Lattice QCD - phenomenology

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NPQCD INPUT NEEDED

e.g.

$$\underbrace{\frac{d\Gamma(B\to\pi e\nu)}{dq^2}}_{\text{measure exp.}} = \overbrace{|V_{ub}|^2}^{\text{CKM}} \underbrace{\frac{G_F^2}{192\pi^3 m_B^3}\lambda^{3/2}(q^2)}_{\text{kinematics}} \underbrace{|F_+(q^2)|^2}_{\text{compute th.}}$$

- *Impressive statistics $\Rightarrow$ better experimental input*
- *Theory input: quantities that carry info on NPQCD*

(decay constants, form factors, bag parameters etc.)

We do not understand the non-perturbative QCD dynamics

HIGH PRECISION RESULTS CUM GRANO SALIS

Theory Tools: LCSR

→ *Correlator*

$$i \int d^4x e^{iqx} \langle \pi(p) | T\{V_\mu(x), P_B(0)\} | 0 \rangle$$

- *Borel transformation of single dispersion relation (2 extra parameters!)*
- *New input: distribution amplitudes*

$$\langle \pi(p) | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | 0 \rangle = -i p_\mu f_\pi \int_0^1 du e^{ipu.x} \Phi_\pi(u) + \ldots$$

$$\Phi_\pi(u, \mu) = 6u(1-u) \left[1 + a_{2n}(\mu) C_{2n}^{3/2}(2u-1) \right]$$

Moments a_{2n} *are non-perturbative input parameters! Higher twist DA's*

$\Leftarrow$ *more input stuff needed*

- *in heavy* $\longrightarrow$ *light decays, this is how it was first noticed that all form factors at* $q^2 \to 0$ *scale like* $m_Q^{3/2} F(0) \to \text{const.}$

Theory Tools: LCSR

→ *Good*

- *semi-analytic approach that provides numerical predictions for the form factors and decay constants*
- *HQ scaling laws satisfied*
- *essentially insensitive to radiative corrections & net effect of higher twist DA's is small*

→ *Less good*

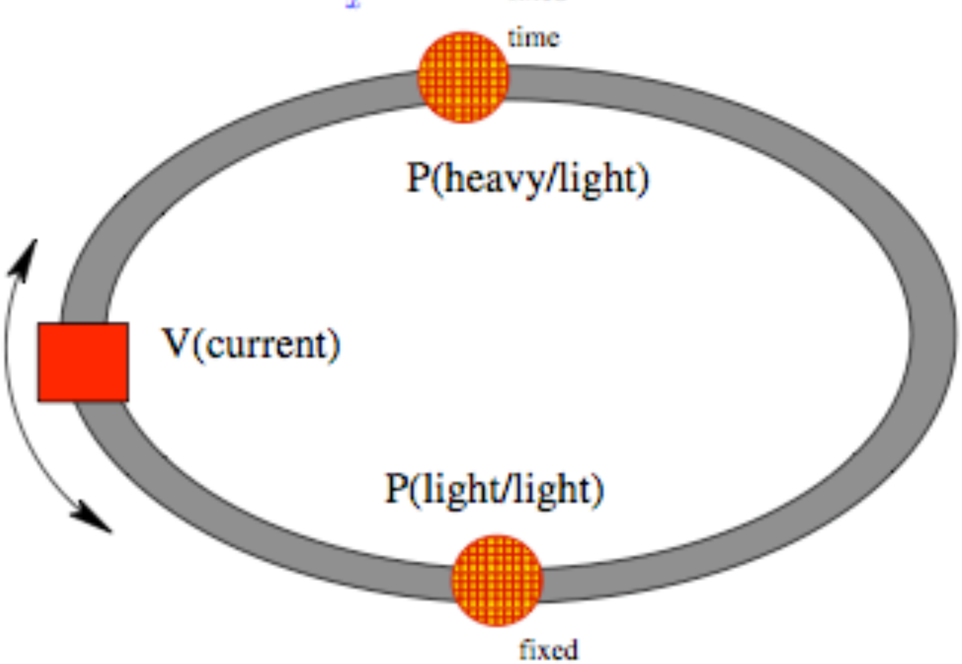
- *Bunch of NP-input parameters* ($a_{2n}^{\pi/K}$, a_{2n+1}^{K}, $\tau = 3, 4$ *DA's*)
- *Borel and duality onset parameters are not QCD*
- *What to say about the systematic uncertainties?*
- *LCSR need a large scale*
  $E_\pi \sim m_B/2$ *in* $B \to \pi$ *at* $q^2 \to 0$, *in contrast to* $D \to \pi$ *decay*

How do we compute $B \to \pi$ form factors?

$$\langle \pi^-(p)|\bar{b}\gamma_\mu u|B^0(p_{_B})\rangle = \left(p_B + p - q\frac{m_B^2 - m_\pi^2}{q^2}\right)_\mu F_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu F_0(q^2)$$

$$C_\mu^{(3)}(t;\vec{q},\vec{p}_{_H}) = \left\langle \sum_{\vec{x},\vec{y}} e^{i(\vec{q}\vec{y} - i\vec{p}_{_H})\vec{x}} \underbrace{(\bar{q}\gamma_5 q)_0}_{\text{P(light/light)}} \underbrace{(\bar{Q}\gamma_\mu q)_{\vec{y},t}}_{V_\mu} \underbrace{(\bar{Q}\gamma_5 q)^\dagger_{\vec{x},t_F}}_{\text{P(heavy/light)}} \right\rangle$$

$$C_{qq}^{(2)}(t;\vec{p}_{_H} - \vec{q}) = \left\langle \sum_{\vec{x}} e^{i(\vec{p}_{_H} - \vec{q})\vec{x}} (\bar{q}\gamma_5 q)_0 (\bar{q}\gamma_5 q)^\dagger_{\vec{x},t} \right\rangle; \qquad C_{Qq}^{(2)}(t;\vec{p}_{_H}) = \left\langle \sum_{\vec{x}} e^{i\vec{p}_{_H}\vec{x}} (\bar{Q}\gamma_5 q)_0 (\bar{Q}\gamma_5 q)^\dagger_{\vec{x},t} \right\rangle$$



Operators sufficiently separated!

Matrix element $\Leftrightarrow$ plateau of the ratio

$$R_\mu(t) = \frac{C_\mu^{(3)}(t)}{C_{qq}^{(2)}(t) C_{Qq}^{(2)}(t_{_F} - t)}$$

$$\to \langle P(p_{_H} - q)|V_\mu|H(p_{_H})\rangle$$

Explore as many kinematical configurations $(\vec{p}_{_H}, \vec{q})$ as possible

How do we compute $B \to \pi$ form factors?

Very simple strategy

1. Generate an SU(3) gauge field configuration U (MC)
2. $\forall\, t \in [0, T)$, compute the correlation functions

$$C^{(3)}_{\mu}(t)_{U} \qquad C^{(2)}_{qq}(t)_{U} \qquad C^{(2)}_{Qq}(t)_{U}$$

3. Repeat 1. and 2. for $N_{\mathrm{conf.}}$ independent U's and compute the ratio

$$R_{\mu} \overset{t_F \gg t \gg 0}{\Longrightarrow} \langle P(p_H - q)|V_{\mu}|H(p_H)\rangle$$

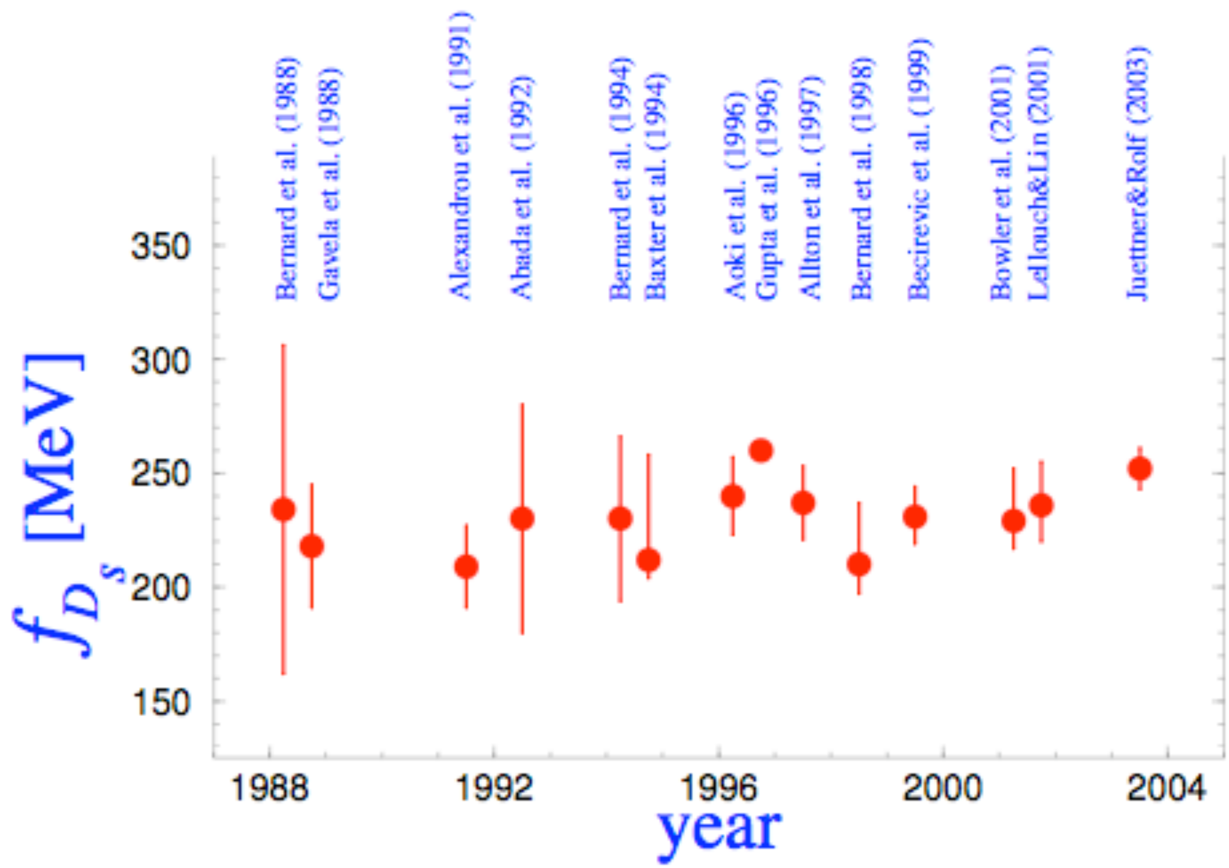
$\Rightarrow F_0(q^2), F_+(q^2)$ for $H_{Qq} \to P_{qq}$

4. Do 2. and 3. for several light quarks q and several heavy quarks Q

However,

$$m_c \ll \pi/a\,,\ \text{but}\quad m_b \not\ll \pi/a \;\to\; m_c \leq m_Q < m_b \oplus m_d < m_q \leq m_s$$

