Lattice QCD (intro)

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Why is it important?

Gauge sector of the Standard Model (SM)

- *Completely fixed by the gauge invariance once the particle content (and its quantum numbers) are fixed*
- *Tested with great precision at the collider experiments at energy scales* $\sim \mathcal{O}(100\text{GeV})$

Flavor sector of SM

- *SM (Yukawa) couplings not fixed by symmetry*
- *What underlying symmetry is so badly broken that the quark masses cover 5 orders of magnitudes?* [$m_{u,d} \sim \mathcal{O}(1\text{MeV}) \;\rightarrow\; m_t \sim \mathcal{O}(100\text{GeV})$]
- *Why there are complex Yukawa couplings giving rise to the CP violation in SM?*

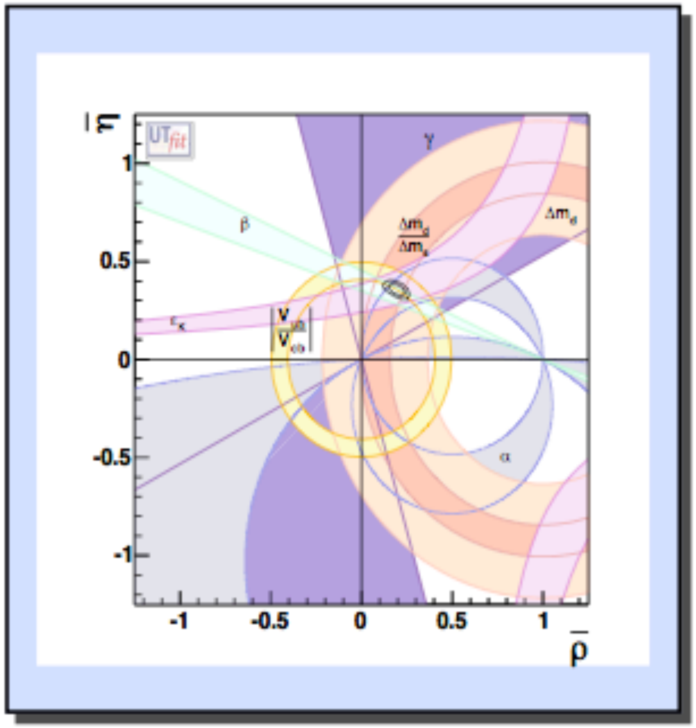
Why is it important?

Flavor sector of the Standard Model (SM)

- *Yukawa couplings are the only source of both the flavour structures and the CP violating phenomena.*
- *Structure of Yukawa couplings: hierarchical or democratic?*
  Can't tell: SM phenomenology the same in both cases.
  But in SUSY extensions, new non-SM interactions can distinguish.
- *Absolute values and ratios of quark masses are helpful.*
- *Practical issues: fundamental parameters enter all predictions of relevance to phenomenology.*

Way to get to m_q, m_Q and V_{ij} goes through LQCD

CKM unitarity triangle analysis



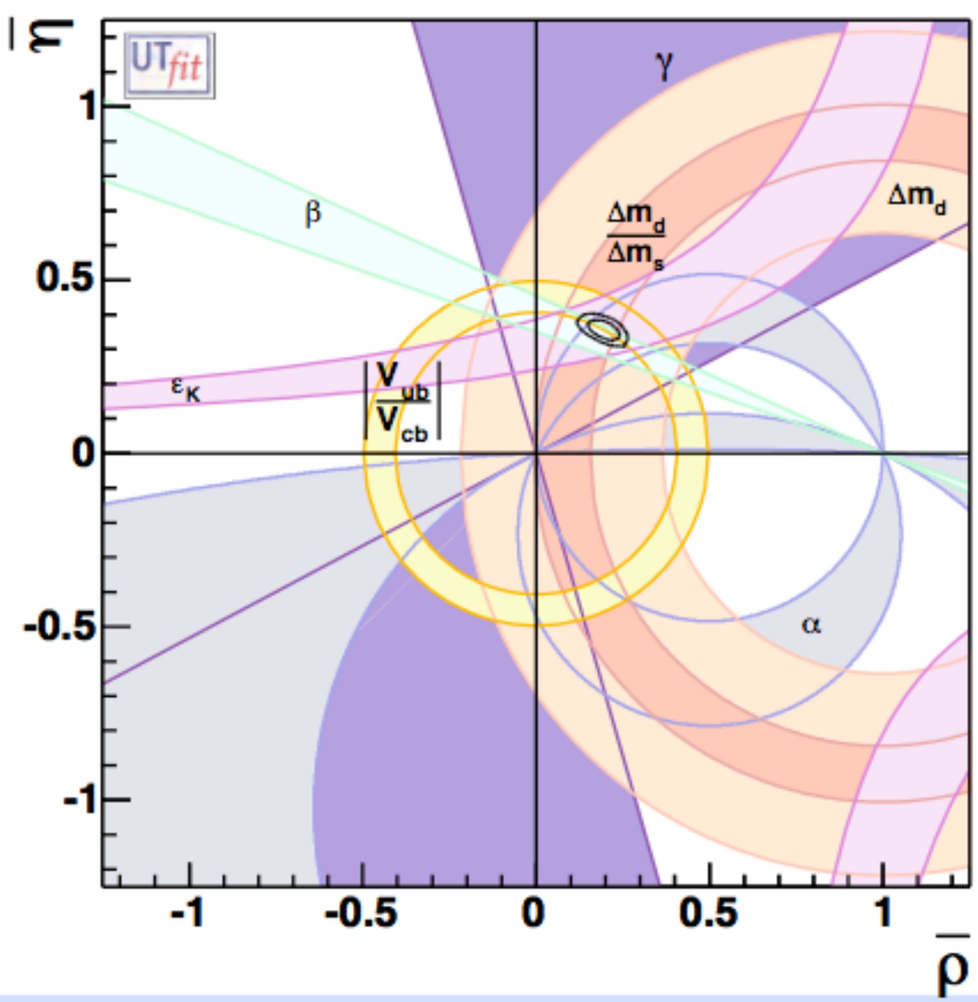
- The main constraints:

$$|\varepsilon_K| = C_K\, \hat{B}_K\, A^2\lambda^6\, \bar{\eta}\left[A^2\lambda^4\,(1-\bar{\rho})\; F_{tt} + F_{tc}\right]$$

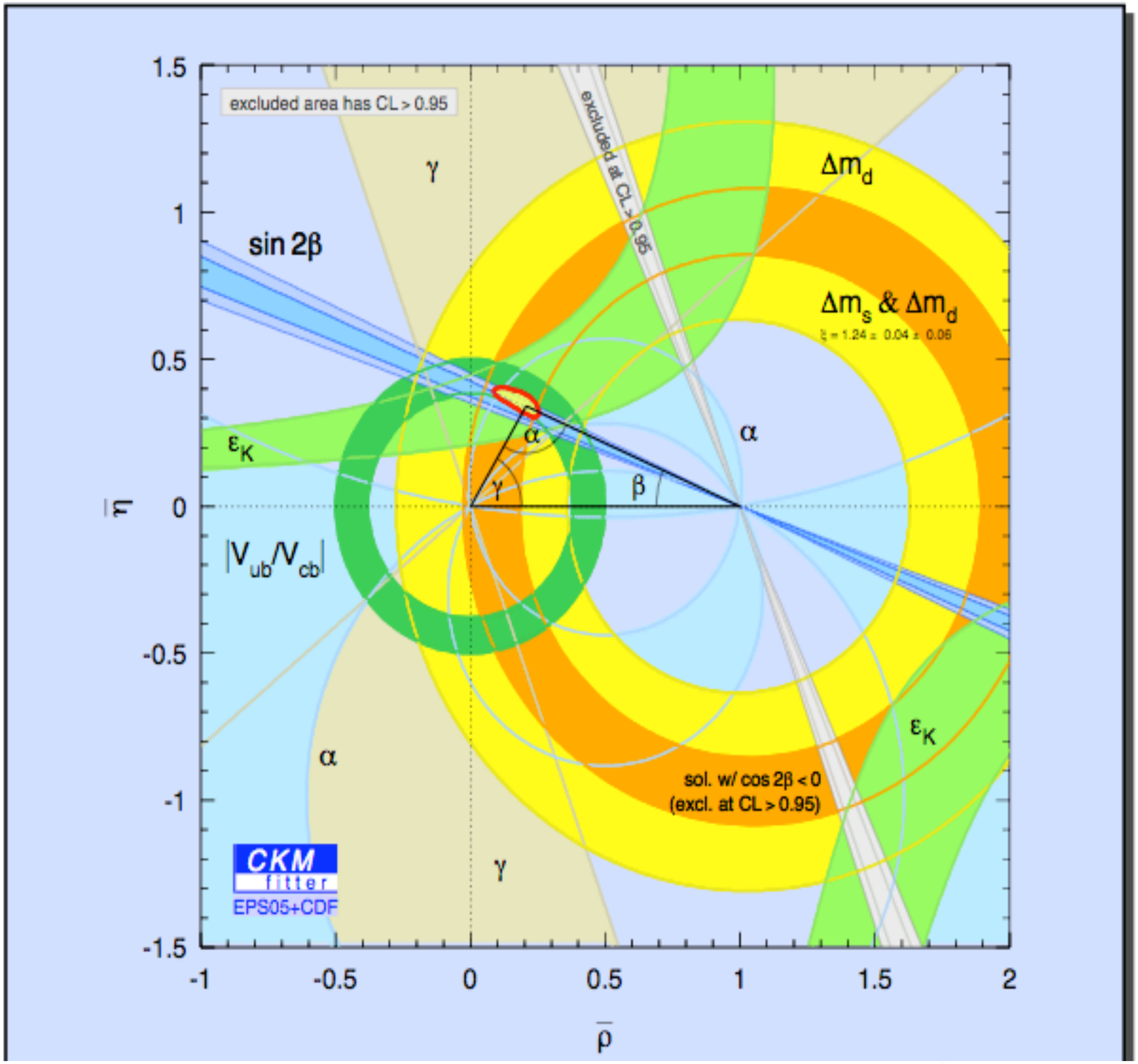
$$\Delta m_d = C_B\, m_{B_d}\, f_{B_d}^2 \hat{B}_{B_d}\, A^2\lambda^6\, [(1-\bar{\rho})^2 + \bar{\eta}^2]$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \frac{f_{B_d}^2 \hat{B}_{B_d}}{f_{B_s}^2 \hat{B}_{B_s}}\, \lambda^2\, [(1-\bar{\rho})^2 + \bar{\eta}^2] \quad \text{and} \quad \left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

CKM unitarity triangle analysis by UTfit (Bayesian)



CKM unitarity triangle analysis by CKM-fitter (frequentist)



Phenomenologically interesting

CP violation studies and the search for physics beyond Standard Model:

- $|V_{us}|$ — $K \to \ell\nu$, $K \to \pi\ell\nu$
- $|V_{cs}|$ — $D_s \to \ell\nu$, $D \to K\ell\nu$
  $D \to K^*\ell\nu$, $D_s \to \phi\ell\nu$
- $|V_{cd}|$ — $D \to \ell\nu$, $D \to \pi\ell\nu$
  $D \to \rho\ell\nu$, $D_s \to K^{(*)}\ell\nu$
- $|V_{cb}|$ — $B_c \to \ell\nu$, $B \to D^{(*)}\ell\nu$
  $B_s \to D_s^{(*)}\ell\nu$
- $|V_{ub}|$ — $B \to \ell\nu$, $B \to \pi\ell\nu$
  $B \to \rho\ell\nu$, $B_s \to K^{(*)}\ell\nu$
- $|V_{ts}/V_{td}|$ — $B \to K^*\gamma / B \to \rho\gamma$

- $K^0 - \bar{K}^0$ & $B^0 - \bar{B}^0$ mixing amplitudes

Flavor physics at LHC

Check UT through B_s-modes

- $B_s \to [D_s^{(*)}, K^{(*)}, B]\ell\nu$ *(and excitations)*
- $B_s \to J/\psi\,\phi$
- *many non-leptonic modes (esp. γ)*

FCNC

- $B_s \to \phi\gamma$, $B_s \to K^*\gamma$
- $B_s - \overline{B}_s$ *mixing*
- $B_s \to \mu^+\mu^-$
  $BR^{\rm sm} = 3.4(5) \times 10^{-9}$ *Vs.* $BR^{\rm susy} \propto \tan^6\beta$
- $B_s \to KK$

