Primordial non-gaussianities or relativistic effects in Large Scale Structures?

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Based on Castiblanco 1811.05452, Calles 1912.13034
Collaborators: J. Adamek, J. Calles, L. Castiblanco, R. Gannouji, T. Montandon, J. Noreña.

Candidate at MdC 4309, Résumé

- Currently postdoc @APC (Paris)
- 2017-2019 postdoc @Valparaiso (Chile)
- 2013-2017 PhD @Roma (Sapienza)

Main teaching activities

- 2019 Differential geometry for GR (20h, Master)
- 2015 Cosmology (12h, PhD)

Research

- inflation: presence of electric field, application to magnetogenesis.
- models of universes: inhomogeneous cosmologies, interacting dark energy, repulsive baryon cosmology.
- large scales structures: relativistic effects and non-gaussianities (today).

Candidate at MdC 4309, Vision

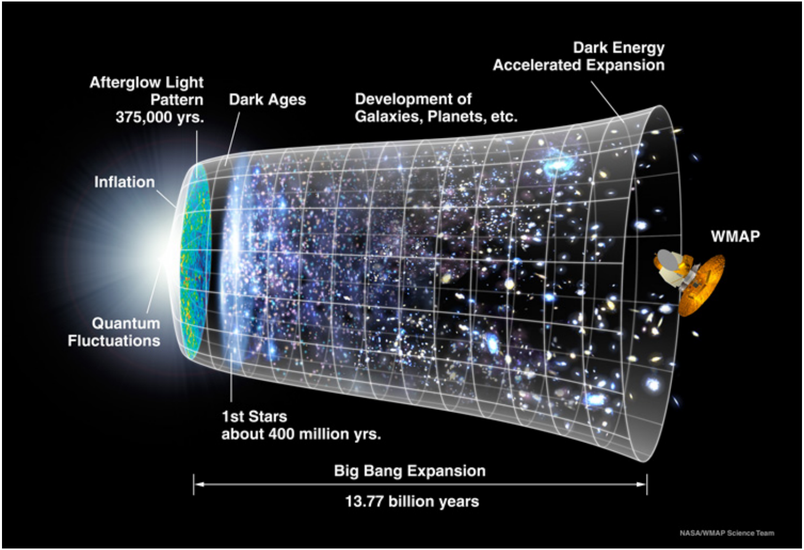
Teaching prospects

- good knowledge of GR for cosmology, make links with students from academics exercises to real world research.
- models of universe, in particular repulsive baryon. Nice exercice to understand the cosmological model. Supernovae: Even possible at high school level!
- Teach though computing science and statistics. Tutorials for CLASS, N-body simulations *eg.* `RAMSES`).

Research prospects

- Nature of dark matter on galactic scales with N-body simulations. MOND like-behavior Chardin 2102.08834. Dipolar dark matter (Blanchet 0901.3114).
- Bring expertise in large scale structures, N-body simulations.
- Explore extra dimensions cosmologies.

Large Scale Structures (LSS) formation



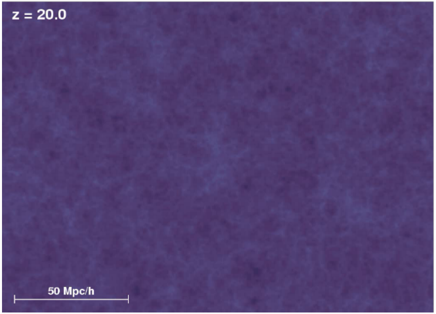
Cosmological structures formation

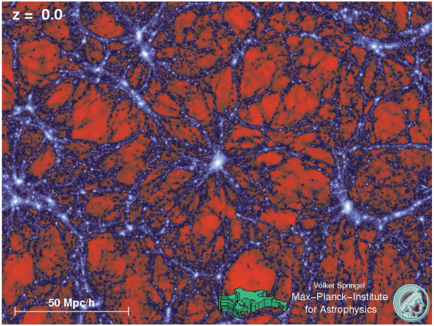
Fluids mechanics in an expanding universe.

Large Scale Structures (LSS) formation

In LSS, split between large scales *background* (expanding universe, well defined mean density) and intermediate scales *perturbations* (density differs little from background).