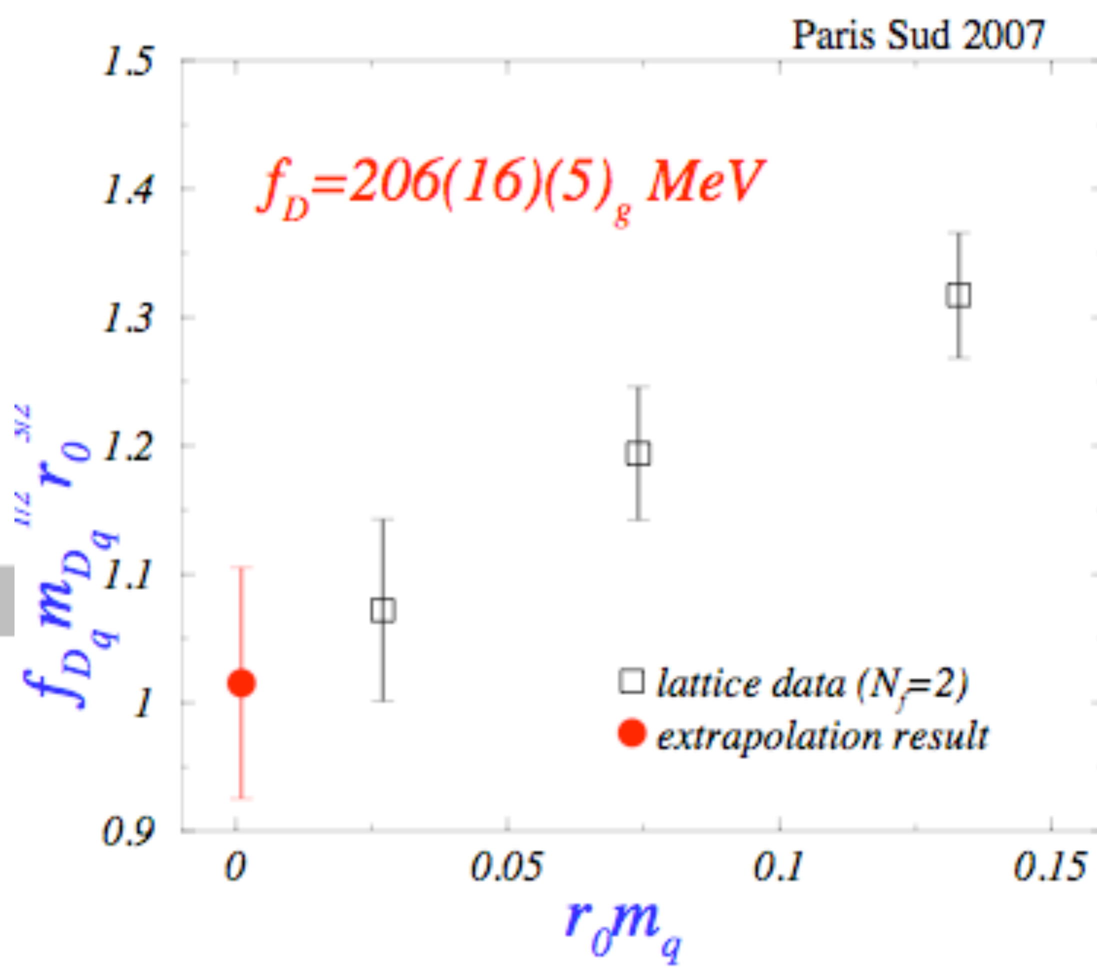
Parenthèse: "cs"-physics



Recent unquenched $N_f = 2 + 1$ *(staggered) results:*

$$f_{D_s}^{FNAL} = 249 \pm 3^{\text{stat.}} \pm 16 \text{ MeV}$$

$$f_{D_s}^{NRQCD} = 290 \pm 20^{\text{stat.}} \pm 41 \text{ MeV}$$



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f_D=206(16)(5)$_g$ MeV
1.5
1.4
1.3
1.2
1.1
1
0.9
0
0.05
0.1
0.15
$f_{D_q} m_{D_q}^{1/2} r_0^{3/2}$
$r_0 m_q$
lattice data (N_f=2)
extrapolation result

Problem 1: Heavy quark

Lattices not fine enough to accomodate $m_b \to$ `4 ways out`

- `QCD` *with propagating quarks that are accessible:* *extrapolate to* $1/m_B$ *by using the heavy quark scaling laws*
- `HQET` *(static limit)* $m_b \to \infty$: $\mathcal{L}_{\rm HQET} = Q^\dagger D_4 Q$
  - *non-perturbative renorm. devised*
  - *for small* E_π *it might help constraining extrapolation of QCD accessible FF's*
- `NRQCD` *(static limit +* $1/m_b$ *terms which are cut-off as* $m_Q v \ll m_Q$*)* : $\mathcal{L}_{\rm NRQCD} = Q^\dagger \left(D_4 - (\vec{D}^2 + \vec{\sigma}\cdot\vec{B})/2m_Q \right) Q$
  - *expansion in* $1/(am_Q) \Rightarrow$ *no continuum limit*
  - *problems in including terms* $\propto 1/m_Q$ *in renormalisation/matching*
- `Fermilab`: *use the full QCD action and go over the cut-off; redefine masses and reinterpret the theory in terms of* $1/m_Q$ *expansion; separation of scales and renormalisation may be problematic*

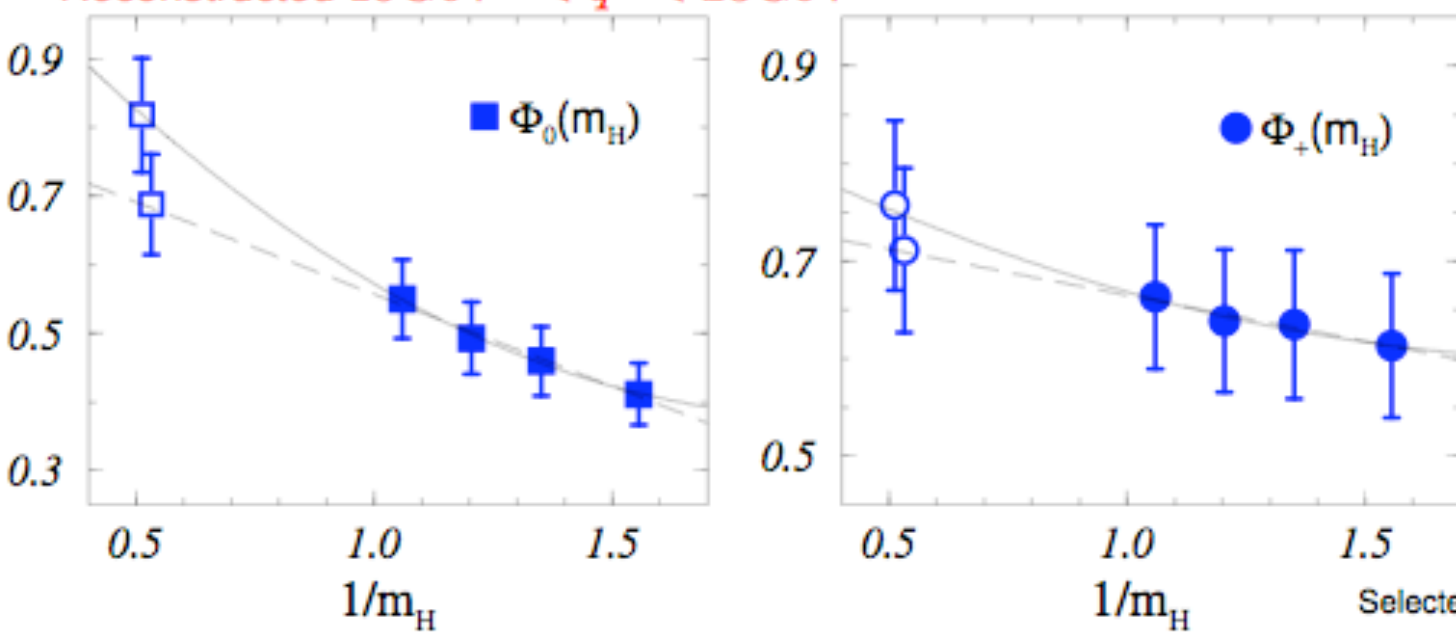
None is fully satisfactory!
All should be used and check the consistency of results

Problem 2: Accessible q^2's

♣ QCD with propagating heavy quark:
$q^2 \to vp \equiv E_P = (m_H^2 + m_P^2 - q^2)/2m_H$
at fixed ("small") E_P *extrapolate* $F_{+,0}^{H\to P}$ *in* $1/m_H$ *to* $1/m_B$
by using the heavy quark scaling laws

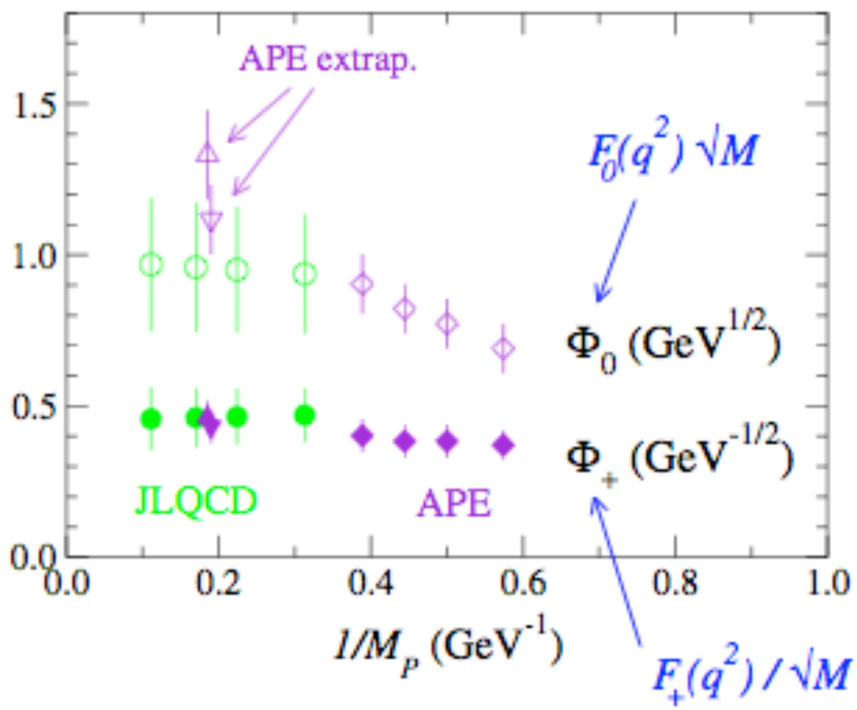
$$\Phi_i(m_H) \;=\; \left\{F_0(vp)\sqrt{m_H}, \frac{F_+(vp)}{\sqrt{m_H}}\right\} = a_i^{(0)} + \frac{a_i^{(1)}}{m_H} + \frac{a_i^{(2)}}{m_H^2} \longrightarrow \Phi_i(m_B)$$

Reconstructed $15\mathrm{GeV}^2 < q^2 < 23\mathrm{GeV}^2$

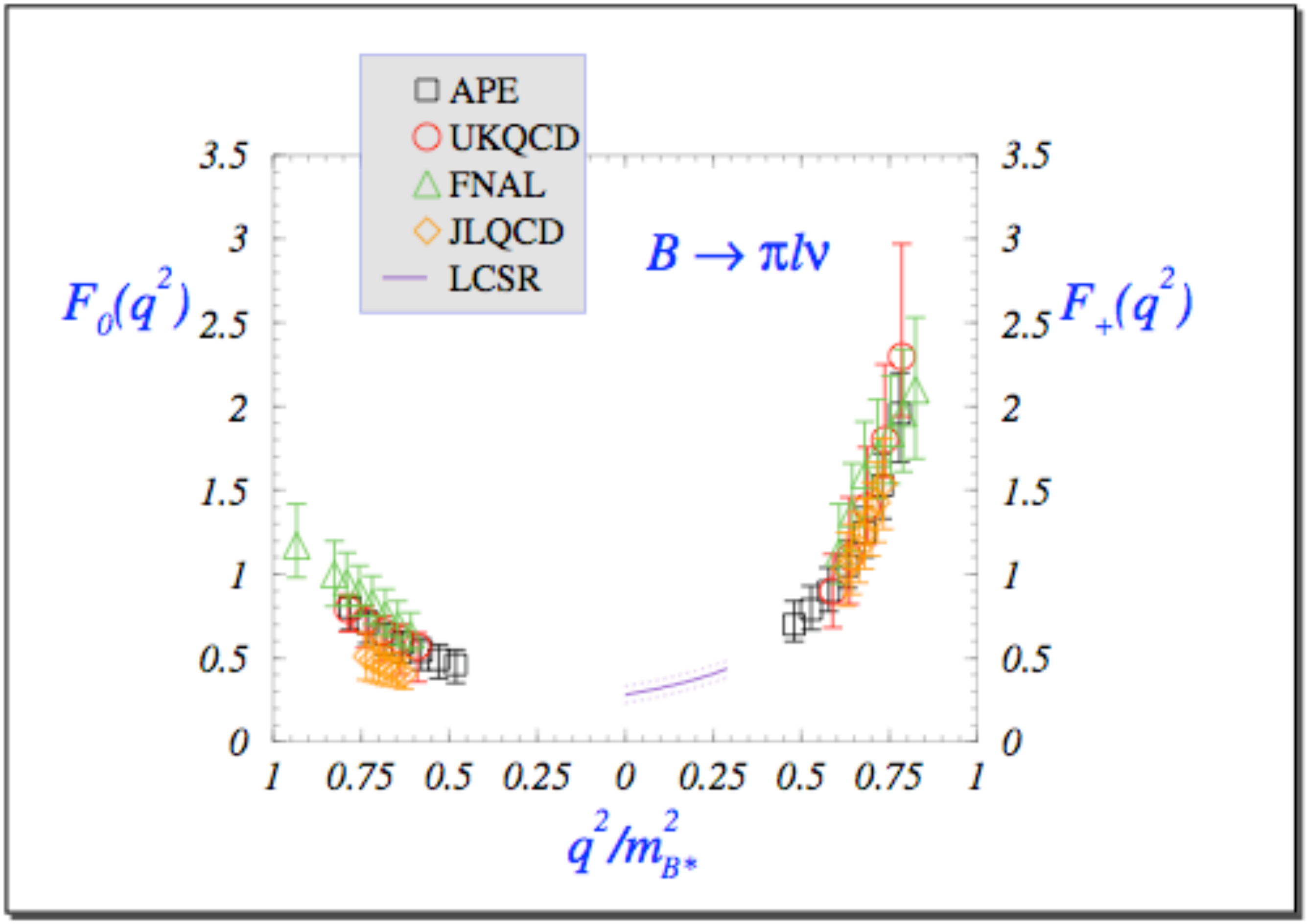


Problem 2: Accessible q^2's

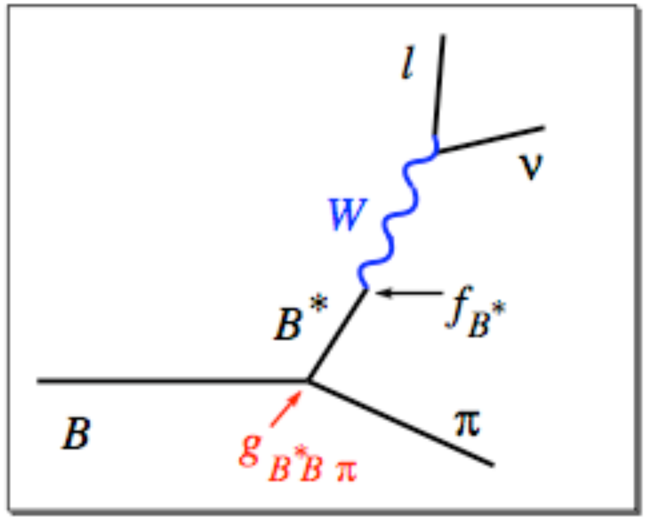
- NRQCD heavy quark action:
  Opposite situation as large $1/(am_H)$ *is dangerous*
  Reconstructed $16\mathrm{GeV}^2 < q^2 < 26\mathrm{GeV}^2$



Resulting in...



q^2–shapes and why even bother



- Crossing symmetry and polology

Kinematical region large $[0 \leq q^2 \leq (m_B - m_\pi)^2]$, pole $m^2_{B^*}$ below cut.