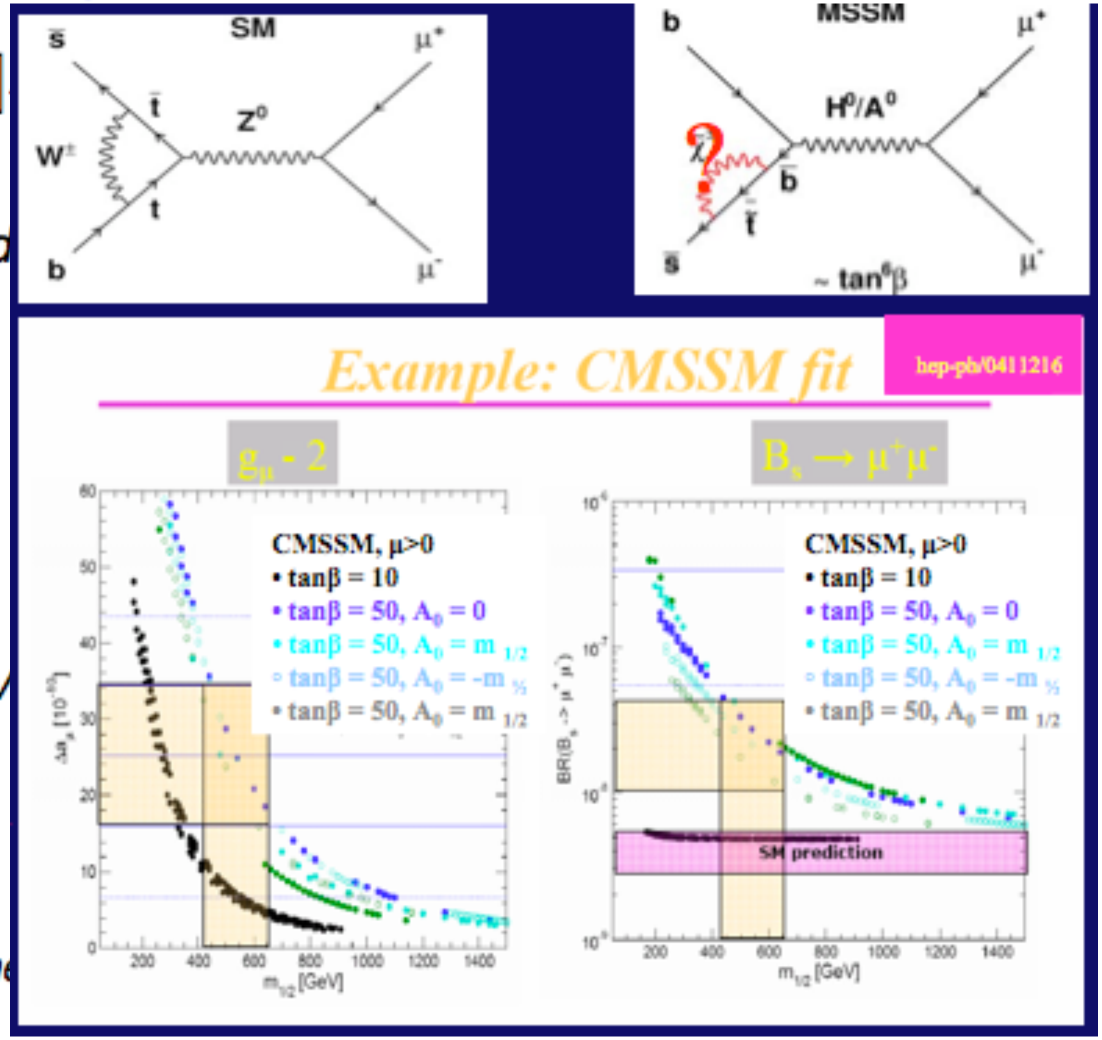
B_c modes

- *LHCb =* 10^9 B_c*'s/year* $\to$ *new physics scenarios*

Flavor physics at LHC

- Check UT through B_s-modes
  - $B_s \to [D_s^{(*)}, K^{(*)}, B]$
  - $B_s \to J/\psi\ \phi$
  - *many non-leptonic mod*
- FCNC
  - $B_s \to \phi\gamma$, $B_s \to K^*\gamma$
  - $B_s - \overline{B}_s$ *mixing*
  - $B_s \to \mu^+\mu^-$

    $BR^{sm} = 3.4(5) \times 10^{-9}$ *V*
  - $B_s \to KK$
- B_c modes
  - *LHCb* $= 10^9$ B_c*'s/year* $\to$ *ne*



How do hadrons arise from QCD?

- Very simple lagrangian

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{q=u,d,s,\ldots} \bar{q} \left\{ \gamma_\mu \left(\partial_\mu + g A^a_\mu t^a \right) + m_q \right\} q$$

yet amazingly rich and complex structure of strongly interacting matter.

- Atoms, molecules, solids,...
  - Constituents can be removed
  - Exchanged boson generating interactions subsumable into static potential ($\gamma \to$ coulombic, $\pi \to$ N-N potential)
  - most of mass from fermion constituents
- Hadrons (mesons, baryons)
  - Quarks are confined
  - Gluons are essential degrees of freedom; they carry about a half of the nucleon momentum; non-perturbative topological excitations
  - most of mass generated by interactions

Main goal is to compute...

$$\langle 0|\, \phi(x)\phi(x')\phi(x'')...\, |0\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},A_\mu]\phi(x)\phi(x')\phi(x'')...e^{iS_{QCD}}$$

with $\quad Z = \int \mathcal{D}[\psi,\bar{\psi},A_\mu]e^{iS_{QCD}}$

$\rightarrow$ *Minkowski to Euclidean space* $e^{iS} = e^{-S}$ *etc.*

$\rightarrow$ *Functional integral handled by MC on discrete space-time*

$\rightarrow$ *Suitable choice of* $\phi(x)\phi(x')... \rightarrow$ *desired physics info*

In practice we are interested in two- and three-point functions

On 2-pt functions

$$C_2(t) = \int d^3x e^{i\vec{p}\vec{x}} < 0|\Phi(\vec{x},t)\Phi^\dagger(\vec{0},0)|0 >$$

$\Phi(x)$- *interpolating operator for the hadron state (h) which we want to study*

$$\begin{aligned} C_2(t) &= \sum_n \int d^3x e^{i\vec{p}\vec{x}} \langle 0|\Phi(\vec{x},t)|n\rangle\langle n|\Phi^\dagger(\vec{0},0)|0\rangle \\ &= \int d^3x e^{i\vec{p}\vec{x}} \langle 0|\Phi(\vec{x},t)|h\rangle\langle h|\Phi^\dagger(\vec{0},0)|0\rangle + \ldots \\ &= \frac{1}{2E} e^{-iEt} \left| \langle 0|\Phi(\vec{0},0)|h\rangle \right|^2_{E=\sqrt{\vec{p}^2+m_h^2}} + \ldots \end{aligned}$$

→ *Minkowski to Euclidean space:* $iEt \to Et$

→ *Fit* $C_2(t)$ *to extract matrix element and hadron mass (*$|\vec{p}| = 0$*)*

→ *e.g.* $\Phi = \bar{u}\gamma_\mu\gamma_5 s \Rightarrow m_K$ *and* $\langle 0|\bar{u}\gamma_\mu\gamma_5 s|K\rangle = m_K f_K$

On 3-pt functions

$$
\begin{aligned}
C_3(t, t_x) &= \int d^3x d^3y e^{i(\vec{p}.\vec{x}+\vec{q}.\vec{y})} \langle 0|\Phi'(\vec{x}, t_x)\mathcal{O}(\vec{y}, t)\Phi^\dagger(\vec{0}, 0)|0\rangle \\
&\simeq \frac{e^{-Et}}{2E}\frac{e^{-E'(t_x-t)}}{2E'}\langle 0|\Phi'(\vec{0}, 0)|h_2(\vec{p})\rangle \times \\
&\quad \langle h_2(\vec{p})|\mathcal{O}(\vec{0}, 0)|h_1(\vec{p}+\vec{q})\rangle\langle h_1(\vec{p}+\vec{q})|\Phi^\dagger(\vec{0}, 0)|0\rangle
\end{aligned}
$$

$E' = \sqrt{\vec{p}^2 + m_{h_2}^2}$ and $E = \sqrt{(\vec{p}+\vec{q})^2 + m_{h_1}^2}$

→ *Combining with 2-pt functions* ⇒ *extract the transition matrix elements*

→ *Matrix elements of* $\Delta F = 2$ *operators*

→ *e.g. set* $|\vec{p}| = |\vec{q}| = 0$

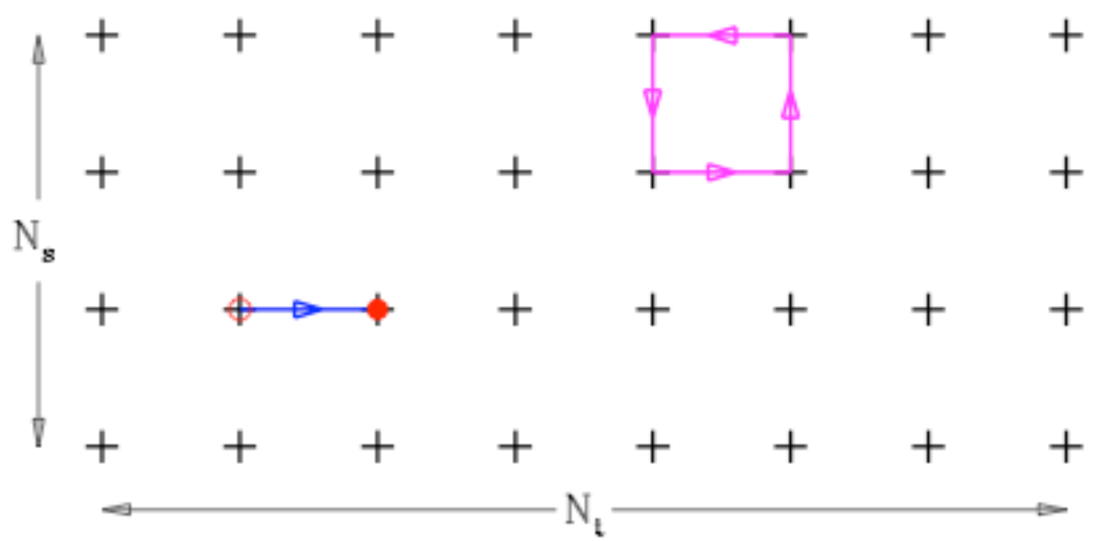
$\Phi = \bar{d}\gamma_5 s$, $\Phi' = \bar{s}\gamma_5 d$, and $\mathcal{O} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$

$\Rightarrow \langle \bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle = \frac{8}{3}f_K^2 m_K^2 B_K$

1. Lattice actions for QCD

Minkowski space-time, continuum $\longrightarrow$ Euclidean space-time, discretised

Lattice spacing $a, \quad a^{-1} \sim \Lambda_{\mathrm{UV}}, \quad x_\mu = n_\mu a$
Finite volume $L^3 \cdot T, \quad N_s = L/a, \quad N_t = T/a$



(anti)quarks:	$\psi(x), \overline{\psi}(x)$	lattice sites
gluons:	$U_\mu(x) = \mathrm{e}^{aA_\mu(x)} \in \mathrm{SU}(3)$	links
field tensor:	$P_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)$	"plaquettes"

In lattice QCD the (non-Abelian) gauge field is represented by an $\mathrm{SU}(3)$ matrix:

$$U_\mu(x) \in \mathrm{SU}(3), \qquad \text{(link variable)}$$

Gauge transformation:

$$U_\mu(x) \longrightarrow g(x) U_\mu(x) g(x+a\hat{\mu})^{-1}, \qquad g(x),\, g(x+a\hat{\mu}) \in \mathrm{SU}(3)$$

Let $A_\mu^{\mathrm{cont}}(x)$ be a given gauge potential in the continuum:

$$U_\mu(x) = \mathrm{e}^{aA_\mu^{\mathrm{cont}}(x)}, \qquad A_\mu^{\mathrm{cont}}(x) = \lim_{a\to 0} \frac{1}{a}\Big(U_\mu(x) - 1\Big)$$

Formulate expressions for the QCD action in terms of link variables and fermionic fields

Lattice action: $\quad S[U,\overline{\psi},\psi] = S_{\mathrm{G}}[U] + S_{\mathrm{F}}[U,\overline{\psi},\psi]$

Wilson "plaquette" action for Yang-Mills theory

[Wilson 1974]

$$S_{\rm G}[U]=\beta\sum_x\sum_{\mu<\nu}\left(1-\tfrac{1}{3}{\rm Re}\,{\rm Tr}\,P_{\mu\nu}(x)\right),\quad \beta=6/g_0^2,\quad \text{(gauge invariant)}$$

$$P_{\mu\nu}(x)=U_\mu(x)U_\nu(x+a\hat\mu)U_\mu^\dagger(x+a\hat\nu)U_\nu^\dagger(x)$$

For small lattice spacings:

$$S_{\rm G}[U]\longrightarrow-\frac{1}{2g_0^2}\int {\rm d}^4x\,{\rm Tr}\left[F_{\mu\nu}(x)F_{\mu\nu}(x)\right]+{\rm O}(a)$$

Proof: insert $U_\mu(x)={\rm e}^{aA_\mu(x)}$ into $P_{\mu\nu}$ and Taylor-expand in a.

N.B. Discretisation not unique!

