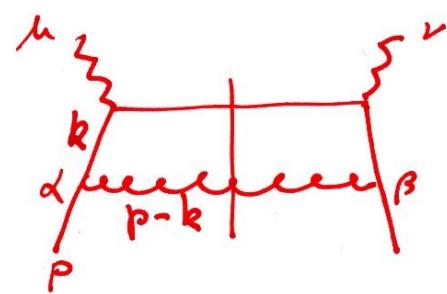
$$F_1(x_{Bj}) = e_q^2 G(x_{Bj})$$

$$F_2(x_{Bj}) = e_q^2 x_{Bj} G(x_{Bj})$$

$$\int \frac{d\vec{p}_\perp^2}{2} G(p^2, \vec{p}_\perp^2, z) = G(p^2, z)$$

$$\int dp^2 G(p^2, z) = G(z)$$

$$x_{Bj} F_1(x_{Bj}) = F_2(x_{Bj}) \quad (\text{Callan-Gross})$$



$$p^2 = -\mu^2$$

$$M_{(1)}^{\mu\nu} = e_q^2 g^2 \underbrace{\int \frac{d^4 k}{(2\pi)^4}}_{2\pi \delta((p-k)^2) 2\pi \delta((k+q)^2)} \frac{1}{2} \text{Tr} (p^\rho \gamma^\alpha k^\beta \gamma^\mu (k+q)^\nu k^\rho \gamma^\lambda) \frac{1}{k^4} d(p-k) \underbrace{\frac{2\beta}{N}}_{\frac{N^2-1}{2N} = \frac{4}{3} = C_F} \text{Tr} \left( \frac{\lambda^\alpha}{2} \frac{\lambda^\beta}{2} \right)$$

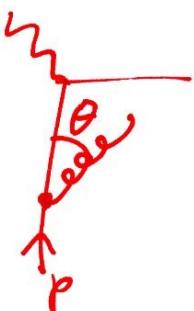
$$\int \frac{d^4 k^2}{|k^2|} (1 + \sigma(k^2))$$

$$a^\mu \overline{b}^\nu = - \frac{i \delta^{ab}}{k^2 + i\epsilon} d^{\mu\nu}(k)$$

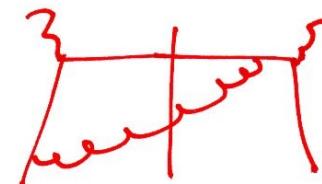
$$d^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu c^\nu + k^\nu c^\mu}{k \cdot c}$$

$$k^\mu d_{\mu\nu}(k) = - \frac{k^2 c_\nu}{k \cdot c}$$

$g^{\mu\nu} d_{\mu\nu}(k) = 2$   
 $\rightarrow$  pas de ghost



$$\frac{\partial \theta}{\theta^\zeta} \theta^\zeta = \frac{d\theta}{\theta}$$



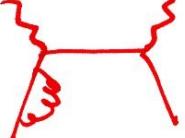
pas de divergence  
colinéaire

$$M_{(1)}^{\mu\nu} = e_q^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{2Q^2} \int \frac{dk^2}{k^2} \frac{1+x^2}{1-x+\Delta} \text{Tr}(T_F f^\mu(k+q) f^\nu)$$

$$x = \frac{Q^2}{2p \cdot q} \quad k = xp$$

$$\Delta = \sigma(k^2/Q^2)$$

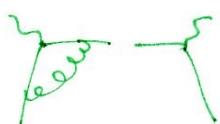
$$F_1^{(p)} = e_q^2 \left( \delta(1-x) + \frac{\alpha_s}{2\pi} C_F \int \frac{dk^2}{k^2} \frac{1+x^2}{1-x+\Delta} \right)$$

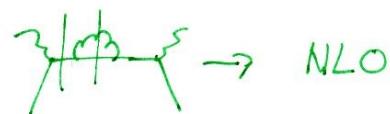

 $\rightarrow -\delta(1-x) \int \frac{dk^2}{k^2} \int_0^1 dx' \frac{1+x'^2}{1-x'+\Delta}$

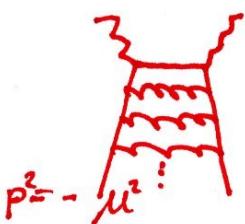
$$\left\{ F_1^{(p)} = e_q^2 \left( \delta(1-x) + \frac{\alpha_s}{2\pi} C_F \left( \frac{1+x^2}{1-x} \right)_+ \log \frac{Q^2}{\mu^2} \right) \right.$$

$$F_1^h(x_{Bj}, Q^2) = \int_0^1 \frac{dz}{z} G(z) F_1^{(p)}\left(\frac{x_{Bj}}{z}, Q^2\right) = \int_0^1 dz G(z) \int_0^1 du F_1^{(p)}(u, Q^2) \delta(u z - x_{Bj})$$

$$\int_0^1 dx \left\{ f(x) \left( \frac{1+x^2}{1-x+\Delta} \right) - \delta(1-x) \left( \int_0^1 \frac{1+x'^2}{1-x'+\Delta} \right) \right\} = \int_0^1 dx \left( f(x) - f(1) \right) \frac{1+x^2}{1-x+\Delta} \stackrel{\Delta=0}{=} \int_0^1 dx f(x) \left( \frac{1+x^2}{1-x} \right)_+$$



  $\rightarrow \text{NLO}$



$$\int_0^1 dx \times^{n-1} F_i(x, Q^2) = F_i(n, Q^2) = e_q^2 \sum_{k=0}^{\infty} \left( \frac{\alpha_s}{2\pi} P_{qq}(n) \right)^k \frac{1}{k!} \left( \log \frac{Q^2}{\mu^2} \right)^k = e_q^2 e^{\mu^2}$$

$\int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{d\alpha_s(k^2)}{2\pi} P_{qq}(n)$

$$P_{qq}(n) = C_F \int_0^1 dx x^{n-1} \left( \frac{1+x^2}{1-x} \right)_+$$

$$F_i^h(n, Q^2) = e_q^2 \int dp^2 G(p^2, n) e^{\int_{-p^2}^{Q^2} \dots} = e_q^2 \int dp^2 G(p^2, n) e^{\int_{Q_0^2}^{Q^2} \dots}$$

$$F_i^h(n, Q^2) = F_i(n, Q_0^2) e^{\int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s}{2\pi} P_{qq}(n)}$$

$$F_i^h = e_q^2 q_h(n, Q^2)$$

$$Q^2 \frac{\partial q_h(n, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(n) q(n, Q^2)$$

$$P_{qq}(1) = C_F \int_0^1 dx \left( \frac{1+x^2}{1-x} \right)_+ = 0$$

$$Q^2 \frac{\partial q(1, Q^2)}{\partial Q^2} = 0$$

$$P_{qq}(2) = \frac{4}{3} C_F$$

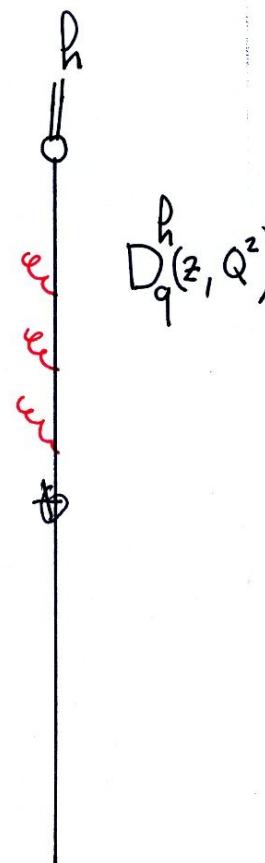
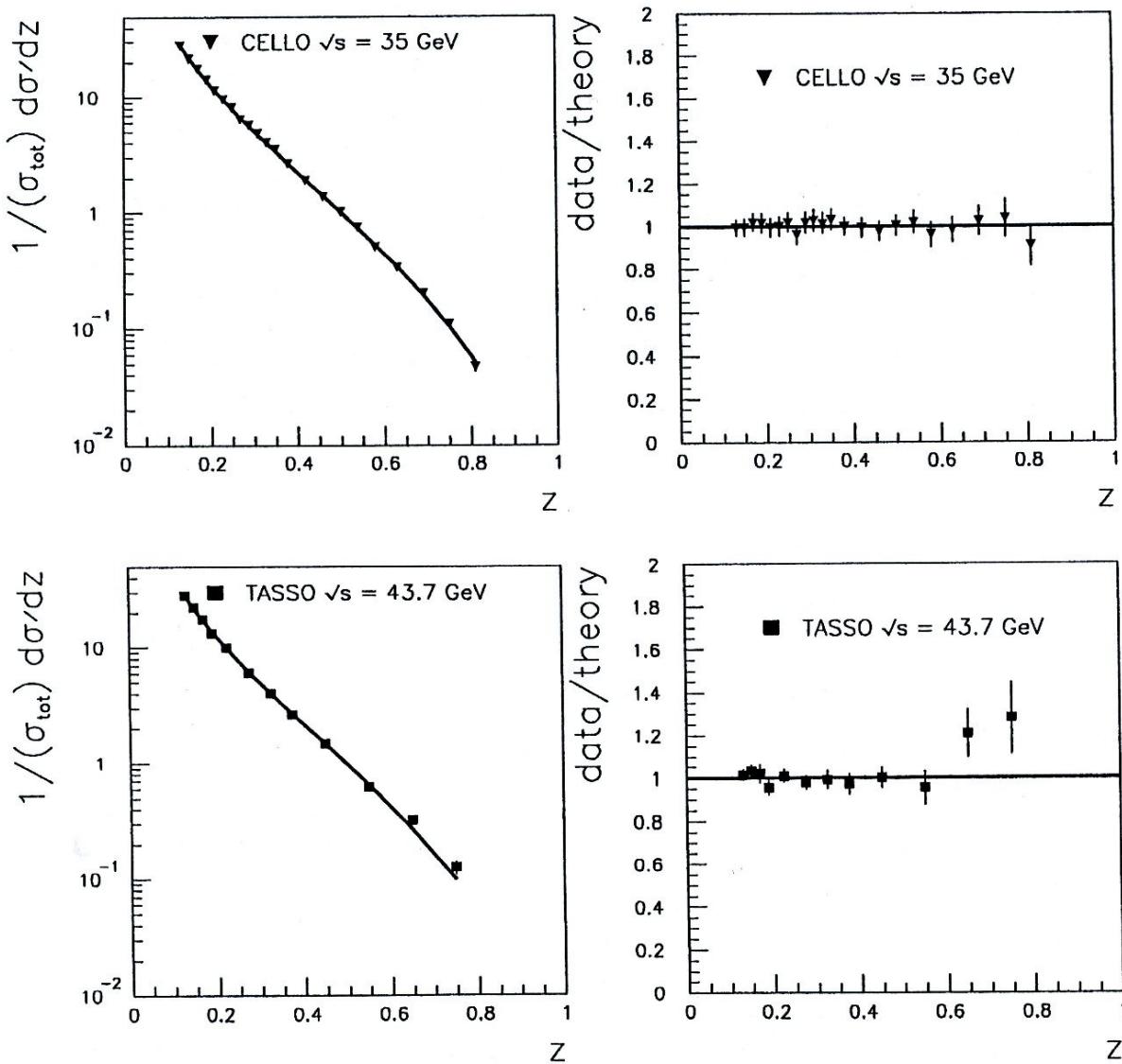
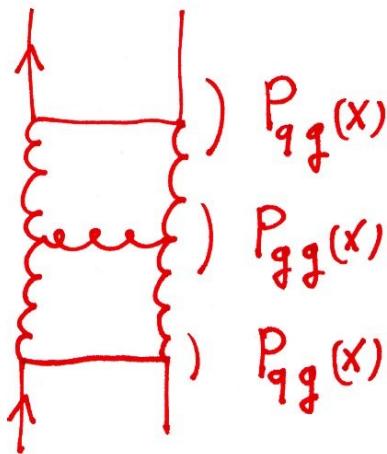