Cosmic structures grow out of tiny initial fluctuations.





Newtonian structure formation

- Study of LSS on scales smaller than the Hubble scale (3000 h^{-1} Mpc).
- typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$.
- Linear fluids mechanics in an expanding universe: success story (cf. CMB).

Epic Battle: Newton vs Einstein

For CDM (non-relativistic matter):

- On background level (FLRW): Newton and Einstein agree.
- For linear (scalar) perturbations: Newton and Einstein agree.
- In the non-linear regime: small scales: *Newton and Einstein agree*.

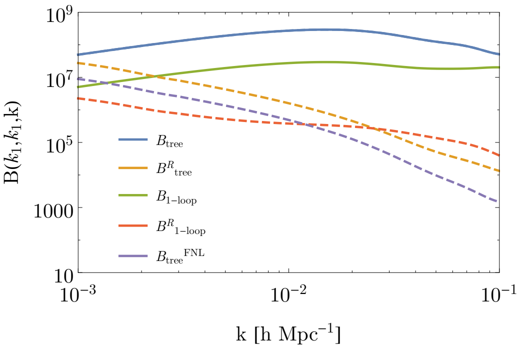
A case for Einstein

Relativistic structure formation

- Relativistic matter content of the universe (neutrinos, cosmic strings, DDE).
- Gravity has 6 degrees of freedom (2 scalars, 2 vectors and 2 tensors)
- Backreaction: how non-linear evolution impacts means quantities.
- Observations are made on the relativistic perturbed light cone.

I argue (Castiblanco 1811.05452)

The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.



Bispectrum: generalities

I argue (Castiblanco 1811.05452)

The bispectrum in the squeezed limit at 1-loop receives relativistic corrections due to the dynamics of the CDM field of the same order than Newtonian results.

Power spectrum vs Bispectrum

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2) P(k_1, t)\,, \tag{1}$$

$$\langle \delta(\boldsymbol{k}_1, t)\delta(\boldsymbol{k}_2, t)\delta(\boldsymbol{k}_3, t)\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3, t)\,. \tag{2}$$

Note that the bispectrum couples the scales !!

Constraints

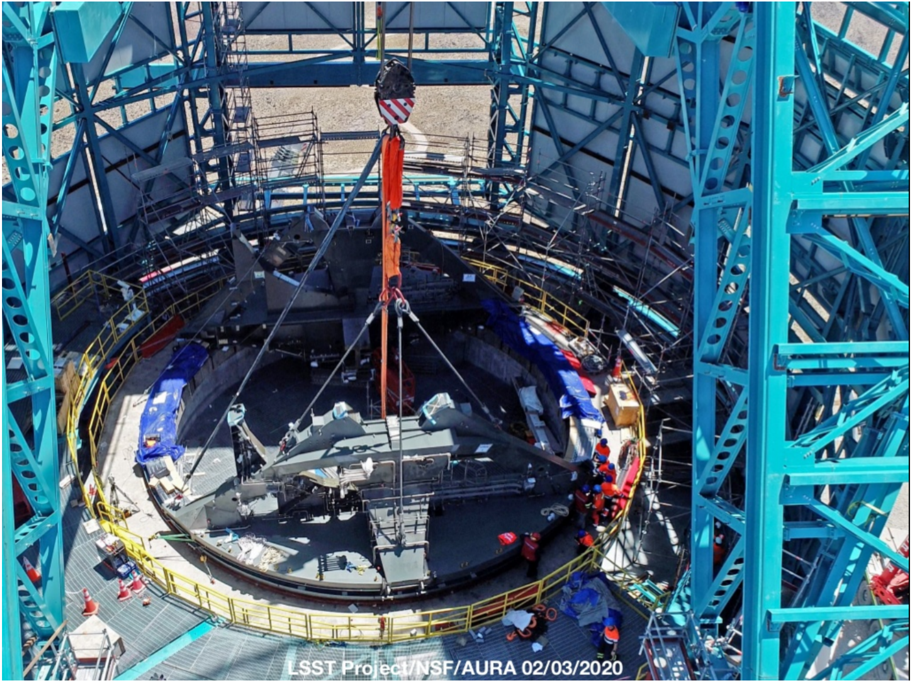
$f_{\mathrm{NL}} = 37 \pm 20$ (WMAP 1212.5225),
$f_{\mathrm{NL}} = -0.9 \pm 5.1$ (Planck 1905.05697). Compatible with zero at 2σ.
Could LSS improve those constraints?