Gauge Action Choice

- The standard Wilson action:

$$S_g = \frac{\beta}{3} \mathrm{Re}\,\mathrm{Tr} \left\langle 1 - \square \right\rangle$$

- The Symanzik improved action.

$$S_g = \frac{\beta}{3} \mathrm{Re}\,\mathrm{Tr} \left[c_0 \left\langle 1 - \square \right\rangle + 2c_1 \left\langle 1 - \square\!\square \right\rangle + \frac{4}{3} c_2 \left\langle 1 - \boxdot \right\rangle \right]$$

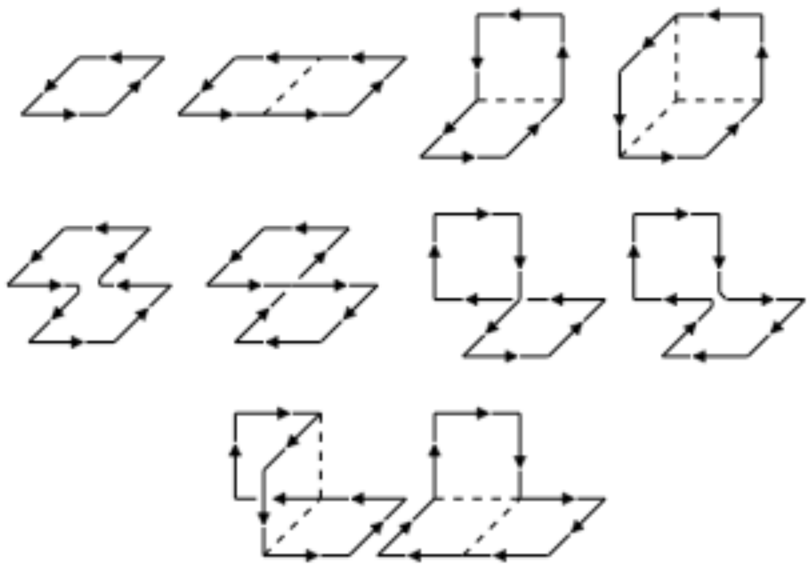
c_0, c_1 and c_2 chosen to cancel the $O(a^2)$ error at one loop.

- RG-improved

$$S_g = \frac{\beta}{3} \mathrm{Re}\,\mathrm{Tr} \left[(1 - 8c_1) \left\langle 1 - \square \right\rangle + c_1 \left\langle 1 - \square\!\square \right\rangle \right]$$

  - $c_1 = -0.331$ Iwasaki
  - $c_1 = -1.4067$ DBW2 [de Forcrand 1999]

Glueball interpolating operators



- contamination of higher spin states;

  e.g. 0^{++} can mix with 4^{++}

- Matrix correlators:

$$C_{ij}(x_0) = \sum_{\vec{x}} \langle O_i(x) O_j(0) \rangle$$

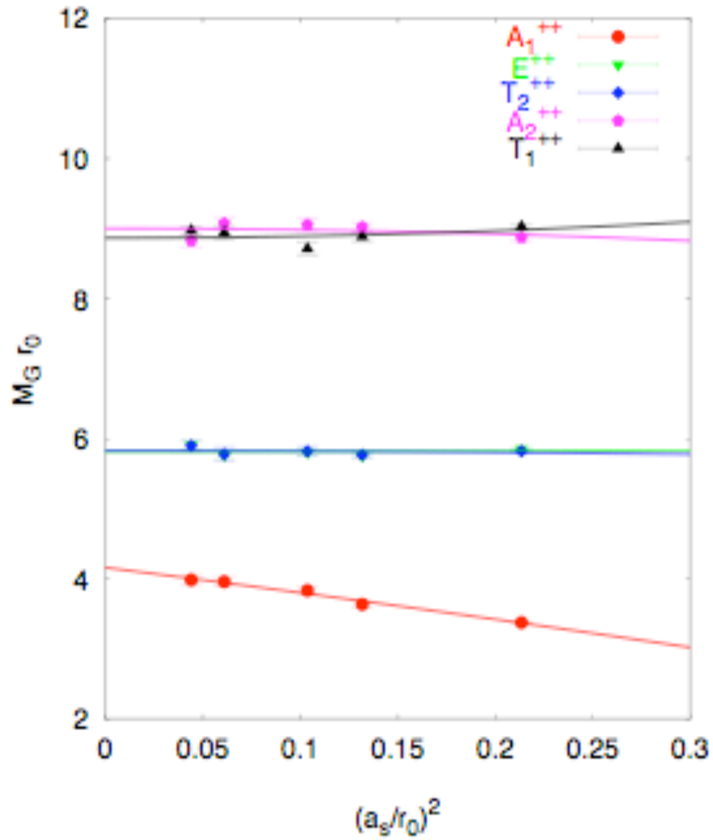
- $\{O_1, \ldots, O_n\}$:

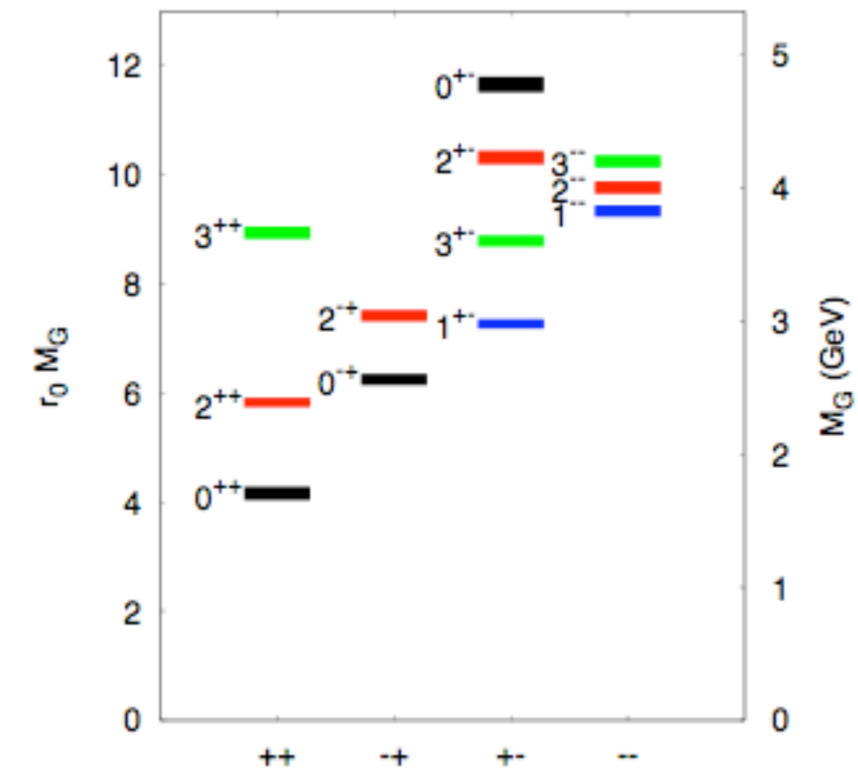
  basis of interpolating operators for given irrep. of the hypercubic group

$\rightarrow$ Recover a given spin-parity in the continuum limit

Glueball spectrum in quenched QCD

[Chen et al., hep-lat/0510074]





$$m_{0^{++}} = 1710(50)(80)\,\text{MeV}, \qquad m_{2^{++}} = 2390(30)(120)\,\text{MeV}$$

$$m_{0^{-+}} = 2560(35)(120)\,\text{MeV}$$

- No effects due to dynamical quarks or glueball-meson mixing

Fermionic part:

- Discretised version of the covariant derivative:

$$\nabla_\mu \psi(x) \equiv \frac{1}{a}\Big(U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)\Big)$$
$$\nabla^*_\mu \psi(x) \equiv \frac{1}{a}\Big(\psi(x) - U^\dagger_\mu(x-a\hat{\mu})\psi(x-a\hat{\mu})\Big)$$

- "Naive" discretisation of fermionic part $S_{\rm F}$:

$$D_{\rm naive} + m_f = \frac{1}{2}\gamma_\mu\left(\nabla_\mu + \nabla^*_\mu\right) + m_f$$

$$\widetilde{D}_{\rm naive}(p) = i\gamma_\mu \frac{1}{a}\sin(ap_\mu) = i\gamma_\mu p_\mu + {\rm O}(a^2) \quad \text{(free theory)}$$

→ $\widetilde{D}_{\rm naive}(p)$ vanishes for $p_\mu = 0, \pi/a$

→ produces $2^4 = 16$ poles in fermion propagator of flavour f

→ 16-fold degeneracy of fermion spectrum: **Fermion doubling problem**

Fermionic discretisations:

a. Wilson fermions *[Wilson 1974/75]*

b. Staggered (Kogut-Susskind) fermions *[Kogut+Susskind 1975]*

c. Overlap/Domain Wall fermions *[Kaplan '92, Furman+Shamir '96, Neuberger '98]*

d. "Perfect"/Fixed point actions *[Hasenfratz+Niedermaier '93/'98]*

c.+d.: Ginsparg-Wilson fermions *[Ginsparg+Wilson 1982, Lüscher 1998]*

Wilson fermions

- Add a term to $D_{\rm naive}$ which formally vanishes as $a \to 0$:

$$D_{\rm w} + m_f = \frac{1}{2}\gamma_\mu \left(\nabla_\mu + \nabla_\mu^*\right) + ar\nabla_\mu^*\nabla_\mu + m_f$$

$$\widetilde{D}_{\rm w}(p) = i\gamma_\mu \frac{1}{a}\sin(ap_\mu) + \frac{2r}{a}\sin^2\left(\frac{ap_\mu}{2}\right) \quad \text{(free theory)}$$

⇒ Mass of doubler states receives contribution $\propto r/a$: pushed to cutoff scale

:-) Complete lifting of degeneracy

:-(Explicit breaking of chiral symmetry:

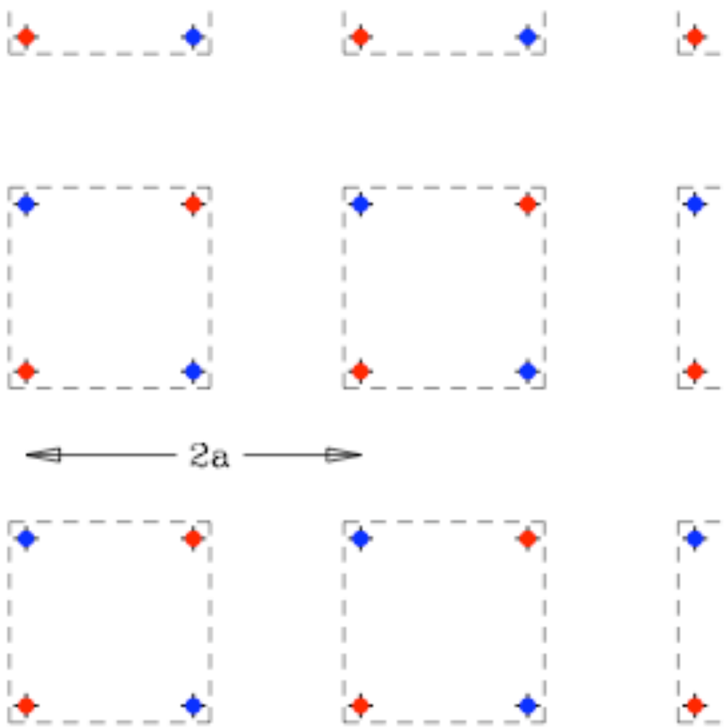
even for $m_f = 0$ the action is no longer invariant under

$$\psi(x) \to {\rm e}^{i\alpha\gamma_5}\psi(x), \qquad \overline{\psi}(x) \to \overline{\psi}(x){\rm e}^{i\alpha\gamma_5}$$

Mostly acceptable, but makes things more complicated

Staggered (Kogut-Susskind) fermions

- Reduce d.o.f. by distributing single spinor components over corners of hypercube



:-(Only partial lifting of degeneracy:

$$16 \longrightarrow 4$$

$\Rightarrow$ 4 "tastes" per physical flavour

- Flavour symmetry broken: gluons mix "tastes"

:-) Remnant of chiral symmetry: global $\mathrm{U}(1) \otimes \mathrm{U}(1)$

Chiral Symmetry on the Lattice

- Lattice regularisation: incompatible with chiral symmetry?

  Either: fermion doubling problem

  Or : explicit chiral symmetry breaking: $\{\gamma_5, D\} \neq 0$

  [Nielsen+Ninomiya 1979]

- Chiral symmetry at non-zero lattice spacing realised if

  [Ginsparg & Wilson 1982, Lüscher 1998]

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Explicit construction: Neuberger-Dirac operator *[Neuberger 1998*

$$D_{\mathrm{N}} = \frac{1}{a}\left\{1 - \frac{A}{\sqrt{A^\dagger A}}\right\}, \quad A = 1 - aD_{\mathrm{w}}$$

D_{w} : massless Wilson-Dirac operator

$$S_{\mathrm{F}}[U,\overline{\psi},\psi] = a^4 \sum_x \overline{\psi}(x)[D_{\mathrm{N}}\psi](x)$$ – No fermion doublers!