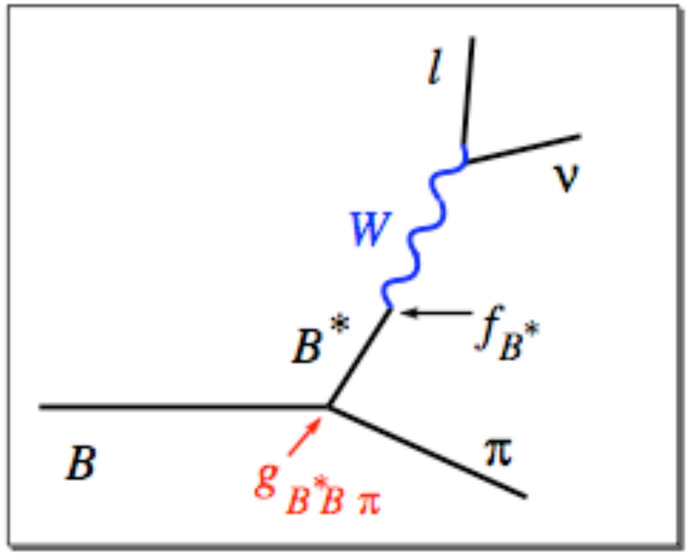
- HQET (HQS) helps when recoiling pion is soft $q^2 \simeq q^2_{max}$

$$F_+(q^2 \simeq q^2_{max}, m_H) \sim \sqrt{m_H} \qquad F_0(q^2 \simeq q^2_{max}, m_H) \sim 1/\sqrt{m_H}$$

- LCSR/LEET/SCET help when recoiling pion is very energetic

$$F_+(q^2) = \zeta_P(m_H, E) \quad F_0(q^2) = \frac{2E}{m_H}\zeta_P(m_H, E) \quad F_{+,0}(q^2 \approx 0) \sim \frac{\sqrt{E}}{m_H^2} \sim m_H^{-3/2}$$

q^2–shapes and why even bother



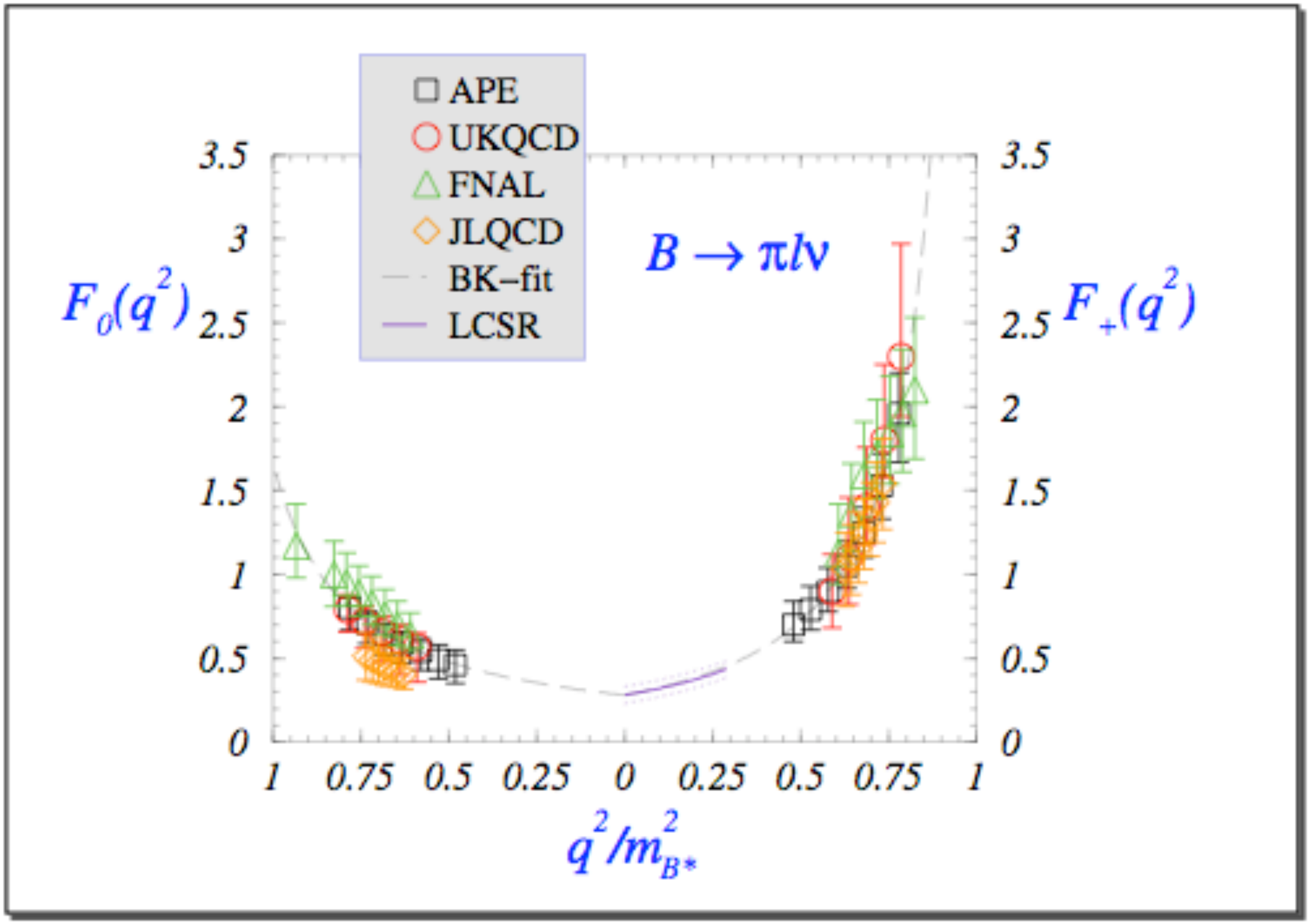
F_0 polelike : $m^2_{0^+\ \mathrm{eff.pole}} = m^2_{B^*}\beta$
F_+ two poles : $m^2_{1^-\ \mathrm{pole}} = m^2_{B^*}$, $m^2_{1^-\ \mathrm{eff.pole}} = m^2_{B^*}/\alpha$

$$F_+(q^2) = \frac{C(1-\alpha)}{(1-q^2/m^2_{B^*})(1-\alpha q^2/m^2_{B^*})}$$

$$F_0(q^2) = \frac{C(1-\alpha)}{1-q^2/(\beta m^2_{B^*})}$$

N.B. $C = g_{B^*B\pi} f_{B^*}/2m_B$
We want to learn from data how to saturate FF's!

More on results...



Remedy problem 2?!

- **QCD with propagating quarks:**
  Directly accessed $F_{+,0}^{H\to P}$ *ARE around* $q^2 \approx 0 \to$ *use* $F_{+,0}^{H\to P}(0)m_H^{3/2}$ *scaling law.*
  Compare APE Vs. LCSR

$$
\begin{aligned}
F_{+,0}^{\text{latt.}}(0) &= \frac{3.1(5)\ \text{GeV}^{3/2}}{m_H^{3/2}}\left[1-\frac{0.98(9)\ \text{GeV}}{m_H}\right] \\
F_{+,0}^{\text{lcsr}}(0) &= \frac{3.2\ \text{GeV}^{3/2}}{m_H^{3/2}}\left[1-\frac{1.3\ \text{GeV}}{m_H}\right]
\end{aligned}
$$

  $\mathcal{O}(1/m_H)$*-corrections large – similar in magnitude to the ones that appear in the calculation of* f_B*!*

- **mNRQCD:**
  NRQCD in the brick frame $\longrightarrow$ *low* q^2*'s. Implementation difficulties.*

Important for $B_s \to \phi\gamma$ at LHCb

Parenthèse : $B_s \to \phi\gamma, B \to K^*\gamma, \rho\gamma$

$$\langle K^*(p', e_\lambda)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|B(p)\rangle = c^{(1)}_{\mu\nu}T_1(q^2) + c^{(2)}_{\mu\nu}T_2(q^2) + c^{(3)}_{\mu\nu}T_3(q^2)$$

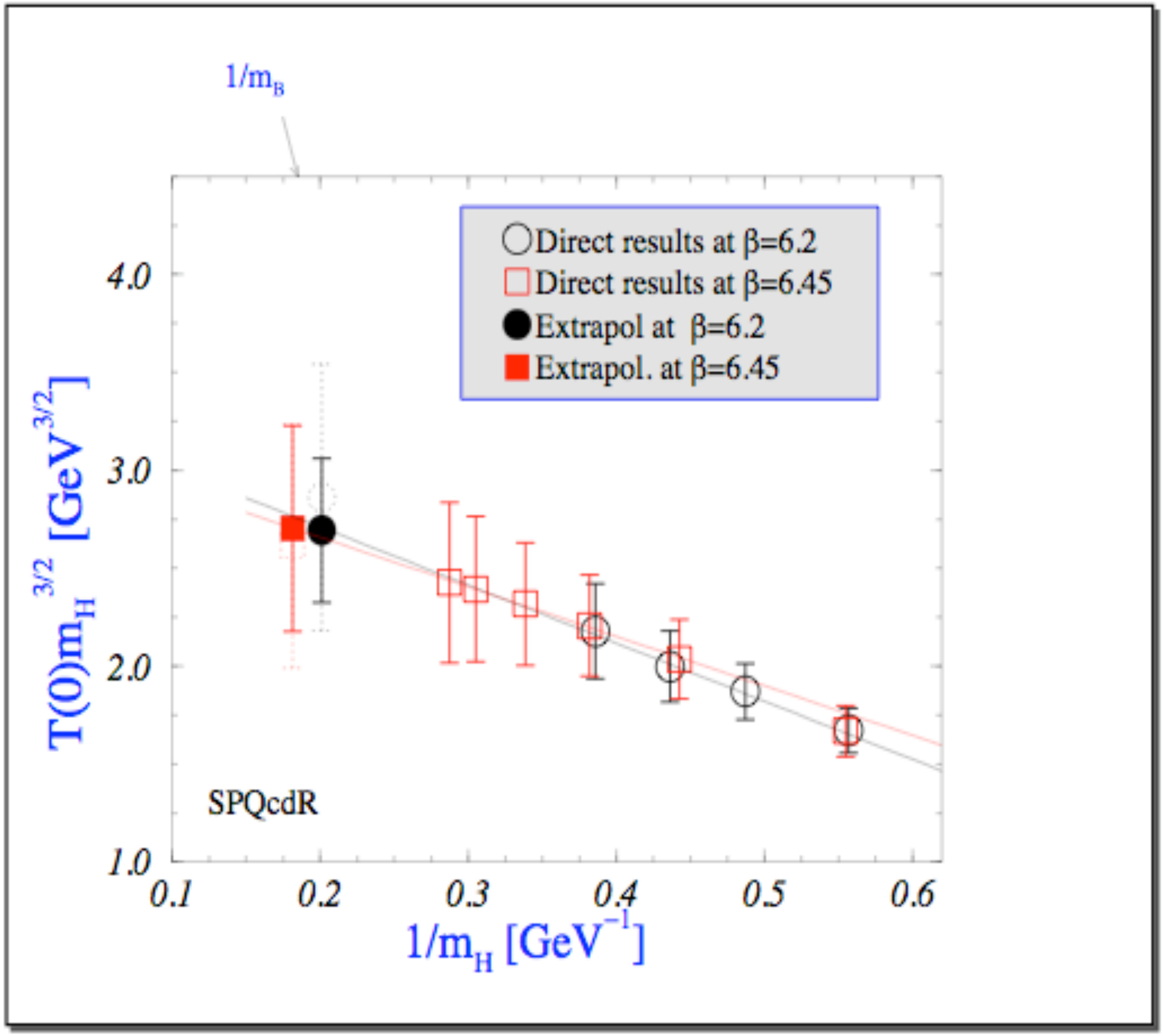
$c^{(1,2,3)}$-*known kinematical factors* $f(p, p', e_\lambda, m_{K^*}, m_B)$
$T_{1,2,3}(q^2)$ - *form factors relevant to* $B \to K^*\ell^+\ell^-$
On-shell photon ($q^2 = 0$): $c^{(3)} = 0$ *and* $T_1(0) = T_2(0)$
A bit complicated details, but the situation is very similar to $B \to \pi$

$$T_1(q^2) = \zeta_\perp(m_H, E) \qquad T_2(q^2) = \frac{2E}{m_H}\zeta_\perp(m_H, E)$$

$$T_1(q^2 \approx 0) \simeq T_2(q^2 \approx 0) \sim \sqrt{E}/m_H^2 \sim m_H^{-3/2}$$

$$\boxed{T(0, m_H)m_H^{3/2} = a_0 + a_1/m_H + a_2/m_H^2}$$

Results...