SPOILER ALERT: yes... Will reach $\sigma(f_{\mathrm{NL}}) = \mathcal{O}(1)$

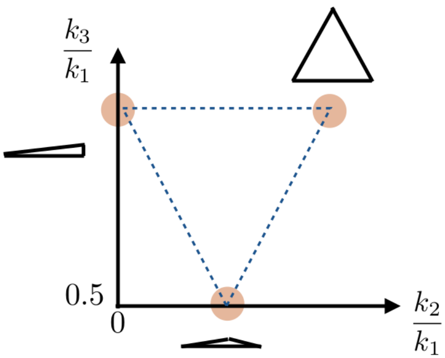
LSS: $N^{\mathrm{LSS}}_{\mathrm{modes}} \sim V k^3_{max} \sim 10^{10}$: $V = (10^4 \mathrm{Mpc}/h)^3$; $k_{max} = 0.5h.\mathrm{Mpc}^{-1}$.
CMB: $N^{\mathrm{CMB}}_{\mathrm{modes}} \sim S k^2_{max} \sim 10^7$.

In the future, data will be amazing but theory and analysis can be improved.


LSST — now NSF Vera C. Rubin Observatory
- Cerro Pachón, Chile (2,663 m / 8,737 ft)
- 8.4m / 27-ft mirror
- Cover entire southern sky every few nights
- 10 year survey over 18,000 deg2
- 37 billion stars and galaxies
- First light 2021, full operations 2022-2032
LSST Project/NSF/AURA 01/31/2020


LSST Project/NSF/AURA 02/03/2020

Bispectrum: generalities



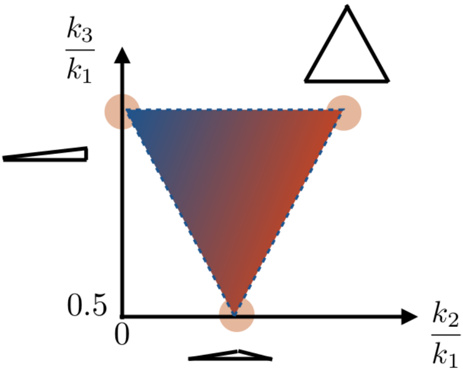


Image credit: J. Noreña

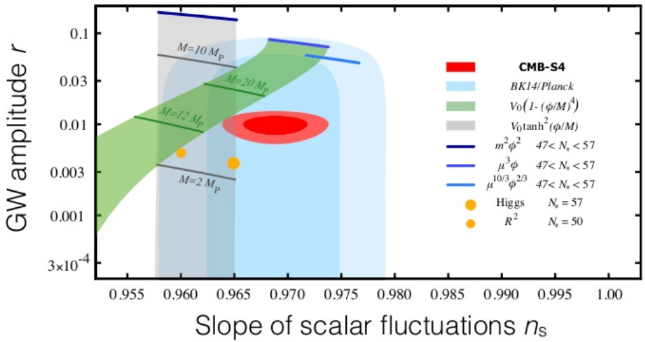
The red zone is degenerated with non-linear growth, biasing and astrophysics.

The blue zone, the the squeezed limit is believed to be much more solid.

Bispectrum for Fundamental physics: Inflation

The squeezed limit contains model independent information about the physics during inflation.

Energy scale at which inflation occurs is unknown and can range across 10 orders of magnitude. Quantum fluctuations imprint into the *full* gravitational fields of the universe $\rightarrow$ Production of gravitational waves! Potential observation for highest energy model of inflation ($>10^{16}$ Gev) through interaction with polarization of CMB photons (B-modes).



CMS-S4 Science book 1610.02743

$$\left(\frac{r}{0.01}\right) \simeq \frac{V^{1/4}}{10^{16}\text{Gev}}$$

Bispectrum for Fundamental physics

Models with energy scale below 10^{16} Gev have no observable primordial gravitational waves. Class these models using **primordial non-gaussianities**: complements GW searches (Meerburg 1903.04409).