- Invariance under infinitesimal chiral transformations:

$$\psi \to \psi + \epsilon\delta\psi, \quad \delta\psi = \gamma_5(1 - \tfrac{1}{2}aD)\psi$$
$$\overline{\psi} \to \overline{\psi} + \delta\overline{\psi}\epsilon, \quad \delta\overline{\psi} = \overline{\psi}(1 - \tfrac{1}{2}aD)\gamma_5$$

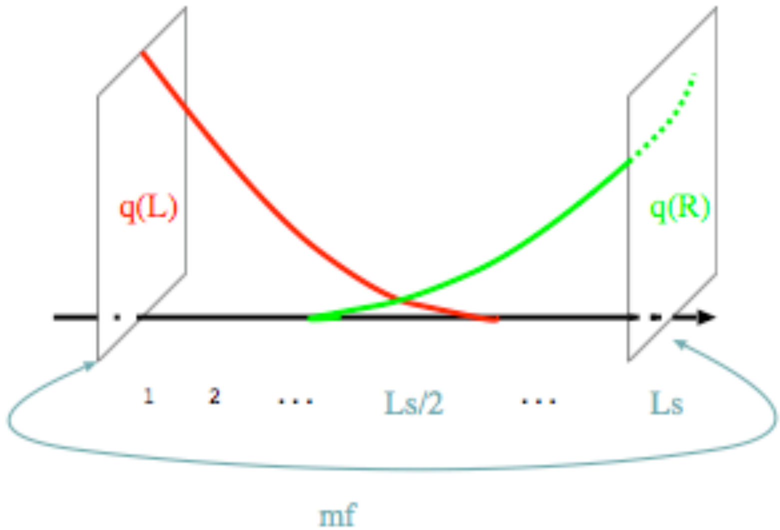
- D_{N} satisfies the Atiyah-Singer index theorem:
  [Hasenfratz, Laliena & Niedermayer 1998]

$$\text{index}(D_{\mathrm{N}}) = a^5 \sum_x \frac{1}{2} \mathrm{Tr}\,(\gamma_5 D_{\mathrm{N}}) = n_- - n_+$$

→ D_{N} exhibits $|n_- - n_+|$ exact zero modes

- But: numerical implementation of D_{N} expensive

Domain Wall Fermions



- Domain Wall Fermions preserve flavour symmetry and have greatly reduced chiral symmetry breaking.
  - at the expense adding a extra, fifth, dimension.

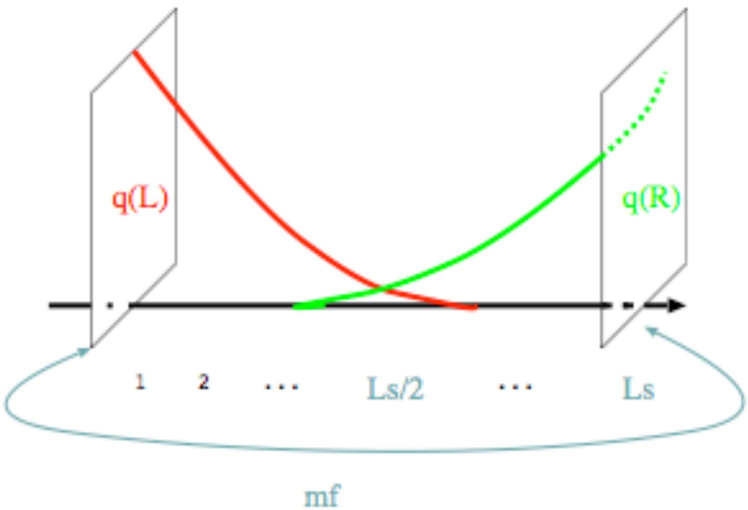
- The nearest neighbour derivative in the 5th dimension distinguishes left- and right-handed fermions

$$
\begin{aligned}
D_{DWF} = -\gamma_\mu \frac{1}{2}\left(\nabla_\mu^+ + \nabla_\mu^-\right) + \frac{1}{2}\nabla_\mu^- \nabla_\mu^+ + M_5 & \quad \text{4d piece} \\
+P_L \partial_5^+ - P_R \partial_5^- & \quad \text{5d piece}
\end{aligned}
$$

$L_s = 4$ case:

$$
\partial_5^+ \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \frac{1}{a_5} \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}
$$

Domain Wall Fermions



- Define 4d quark fields on the wall

$$q_x = P_L \Psi_{x,0} + P_R \Psi_{x,L_s-1}$$

- Couple the two walls with a mass term

$$m_f \bar{q} q$$

- For finite L_s chiral symmetry is broken, leading to an additive renormalisation of the mass

$$m_f \to m_f + m_{\text{res}}$$

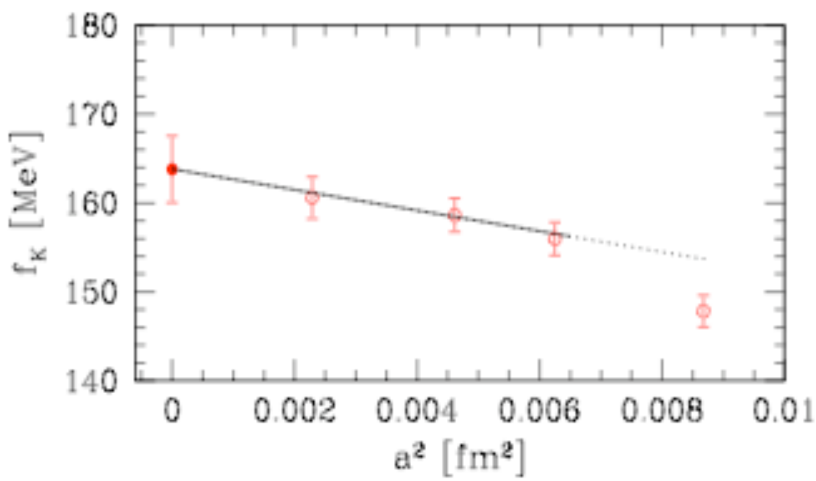
plus mixing between operators in different chiral multiplets

- $m_{res} \to 0$ as $L_s \to \infty$; The cost in computer time $\propto L_s$
  - Need small m_{res} (few MeV) for reasonable L_s ($O(10)$)

Continuum limit

$$\langle\Omega\rangle = \langle\Omega\rangle^{\mathrm{lat}} + O(a^p), \quad p \in \mathbb{N}, \qquad \text{lattice artefacts}$$

fermion discretisation	
Wilson	$\mathrm{O}(a)$
Improved Wilson	$\mathrm{O}(\alpha_s a)$, $\mathrm{O}(a^2)$
Staggered	$\mathrm{O}(a^2)$
DWF, Neuberger	$\mathrm{O}(a^2)$



- Classical continuum limit: $a \to 0$
- QFT: adjust the bare parameters as cutoff is removed, whilst keeping "constant physics"
- $a\Lambda = \left(b_0 g_0^2\right)^{-b_1/2b_0^2} \mathrm{e}^{-1/2b_0 g_0^2} \ldots$

  Continuum limit: $\beta = 6/g_0^2$

  $a \to 0 \quad \Leftrightarrow \quad \beta \to \infty$
- Perform simulations at several values of β and extrapolate to $a = 0$.

Work of various collaborations differs mainly in the choice of fermion action S_F:

$$S_{QCD} = \int \mathrm{d}^4x\, \mathcal{L}_{QCD}, \quad S_{QCD} = S_G + S_F$$

S_F	Chiral Symmetry	Flavor Symmetry	Simulation
Overlap	Exact	OK	Very Expensive
Domain Wall	Residual Mass	OK	Expensive
Wilson/Clover	To $O(a^2)$	OK	Fast
Twisted Mass	To $O(a^2)$	Broken	Fast *
ϟ Staggered ϟ	$U(1) \times U(1)$ Valence quarks	?	Fast *

Various gauge actions S_G

* No advantage over Wilson

Helps to get systematic errors under control

All symmetries (chiral & flavor) are supposed to be recovered in the continuum limit

Improved staggered quarks

- numerically more efficient than Wilson fermions

but: 4-fold degeneracy of fermion spectrum: 4 "tastes" per flavour

taste symmetry violations at $\mathrm{O}(a^2)$ can be reduced → "improved"

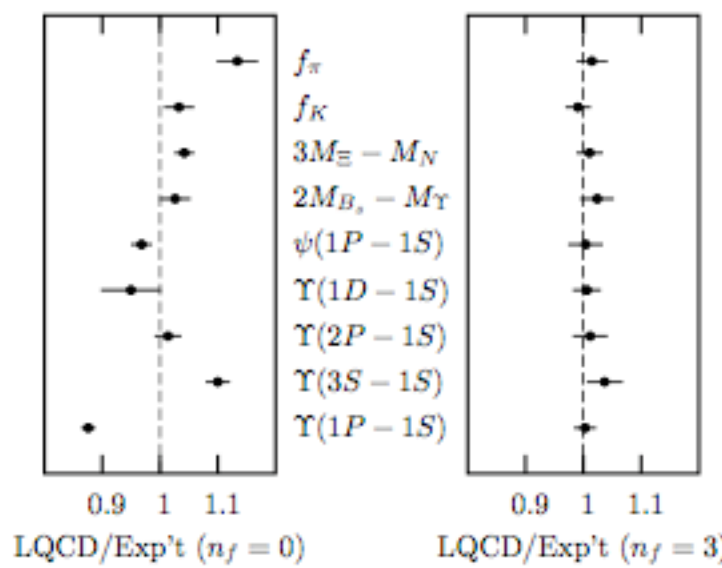
→ Take fractional powers of fermion determinant: "fourth root trick"

$2+1$ light flavours: $\{\det(D_{\mathrm{stag}}+m_{u,d})\}^{1/2} \times \{\det(D_{\mathrm{stag}}+m_s)\}^{1/4}\,\mathrm{e}^{-S_\mathrm{G}}$

- Does this correspond to a local field theory?

- Does this lead to the correct continuum limit?

→ under debate . . .



[Davies et al.,
hep-lat/0304(

Before continuing...

- Lattice 2005: *"Is staggered QCD really QCD or just a model of QCD?"*-S.Dür
- Lattice 2006: *"Not bad, just ugly"*-S.Sharpe
- Bernard et al 2006: *The fourth root trick corresponds to a non-local theory at $a \neq 0$, but argue that the non-local behavior is likely to go away in the continuum limit."*
- So what do you do about the non-locality and renormalisability?
- SChPT is used to extract f_π and f_K: fit contains more than 50 parameters!
- renormalisation is only perturbative (c.f. lesson from m_s!)
- MILC and HPQCD made a great effort to make the best out of SQCD, BUT the use of other actions is indispensable!

Strong coupling constant

Benchmark calculation

– Compute $\alpha_s^S(a)$ $\quad$ S: 'lattice' scheme $\quad$ $N_f = 0, 2, 3$

– Convert $\alpha_s^S(a)$ to $\alpha_s^{\overline{MS}}(\mu)$, $\quad \mu^2 \gg 5\,\mathrm{GeV}^2$ $\quad$ $\Lambda^{(0)}_{\overline{MS}}, \Lambda^{(2)}_{\overline{MS}}, \Lambda^{(3)}_{\overline{MS}}$

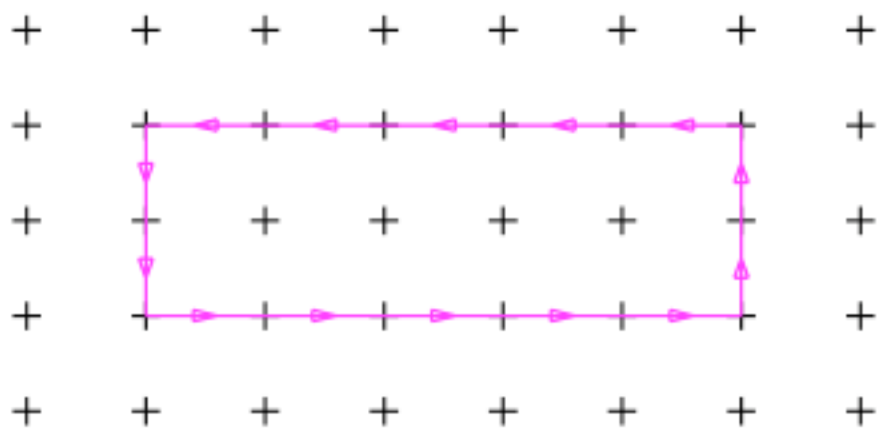
Obtain higher Λ's by 3-loop matching :

– $\alpha^{\overline{MS}}_{s\,(N_f=4)}(m_c) = \alpha^{\overline{MS}}_{s\,(N_f=3)}(m_c) - \frac{11}{72\pi^2}\,\alpha^{\overline{MS}}_{s\,(N_f=3)}(m_c)^3 + \cdots$ $\quad$ $\Lambda^{(4)}_{\overline{MS}}$

– $\alpha^{\overline{MS}}_{s\,(N_f=5)}(m_b) = \alpha^{\overline{MS}}_{s\,(N_f=4)}(m_b) - \frac{11}{72\pi^2}\,\alpha^{\overline{MS}}_{s\,(N_f=4)}(m_b)^3 + \cdots$ $\quad$ $\Lambda^{(5)}_{\overline{MS}}$

