Gravitational birefringence of light

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to the memory of Christian Duval, Sep'18
to the memory of Vaughan Jones, Sep'20

4 similarities between optics & general relativity

- lensing in matter with optical gradient

• lensing in tidal forces

Einstein 1919: Dann könnt' mir halt der liebe Gott leid tun.

(Then I would have to pity the dear Lord.)

- electro-magnetic waves

• gravitational waves

Brinkmann 1925, Einstein & Rosen 1937:

exact plane wave solution

- birefringence in matter with optical gradient

  Fedorov 1955, Imbert 1972

• birefringence in tidal forces

2 chapels: Pirani 1957, Tulczyjew 1959

- photons

• gravitons

Birefringence in *isotropic* matter with optical gradient

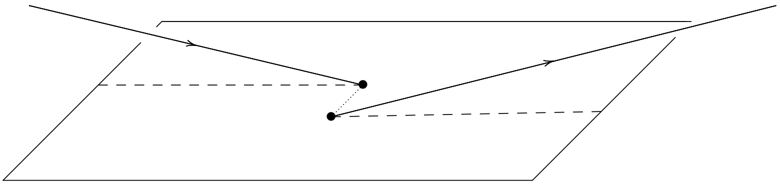

Figure: The Fedorov-Imbert effect for reflection: A plane glass surface reflects an incoming, circularly polarized light beam. The dashed lines show the projections of the beams on the surface. The dotted line (between the blobs) is the offset between incoming and reflected beams. It is of the order of the wave-wength of the light.

Hosten & Kwiat, “Observation of the Spin Hall Effect of Light via weak measurements”, Science **319** (2008) 787.

K. Bliokh, Niv, Kleinert & Hasman, “Geometrodynamics of spinning light”, Nature Photonics **2** (2008) 748.

Birefringence in Schwarzschild's metric (Tulczyjew)

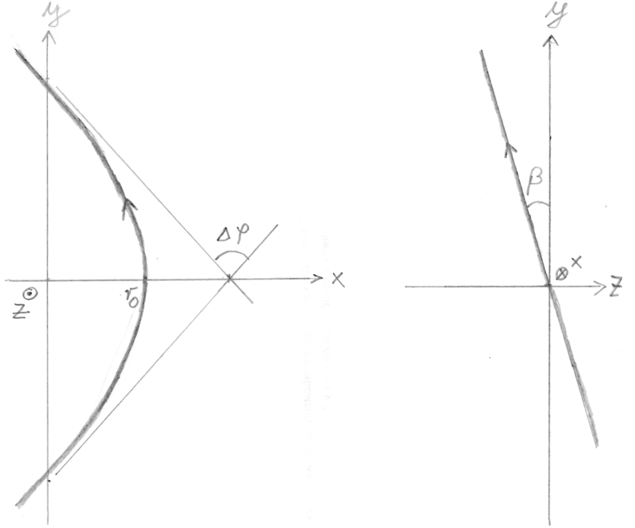

Figure: The trajectory of a photon with positive helicity, $\chi = +1$,
$\Delta\varphi$ = scattering angle, β = angle out of geodesic plane

$r_0 :=$ perihelion $> r_\odot$,
$\lambda_0 :=$ wavelength at perihelion, $\chi :=$ helicity $= \pm 1$.

- Scattering angle of light $\Delta\varphi \sim 4GM_\odot/r_0$ ($\sim$ 1 arc" for $r_0 \sim r_\odot$)
- Angle out of geodesic plane $\beta \sim -\chi\lambda_0/(2\pi\, r_0)$
- β depends on λ_0 ('rainbow effect'), but not on M.

!! The projection of the photon on the geodesic plane moves with the speed of light c.

!! Stable numerical and perturbative solutions exist only if $s_0^\perp = 0$.

Duval, Marsot & Schücker 2018

Upper limit from VLBI of radio-sources ‘close’ to the Sun

Harwit, Lovelace, Dennison, Jauncey & Broderick, Nature **249** 1974

Abstract:

“An upper limit is determined for the difference in the deflection of beams of orthogonally polarised radiation passing through the Sun’s gravitational field. A null result is anticipated by present theories of gravitation but this prediction has never been tested.”