Figure 5: NLO inclusive charged particle production in  $e^+e^- \rightarrow hX$  collisions with optimized scales and with fragmentation functions obtained here (formula 8) compared to data at  $\sqrt{s} = 35$  GeV from the CELLO collaboration [20] and at  $\sqrt{s} = 44$  GeV from the TASSO collaboration [21].



$$\sum_b \int dx \times P_{ba}(x) = 0 \Rightarrow \frac{\partial}{\partial Q^2} \left( \sum_i q_i^{(2)} + \sum_{\bar{i}} \bar{q}_{\bar{i}}^{(2)} + g^{(2)} \right)$$

$$Q^2 \frac{\partial q_i(n, Q^2)}{\partial Q^2} = \frac{\alpha_s}{2\pi} \sum_{j=1}^{N_F} P_{ij}(n) q_j + \frac{\alpha_s}{2\pi} \sum_{\bar{j}} P_{i\bar{j}} \bar{q}_{\bar{j}} + \frac{\alpha_s}{2\pi} P_{ig} g$$

$$Q^2 \frac{\partial \bar{q}_{\bar{i}}(n, Q^2)}{\partial Q^2} = \frac{\alpha_s}{2\pi} \sum_j P_{\bar{i}j} q_j + \frac{\alpha_s}{2\pi} \sum_{\bar{j}} P_{\bar{i}\bar{j}} \bar{q}_{\bar{j}} + \frac{\alpha_s}{2\pi} P_{\bar{i}g} g$$

$$Q^2 \frac{\partial g(n, Q^2)}{\partial Q^2} = \frac{\alpha_s}{2\pi} \sum_j P_{gj} q_j + \frac{\alpha_s}{2\pi} \sum_{\bar{j}} P_{g\bar{j}} \bar{q}_{\bar{j}} + \frac{\alpha_s}{2\pi} P_{gg} g$$

Dokshitzer

Gribou

## Lipatous

A. Tatarelli

Paris

↳ DGLAP

$$P_{qg}^{(c)} = \frac{1}{2} \left( x^2 + (1-x)^2 \right) \quad (c) \rightarrow LL : \quad \sum (Q_s Q^2) \log(Q^2)^n$$

$$P_{gg}^{(o)} = C_F \left(1 + (1-x)^2\right)/x$$

$$P_{gg}^{(c)} = 2 N_c \left\{ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \frac{11N_c - 2N_F}{2N_c} \delta(1-x) \right\}$$

$$q_i^{(+)} = q_i + \bar{q}_i$$

$$Q^2 \partial q_i^{NS} = \frac{\omega_s}{2\pi} P^{NS} q_i^{NS}$$

$$q^{(+)} = \sum_i q_i^{(+)}$$

$$q_i^{NS} = q_i^{(+) - \frac{q_f^{(+)}}{N_p}}$$

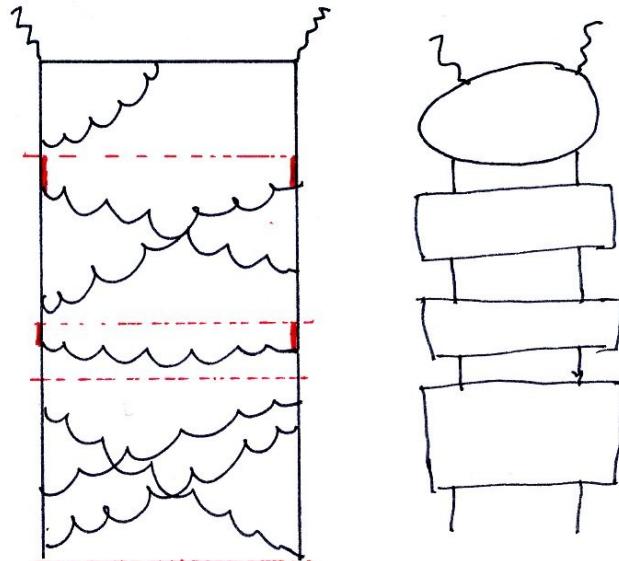
$$P^{(o)N3} = P_{99}^{(o)} = C_F \left( \frac{1+x^2}{1-x} \right)$$

## → Théorie vectorielle

NLO, NNLO

$$(\alpha_s(Q^2) \log \frac{Q^2}{\mu^2})^n \rightarrow LL$$

$$\alpha_s^n \log^k \frac{Q^2}{\mu^2} \quad k < n$$



$$\rightarrow [ \text{rect} ] + [ \text{rect} ] \quad \int \frac{dk^2}{k^2} k^2 P_{qq}$$

$$P_{qq} = \frac{\alpha_s}{2\pi} P_{qq}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{qq}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{qq}^{(2)}$$

$$F_1(n_1 Q^2) = \sum_{q,\bar{q}} C_{1,q}(n_1 Q^2) q(n_1 Q^2) + C_{1,g}(n_1 Q^2) g(n_1 Q^2)$$



## Schéma de Factorisation

$$\sum_{P^2=0} + \sum_{\text{free}} + \sum_{\text{ext}} + \sum_{P^+} + \sum_{P^-} + \sum_{\text{int}} - \sum_{\text{g}} - \sum_{\text{f}} - \sum_{\text{o}}$$

$$h = 4 - 2\varepsilon \quad (1) \varepsilon > 0 \rightarrow \text{uv}$$

(2)  $\varepsilon < 0 \rightarrow \text{IR} + \text{collinéaire}$

$$\frac{1}{x} F_2^{(p)}(x, Q^2) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi} f_2(x) - \frac{1}{\varepsilon} \frac{\alpha_s}{2\pi} P_{qg}^{(0)} \left( \frac{4\pi\mu^2}{Q^2} \right)^\varepsilon \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

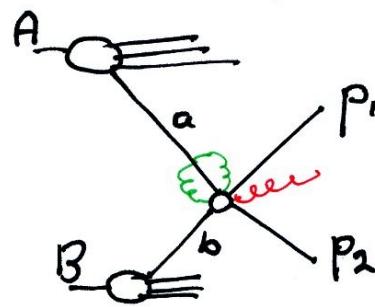
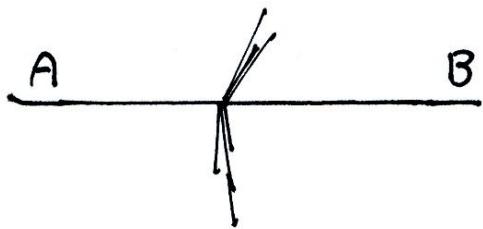
$$f_2(x) = C_F \left[ (1+x^2) \left( \frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \left( \frac{1}{1-x} \right)_+ - \frac{1+x^2}{1-x} \ln x + 3 + 2x - \delta(1-x) \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \right]$$

$$F_2^{(p)}(n-1, Q^2) = \left[ 1 + \frac{\alpha_s}{2\pi} f_2(n) + \frac{\alpha_s}{2\pi} P_{qg}^{(0)}(n, \log \frac{Q^2}{M^2}) \right] \left[ 1 - \frac{\alpha_s}{2\pi} \left( \frac{1}{\varepsilon} + \ln 4\pi \right) P_{qg}^{(0)} + \frac{\alpha_s}{2\pi} P_{qg}^{(0)} \log \frac{M^2}{\mu^2} \right] + O(\varepsilon)$$