α_s from heavy quarkonia

- Definition: $\alpha_V(q) = -\frac{3}{16\pi^2} q^2 V(q)$ (heavy quark potential)



$\langle W_{nm} \rangle \propto \mathrm{e}^{-V(r)t}$

$r = na,\ t = ma$

$n \cdot m$: area of Wilson loop

- Perturbative expansion:

$$-\ln \langle W_{nm} \rangle = c_{nm}^{(1)} \alpha_V(q^*) \left\{ 1 + c_{nm}^{(2)} \alpha_V(q^*) + c_{nm}^{(3)} \alpha_V(q^*)^2 + \mathrm{O}(\alpha_V^3) \right\}$$

$c_{11}^{(1)}, c_{11}^{(2)}$: known in perturbation theory

q^* : "characteristic" momentum scale

- Conversion to $\overline{\mathrm{MS}}$-scheme:

$$\alpha_{\overline{\mathrm{MS}}}(\mathrm{e}^{-5/6}q) = \alpha_V(q) + \frac{2}{\pi}\alpha_V(q)^2 + \ldots$$

- Calibration:

$$q\,[\mathrm{GeV}] = \frac{(aq)}{(a\Delta^{\Upsilon}_{1\mathrm{S}-1\mathrm{P}})}\,\Delta^{\Upsilon}_{1\mathrm{S}-1\mathrm{P}}\,[\mathrm{GeV}]$$

(radial $(1S-2S)$ or orbital $(1S-1P)$ splitting in Υ-system)

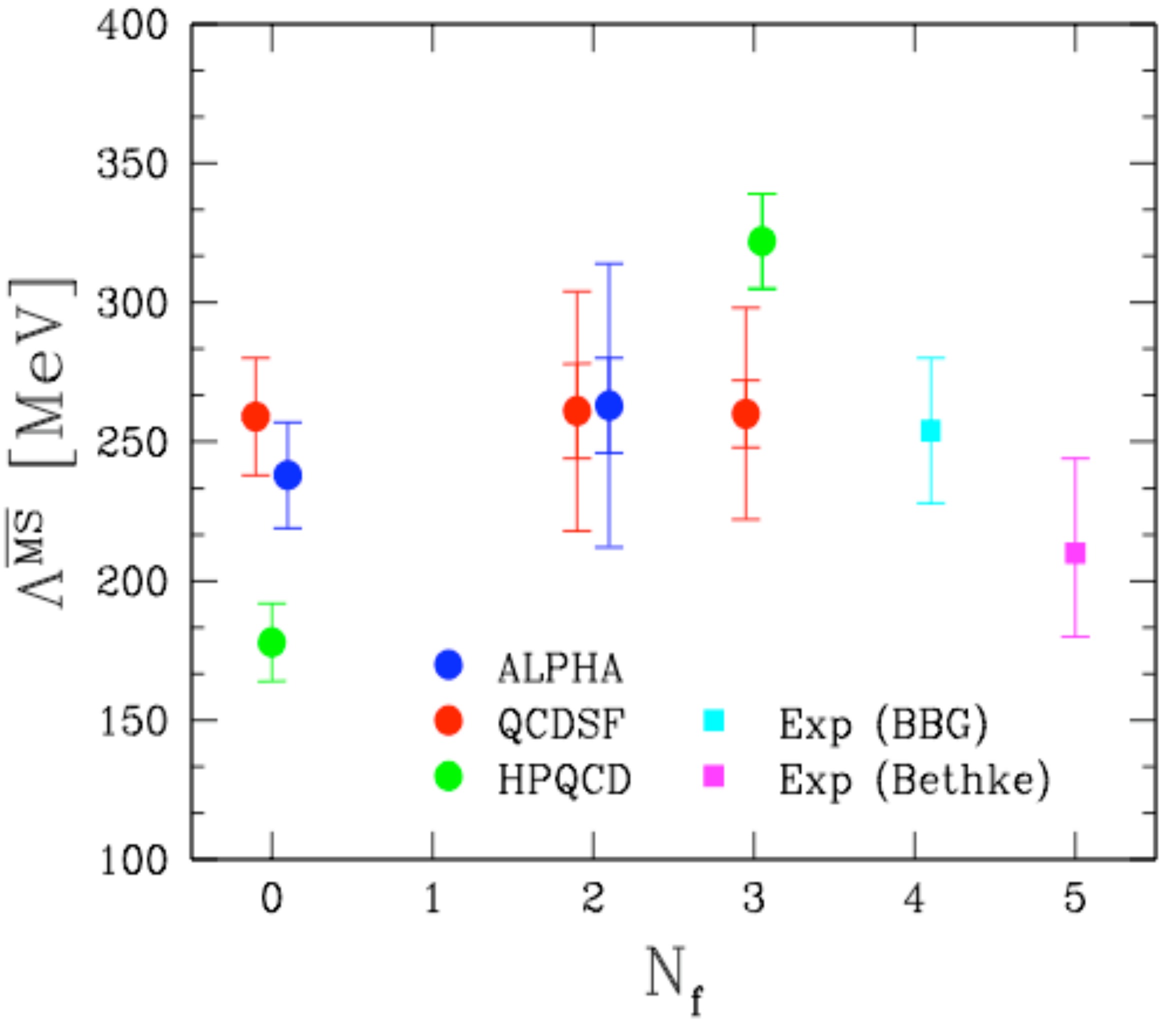
- Matching & running: purely perturbative

Quark thresholds, e.g. for $N_\mathrm{f}=3$ dynamical flavours:

$$\alpha^{(3)}_{\overline{\mathrm{MS}}}(7.5\,\mathrm{GeV}) \longrightarrow \alpha^{(3)}_{\overline{\mathrm{MS}}}(m_c) = \alpha^{(4)}_{\overline{\mathrm{MS}}}(m_c)$$

$$\longrightarrow \alpha^{(4)}_{\overline{\mathrm{MS}}}(m_b) = \alpha^{(5)}_{\overline{\mathrm{MS}}}(m_b) \longrightarrow \alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z)$$

Results (can we learn from this?)



Light quark masses

$$\partial_\mu A^\mu = (m_1 + m_2)P$$

- Calculate the two-point correlation function

$$\langle 0 \,|\, P^l(t) P^{s\,\dagger}(0) \,|\, 0 \rangle = \frac{Z_P^l Z_P^s}{2m_P} \left\{ \exp(-m_P t) + \exp(-m_P(L_t - t)) \right\} \,.$$

- m_P is the mass of the pseudoscalar meson.
- The superscripts l and s stand for *local* and *smeared* respectively.

Optional

- The $Z_P^{l,s}$'s are the matrix elements $\langle 0 | P^{l,s}(0) | P \rangle$.

$$\partial_\mu A^\mu = 2m_q P$$

$$\langle\, 0\,|\, P^l(t) P^{s\,\dagger}(0)\,|\, 0\,\rangle = \frac{C_P^l C_P^s}{2m_P}\,\{\exp(-m_P t) + \exp(-m_P(L_t - t))\}\ .$$

- Calculate also

$$\langle\, 0\,|\, A_4^l(t) P^{s\,\dagger}(0)\,|\, 0\,\rangle = \frac{C_A^l C_P^s}{2m_P}\,\{\exp(-m_P t) - \exp(-m_P(L_t - t))\}\ .$$

- Thus we obtain

$$m_q^{(0)\,\mathrm{AWI}} \equiv \frac{m_P C_A^l}{2C_P^l}\ .$$

- Now we would like the mass in some standard renormalization scheme, and the axial current and pseudoscalar density are both multiplicatively renormalizable. The renormalization constants can be fixed and we obtain the masses.

Non-Perturbative Renormalization

• Lattice computations are of bare quantities with a^{-1} as the ultraviolet cut-off.

• As an example consider a local operator, such as the pseudoscalar density $\bar{\psi}_1\gamma^5\psi_2$.

In lattice simulations we compute

$$\langle f|O_B(a)|i\rangle\,,$$

whereas we would like to know

$$\langle f|O_R(\mu)|i\rangle\,,$$

in some standard renormalization scheme R.

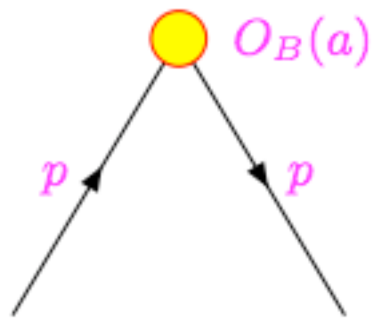
The long distance physics is the same in both.

• For sufficiently large scale, a^{-1} and $\mu \gg \Lambda_{\text{QCD}}$, the relation between these two matrix elements can be determined in perturbation theory, but the coefficients in Lattice PT are frequently large.

- It is possible to perform the renormalization non-pertubatively, eliminating the need for lattice perturbation theory.

For example (there are more sophisticated schemes), let us define the renormalized operator O_R as being the one whose matrix element between quark states, at some scale $p^2 = \mu^2$ and in some gauge (the Landau gauge say) is the tree-level one. We compute

$$\langle\, p | O_B(a) | p \,\rangle =$$



and determine the renormalization constant $Z_O(a\mu)$ by requiring that

$$Z_O(a\mu)\,\langle\, p | O_B(a) | p \,\rangle_{p^2=\mu^2} \;=\; \text{tree level value.}$$

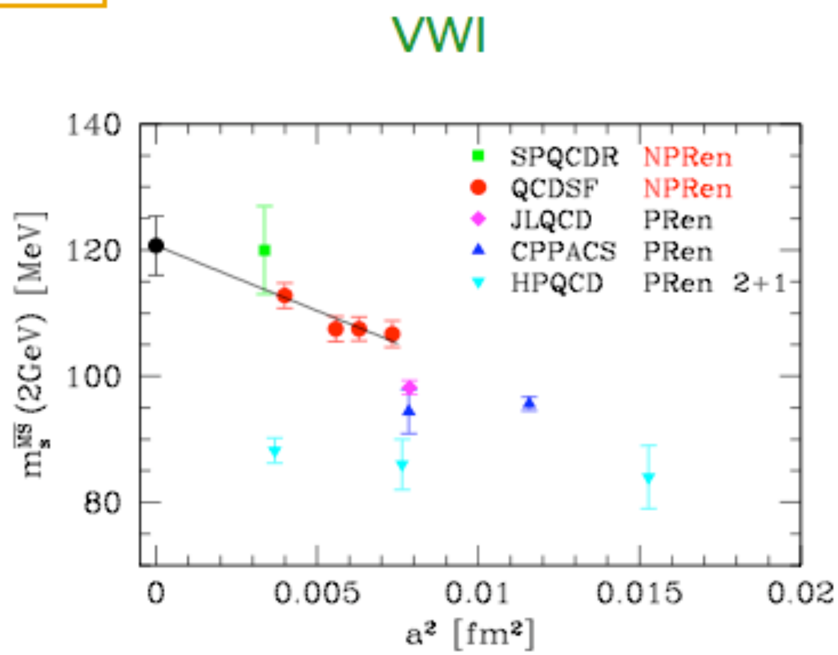
- The renormalized operator

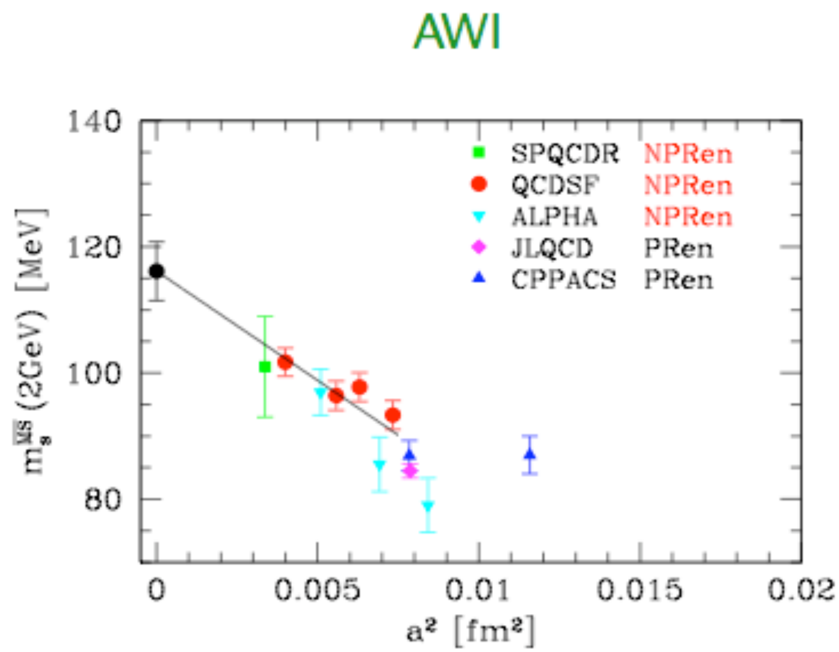
$$O_R^{RI\;Mom}(\mu) \equiv Z_O(a\mu)\,O_B(a)$$

is independent of the regularization (RI) and can be used in hadronic matrix elements.