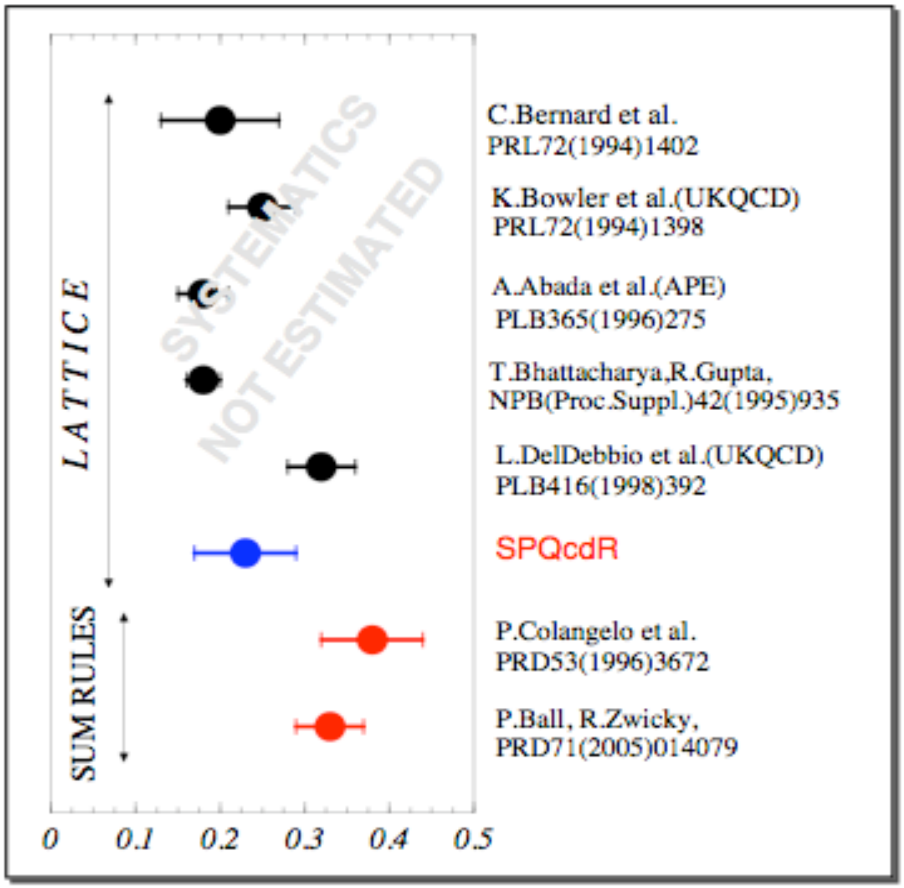


1/m_B
Direct results at β=6.2
Direct results at β=6.45
Extrapol at β=6.2
Extrapol. at β=6.45
SPQcdR
4.0
3.0
2.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
T(0)m_H^{3/2} [GeV^{3/2}]
1/m_H [GeV^{-1}]

Results...

$$\text{LCSR} \;:\; T^{B\to K^*}(0) = 0.33(5), \quad \frac{T^{B\to K^*}(0)}{T^{B\to\rho}(0)} = 1.17(9)$$

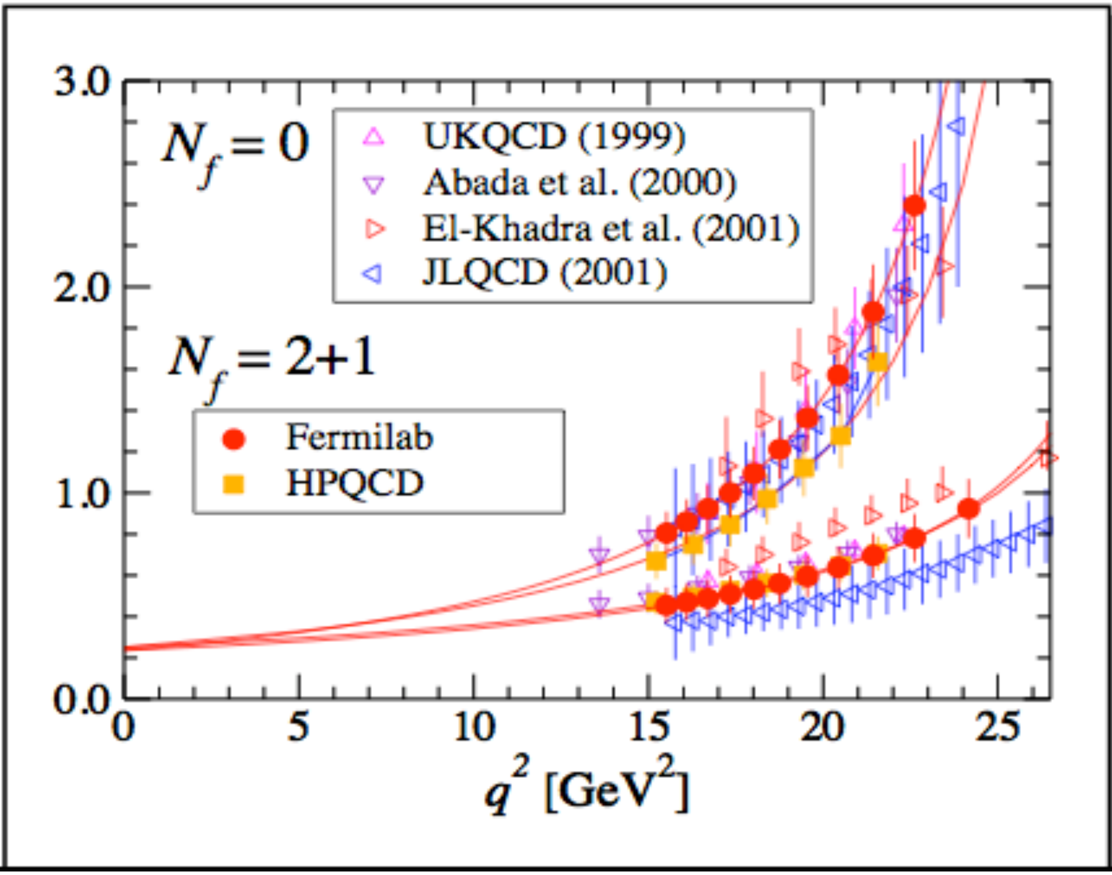
$$\text{lattice} \;:\; T^{B\to K^*}(0) = 0.25(6), \quad \frac{T^{B\to K^*}(0)}{T^{B\to\rho}(0)} = 1.1(1)$$



Back to $B \to \pi$: Precision LQCD on form factors?!

Cleaning many different sources of systematics:

- Unquenched studies (so far only staggered light quarks attempted)
- Renormalisation and matching (nonperturbatively)
- Chiral extrapolation and infinite volume
- Taking continuum limit (?)



Problem 3:Chiral extrapolation

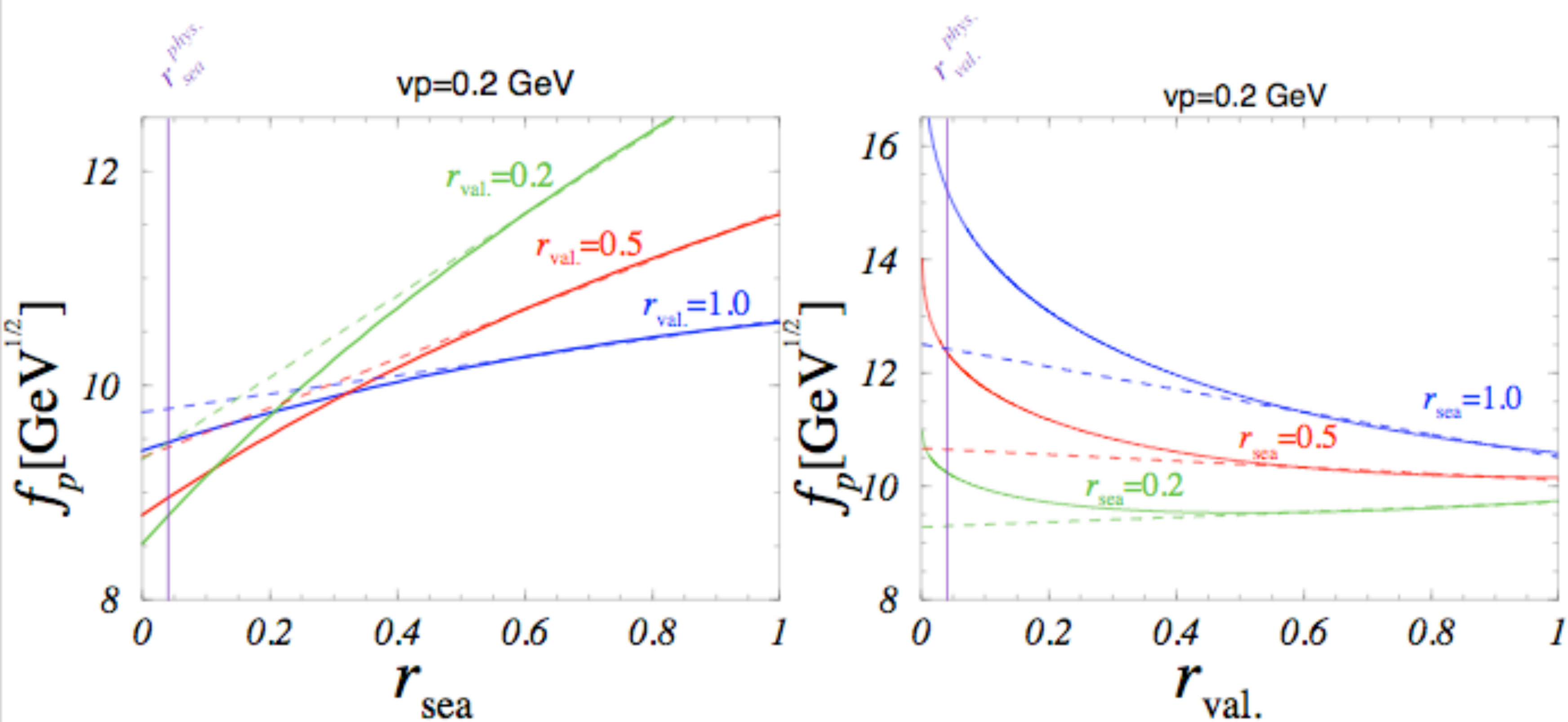
- *light quark accessible from the lattice* $r = m_q/m_s^{phys.} \gg r_{u/d} \simeq 1/25$
- *we usually do it assuming* $F_{+,0} = \alpha + \beta \cdot r + \gamma \cdot r^2$
- In unquenched studies*: Worry about the chiral logs*
- *In partially unquenched studies* $r_{\rm sea} \neq r_{\rm val.}$

$$\begin{aligned}
\delta F_+^{\chi-Loop} &= \frac{1}{(4\pi f)^2}\Big[\Big(-2g^2\frac{M_S^2}{(vp)^2}+1+3g^2\Big)M_V^2\ln(M_V^2) \\
&\quad -\frac{1+3g^2}{2}M_S^2\ln(M_V^2) - 4\pi g^2\frac{M_S^2}{vp}M_V\Big] + C_0'^p + C_2'^p M_V^2 + \dots\,, \\
\delta F_0^{\chi-Loop} &= \frac{1}{(4\pi f)^2}\frac{1+9g^2}{6}(2M_V^2 - M_S^2)\ln(M_V^2) + C_0'^v + C_2'^v M_V^2 + \dots\,,
\end{aligned}$$