$$F_2^{(p)}(n-1, Q^2) = C_{2,q}(n, M^2, Q^2) q_p(n, M^2)$$

Factorisation  $\bar{MS}$ , échelle de factorisation  $M$

# Réactions à grande impulsion transverse



$$p_a = x_a P_A$$

$$\hat{s} = (p_a + p_b)^2 = x_a x_b S$$

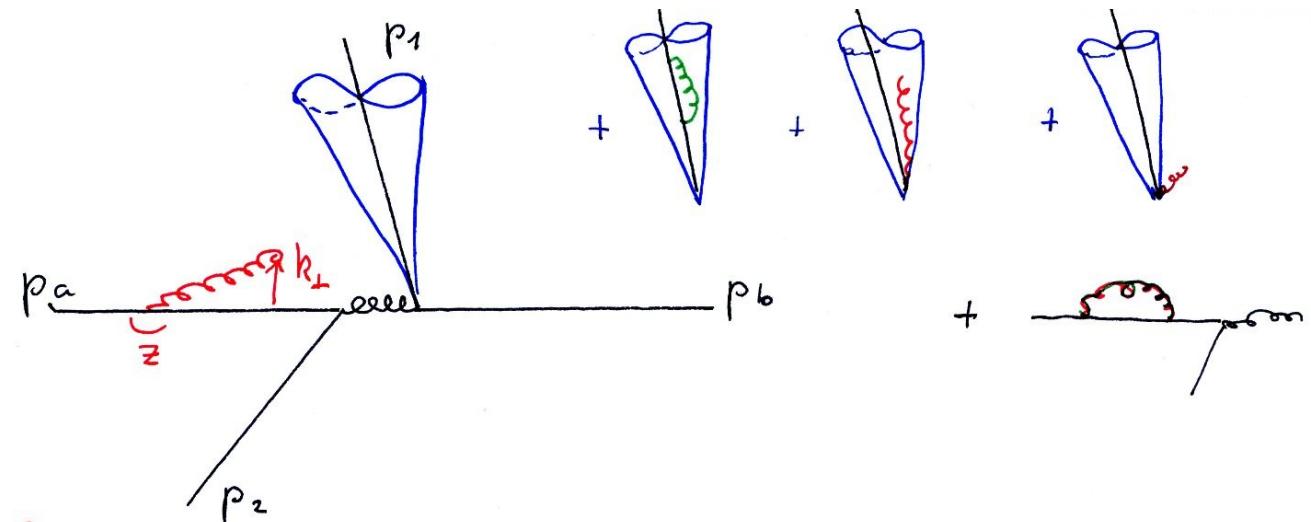
$$S = 2 P_A \cdot P_B$$

$$d\sigma = \sum_{a,b} \left\{ dx_a G_{a/A}(x_a) dx_b G_{b/B}(x_b) \left\{ \frac{1}{2\hat{s}} |\mu_{ab}^B|^2 \prod_{i=1}^2 \frac{d^4 p_i \delta(p_i^2)}{(2\pi)^3} (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2) \right. \right.$$

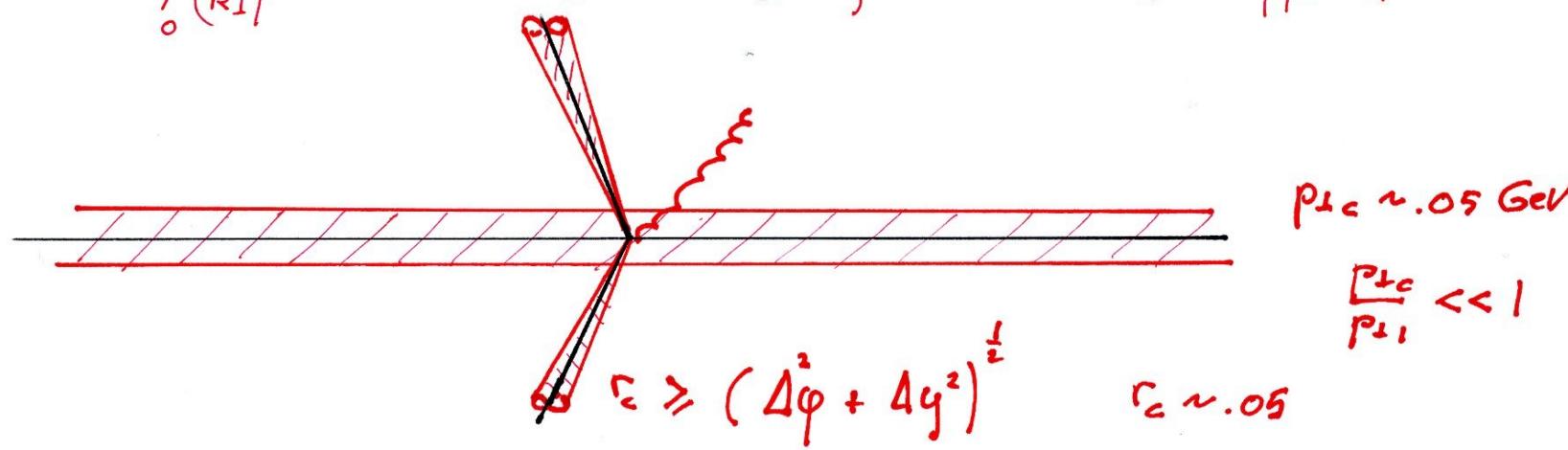
$$\left. \left. + \frac{1}{2\hat{s}} |\mu_{ab}^{(2 \rightarrow 3)}|^2 \prod_{i=1}^2 \frac{d^4 p_i \delta(p_i^2)}{(2\pi)^3} (2\pi)^4 \delta^4(p_a + p_b - \sum_{i=1}^3 p_i) \right. \right. + \frac{1}{2\hat{s}} |\mu_{ab}^\nu|^2 \prod_{i=1}^2 \frac{d^4 p_i \delta(p_i^2)}{(2\pi)^3} (2\pi)^4 \delta(p_a + p_b - p_1 - p_2) \right\}$$

$$\int d^4 p_i \delta(p_i^2) = \int \frac{d\vec{p}_i}{2p_i^0} = \frac{1}{2} \int d\vec{p}_{\perp i} dy \rightarrow d\sigma / d\vec{p}_{\perp} dy \quad (p_z = p_{\perp} \sin y)$$

$$\int d^4 p_2 \delta(p_2^2) \delta^4(p_a + p_b - p_1 - p_2) = \delta((p_a + p_b - p_1)^2)$$



$$\mu^2 \epsilon g^2(\mu) \left\{ dz P_{qq}(z) \cdot \Gamma^B(z) \right\} \int_0^{p_{\perp 1}} \frac{dk_{\perp}}{(k_{\perp})^{1+2\epsilon}} = \frac{g^2(\mu)}{2} \left\{ -\frac{1}{\epsilon} + \ln \frac{M^2}{\mu^2} \right\} P_{qq} \otimes \Gamma^B + \frac{g^2(\mu)}{2} \ln \frac{p_{\perp 1}^2}{M^2} P_{qq} \otimes \Gamma$$



$2 \rightarrow 2$  (Born virtuelle füher) +  $2 \rightarrow "2"$  +  $2 \rightarrow 3$  (ext.)

## The PHOX Family

This site aims to make available **NLO** FORTRAN codes allowing users to compute **single and double inclusive large pt cross sections** for reactions involving **photons, hadrons and jets**. The production of massive heavy quarks is not described by these codes in which a massless approximation is used.

These codes are event generators at the parton level and they are all built according to the same scheme. Please read the [technical presentation page](#) for a description of the method. This approach is flexible and allows the users to impose almost any experimental cuts, jet definitions, cross section definition via a histogram package. However, please read also the [warnings](#) about the limitations of the codes.

The program packages contain the **MRST99**, **MRST01**, **CTEQ5** and **CTEQ6** parton distributions for the proton. The photoproduction programs also include the **AFG** and the [new AFG04](#) parton distributions for the photon. The option to link any parton distribution from the **PDFLIB** is also provided, but note that using the local grids is faster.

The fragmentation functions provided are

- . Bourhis et al. for photons (*Eur. Phys. J. C2 (1998) 529* )
- . Bourhis et al. for charged hadrons (*Eur. Phys. J. C19 (2001) 89* )
- . Kniehl, Kramer, Pötter for hadrons (*Nucl. Phys. B582 (2000) 514* )
- . Binnewies, Kniehl, Kramer for hadrons ( *Phys.Rev. D52 (1995) 4947; Phys.Rev. D53 (1996) 3573* )
- . Kretzer for hadrons (*Phys. Rev. D 62 (2000) 054001*)

- **DIPHOX** (acronym for DI-PHoton/hadron X sections,  $h_i$  denotes hadrons)

- $h_1 \ h_2 \rightarrow \gamma \ \gamma + X$

- $h_1 \ h_2 \rightarrow \gamma \ h_3 + X$

- $h_1 \ h_2 \rightarrow h_3 \ h_4 + X$

- **JETPHOX** (acronym for JET-PHoton/hadron X sections) (available soon)

- $h_1 \ h_2 \rightarrow \gamma \ \text{jet} + X$  and  $h_1 \ h_2 \rightarrow \gamma + X$  (see below)

- $h_1 \ h_2 \rightarrow h_3 \ \text{jet} + X$  and  $h_1 \ h_2 \rightarrow h_3 + X$  (see below)

- $h_1 \ h_2 \rightarrow \text{jet jet}$  (under construction)

- **EPHOX** (acronym for Electron-Proton-initiated PHoton/hadron X sections,  $h_1$  is typically a proton)

- $\gamma \ h_1 \rightarrow \gamma \ \text{jet} + X$  and  $\gamma \ h_1 \rightarrow \gamma + X$