Theorem: (Consistency relations), Maldacena 0210603

If only one light scalar field is active during inflation, the behavior of the three-point correlation function, in the squeezed limit, is entirely fixed by the two-point correlation function.

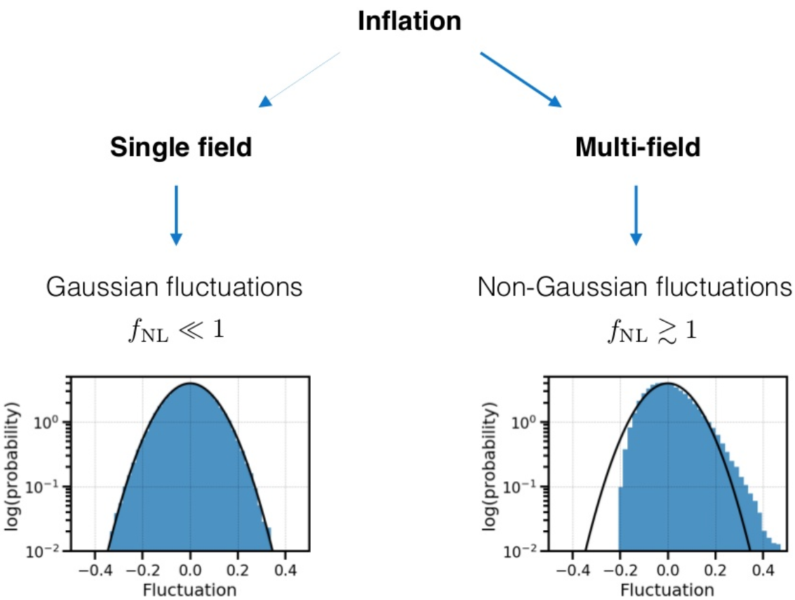
Single field predicts $f_{\mathrm{NL}} \simeq \frac{5}{12}(1 - n_S) \simeq 0.02$. A detection of $f_{\mathrm{NL}} \gg 0.02$ rules out all single inflation.

Way out of the theorem:

- Several fields active during inflation Sugiyama 1101.3636
- higher spin Arkani-Hamed 1503.08043
- 'modified' gravity Tahara 1805.00186
- anisotropic inflation Emaml 1511.01683
- electromagnetic field Chua 1810.09815 Stahl 1507.01686

These theorems also apply to the late universe (Creminelli 1309.3557)
$\rightarrow$ probe the early universe with LSS observables.

Conclusions



Slide from M. Schmittfull.

Conclusions

Motivations

- While most of LSS do not need relativity, the bispectrum couples scales, its non-linear evolution has to be calculated within GR.
- The bispectrum in the squeezed limit is 'protected' from astrophysical effects (equivalence principle)
- In LSS, the bispectrum can be used to probe early universe physics.
- The next generation of LSS experiments should be able to measure $f_{\mathrm{NL}} = \mathcal{O}(1)$.

General Relativity: diffeomorphism invariance

Perturbations around a FLRW universe

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2 \left[(1-2\psi)\delta_{ij} + h_{ij}\right] dx^i dx^j . \qquad (3)$$

Poisson gauge

- $\delta^{ij}\omega_{i,j} = \delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0.$
- Velocity of the fluid:

$$u^\mu = \left(1 - \phi + \frac{a^2 v^2}{2}, v^i\right).$$

- Physical interpretation simple.
- Gauge used for relativistic N-body simulations `gevolution` (Adamek 1604.06065).

Synchronous-Comoving gauge

- $\delta^{ij}h_{ij} = \delta^{jk}h_{ij,k} = 0$ and $u^0 = 1.$
- Velocity of the fluid:

$$u^\mu = \left(1, -\frac{(1+2\psi)\partial_i\omega + w_i}{a^2(t)}\right),$$

where $\omega_i \equiv \partial_i\omega + w_i$.

- Gauge relevant when it comes for observation: use the time measured by a local observer.

Weak Field Approximation

Typically $v \sim 10^{-2}$, $\phi \sim 10^{-5}$, but: $\delta = \frac{2}{3(aH)^2}k^2\phi \sim \frac{0.1\text{Mpc}^{-1}}{10^{-6}\text{Mpc}^{-1}}\phi \sim 1.$
$\rightarrow$ Work perturbatively in v and ϕ but full non-linear in δ.