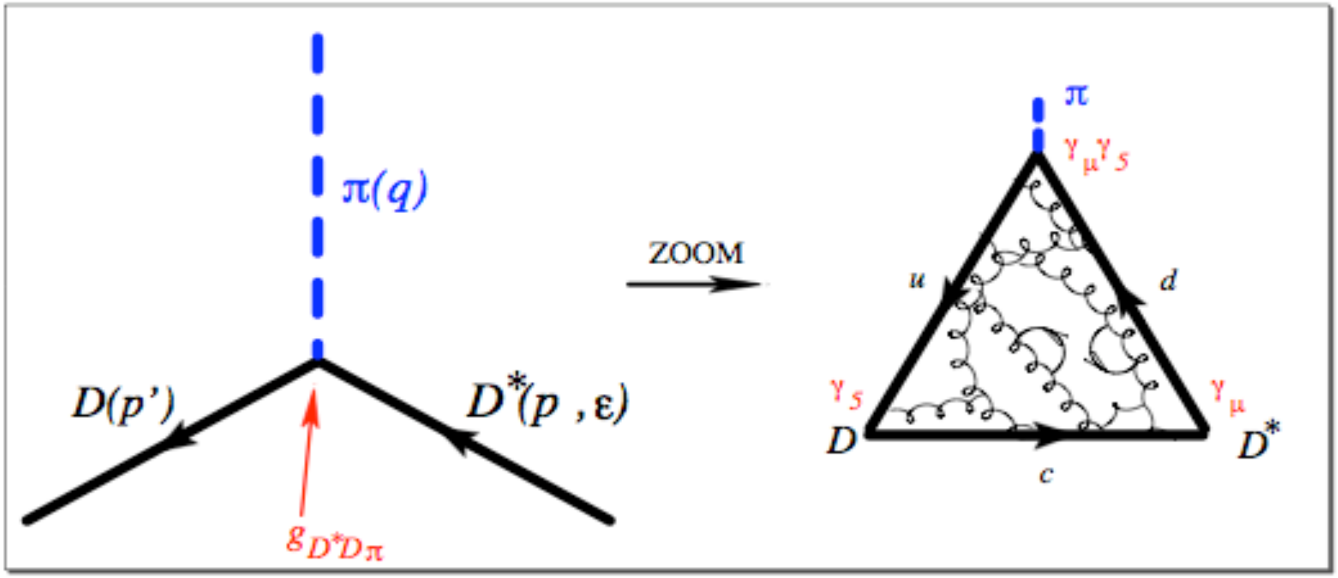
where $M_S = 2B_0 m_s^{phys.} r_{\rm sea}$, $M_V = 2B_0 m_s^{phys.} r_{\rm val.}$,
$C'^{p,v}_{0,2}$ *functions of* vp *and* M_S.

Problem 3:Chiral extrapolation

- Small errors possible IFF the extrapolation is made first in $r_{\rm sea} \to r_{u/d}^{phys.}$, and then in $r_{\rm val} \to r_{u/d}^{phys.}$! Numerically costly
- When working with $N_f = 2+1$ using ChPT can be extra dangerous as the validity of ChPT with $N_f = 2+1$ is not yet established.



Remarks 2



We computed $g_{D^*D\pi} = 2g_c\sqrt{m_D m_{D^*}}/f_\pi$, on the lattice and obtained

$$g_c = 0.67(8)(5) \qquad [g_c^{\rm CLEO} = 0.61(7)]$$

whereas from $F^{D\to\pi}(q^2 \approx q^2_{\rm max})$ we get

$$g_c = 0.43(7)\left(^{+7}_{-0}\right)$$

Single pole never saturates the form factor?

All above values larger than LCSR prediction $g_c \simeq 0.35$

Saturation of $F_+^{D\to\pi}(q^2)$ by the nearest pole depends on $g_{D^*D\pi}$

Is the value for $\Gamma(D^{*+})$ by CLEO reliable? Can it be checked?

Combine D and B semileptonic decays

- *Need*

$$T_{\rm bin} = \int_{q^2_{\rm min}}^{q^2_{\rm max}} d\Gamma(B \to \pi\ell\nu) = \int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{G_F^2 |V_{ub}|^2}{192\pi^2 m_B^3} \lambda^{3/2}(q^2)|F_+(q^2)|^2$$

where

$$\lambda(q^2) = [q^2 - (m_B + m_\pi)^2][q^2 - (m_B - m_\pi)^2] = 4m_B^2[(vp)^2 - m_\pi^2]$$

($q^2 = m_B^2 + m_\pi^2 - 2m_B vp$) so that

$$\frac{d\Gamma}{dvp}(B \to \pi\ell\nu) = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|\vec{p}_\pi|^3|F_+(vp)|^2$$

- *Take the ratio with the corresponding D-mode*

$$R(vp) \equiv \left.\frac{d\Gamma(B \to \pi\ell\nu)/d(vp)}{d\Gamma(D \to \pi\ell\nu)/d(vp)}\right|_{vp-\rm fixed} = \frac{|V_{ub}|^2}{|V_{cd}|^2}\left|\frac{F_+^{B\to\pi}(vp)}{F_+^{D\to\pi}(vp)}\right|^2$$

Combine D and B semileptonic decays

- *Form factor*

$$F_+^{B\to\pi}(q^2) \to F_+(vp) = \sqrt{M}\left(f_+^{(0)}(vp) + f_+^{(1)}(vp)/M + \dots\right)$$

$$\Rightarrow\ R(vp) = \frac{|V_{ub}|^2}{|V_{cd}|^2}\frac{m_B}{m_D}\left|1 + \frac{f_+^{(1)}(vp)}{f_+^{(0)}(vp)}\left(\frac{1}{m_B} - \frac{1}{m_D}\right) + \dots\right|^2$$

- *2 questions:*
  - *Are there* (vp)*'s accessible experimentally from* $B \to \pi$ *and from* $D \to \pi$ *simultaneously?*
  - *What is doable on the lattice?*

$$q^2_{D^0\to\pi^+} \in (0, (m_{D^0} - m_{\pi^+})^2] = (0, 2.975]\ \mathrm{GeV}^2 \ \to\ vp \in [0.14,\ 0.94)\ \mathrm{GeV}$$
$$q^2_{B^0\to\pi^+} \in (0, (m_{B^0} - m_{\pi^+})^2] = (0, 26.4]\ \mathrm{GeV}^2 \ \to\ vp \in [0.14,\ 2.64)\ \mathrm{GeV}$$

Combine D and B semileptonic decays

- **A1** *I went through experimental papers and...*

  GOOD: I checked and the common region in (vp) *DOES exist b/c*

$$
\begin{aligned}
(vp)^{D\to\pi}_{\min} &\quad\to\quad q^2_{B\to\pi} = 26.4\,\mathrm{GeV}^2 \\
(vp)^{D\to\pi}_{\max} &\quad\to\quad q^2_{B\to\pi} = 18.0\,\mathrm{GeV}^2
\end{aligned}
$$

  BAD: High $q^2_{\bar{B}^0\to\pi^+}$*-bin(s) essential*

- **A2** *Could be done on the lattice (cf.quenched latt.result)*

vp [GeV]	$q^2_{D\to\pi}$ [GeV^2]	$q^2_{B\to\pi}$ [GeV^2]	$f_+^{(1)}(vp)/f_+^{(0)}(vp)$ [GeV]	$R(vp)\times\frac{m_D\lvert V_{cd}\rvert^2}{m_B\lvert V_{ub}\rvert^2}$
0.55	1.45	22.08	$-0.27(6)$	1.24(9)
0.69	0.92	20.60	$-0.25(8)$	1.21(9)
0.83	0.40	19.20	$-0.27(8)$	1.23(10)
0.96	-0.08	17.75	$-0.23(8)$	1.19(9)

Combine D and B semileptonic decays

- *Always keep in mind that FF's are harder to compute than decay constants (more potentially dangerous systematic uncertainty present)*
- *Chiral corrections to* $1/M$ *slope are very difficult to compute, plus HMChPT becomes less adequate as* vp *increases*
- *Seems that* $R(vp) \times (m_D|V_{cd}|^2)/(m_B|V_{ub}|^2)$ *is nearly flat and around* 1.2 *(MUST BE CHECKED IN UNQUENCHED ENVIRONMENT!)*
- *Side remark: testing LQCD weak matrix elements results w/o any CKM assumption*

$$\frac{\mathrm{Br}(B^+ \to \tau^+ \nu_\tau)/\mathrm{Br}(D^+ \to \mu^+ \nu_\mu)}{R(vp)}$$

$|V_{ub}/V_{cd}|$ *cancel!*

f_{B_s}/f_{B_d} from D-decays

- *SU(3) breaking ratio of HL decay constant*

$$\frac{\Phi_s(m_b)}{\Phi_{u/d}(m_b)} \equiv \frac{\sqrt{m_{B_s}} f_{B_s}}{\sqrt{m_{B_{u/d}}} f_{B_{u/d}}} = \frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}} + \frac{\phi_s^{(1)}/\phi_s^{(0)}}{\phi_{u/d}^{(1)}/\phi_{u/d}^{(0)}} \times \frac{1}{m_B} + \dots$$

Mess with chiral extrapolation of f_{B_s}/f_B propagates to

$$\xi^2 = \frac{f_{B_s}^2}{f_{B_d}^2}\frac{B_{B_s}}{B_{B_d}}$$

b/c χ-log term $\propto (1+3g^2) \approx 2$

$$\left(\frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}}\right)^{\text{ChPT}} = 1 + \frac{1+3g^2}{(4\pi f)^2}\frac{3}{4} m_\pi^2 \log(m_\pi^2) + \text{“irrelevant” terms}$$

n.b. χ-log term $\propto (1-3g^2) \approx 0$ in B_{B_s}/B_{B_d} case

f_{B_s}/f_{B_d} from D-decays

- Argued that when combined with f_K/f_π in double ratio, the chiral logs wash out and from the lattice data one verify

$$f_K/f_\pi \approx f_{D_s}/f_D \approx f_{B_s}/f_B$$

Indeed HPQCD see very small deviation from the flat m_q dependence (reduced uncertainties due to chiral extrapolation) and obtain

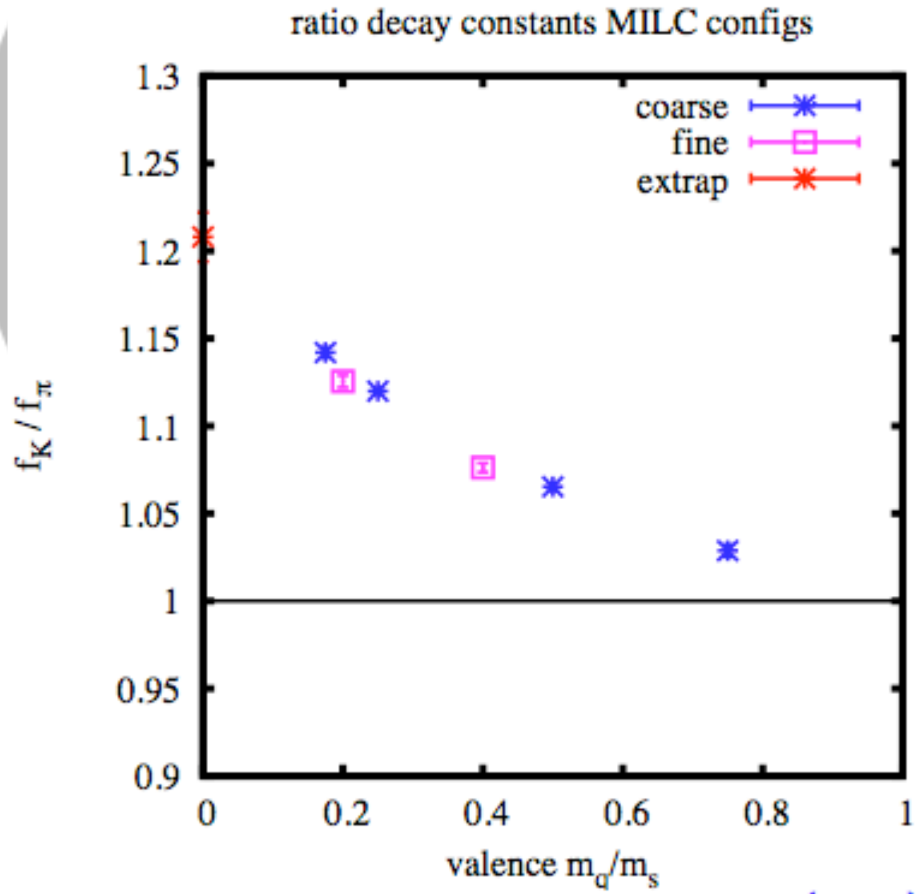
$$(f_{B_s}/f_{B_d})/(f_K/f_\pi) = 1.019(11)$$