Quark masses

m_s





$$m_s^{\overline{MS}}(2\,\mathrm{GeV}) = 118(5)\,\mathrm{MeV}$$

A reliable calculation requires

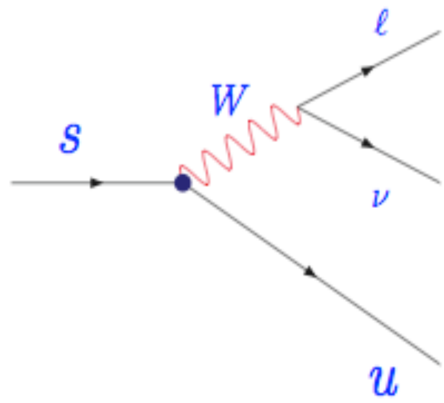
– Nonperturbative renormalization

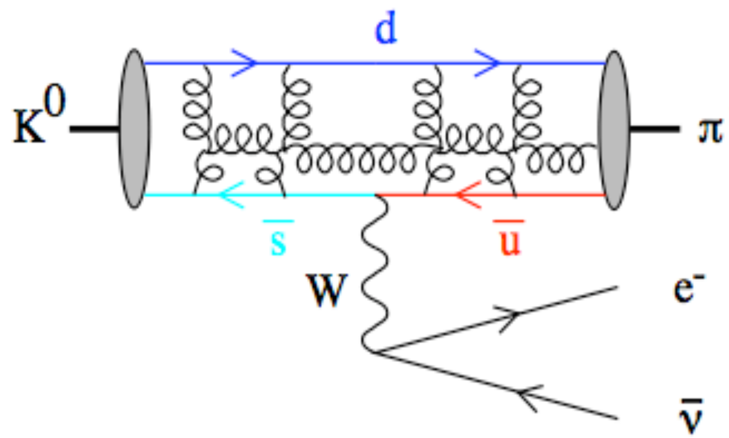
– Continuum extrapolation

Recent Compilation of (Unquenched) Lattice Results

Reference	m_s	$\hat{m}$
HPQCD, MILC and UKQCD	$76 \pm 3 \pm 7$ MeV	$2.8 \pm .1 \pm .3$ MeV
HPQCD, MILC and UKQCD Update including 2-loop Z's	$86 \pm 3 \pm 4$ MeV	$3.2 \pm .1 \pm .2$ MeV *
CP-PACS & JLQCD (K-input)	80.4 ± 1.9 MeV	$3.05 \pm .06$ MeV
CP-PACS & JLQCD (Φ-input)	89.3 ± 2.9 MeV	$3.04 \pm .06$ MeV
SPQR (VWI)	111 ± 6 MeV	$4.8 \pm .5$ MeV
SPQR (AWI)	103 ± 9 MeV	$4.5 \pm .5$ MeV
QCDSF & UKQCD	$119 \pm 5 \pm 8$ MeV	$4.7 \pm .2 \pm .3$ MeV
Alpha	97 ± 22 MeV	—

$|V_{us}|$ (λ)





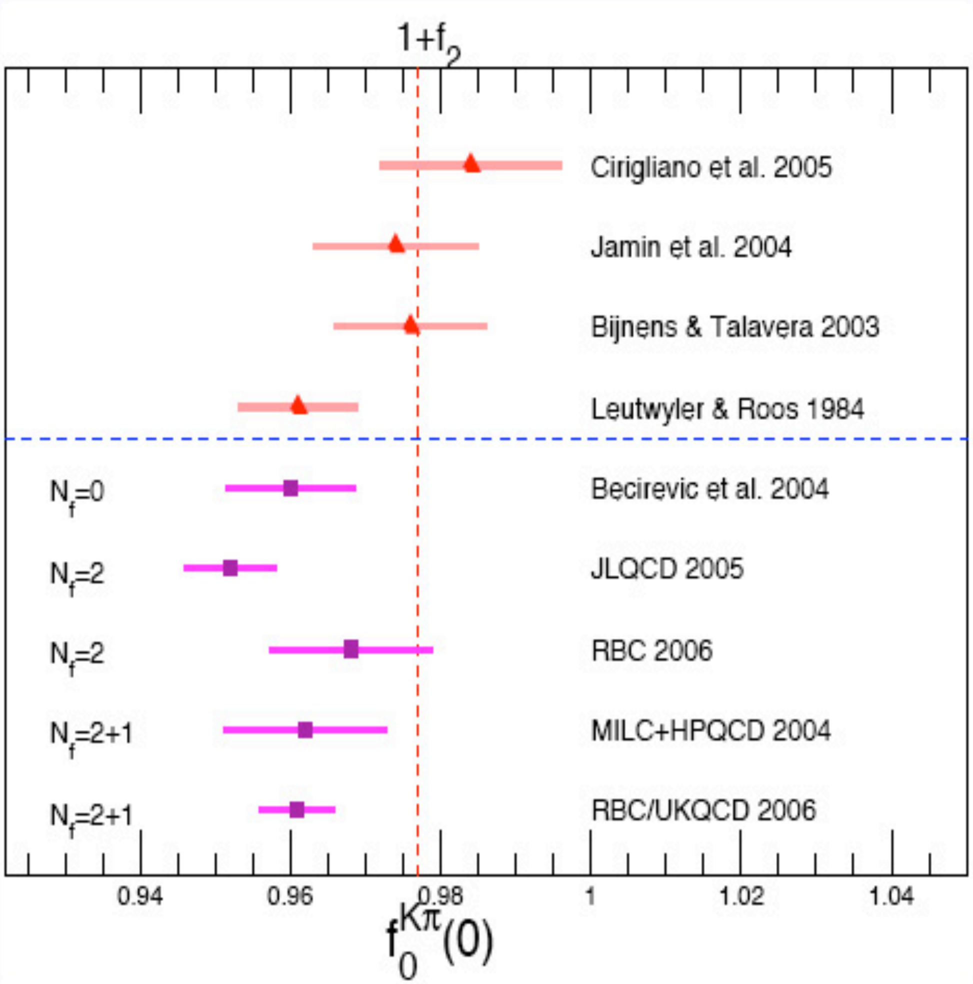
$$\langle\pi^-(p)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = \left(p_K + p - q\frac{m_K^2 - m_\pi^2}{q^2}\right)_\mu F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} q_\mu F_0(q^2)$$

$F^{K\to\pi}(0) = ?$

- *CVC* $\to$ *normalisation in SU(3) limit:* $F(0) = 1 \otimes$ *AGT* $\mathcal{O}[(m_s - m_u)^2]$
- *ChPT to* $\mathcal{O}(p^6)$: $F^{K^0\to\pi^-}(0) = 1 + f_{p^4} + f_{p^6}$ *AND* $f_{p^4} = -0.0227$
- $f_{p^6} = ?$ *: high precision possible from double ratio*

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle\,\langle K|\bar{u}\gamma_0 s|\pi\rangle}{\langle K|\bar{s}\gamma_0 s|K\rangle\,\langle\pi|\bar{u}\gamma_0 u|\pi\rangle} = \frac{(m_K + m_\pi)^2}{4 m_\pi m_K}\left(F_0(q^2_{\mathrm{max}})\right)^2$$

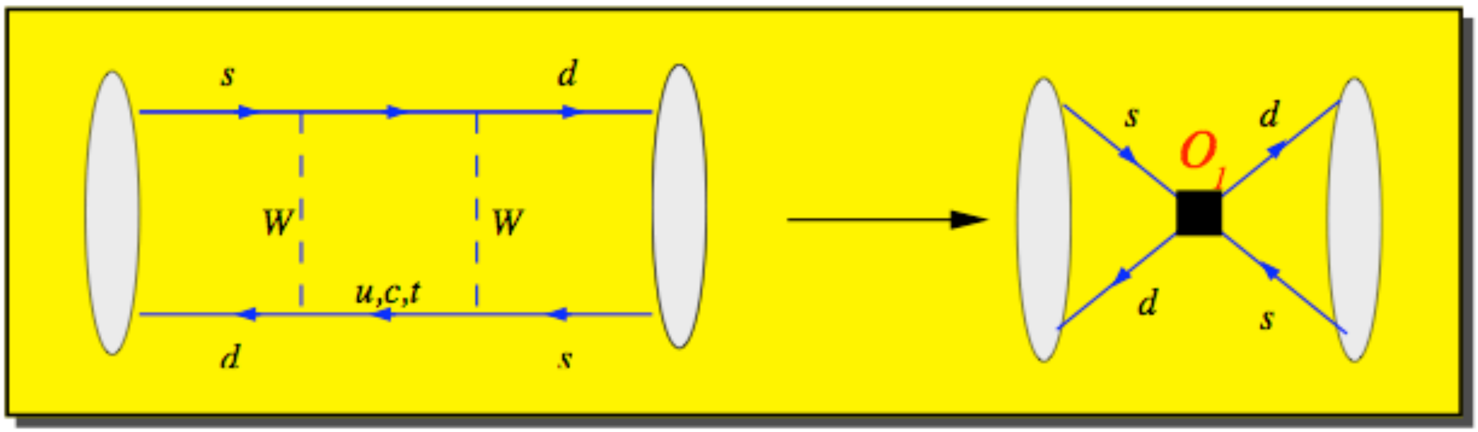
- *Plus one momentum injection to e.g. pion* $\to$ *to* $F(0)$
- *Mass dependence (ChPT, QChPT and PQChPT formulas available)*
- *Results:*
  - $F(0) = 0.960(5)(7)$ *SPQcdR (2004)*
  - $F(0) = 0.962(6)(9)$ *Fermilab (2005)*
  - $F(0) = 0.962(6)$ *JLQCD (2005)*
  - $F(0) = 0.955(12)$ *RBC (2006)*



- tension between χPT and lattice?
  $\rightarrow$ need to reduce error bars
- seems independent of N_f
- in many cases Leutwyler & Roos (1984!) is still used to determine $|V_{us}|$ (cf. PDG)

On $K^0 - \bar{K}^0$ mixing

$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C(\mu) \cdot \langle \bar{K}^0 | \overbrace{\bar{s}\gamma_\mu(1-\gamma_5)d \; \bar{s}\gamma_\mu(1-\gamma_5)d}^{O_1(\mu)} | K^0 \rangle$$



- $C(\mu)$ info on SD dynamics: perturbation theory
- low energy QCD dynamics

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

- AD to 2-loops in $\overline{\text{MS}}$, RI/MOM and SF schemes
- NPR essential but difficult with Wilson quarks (with staggered NPR impossible while perturb.corrs. huge $\rightarrow$ HPQCD-2006 : $B_K = 0.62(2)(4)(13)$)

B_K with Wilson quarks - bad⟶better

:-(Wilson term explicitely breaks chiral symmetry: CP⊗S ($s \leftrightarrow d$) symmetry, but no chirality → additive renormalisation. Mixing with 4 other parity even operators ...spent years computing Z_{ij}'s and Δ_{ij}'s in RI/MOM)

$$\begin{aligned} O_1 &= V \times V + A \times A \\ O_2 &= V \times V - A \times A \\ O_3 &= S \times S - P \times P \\ O_4 &= S \times S + P \times P \\ O_5 &= T \times T \end{aligned}$$

$$O_1(\mu) = Z_1(a\mu) \left\{ O_1(a) + \sum_{k=2}^{5} \Delta_k(a) O_k(a) \right\} \qquad (\partial\Delta/\partial\mu = 0)$$

- spurious mixing in PV sector absent
- Ward Identity on the m.e.of the PV operator leads to the m.e. of the PC one. Rotation around the 3^{rd} axis in the isospace:

$$\begin{aligned} \delta u &= \gamma_5 u & \delta d &= -\gamma_5 d \\ \delta \bar{u} &= \bar{u}\gamma_5 & \delta \bar{d} &= -\bar{d}\gamma_5 \end{aligned} \qquad (m_u = m_d \equiv m)$$

$$2m\langle \sum_{\vec{x},\vec{y},\vec{z},t_z} \Pi(\vec{z},t_z)\ P(\vec{x},t_x)\ O_{VA+AV}(0)\ P(\vec{y},t_y)\rangle = 2\langle \sum_{\vec{x},\vec{y}} P(\vec{x},t_x)\ O_{VV+AA}(0)\ P(\vec{y},t_y)\rangle$$

Why tmQCD?

Consider the continuum action of a doublet of massless quarks

$$S_{\rm f} = \int \mathrm{d}^4x \; \bar{\psi}(x)\partial_\mu\gamma_\mu\psi(x)$$

The massless action is symmetric under chiral transformations

$$\begin{aligned}\psi \to \psi' &= \exp(i\omega_{\rm A}^a\gamma_5\tau^a/2)\psi \\ \bar{\psi} \to \bar{\psi}' &= \bar{\psi}\exp(i\omega_{\rm A}^a\gamma_5\tau^a/2)\end{aligned}$$

When introducing a quark mass term the choices $\bar{\psi}\psi$ or

$$\bar{\psi}'\psi' = \bar{\psi}\exp(i\omega_{\rm A}^a\gamma_5\tau^a)\psi = \cos(\omega_{\rm A})\;\bar{\psi}\psi + i\sin(\omega_{\rm A})u_{\rm A}^a\;\bar{\psi}\gamma_5\tau^a\psi$$

are equivalent!