Equation of motion

Conservation of the energy momentum tensor + Einstein equation

$$\nabla_\mu(\rho u^\mu) = 0, u^\mu \nabla_\mu u^\nu = 0, G_{\mu\nu} = T_{\mu\nu}. \tag{4}$$

Full non-linear equations: Euler + conservation of mass

$$\dot\delta + \theta = -\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \alpha(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \delta(\boldsymbol{k}_2) + \mathcal{S}_\delta[\delta, \theta],$$

$$\dot\theta + 2H\theta + \frac{3H^2}{2}\delta = -2\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}_{12}) \beta(\boldsymbol{k}_1, \boldsymbol{k}_2) \theta(\boldsymbol{k}_1) \theta(\boldsymbol{k}_2) + \mathcal{S}_\theta[\delta, \theta].$$

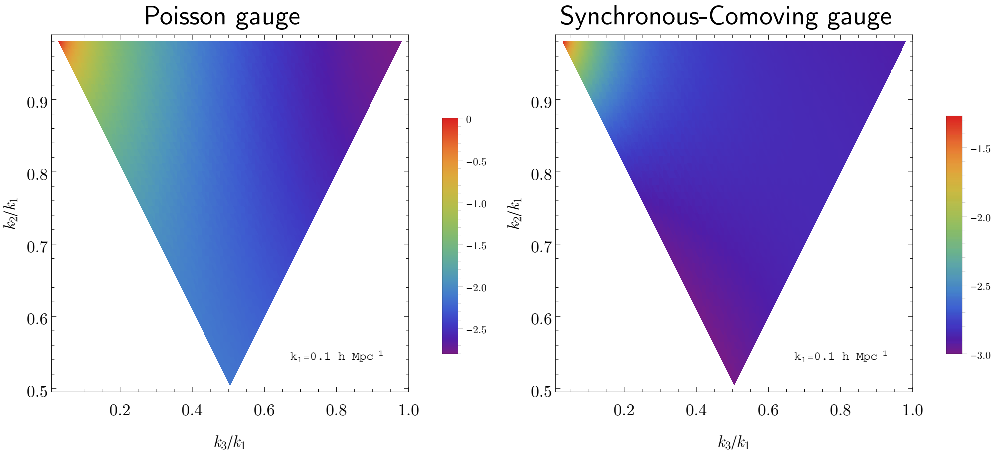
$\theta \equiv \partial_i v^i$. Use G^0_i to include frame dragging effects (ω_i) and G^0_0 for potentials ϕ, ψ. $S_{\delta/\theta}$ are the relativistic corrections: eg. $\sim \dot\delta\delta/k^2$.

Perturbation theory: take $\delta \ll 1$

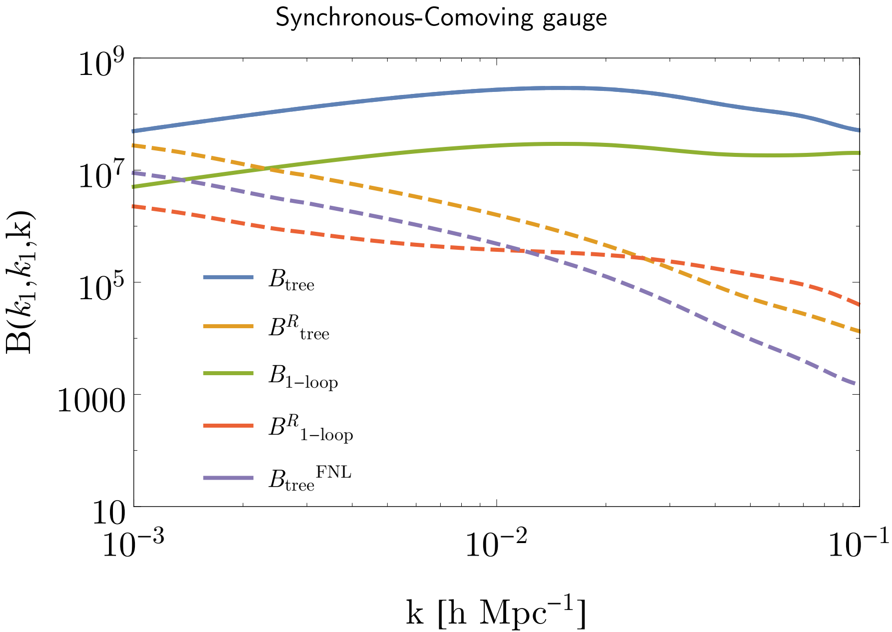
$$\delta(\boldsymbol{k}, t) = \sum_{n=1}^{\infty} a^n(t) \int_{\boldsymbol{k}_1..\boldsymbol{k}_n} \left[F_n(\boldsymbol{k}_1, .., \boldsymbol{k}_n) + a^2(t) H^2(t) F_n^R(\boldsymbol{k}_1, .., \boldsymbol{k}_n) \right] \delta_l(\boldsymbol{k}_1)..\delta_l(\boldsymbol{k}_n) \,.$$