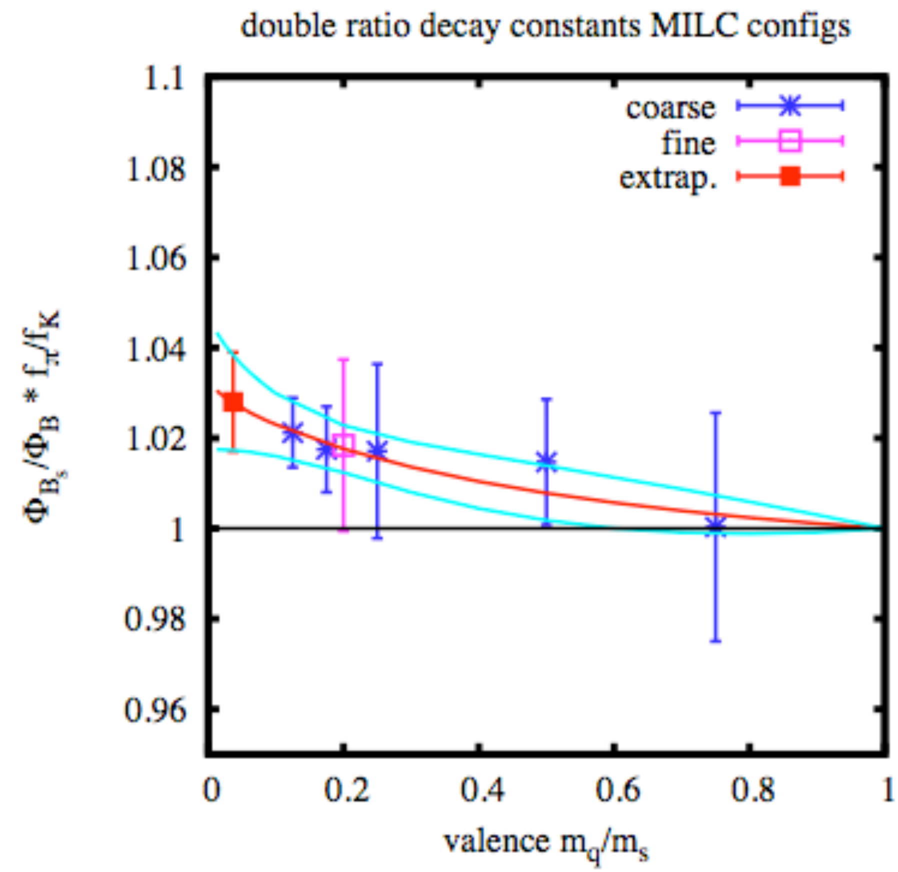
which with MILC's

$$\frac{f_K}{f_\pi} = 1.208(2)\,\binom{7}{14} \Rightarrow \frac{f_{B_s}}{f_{B_d}} = 1.23(2)$$

Try double ratio to f_K/f_π

Much flatter chiral extrapoln





MILC results - $f_K/f_\pi = 1.208(2)\left(\begin{smallmatrix}+7\\-14\end{smallmatrix}\right)$

yield $V_{us} = 0.2223(26)$

Competitive with PDG from SL decay

Sugar, MILC, LAT06

$f_{B_s}/f_B \times f_\pi/f_K = 1.019(11)$

f_{B_s}/f_B Total error 2%

Becirevic et al,hep-ph/0211271

f_{B_s}/f_{B_d} from D-decays

- *Compare:*

$$\frac{\Phi_s(m_b)}{\Phi_{u/d}(m_b)} = \frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}} + \frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}} \times \frac{1}{m_B} + \ldots$$

$$\mathcal{R} = \frac{\Phi_s(m_b)/\Phi_d(m_b)}{\Phi_s(m_c)/\Phi_u(m_c)} = 1 + \alpha\left(\frac{1}{m_B} - \frac{1}{m_D}\right) + \ldots$$

- *What has been done on the lattice?*

propagating heavy	$\mathcal{R}^{\mathrm{n_f}=0} = 1.017(17)(??)$
NRQCD heavy	$\mathcal{R}^{\mathrm{n_f}=2} = 1.005(6)\left(^{+29}_{-00}\right)$
Fermilab heavy	$\mathcal{R}^{\mathrm{n_f}=2} = 1.001(6)(10)$

Always Wilson light!

f_{B_s}/f_{B_d} from D-decays

- *THERE IS something "exclusive" that can be computed with $\approx 1\%$ accuracy.*
- *Illustration*

$$\frac{f_{B_s}}{f_{B_d}} = \overbrace{(1.018 \pm 0.006 \pm 0.010)}^{\text{from } \mathcal{R}^{\mathrm{n_f=2}}_{\mathrm{JLQCD}}} \times \overbrace{(1.26 \pm 0.11 \pm 0.03}^{\text{CLEO-c}}$$

$$= 1.28 \pm 0.11 \underbrace{\pm 0.03^{\text{exp.}} \pm 0.01^{\text{latt.}}}_{\text{syst.}}$$

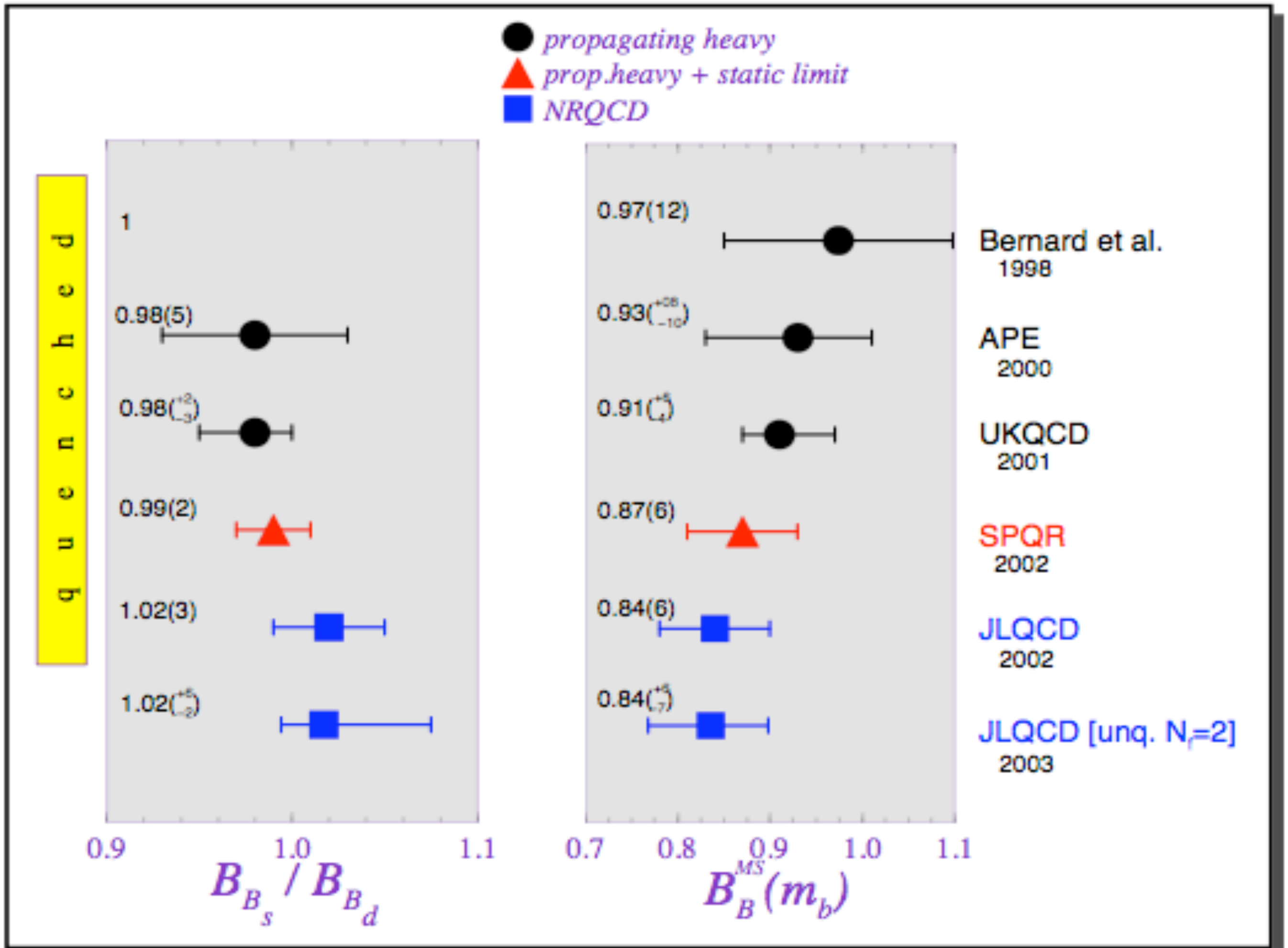
- *What should be done?*
  - *New (better) lattice estimates*

    `many lattice groups working`
  - *NLO chiral-log correction to "α"*

    `almost done`

$B^0 - \bar{B}^0$ results overview

Results presented in the $\overline{\mathrm{MS}}$(NDR) scheme @ $\mu = m_b$



Importantly:

All results – Wilson light quark : subtract spurious mixing!

Doubtful systematic uncertainties??

$O(\mu) = Z(a\mu) O^{latt}(a)$

$$\mathrm{O}_{\Gamma\Gamma} \;=\; \bar{h}\Gamma_1 q \bar{h}\Gamma_2 q \in \{O_{VV+AA},\, O_{SS+PP},\, O_{VV-AA},\, O_{SS-PP}\}$$

No symmetry constraints $\Rightarrow$ 16 independent entries in Z-matrix: z_{ij}
HQS and $O(3) \Rightarrow$ 8 independent constants (Wilsonian case!)

$$Z = \begin{pmatrix} z_{11} & 0 & z_{13} & 2\,z_{13} \\ \frac{-z_{11}+z_{22}}{4} & z_{22} & z_{23} & -z_{13} - 2\,z_{23} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ \frac{2\,z_{31}-z_{32}}{4} & \frac{-z_{32}}{2} & \frac{z_{34}}{4} & z_{33} \end{pmatrix}$$

Chiral symmetry $\Rightarrow$ 4 independent constants

$$\begin{pmatrix} O_{VV+AA} \\ O_{SS+PP} \\ O_{VV-AA} \\ O_{SS-PP} \end{pmatrix}^{Ren} = \begin{pmatrix} z_{11} & 0 & 0 & 0 \\ \frac{-z_{11}+z_{22}}{4} & z_{22} & 0 & \\ 0 & 0 & z_{33} & z_{34} \\ 0 & 0 & \frac{z_{34}}{4} & z_{33} \end{pmatrix} \cdot \begin{pmatrix} O_{VV+AA} \\ O_{SS+PP} \\ O_{VV-AA} \\ O_{SS-PP} \end{pmatrix}^{latt}$$

Manifest chiral $\oplus$ HQ symmetry on the lattice

♠ Neuberger Dirac operator for the light quark ($\{D^{-1}, \gamma_5\} = \gamma_5 a/\rho$)

$$D_N = \frac{1}{a}\rho\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right], \qquad X = D_W - \frac{\rho}{a}$$

D_W is the Wilson-Dirac operator $[D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu(x) + \nabla^*_\mu(x)) - \frac{1}{2}a\,\nabla^*_\mu\,\nabla_\mu]$

$$\Rightarrow\ q(x) \to i\gamma_5 q(x) \qquad \bar{q}(x) \to \bar{q}(x)i\,(1 - \frac{D}{\rho})\gamma_5$$