($\omega_{\rm A}$ denotes the module of $(\omega_{\rm A}^1, \omega_{\rm A}^2, \omega_{\rm A}^3)$ and $u_{\rm A}^a = \omega_{\rm A}^a/\omega_{\rm A}$ is a unit vector)

Why tmQCD?

- The choice of a mass term $\bar{\psi}\psi$ is a mere convention; in general one may pick any other direction in chiral flavour space

- The form of symmetry transformations depends on this choice:
  - by definition, the flavour (isospin) symmetry leaves the mass term invariant:

$$\psi \rightarrow \exp(-i\omega_{\mathrm{A}}^{a}\gamma_5\tau^{a}/2)\exp(i\omega_{\mathrm{V}}^{b}\tau^{b}/2)\exp(i\omega_{\mathrm{A}}^{c}\gamma_5\tau^{c}/2)\,\psi$$
$$\bar{\psi} \rightarrow \bar{\psi}\,\exp(i\omega_{\mathrm{A}}^{a}\gamma_5\tau^{a}/2)\exp(-i\omega_{\mathrm{V}}^{b}\tau^{b}/2)\exp(-i\omega_{\mathrm{A}}^{c}\gamma_5\tau^{c}/2)$$

  - similarly for parity:

$$\psi(x) \rightarrow \gamma_0\exp(i\omega_{\mathrm{A}}^{a}\gamma_5\tau^{a})\,\psi(x_0,-\mathbf{x}), \qquad \bar{\psi}(x) \rightarrow \bar{\psi}(x_0,-\mathbf{x})\,\exp(i\omega_{\mathrm{A}}^{a}\gamma_5\tau^{a})\gamma_0$$

Twisted Mass Lattice QCD

Lattice action for a doublet ψ of mass degenerate light Wilson quarks quarks (Aoki '84)

$$S_f = a^4 \sum_x \bar{\psi}(x) \left(D_{\rm W} + m_0 + i\mu_q \gamma_5 \tau^3 \right) \psi(x)$$

$D_{\rm W}$: Wilson-Dirac operator with/without Sheikholeslami-Wohlert (clover) term

$\mu_{\rm q}$: bare twisted mass parameter

Properties:

- regularisation of QCD with $N_{\rm f} = 2$ mass degenerate quark flavours (see below)

- $\mu_{\rm q} \neq 0 \Rightarrow$ no unphysical zero modes:

$$
\begin{aligned}
&\det\Big(D_{\rm W} + m_0 + i\mu_q\gamma_5\tau^3\Big) \\
&= \det\begin{pmatrix} \gamma_5(D_{\rm W}+m_0)+i\mu_q & 0 \\ 0 & \gamma_5(D_{\rm W}+m_0)-i\mu_q \end{pmatrix} \\
&= \det\left([D_{\rm W}+m_0)]^\dagger[D_{\rm W}+m_0]+\mu_{\rm q}^2\right) > 0
\end{aligned}
$$

- positive and selfadjoint transfer matrix provided μ_q is real and $|\kappa| < 1/6$ $(\kappa = (2am_0+8)^{-1}) \Rightarrow$ unitarity

- flavour symmetry reduced to U(1) with generator $\tau^3/2$

- symmetries: C, axis permutations, reflections with flavour exchange, e.g.

$$
\psi(x) \to \gamma_0\tau^1\psi(x_0,-\mathbf{x}), \qquad \bar\psi(x) \to \bar\psi(x_0,-\mathbf{x})\gamma_0\tau^1
$$

Equivalence between tmQCD and QCD

Classical continuum limit of twisted mass lattice QCD:

$$S_f = \int \mathrm{d}x \; \bar{\psi}(x)\big(\not{D} + m + i\mu_{\mathrm{q}}\gamma_5\tau^3\big)\psi(x).$$

Perform a global chiral (non-singlet) rotation of the fields:

$$\psi' = R(\alpha)\psi, \quad \bar{\psi}' = \bar{\psi}R(\alpha), \quad R(\alpha) = \exp\left(i\alpha\gamma_5\frac{\tau^3}{2}\right).$$

For $\tan\alpha = \mu_{\mathrm{q}}/m$ the action reads:

$$\begin{aligned} S_f' &= \int \mathrm{d}x \; \bar{\psi}'(x)(\not{D} + M)\psi'(x), \qquad M = \sqrt{m^2 + \mu_q^2} \\ \bar{\psi}'\psi' &= \bar{\psi}\exp(i\alpha\gamma_5\tau^3)\psi = \cos(\alpha)\bar{\psi}\psi + i\sin(\alpha)\bar{\psi}\gamma_5\tau^3\psi \end{aligned}$$

corresponds to $\omega_{\mathrm{A}}^a = \alpha\delta^{3a}$ in the previous discussion.

This is where tmQCD becomes useful (ETMC project)

$$S = \sum_x \left[\bar\psi(x)\,(D_W + m_q + i\mu_q\gamma_5\tau_3)\,\psi(x) + \bar s\,(D_W + m_s)\,s\right], \qquad \psi = (u\ d)^T \qquad m_u = m_d$$

is invariant under: $\psi \to \exp(i\alpha\gamma_5\tau_3/2)\psi$, $\bar\psi \to \bar\psi\exp(i\alpha\gamma_5\tau_3/2)$ for $\tan\alpha = \mu_q/m_q$

- Axial rotation of O_{VV+AA} leads to

$$\langle \bar K^0 | O_{VA+AV} | K^0 \rangle^{\alpha=\pi/2}_{\text{tmQCD}} = i \langle \bar K^0 | O_{VV+AA} | K^0 \rangle^{\alpha=0}_{\text{tmQCD}} \equiv i \langle \bar K^0 | O_1 | K^0 \rangle_{\text{QCD}}$$

- Important advantage of tmQCD : no exceptional configurations

$$\det[D_W + m_q + i\mu_q\gamma_5\tau_3] = \det[(D_W + m_q)(D_W + m_q)^\dagger + \mu_q^2] > 0$$

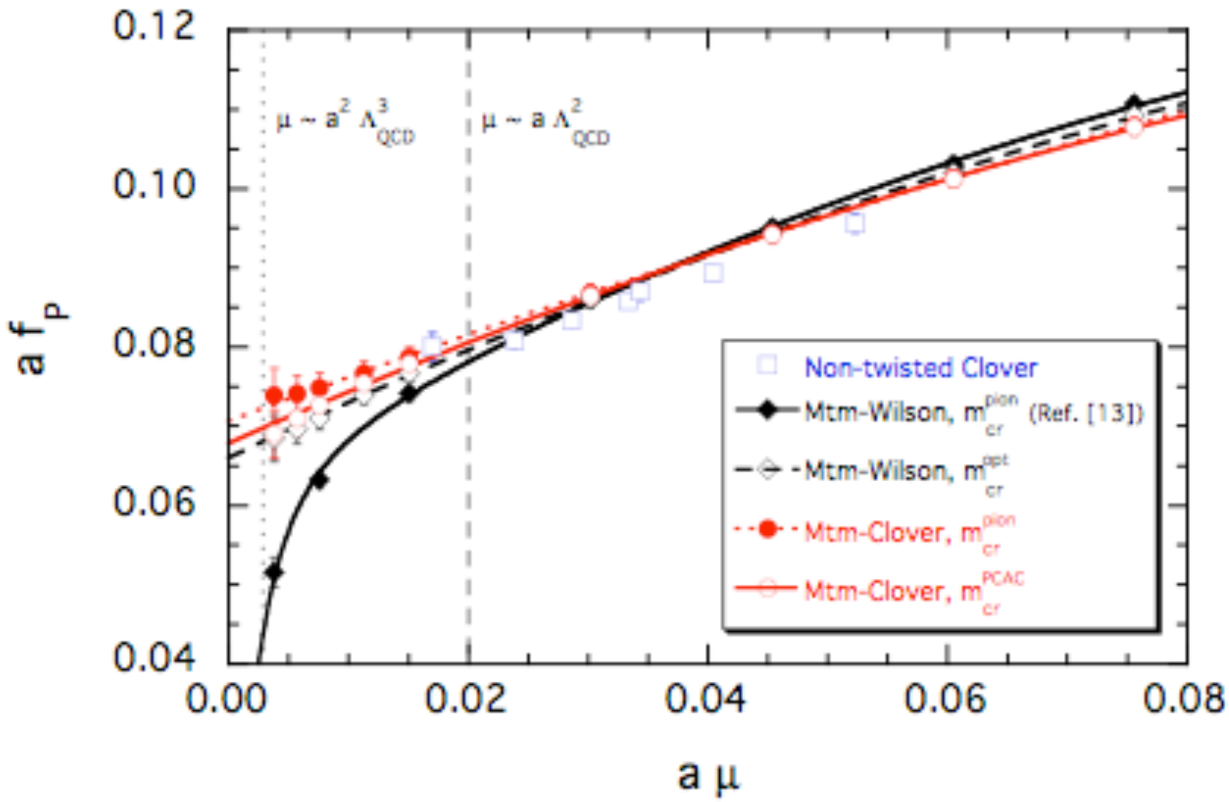
$\Rightarrow$ getting closer to the chiral limit.

- This year Alpha published results from "multi-a" quenched study with NPR (SF):
  (a) consistent with Overlap results $[B_K^{\overline{\text{MS}}/NDR}(2\text{GeV}) = 0.58(3)]$
  (b) raw results consistent with SPQcdR but better accuracy!
  (c) way to go unquenched!

Two comments though...

Small quark mass region might be dangerous due to LtmQCD peculiarity!

Clover term might save the day...



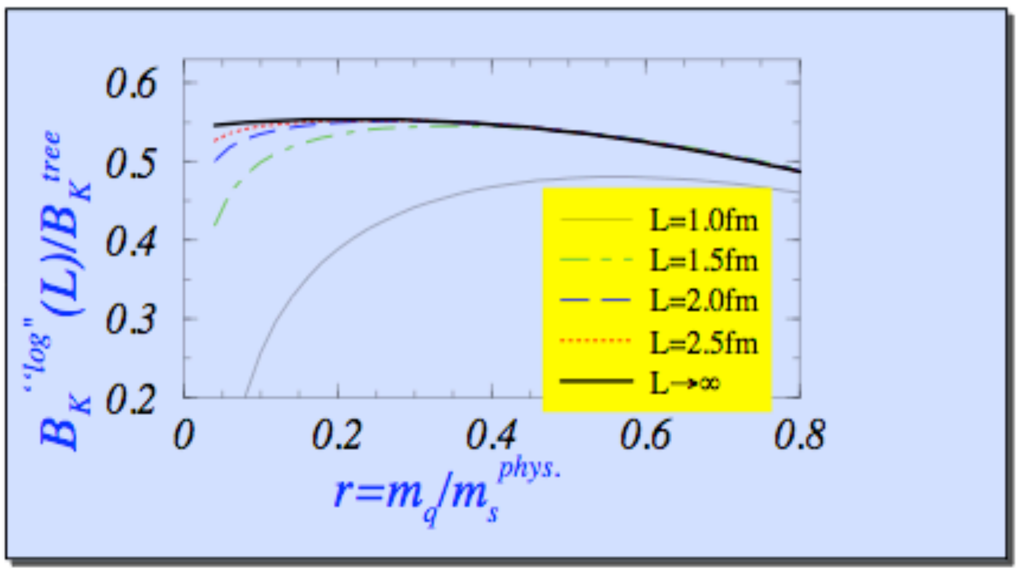
and for every action... (ChPT in finite volume)

Recently derived standard ChPT expressions in the finite and infinite volume

$$\frac{B_K(\infty)}{B_K^{\rm tree}}^{\rm log} = -\frac{1}{2(4\pi f)^2}\left[\frac{m_\pi^2(m_K^2+m_\pi^2)}{m_K^2}\log m_\pi^2 + 4m_K^2\log m_K^2 + \frac{m_\eta^2(7m_K^2-m_\pi^2)}{m_K^2}\log m_\eta^2\right]$$

$$\frac{B_K(L)}{B_K^{\rm tree}}^{\text{"log"}} = -\frac{1}{4f^2L^3}\sum_{\vec q}\left[\frac{m_K^2+m_\pi^2}{m_K^2\omega_\pi} - \frac{2m_K^2}{\omega_K^3} + \frac{7m_K^2-m_\pi^2}{m_K^2\omega_\eta}\right]$$

$\omega_P^2 = m_P^2 + \vec q^{\,2}$ $(P=\pi,K,\eta)$; $\vec q = \frac{2\pi}{L}\vec n$ $(\vec n \in Z^3)$, I set $\Lambda_\chi = 1\ \mathrm{GeV}$