The end !

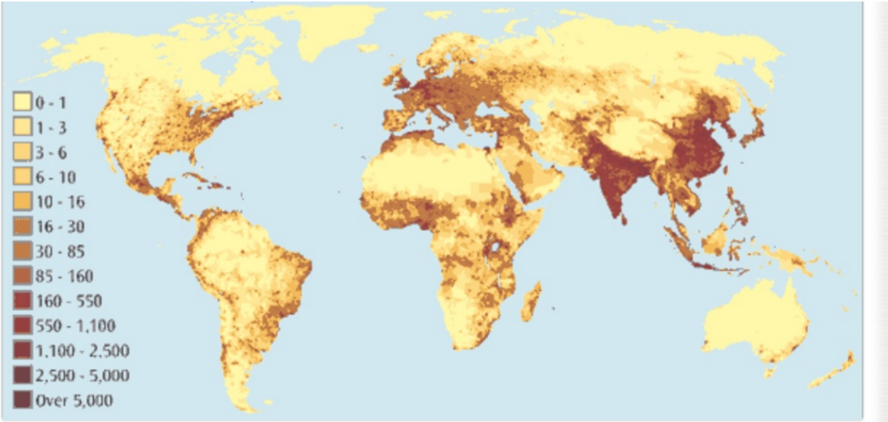
We plot $\frac{B^R(k_1,k_2,k_3)}{B(k_1,k_2,k_3)}$ (1-loop=stopping at $n = 4$ in perturbation theory).



Results


Synchronous-Comoving gauge
10^9
10^7
10^5
1000
10
10^{-3}
10^{-2}
10^{-1}
$B(k_1,k_1,k)$
$k\ [h\ Mpc^{-1}]$
B_{tree}
${B^R}_{tree}$
B_{1-loop}
${B^R}_{1-loop}$
${B_{tree}}^{FNL}$

Human population density



Adopted from Tobias Baldauf

At night

Adopted from Tobias Baldauf

On which geometric quantities the formation of galaxies depends?

Working frame

- Approach à la Effective field Theory: smaller scales are smoothed out and the astrophysical processed are encoded in a handful of *bias* coefficients $b_{\mathcal{O}}$ to be determined (Desjacques 1611.09787).
- Frame of reference of an observer moving with the halo's center of mass ($\rightarrow$ Synchronous-comoving gauge).
- Velocity of dark matter = velocity of halos/galaxies.
- No creation of galaxies.

Build on Umeh 1901.07460, generalized to 4th order:

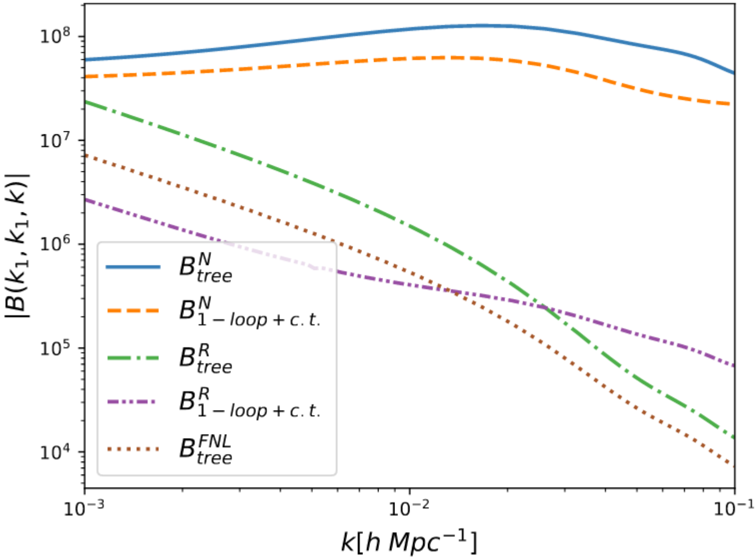
$$\delta_g^{(n)} = a^n \left(F_n^T + \sum b_{\mathcal{O}}^{\mathcal{L}} M_n^{\mathcal{O}} \right) \delta_\ell^n \, , \tag{5}$$