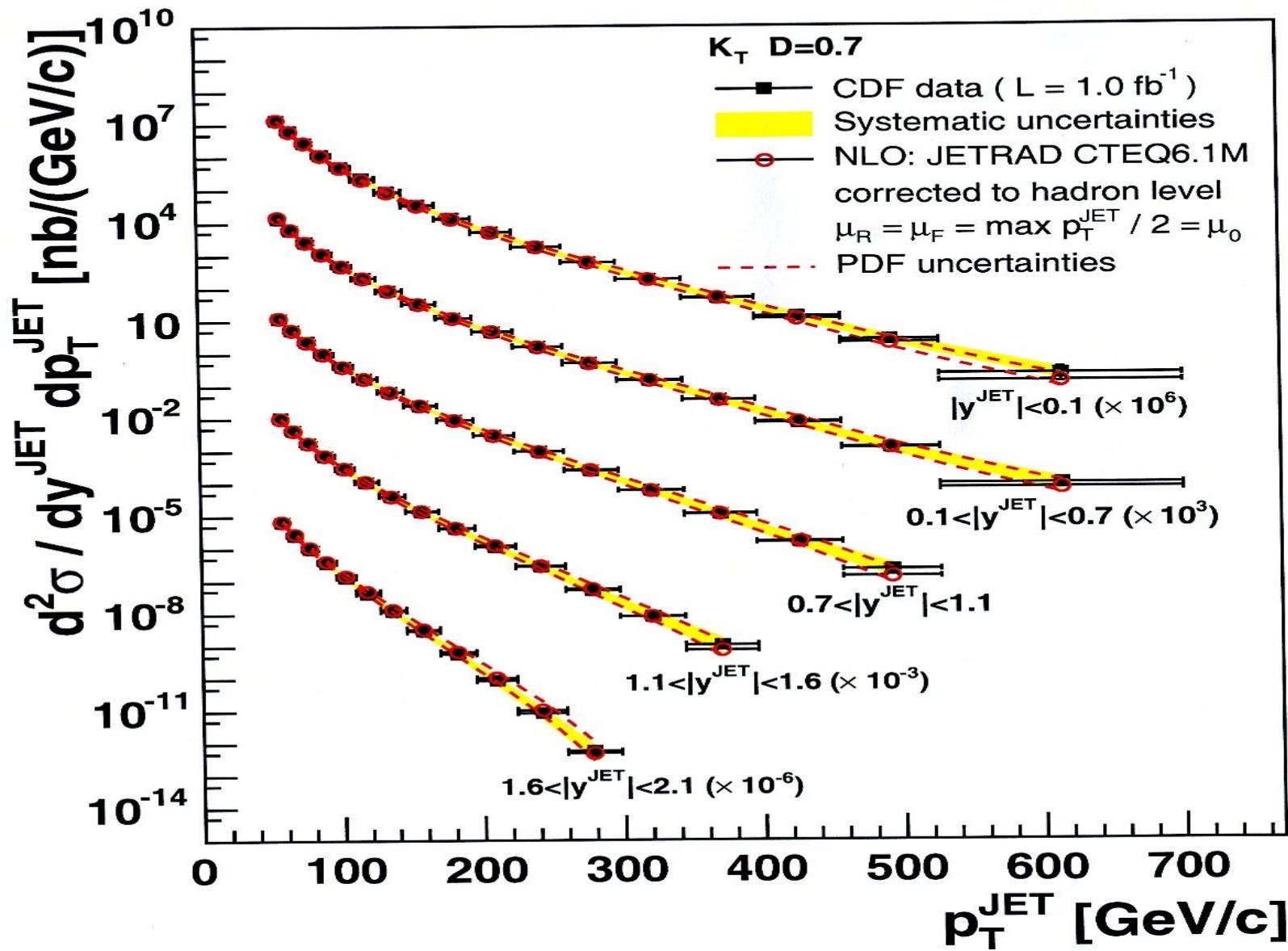
- $\gamma \ h_1 \rightarrow h_2 \ \text{jet} + X$  and  $\gamma \ h_1 \rightarrow h_2 + X$

- $\gamma h_1 \rightarrow \text{jet jet} + X$  (under construction)
- **TWINPHOX** (acronym for Two-photon-INitiated PHOton/hadron X sections)
  - $\gamma \gamma \rightarrow \gamma \text{jet} + X$  and  $\gamma \gamma \rightarrow \gamma + X$  (only with Frixione's isolation criterium)
  - $\gamma \gamma \rightarrow \text{jet jet} + X$  (under construction)

Enjoy the PHOX Family of Programs !

If one wants to compute inclusive cross section for photon/hadron production in hadronic collisions, one can use older programs **JNCNLO**. These programs are faster than JETPHOX but they are purely inclusive, i.e. some experimental cuts, such as isolation cuts, cannot be taken into account.

- P. Aurenche
- T. Binoth
- M. Fontannaz
- J. Ph. Guillet
- G. Heinrich
- E. Pilon
- M. Werlen



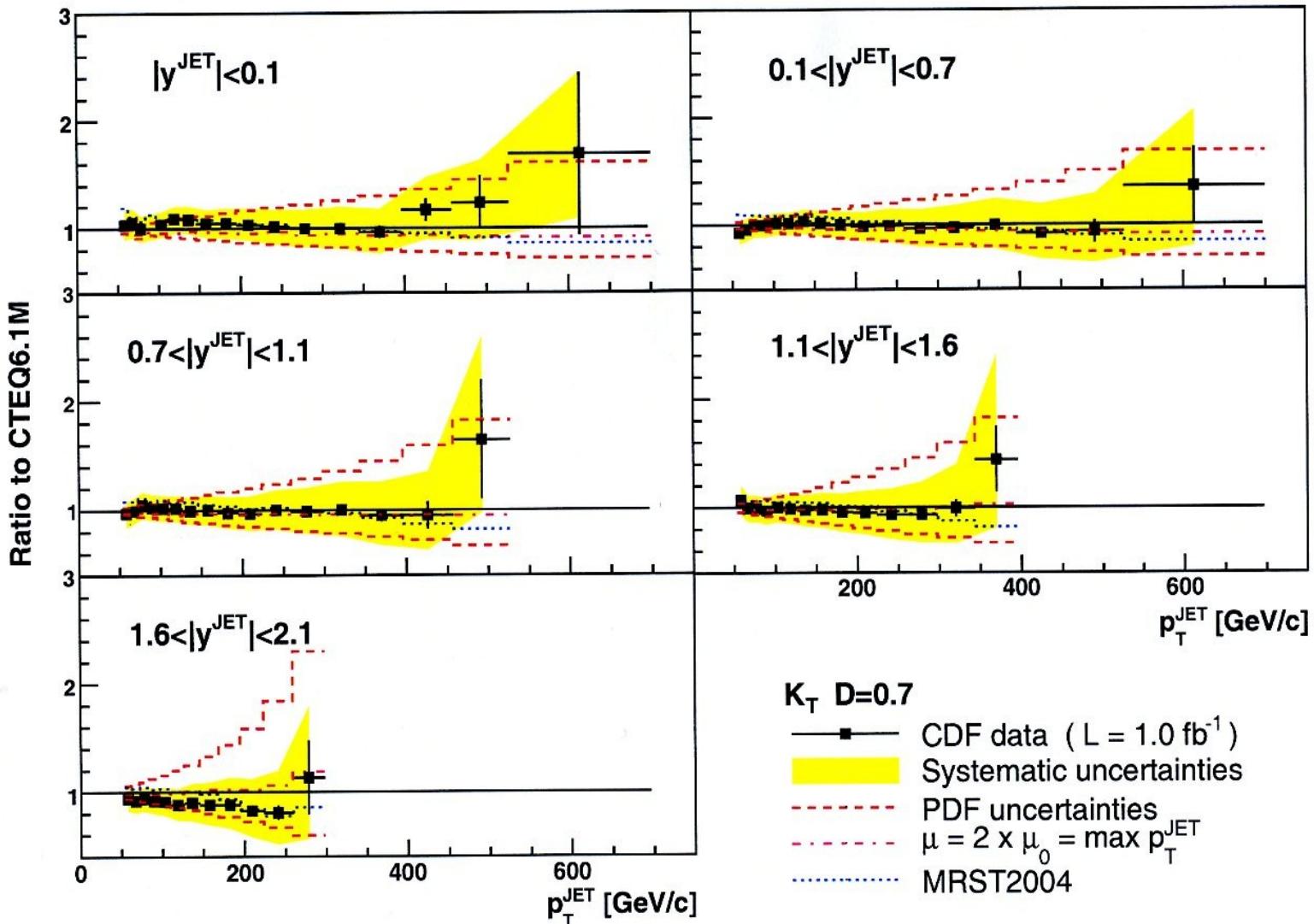


FIG. 10: Ratio data/theory as a function of  $p_T^{\text{jet}}$  in different  $|y^{\text{jet}}|$  regions. The error bars (shaded bands) show the total statistical (systematic) uncertainty on the data. A 5.8% uncertainty on the integrated luminosity is not included. The dashed lines indicate the PDF uncertainty on the theoretical predictions. The dotted lines present the ratio of NLO pQCD predictions using MRST2004 and CTEQ6.1M PDFs. The dotted-dashed lines show the ratios of pQCD predictions with  $2\mu_0$  and  $\mu_0$ .

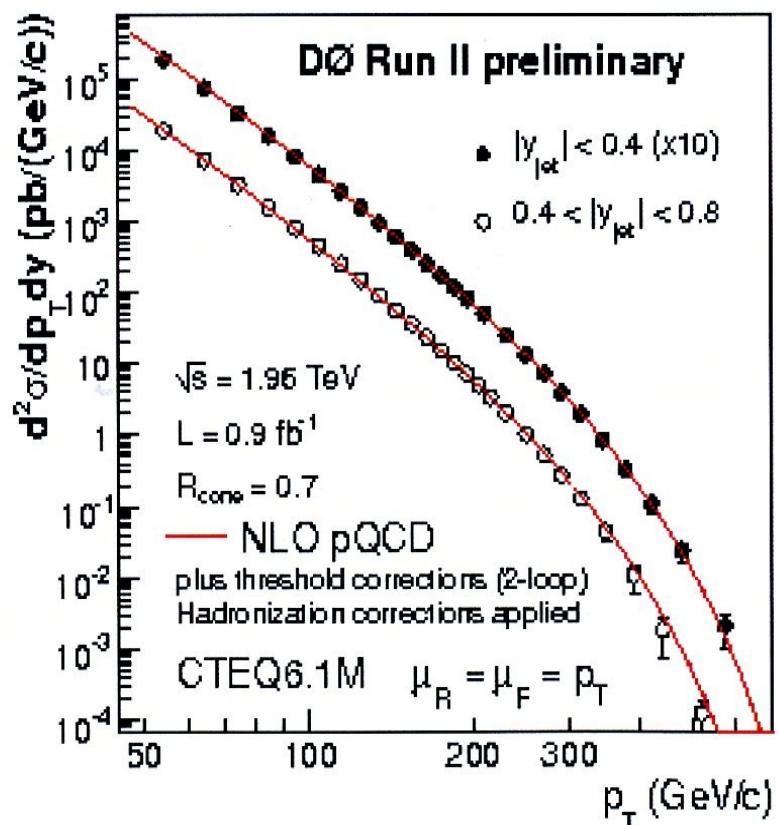


Figure 1: The inclusive jet differential cross section measured in two regions of jet rapidity.