♠ Eichten–Hill backward (forward) derivative action for the static heavy quarks (antiquarks)

$$S_h = \sum_n \left\{\bar{h}^{(+)}(n)\left[h^{(+)}(n) - U_0(n-\hat{0})^\dagger h^{(+)}(n-\hat{0})\right]\right.$$
$$\left. -\bar{h}^{(-)}(n)\left[U_0(n)h^{(-)}(n+\hat{0}) - h^{(-)}(n)\right]\right\}$$

$$\Rightarrow \quad h^{(\pm)}(x) \to e^{i\omega_{ij}\sigma_{ij}} h^{(\pm)}(x)$$

Also Manifest $O(3)$ symmetry

Rotation about the i^{th} axis by $\pi/2$:

$$x_i \rightarrow x_i\,, \quad x_{j\neq i} \rightarrow \epsilon_{ijk} x_k$$

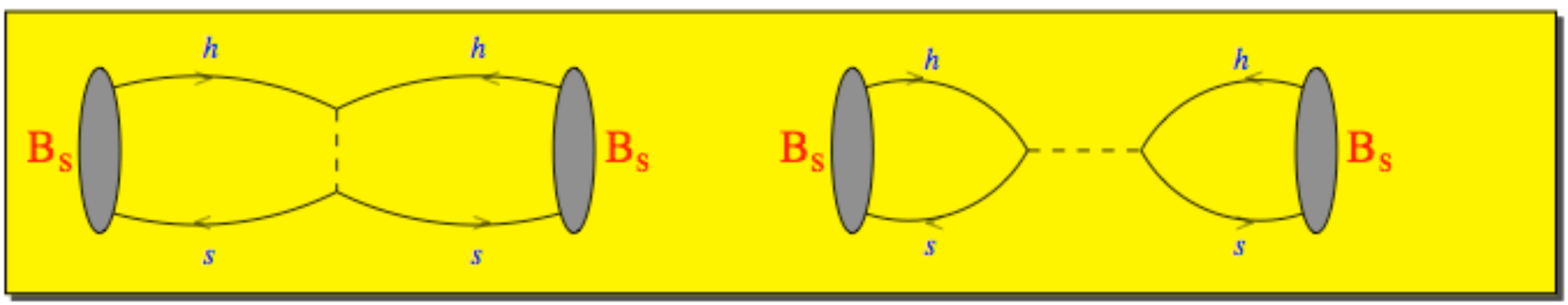
$$q(x)\ (h^{(\pm)}(x)) \rightarrow \frac{(1-\frac{1}{2}\varepsilon_{ijk}\gamma_j\gamma_k)}{\sqrt{2}} q(x)\ (h^{(\pm)}(x))$$
$$\bar{q}(x)\ (\bar{h}^{(\pm)}(x)) \rightarrow \bar{q}(x)\ (\bar{h}^{(\pm)}(x)) \frac{(1+\frac{1}{2}\varepsilon_{ijk}\gamma_j\gamma_k)}{\sqrt{2}}$$

$\Rightarrow$ Renormalisation pattern like in the continuum: NO SPURIOUS MIXING!

INSTEAD OF 16 RENORMALISATION CONSTANTS, ONE ENDS UP WITH ONLY 4!

computable in perturbation theory

`Compute on the lattice:`

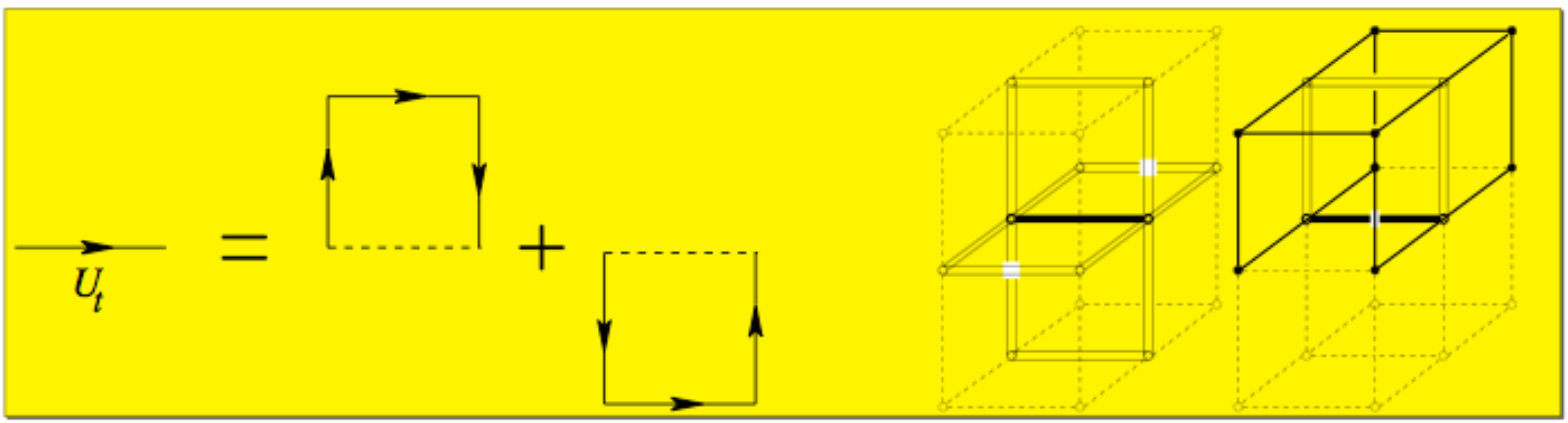


Static HQET on the lattice – Rinascimento

- Heavy quark propagator becomes a Wilson line

$$\mathcal{P}e^{ig\int_0^\tau dt A_0(\vec{0},t)} \rightarrow \prod_{t=1}^{\tau} U_0(t) \rightarrow U_0^{\mathrm{HYP}}(t)$$

"Fattening" "HYP": hypercube



- ONLY Wilson line is HYP-ed. Gauge field configuration and light quark propagator intact.
- spectacular improvement of signal/noise

Numerically...[Orsay - new]

Working with Neuberger quarks is very costly. Avoid FV problems by working with light s-quark

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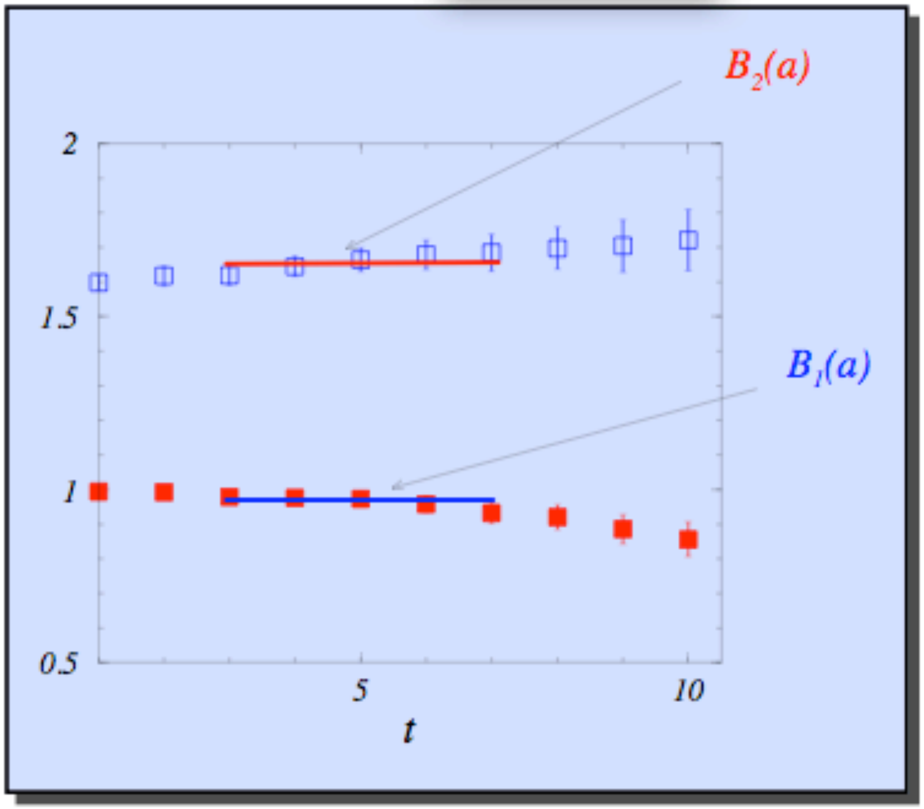


RESULTS IN $\overline{\mathrm{MS}}$(NDR)

- $\beta = 6.0 : B_{B_s}(m_b) = 0.922(12)$
- $\beta = 5.85 : B_{B_s}(m_b) = 0.904(15)$
- *Results a touch bit larger than those previously computed with Wilson fermions.*
- *Check on this!*

Wilson Vs. Overlap

- At $\beta = 6.0$, with Wilson ($\kappa_s = 0.1435$) we obtain

$$\langle O_1^{\rm hqet}(m_b)\rangle = Z_{11}(m_b/a)\langle O_1^{\rm hqet}(a)\rangle \left[1 + z_{13}(a)\frac{\langle O_3^{\rm hqet}(a)\rangle}{\langle O_1^{\rm hqet}(a)\rangle} + z_{14}(a)\frac{\langle O_4^{\rm hqet}(a)\rangle}{\langle O_1^{\rm hqet}(a)\rangle}\right]$$

$$z_{13} = -0.235 \qquad \frac{\langle O_3^{\rm hqet}(a)\rangle}{\langle O_1^{\rm hqet}(a)\rangle} = -1.011(1)$$

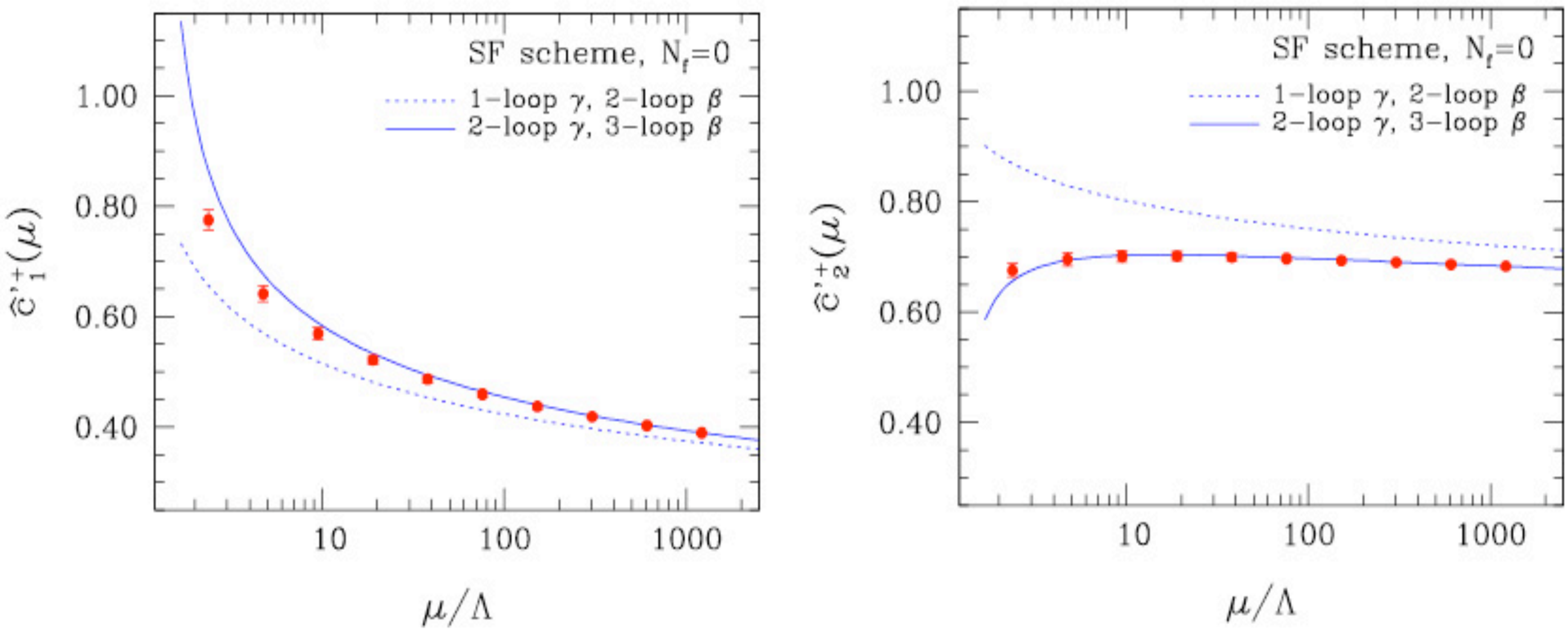
$$z_{14} = -0.470 \qquad \frac{\langle O_4^{\rm hqet}(a)\rangle}{\langle O_1^{\rm hqet}(a)\rangle} = 1.013(2)$$

$$\Rightarrow\ B_{B_s}^{\overline{\rm MS}}(m_b)_{\rm Wilson} = 0.873(5)$$

q to be compared with $B_{B_s}^{\overline{\rm MS}}(m_b)_{\rm Overlap} = 0.922(12)$

- Systematic uncertainty associated to the subtractions is visible but small
- Similar conclusion for all $\Delta B = 2$ operators!