where $F_n^T \equiv F_n + a^2(t) H^2(t) F_n^R$ and $M_n^{\mathcal{O}} \equiv M_n^{\mathcal{O}} + a^2(t) H^2(t) M_n^{\mathcal{O},R}$

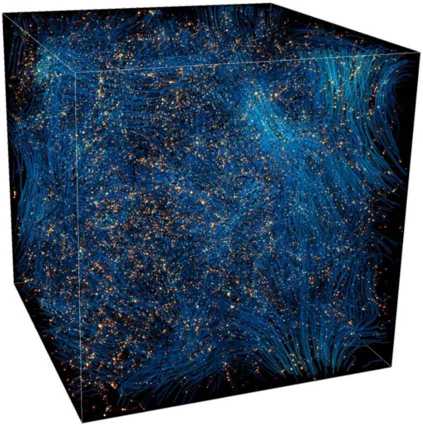
at second order ($n = 2$), we find $\mathcal{O} = \{\delta; \delta^2; s^2\}$ such that: (**Calles** 1912.13034)

$$\delta_g^{(2)} = a^2 \left[\left(1 + \frac{b_1}{a} \right) F_2^T + \frac{1}{2} \left(\frac{b_2}{a^2} - \frac{4}{21} \frac{b_1}{a} \right) + \left(\frac{b_{s^2}}{a^2} - \frac{2}{7} \frac{b_1}{a} \right) s^2 \right] \delta_\ell^2 \, , \tag{6}$$

Results


10^8
10^7
10^6
10^5
10^4
$|B(k_1, k_1, k)|$
B^N_{tree}
$B^N_{1-loop+c.t.}$
B^R_{tree}
$B^R_{1-loop+c.t.}$
B^{FNL}_{tree}
10^{-3}
10^{-2}
10^{-1}
$k[h\ Mpc^{-1}]$

Gevolution: a general relativistic N-body code.



Spin one metric perturbation
Image credit: https://youtu.be/9y6T5CoZgi4

Also based on weak field expansion of general relativity.

For any given $T_{\mu\nu}$ computes the six degree of freedom of GR: ϕ, ψ, ω_i, h_{ij}.

N-body particles ensemble evolved using relativistic geodesic equation

Successfully applied to dark energy, backreaction, ray tracing, neutrinos (not decaying)

Initial conditions are linear

Non-gaussianities and relativistic corrections require non-linear initial conditions.
Michaux 2008.09588 3LTP initial condition, reduce numerical noise, start simulation as late as $z = 12$.

Include our results in gevolution

Working plan

- Use SONG[a] (the second order generalization of CLASS[b]) $\rightarrow$ agrees with my analytic estimates *eg.* Tram 1602.05933
- Generate realizations obeying the new statistics: linear + second order
- Run gevolution
- Measure power spectrum and bispectrum
- Conclude on non-gaussianities

[a] Second Order Non-Gaussianity
[b] the Cosmic Linear Anisotropy Solving System

Generating a non-gaussian field is costly

We want:

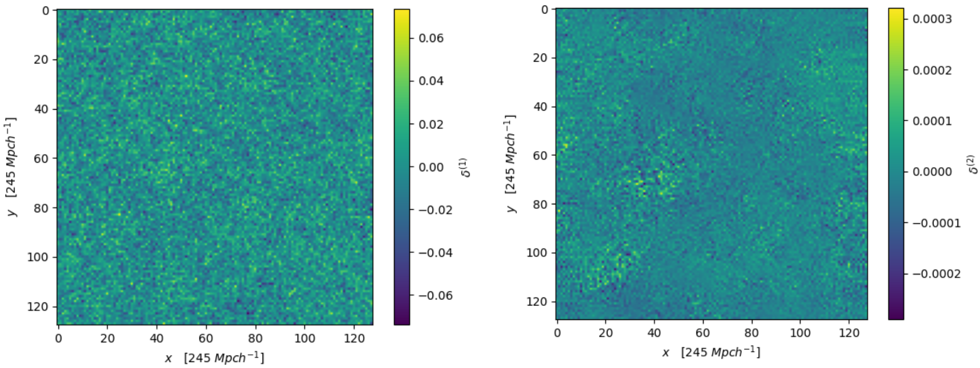
$$\delta^{(2)}(\mathbf{k},t) = \int \frac{d\mathbf{k_1}}{(2\pi)^3} F_2(k_1, |\mathbf{k}-\mathbf{k_1}|, t)\delta^{(1)}(\mathbf{k_1})\delta^{(1)}(\mathbf{k}-\mathbf{k_1}) \tag{7}$$

Need for each **k** to know the value of $\delta^{(1)}(\mathbf{k_1})$ and $\delta^{(1)}(\mathbf{k}-\mathbf{k_1})$. If we work on a grid with N points $\rightarrow$ scales as N^6, a lot!

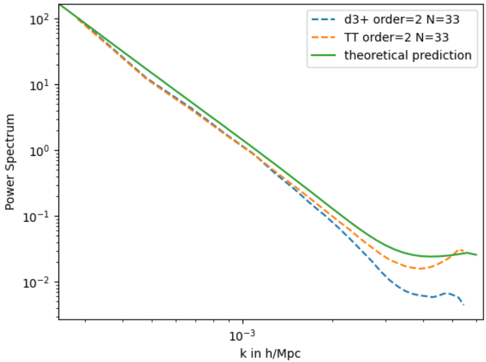
Generate a non-gaussian field

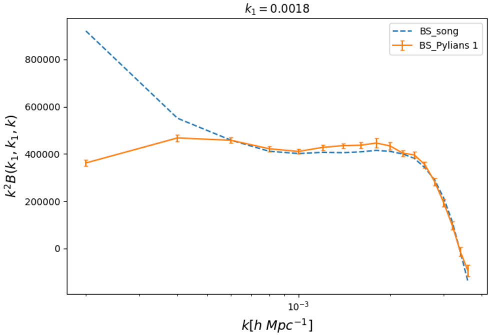
Ignore small scales correlations

Introduce a cutoff scale k_Λ which splits small and large scales: $\delta^{(1)}(\mathbf{k}) = \delta^{(1)}_{\text{small}}(\mathbf{k}) + \delta^{(1)}_{\text{large}}(\mathbf{k})$, plug into (7) and ignore the small $\times$ small contribution. $\rightarrow$ will give the right correlations at larges scales and in the squeezed limit for the bispectrum but cannot be trusted in the general case. Scales as N^3.



Measure of correlation functions in the simulation (preliminary)





Conclusions and next steps

- As the bispectrum couples scales, I calculated the one-loop bispectrum within GR in the weak field approximation for dark matter and galaxies.
- The bispectrum in the squeezed limit is protected from astrophysics and allow to probe early universe physics (primordial non-gaussianities)
- The relativistic contributions are of the same order than a primordial non-gaussianity of the local type.

Cosmological N-body-simulation

- Probe for the first time a bispectrum in a relativistic N-body simulation. Add in the initial conditions a primordial non-gaussian signal and measure it.
- Ray tracing exists within `gevolution`. Include the travel of the photons in a clumpy universe (*cf.* redshift space distortion, finger of god like effects).
- Non-gaussian fields with a modal decomposition (Regan 1108.3813).

Galactic N-body simulations

- Alternative to dark matter and dark energy Chardin 2102.08834.
- Studying dipolar dark matter (Blanchet 0901.3114) with B. Famaey.
- Peeble 2005.07588 suggests to use non-Gaussian initial conditions for galaxies. Recycle my non-Gaussian IC code for galaxies.

Thank you for your attention