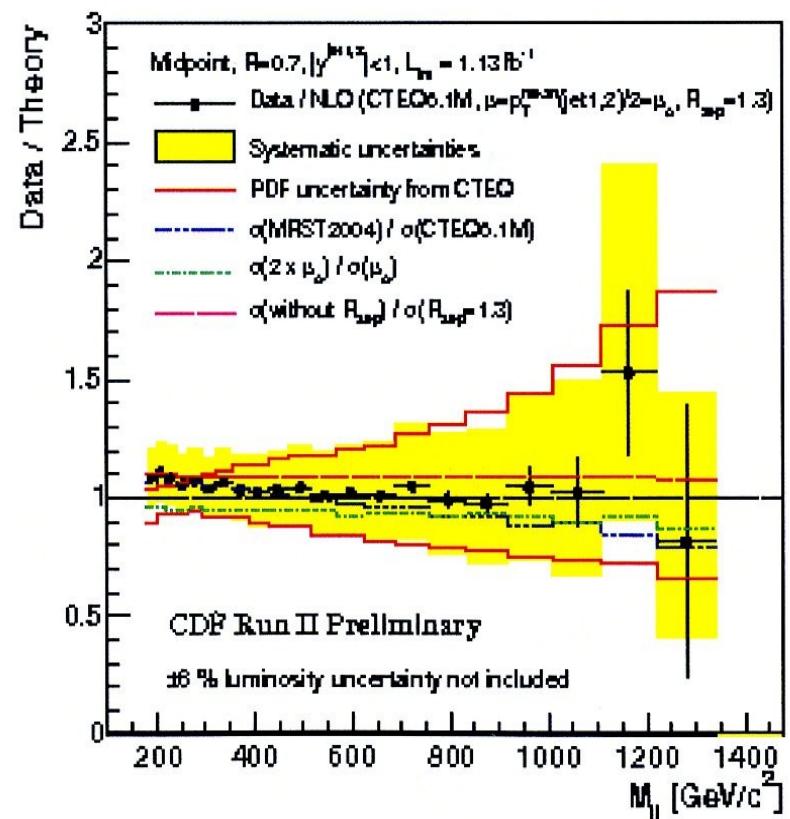
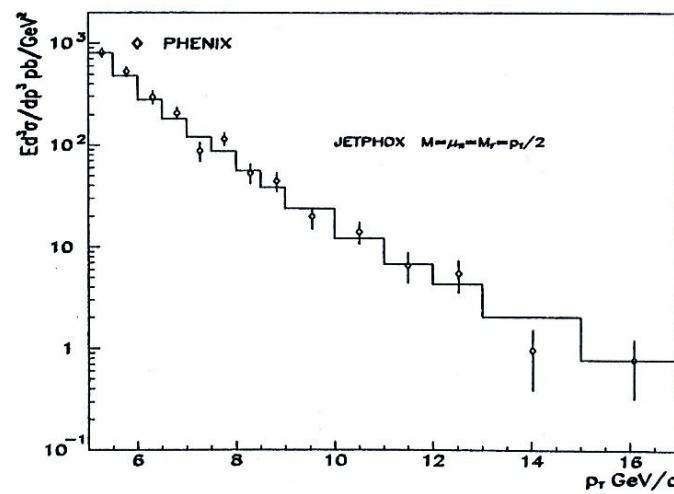
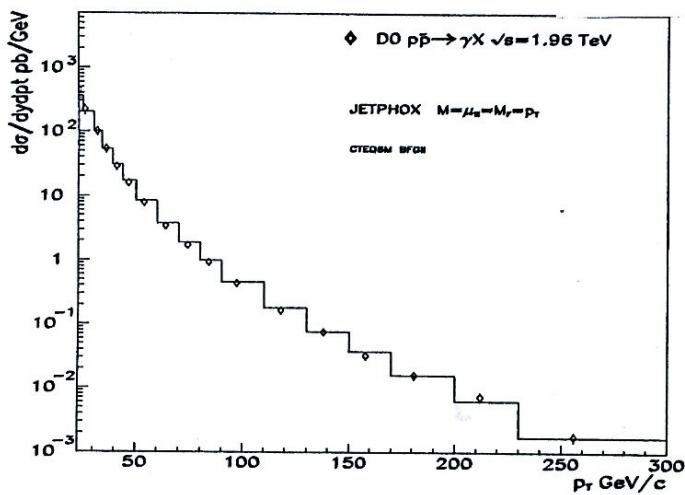
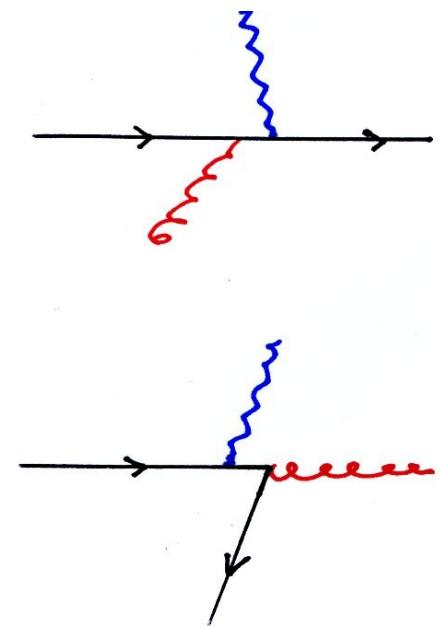
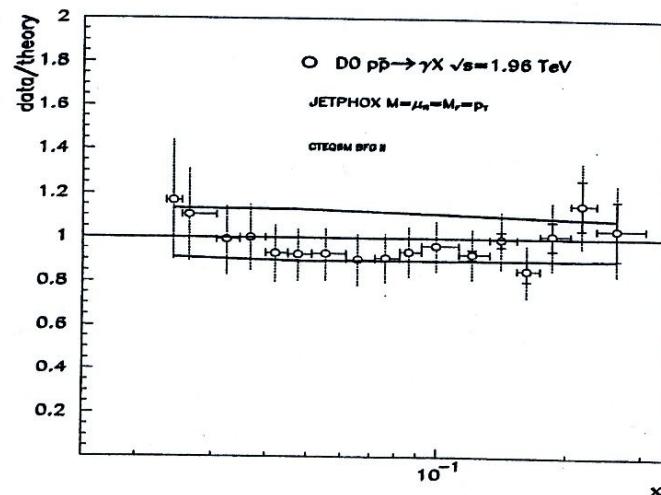


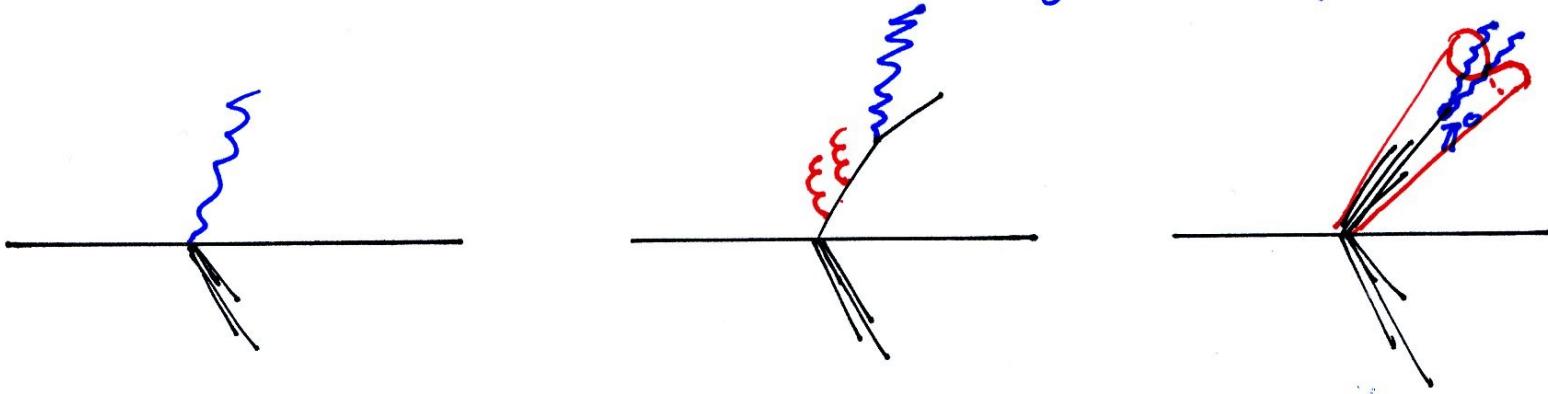
Figure 2: The data/theory ratio for dijet inclusive cross sections as a function of the dijet invariant mass.

$|\eta| < .9$       isolé'  
 $\mu = p_+$



$\sqrt{s} = 200$

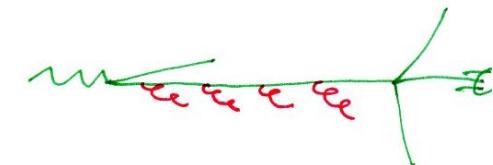
# Photons à grand $p_T$



$$e_q^2 P_{Fq}(z) \ln \frac{p_T^2}{\mu^2} \rightarrow e_q^2 P_{Fq}(z) \ln \frac{p_T^2}{Q_0^2} + D_q^{F, NP}(z)$$

$$\int_{Q_0^2}^{p_T^2} \frac{dk_2^2}{k_2^2} \alpha_s(k_2^2) \left\{ \int_{Q_0^2}^{k_2^2} \frac{dk_1^2}{k_1^2} \right\} = \int_{Q_0^2}^{p_T^2} \frac{dk_2^2}{k_2^2} \alpha_s(k_2^2) \ln \frac{k_2^2}{Q_0^2} \sim \text{Log} \frac{p_T^2}{Q_0^2}$$

$$D_q^F(z, p_T^2) = D_q^P(z) \text{Log} \frac{p_T^2}{Q_0^2} + D_q^{F, NP}(z, p_T^2)$$