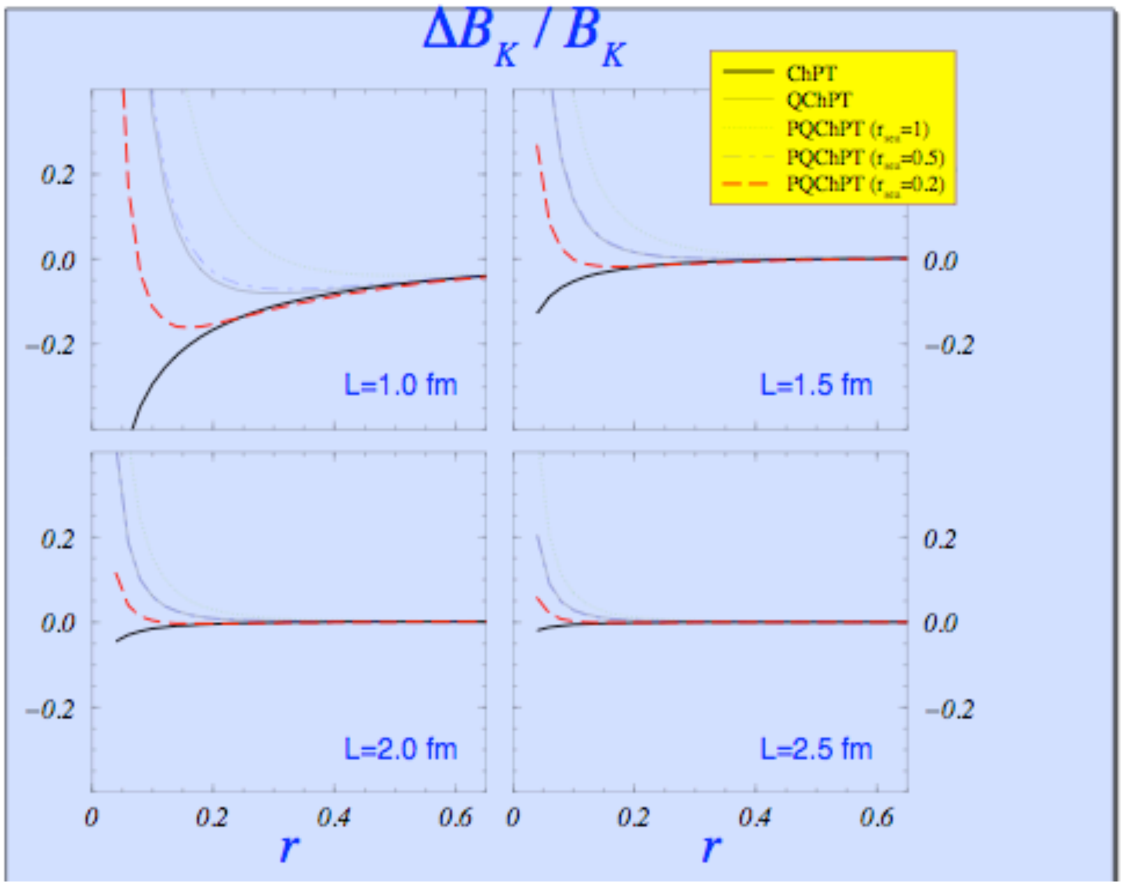


- ♣ Deviation from linearity is NOT evidence for the presence of chiral logs.
- ♣ Smaller the mass, the volume artefacts are more pronounced (fake logs & LEC's)
- ♣ Large physical volumes necessary for this expansion to apply
- ♠ These effects need to be isolated

⇒ FV corrections

$$\frac{B_K(\infty)}{B_K^{\rm tree}} = 1 + \log_\infty + C m_q, \quad \frac{B_K(L)}{B_K^{\rm tree}} = 1 + \log_L + C m_q$$

$$\Rightarrow B_K(L) = B_K(\infty)\,(1 - \log_\infty + \log_L) \quad i.e.\ \frac{\Delta B_K}{B_K(\infty)} = \log_L - \log_\infty$$



♣ *Similar formulas for m_π, f_π, f_K agree with general Lüscher formula and provide further exp. corrections!*

♠ *Assymptotics:*

$$\frac{\Delta B_K}{B_K} \simeq -\frac{3}{2}\frac{m_K^2 + m_\pi^2}{m_K^2} \times \left(\frac{m_\pi}{f}\right)^2 \frac{e_{m_\pi L}}{(2\pi m_\pi L)^{3/2}}$$

Lattice 2007, JLQCD : Unquenched overlap (Nf=2)

Renormalisation multiplicative & non-perturbatively!

B_K

Kaon bag parameter

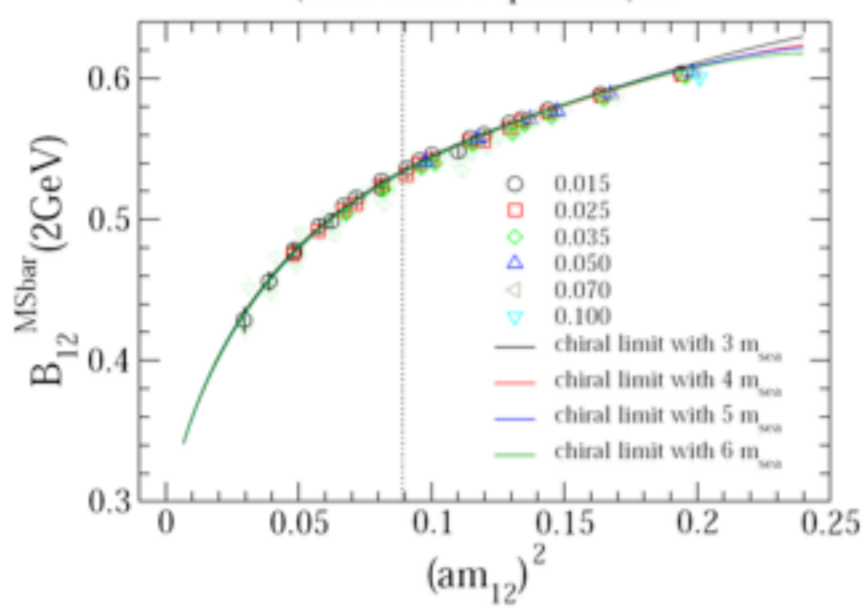
Talk by N.Yamada (Thu,pm)

- Nonperturbative renorm. with RI-MOM scheme
- NLO of PQChPT

Nf=2, a=0.12fm, preliminary result:

$$B_K^{\overline{MS}}(2\,\text{GeV}) = 0.533\,(7)_{\text{stat}}$$

(NLO ChPT + quadratic) fit



Computing platforms in Lattice QCD

- Commercial supercomputers:

  BlueGene/L, SGI Altix, IBM-p690, Hitachi SR8000, NEC Sx6, Fujitsu VPP700,. . .

- Custom made machines:

CP-PACS	$\sim$ 1 TFlop/s	1996	Tsukuba/Hitachi
QCDOC	$\sim$ 10 TFlop/s	2004	CU/UKQCD/Riken/IBM
apeNEXT	$\sim$ 10 TFlop/s	2005	INFN/DESY/Paris-Sud

- PC clusters + fast network:
  - Mass-produced components $\longrightarrow$ cheap
  - Standard software + programming environment

Custom made machines I: apeNEXT

- Developed by INFN/DESY/Paris Sud
- Custom-designed processor

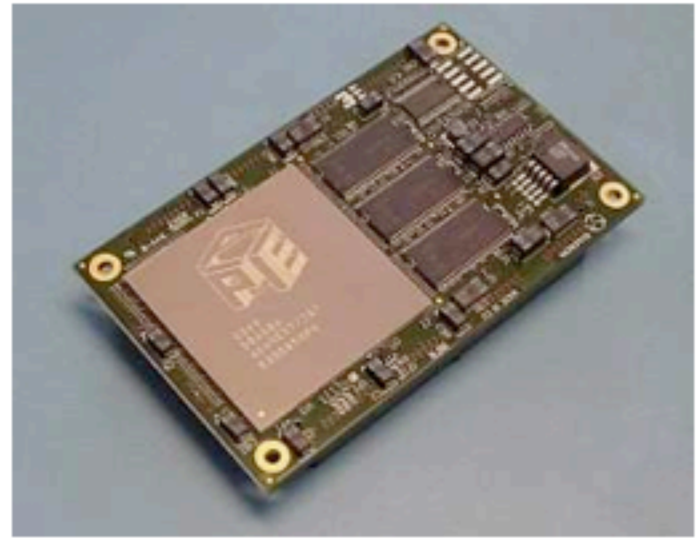
  8 Flops per cycle

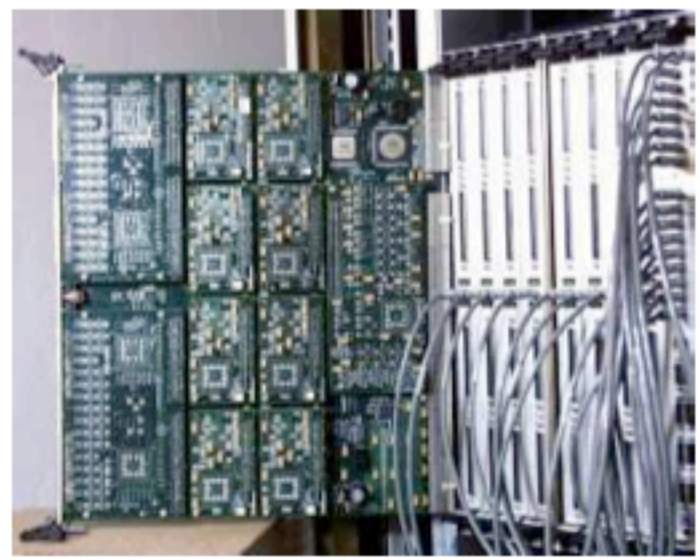
  160 MHz $\Rightarrow$ 1.3 GFlops/s (peak)
- $\approx 40 - 50\%$ efficiency for QCD code
- Installations:

  1 rack = 512 nodes = 0.66 TFlops/s (peak)

INFN	12 racks
Bielefeld	6 racks
DESY	3 racks
Orsay	1 rack

- 0.6 €/MFlops/s (peak)





Custom made machines II: QCDOC

- Developed by Columbia/UKQCD/Riken/IBM
- IBM PowerPC 440 core + 64-bit FPU

  2 Flops per cycle, 400 MHz $\Rightarrow$ 0.8 GFlops/s (peak)
- $\approx 40 - 50\%$ efficiency for QCD code

  (assembly code generator)
- Installations:

  1 rack = 1024 nodes = 0.82 TFlops/s (peak)

Edinburgh	14 racks
DOE	14 racks
Riken/BNL	13 racks
Columbia	2.4 racks
Regensburg	0.5 racks

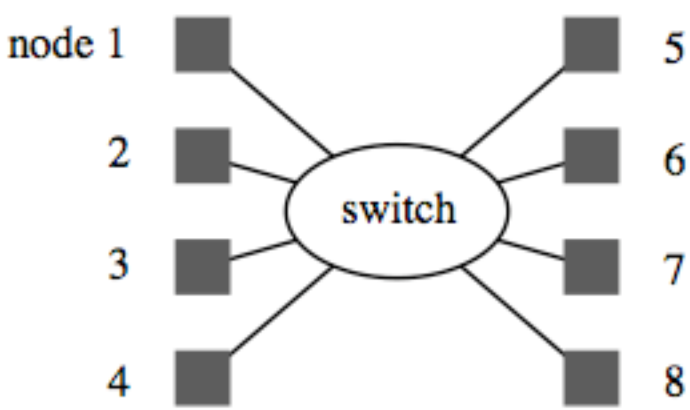
- 0.5 \$/MFlops/s (peak)

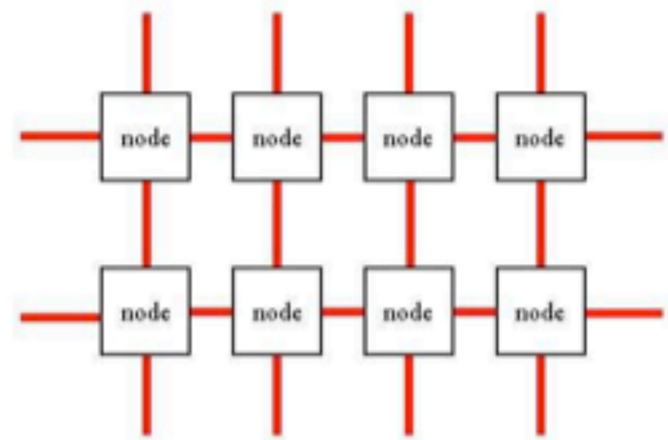
QCDOC installation at Edinburgh

PC clusters

- **CPU:** Intel P4 XEON, AMD Opteron, DualCore
- **Node:** 1 – 2 CPUs, Rambus or DDR memory, local disks,. . .
- **Network:** Myrinet2000 (4 – 8 Gbit/s), Infiniband (10 – 20 Gbit/s) + switch

  GigE (2 Gbit/s) + *"mesh"*





- Typically larger latencies, smaller bandwidths

  ⇒ scalability not as good as for custom made machines

Large installations (Lattice QCD only)

Location	Procs.	Network	[TFlops] (peak)
Wuppertal	1024 Opteron	GigE (2d)	3.7
JLab	384 Xeon	GigE (5d)	2.2
JLab	256 Xeon	GigE (3d)	1.4
FNAL	520 P4	Infiniband	3.4
FNAL	256 Xeon	Myrinet	1.2