$N_f = 0, 2$: With Wilson fermions (in the static approximation) the mixings with operators of wrong chirality can be removed by using tmQCD [MDM, 2004, Palombi et al., 2005]. NP renormalization for the relevant parity odd operators completed in the SF scheme



PT seems to work for $\mu \geq 1\,\text{GeV}$ for both $N_f = 0,\ 2$ [talks by M. Papinutto and C. Pena]

Also preliminary quenched results for the matrix elements [talk by F. Palombi]

LATTICE UV CUT-OFF IS THE LATTICE SPACING $m_q, \Lambda_{\rm QCD} \ll 1/a$, BUT THE HEAVY QUARK IS TOO HEAVY: $m_c < 1/a$, $m_b > 1/a$ [trouble with $\mathcal{O}(a)$ artifacts!]

- remedy 1: use data around charm and extrapolate in $1/m_Q$ to b-quark
  [*poor control over the associated systematic errors* :-(]
- remedy 2: work in the static limit of HQET ($m_b \to \infty$): $\mathcal{L}_{\rm HQET} = h^\dagger D_4 h$
  [*poor signal/noise and missing* $\mathcal{O}(1/m_b^n)$:-(]
- remedy 3: NRQCD (static limit + $1/m_b$ terms which are cut-off as $m_Q v \ll m_Q$) : $\mathcal{L}_{\rm NRQCD} = Q^\dagger \left(D_4 - (\vec{D}^2 + \vec{\sigma}\cdot\vec{B})/2m_Q\right) Q$
  [*expansion in* $1/(am_Q) \Rightarrow$ *no continuum limit* $\oplus$ *renormalisation challenging* :-(]

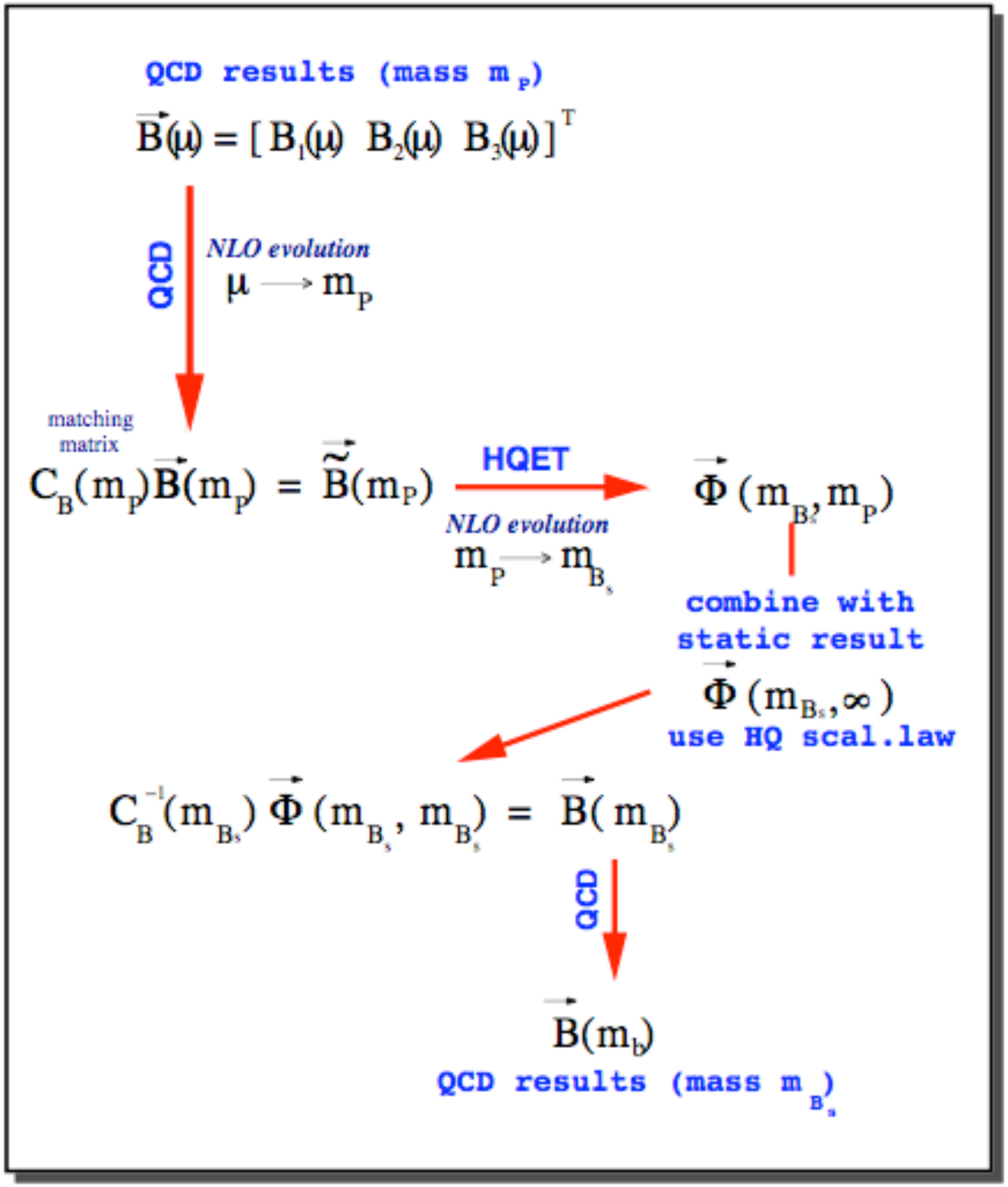
  None of the methods is good enough on its own!
  SPQcdR combine remedy 1 and remedy 2

$$\langle \overline{B}_q | O_1(\mu) | B_q \rangle \equiv \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle = \frac{8}{3} f_{B_q}^2 m_{B_q}^2 B_1(\mu)$$

$$\langle \overline{B}_q | O_2(\mu) | B_q \rangle \equiv \langle \bar{B}_q | (\bar{b}q)_{S-P} (\bar{b}q)_{S-P} | B_q \rangle = -\frac{5}{3} \left(\frac{f_{B_q} m_{B_q}^2}{m_b(\mu) + m_q(\mu)} \right)^2 B_2(\mu)$$

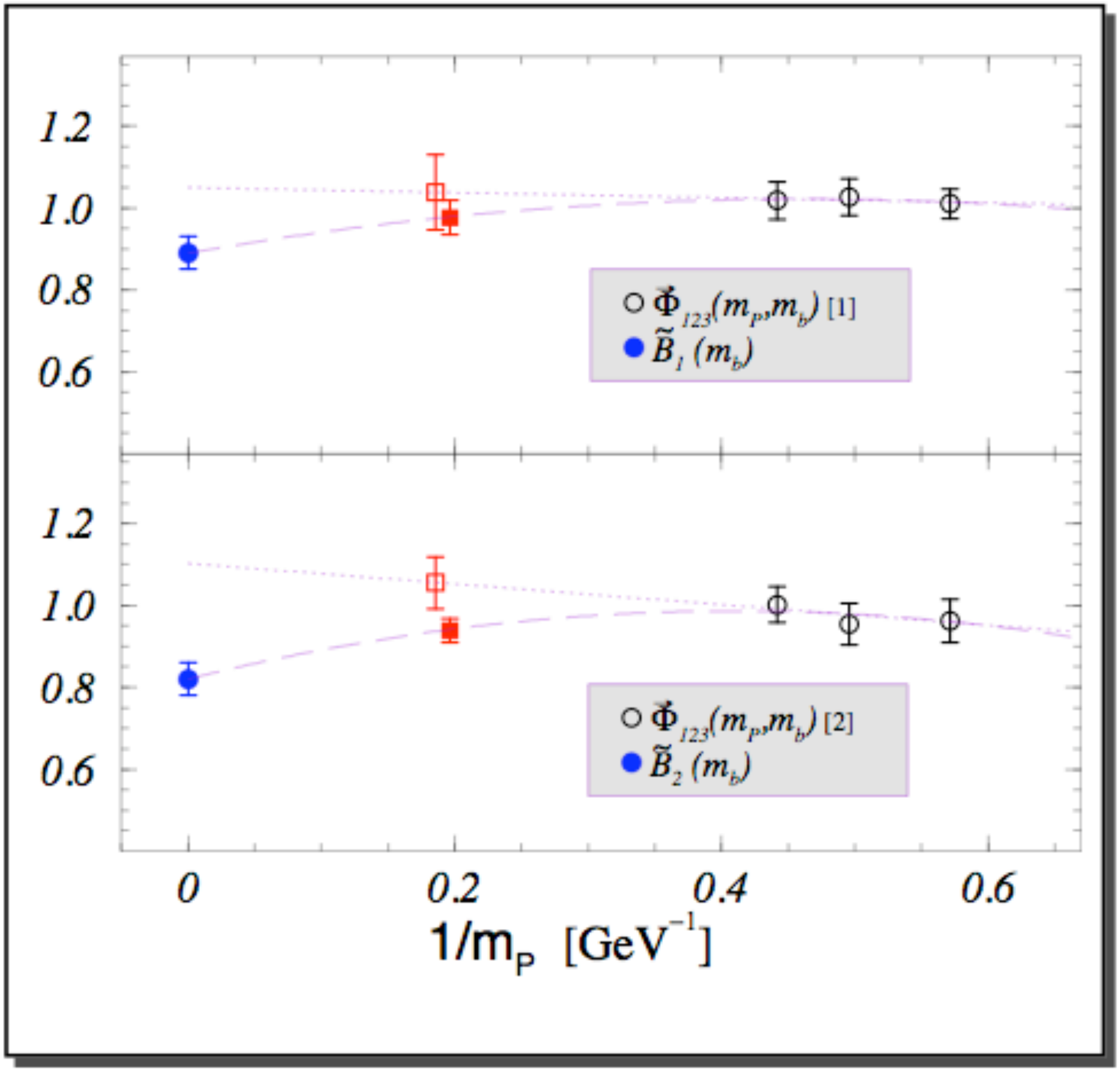
$$\langle \overline{B}_q | O_3(\mu) | B_q \rangle \equiv \langle \bar{B}_q | (\bar{b}^i q^j)_{S-P} (\bar{b}^j q^i)_{S-P} | B_q \rangle = \frac{1}{3} \left(\frac{f_{B_q} m_{B_q}^2}{m_b(\mu) + m_q(\mu)} \right)^2 B_3(\mu)$$

Consistent matching... [SPQcdR]



- *Combine the static HQET results for B-parameters with the full QCD ones $\Rightarrow$ extrapolation $\longrightarrow$ "interpolation"*
- *Perturbative matching of the anomalous dimensions of 4-f QCD and HQET operators made @ NLO in perturbation theory*

Results presented in the $\overline{\mathrm{MS}}$ scheme @ $\mu = m_b$



Resulting in:

$$
\begin{aligned}
B_{B_d}(m_b) &= 0.87(2)(5) \\
B_{B_s}/B_B &= 0.99(2)
\end{aligned}
$$

in the $\overline{\mathrm{MS}}$(NDR) scheme.

systematics controlled?

Fermilab approach revisited

- on-shell improvement à la Symanzik + elliminate $(am_h)^n$
- $$S = \sum_{n,m} \psi_n \left[\gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 - \frac{r_t}{2} D_0^2 + c_B \frac{i}{4} \sigma_{ij} G_{ij} + c_E \sigma_{i0} G_{i0} \right]_{nm} \psi_m$$

  6 dependent parameters which depend on am_h fixed perturbatively
- Christ and Lin take $r_s = r_t = 1$ and propose a method to fix m_0, ζ and $c_E = c_B$ non-perturbatively $\rightarrow$ way to go $\Rightarrow$ precision b-physics on the lattice
- Similar formulation by Aoki et al.
- more coefficients to fix to improve the operators - preferably non-perturbatively
- currently all those coefficients handled perturbatively

Way to go BUT still many points to clarify before the method
-at least in principle- can lead to a % accuracy
$\Rightarrow$ Workshop in Paris - April 2008

Instead of conclusion

- Standard model is in hands of LQCD community (super important)
- research of extentions of SM in FCNC (important)
- We are in the era of massive unquenched lattice QCD computations
- Unquenching solves many old problems but brings in many new ones
- Work in progress on many phenomenologically relevant quantities
  $D_{(s)}$-decays lattice confronting CLEOc and BaBar/Belle $\to$ urgent!
- HPQCD+MILC+Fermilab did impressive work with staggered quarks BUT we must wait for results using other LQCD formulations (light quark actions and heavy quark approaches).
  - JLQCD with overlap dyn.quarks
  - QCDSF, Alpha, Rome2/Cern , Rome/Orsay with Wilson dyn.quarks
  - ETMC with tmQCD
  - UKQCD, RBC/BNL with dyn. domain wall quarks
- Recent tremendous progress is not only due to better machines but also to many clever ways to improve HMC
- Plus more clever ways to confront lattices and experiments by circumventing various sources of systematic uncertainties are always welcome $\to$ benefits from interaction with experimenters