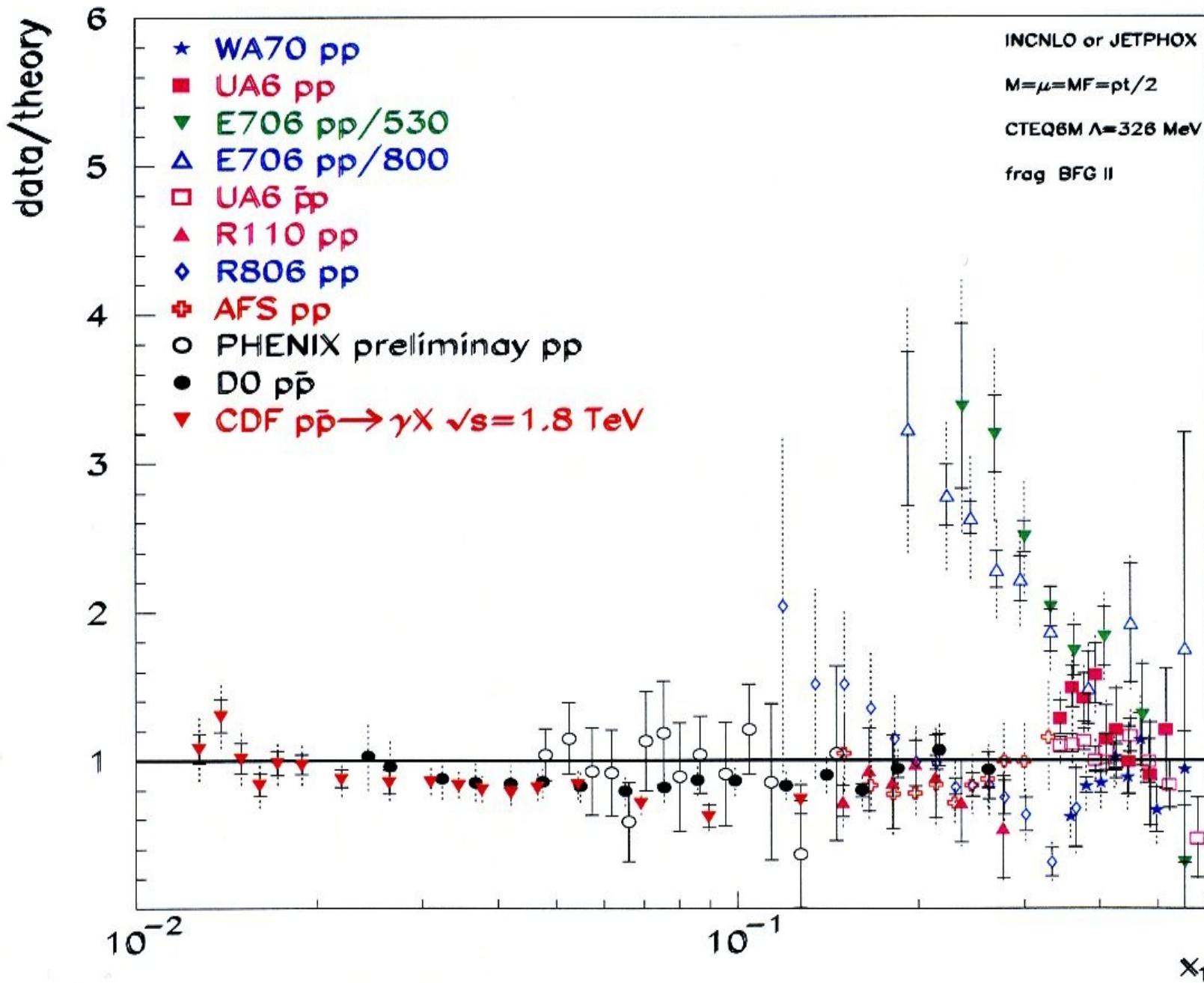
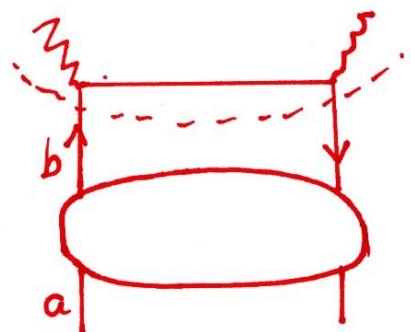
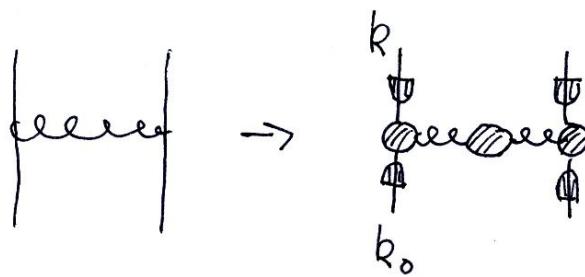


Figure 6: Ratios data /theory for collider and fixed target data with the scale  $\mu = p_t/2$ . For PHENIX



DDT

$$= \uparrow \downarrow + \begin{array}{c} b \\ | \\ \text{eeeeee} \\ | \\ a \end{array} + \begin{array}{c} b \\ | \\ \text{eeeeee} \\ | \\ a \end{array} + \dots = \uparrow \downarrow + \begin{array}{c} b \\ | \\ \text{eeeeee} \\ | \\ c \\ a \end{array}$$



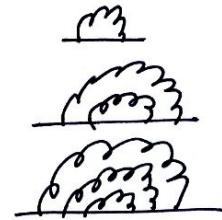
$$\underbrace{g^2(Q^2) d_q(k^2) \Gamma(k^2) d_g(k^2) \Gamma(k^2) d_g(k^2)}_{g^2(k^2)} \frac{d_q(k_0^2)}{d_q(k^2)}$$

$$d_q(k^2, \Gamma_k) \quad \Gamma_k = \frac{(2k \cdot c)}{c^2} \sim Q^2 \quad \frac{1}{1-x+\Delta} \quad \Delta = \frac{|k^2|}{\Gamma_k}$$

$$G_a^b(x, k_0^2, Q^2) = S_a^b \delta(1-x) \frac{d_a(k_0^2)}{d_a(Q^2)} + \sum_c \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \frac{d_b(k^2)}{d_b(Q^2)} \int_0^1 \frac{dz}{z} \tilde{P}_{bc}(z) G_a^c\left(\frac{x}{z}, k^2\right)$$

$$\tilde{P}_{qq} = C_F \frac{1+x^2}{1-x+\Delta}$$

$$\dot{d}_b = Q^2 \frac{\partial d_b(Q^2, \tau_b)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \sum_c \int_0^1 dz z \tilde{P}_{cb}(z) d_b(Q^2, \tau_b)$$



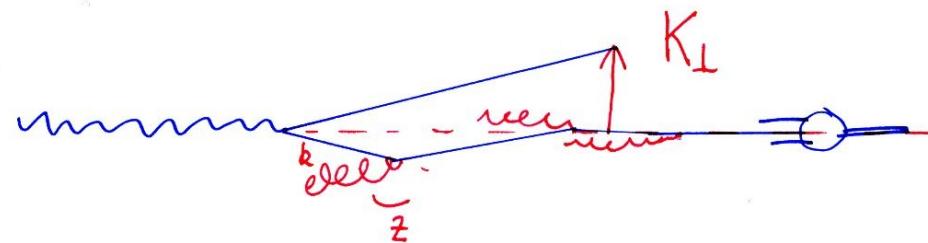
$$\dot{d}_b G_a^b + d_b \dot{G}_a^b = \frac{\alpha_s(Q^2)}{2\pi} d_b(Q^2) \int_0^1 \frac{dz}{z} \tilde{P}_{bc}(z) G_a^c(\frac{x}{z}, Q^2)$$

$$Q^2 \partial G_a^b(x, k_0^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left\{ \sum_c \left( \int_0^1 \frac{dz}{z} \tilde{P}_{bc}(z) G_a^c(\frac{x}{z}, Q^2) - \sum_c \int_0^1 dz z P_{cb}(z) G_a^b(x, Q^2) \right) \right\}$$

DGLAP,  $\alpha_s(k^2)$

$$\frac{\alpha_s(k^2)}{2\pi} \frac{1+z^2}{1-z+\Delta} \frac{d_q(k^2) d_q^{-1}(Q^2)}{k^2} dk^2 dz$$

$$d_q(k^2) = e^{- \int_{k^2}^{T_k} \frac{dx^2}{x^2} \frac{\alpha_s(x)}{2\pi} \sum_c \int_0^1 dz z P_{cq}(z)} \simeq e^{- \frac{\alpha_s}{2\pi} C_F \ln \frac{k^2}{k_0^2}}$$



$$-k^2 = \vec{k}_\perp^2 / (1-z)$$

$$\vec{R}_\perp^2 \leq K_\perp^2$$

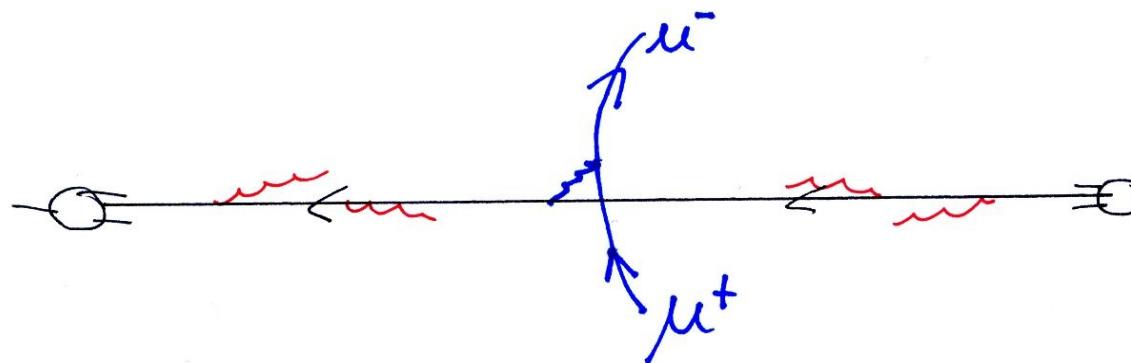
$$\Theta(K_\perp^2 - |k|^2(1-z))$$

$$G_a^b(x, Q^2, K_\perp^2) = G_a^b(x, K_\perp^2, K_\perp^2) \bar{T}_b(K_\perp^2, \bar{\tau}_b)$$

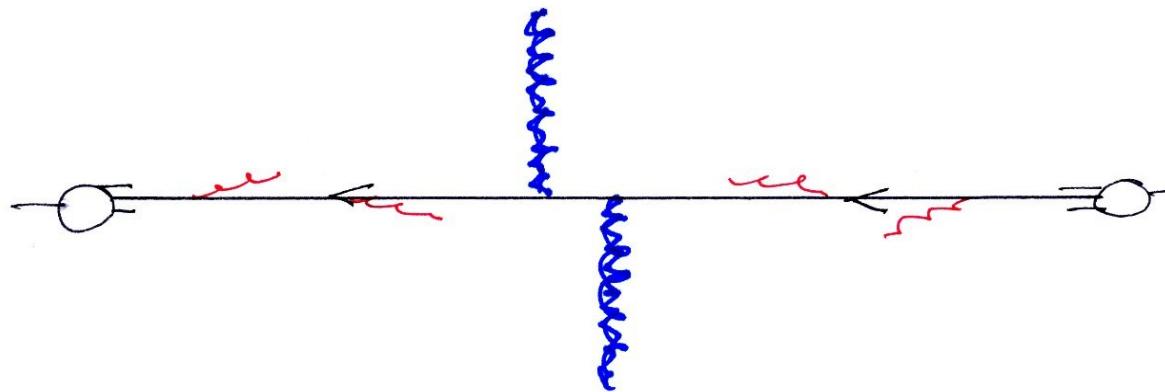
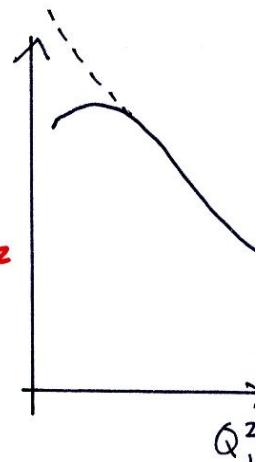
$$\bar{T}_q = e^{- \int_{K_\perp^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k)}{2\pi} \int_{K_\perp^2/k^2}^1 dz P_{qg}(z, \bar{\tau}_b)} \simeq e^{- \frac{\alpha_s}{3\pi} \ln \frac{Q^2}{K_\perp^2}}$$

$$K_\perp^2 \frac{d\bar{\tau}}{d K_\perp^2} \sim \frac{\alpha_s}{3\pi} \ln \frac{Q^2}{K_\perp^2} e^{- \frac{\alpha_s}{3\pi} \ln^2 \frac{Q^2}{K_\perp^2}}$$

## Drell-Yan



$$\frac{d\Gamma}{dQ^2 d\vec{Q}'^2}$$



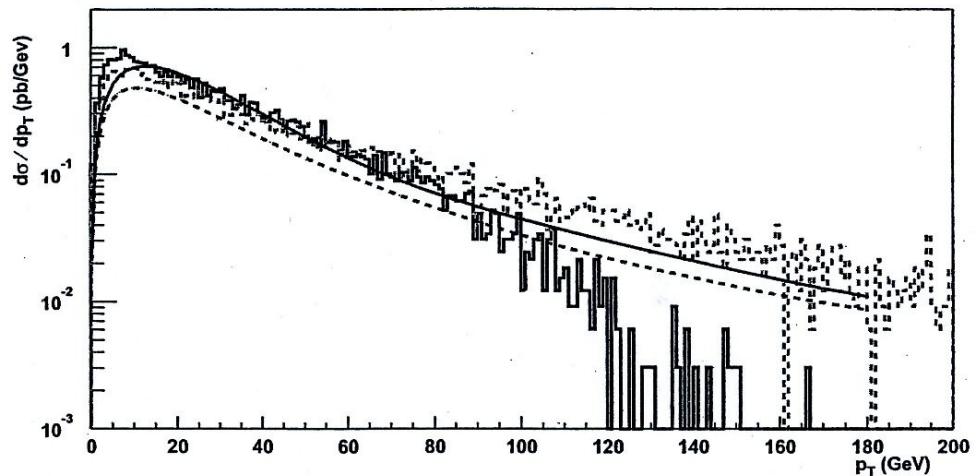
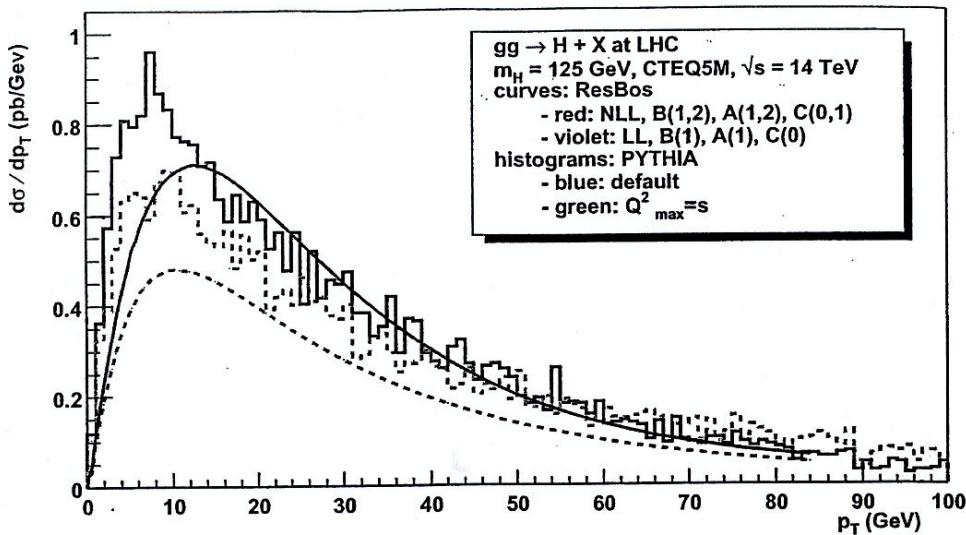
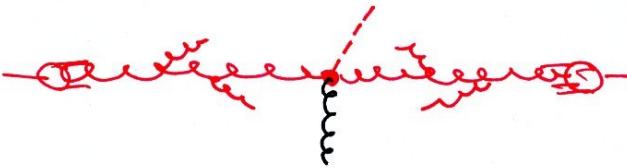


Fig. 6: Higgs boson transverse momentum distributions at the LHC calculated by ResBos (curves) and PYTHIA (histograms). The solid curve was calculated at NLL<sup>16</sup> (including  $A^{(1,2)}$ ,  $B^{(1,2)}$ , and  $C^{(0,1)}$ ). The dashed curve is LL (includes  $A^{(1)}$ ,  $B^{(1)}$ , and  $C^{(0)}$ ). For PYTHIA, the original output with default input parameters rescaled by a factor of  $K = 2$  (solid), and one calculated by the altered input parameter value  $Q^2_{\max} = s$  (dashed) are shown. The lower portion, with a logarithmic scale, also shows the high  $Q_T$  region. In the last frame all are normalized to the solid curve.

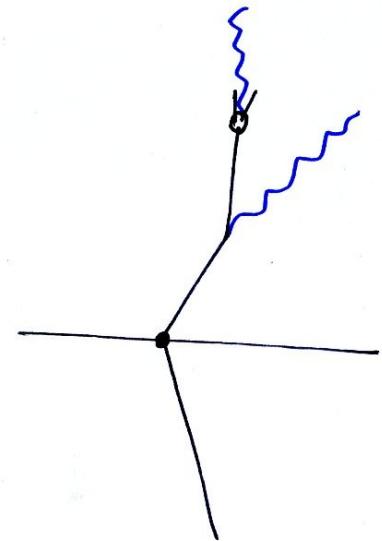
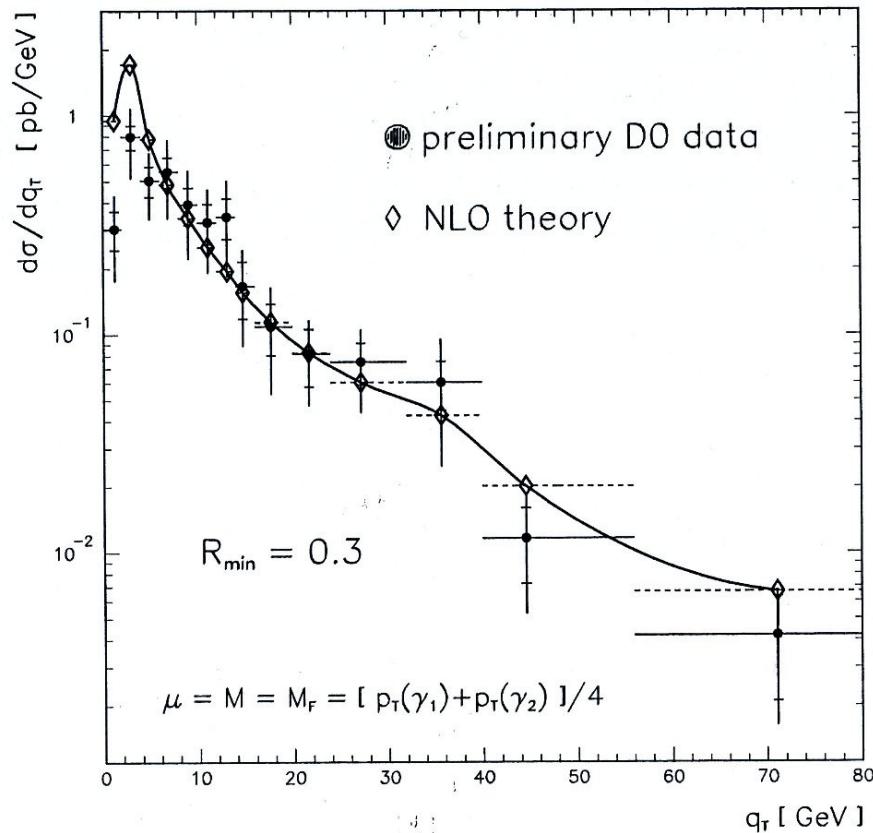
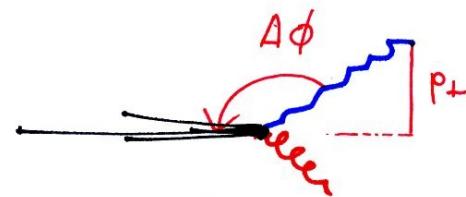
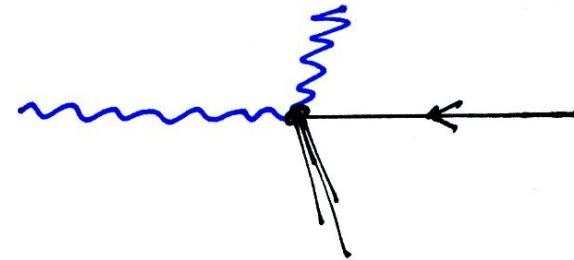


Figure 1:  $q_T$  distribution of photon pairs. Black dots: D0 data [6]; white diamonds: average values of the NLO calculation (DIPHOX code) in the corresponding experimental bins. The curve is a spline interpolation between the theoretical average.

# Photoproduction $\alpha^-$ HERA

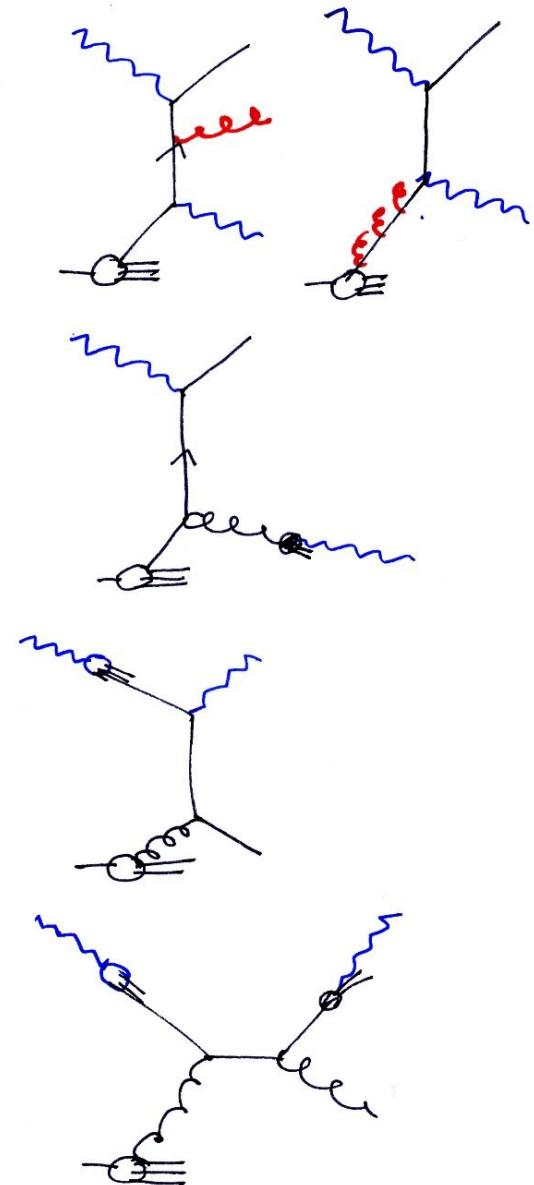


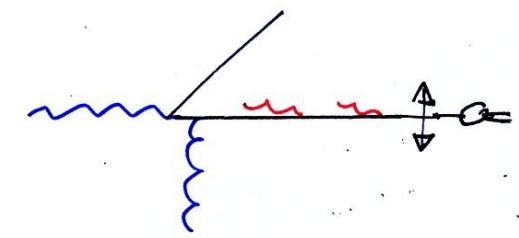
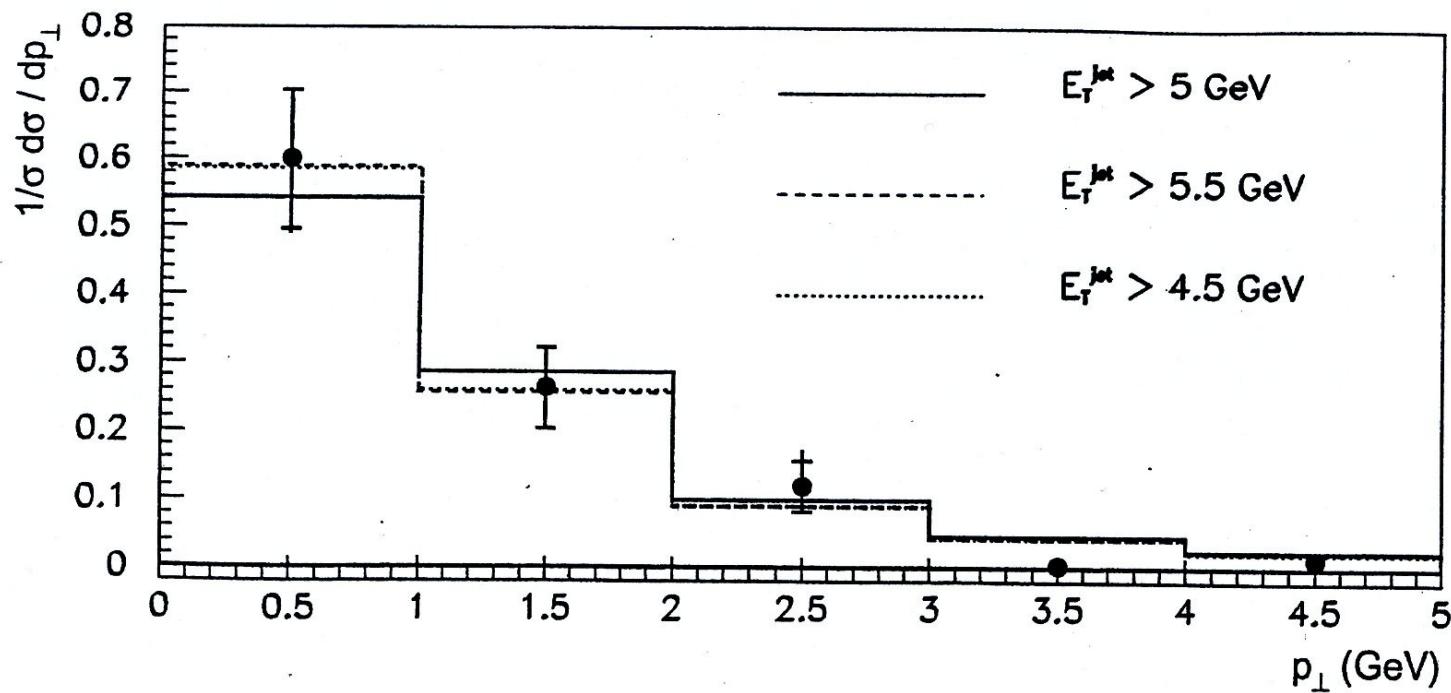
isole'

$E_T^e > 5 \text{ GeV}$

$E_T^{tot} > 5 \text{ GeV}$

ZEUS





Pythia:  $\langle K_\perp^{\text{int}} \rangle = 1.25 \text{ GeV}$

Herwig:  $\langle K \rangle = \phi$

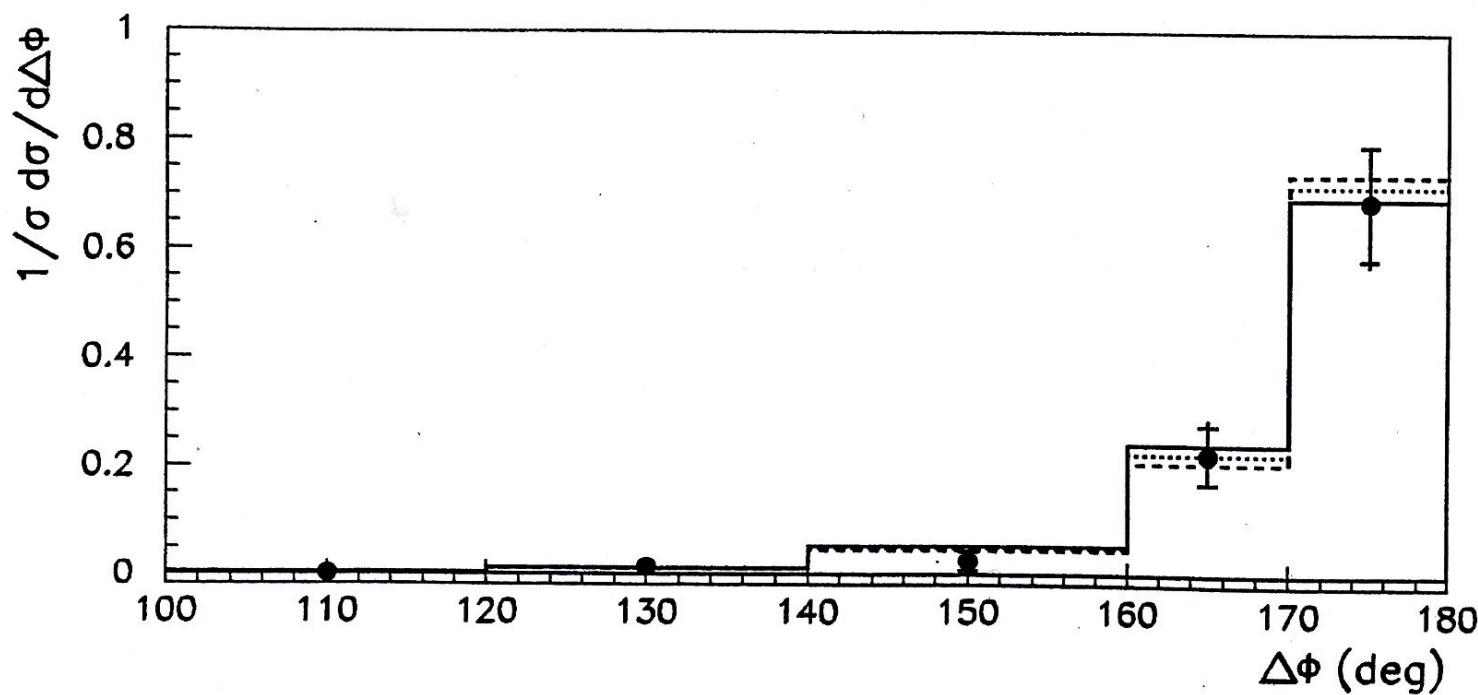


Fig. 12.  
ential in  
and 5.5  
ner and  
statistic