With a baseline of 3900 km and with

$$\lambda_0 = 3.75 \text{ cm}, \quad r_0 = 18.8\, r_\odot,$$

Harwit et al. find an upper limit of

$$|\beta| < 10^{-3} \text{ arc''}.$$

For the same values, general relativity with Tulczyjew’s choice yields

$$\beta \sim \pm\, 3 \cdot 10^{-8} \text{ arc''}.$$

Today

Martin Harwit Feb. 2020:

GRAVITY @ VLT $\lambda_0 = 2 \cdot 10^{-6}$ m, resolution = 10^{-5} arc”,

Event Horizon T $\lambda_0 = 1.5 \cdot 10^{-3}$ m, resolution = $2 \cdot 10^{-5}$ arc”.

Astronomical lenses?

β does not depend on *M*.

Birefringence in a Robertson-Walker metric

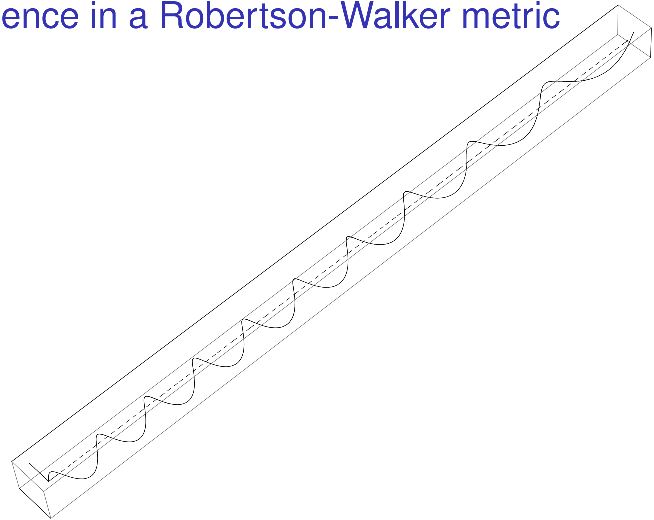

Figure: The trajectory of photons is the helix. The dashed line is the null geodesic. The transverse spin $\vec{s}_e^{\perp}$ at emission time t_e is indicated by the short arrow at the left.

- $R_{\text{helix}}(t) \sim \dfrac{a(t)}{a_e}\,\dfrac{\lambda_e}{2\pi} + O\left(\dfrac{\lambda_e}{2\pi\, a_e}\right)^2,$

- $T_{\text{helix}}(t) \sim \dfrac{a(t)}{a_e}\,\dfrac{\lambda_e}{1+Q},\ Q(t) := \dfrac{-aa''}{a'^2+K},\ K = 0,\, \pm 1.$

- The spin vector $\vec{s}$ rotates with the same period $T_{\text{helix}}(t)$.

!! The projection of the photon on the geodesic moves with the speed of light c $(=1)$. Therefore the speed of the photon on the helix is $\sim \sqrt{2}\,c$.

!! $|\vec{s}^{\perp}|$ is not conserved, but $|\vec{s}^{\perp}|\,\sqrt{a'^2+K}$ is.

An exotic redshift formula in the Lemaître diagram

The photon is a taciturn messenger: it tells us its incoming direction (θ, φ) and its period today T_0. From this period we compute the redshift,

$$z = \frac{T_0 - T_e}{T_e}\,.$$

If we admit that the photon has spin, then it carries one more information: its period of precession $T_{\mathrm{helix}\,0}$ tempting us to try

$$z = \frac{T_{\mathrm{helix}\,0} - T_{\mathrm{helix}\,e}}{T_{\mathrm{helix}\,e}}\,.$$

This assumption leads to an exotic formula for the redshift,

$$z + 1 = \frac{a(t_0)}{a(t_e)}\,\frac{1 + Q(t_e)}{1 + Q(t_0)}\,, \quad Q := \frac{-aa''}{a'^2 + K}\,.$$

Duval, Pasquet, Schücker & Tilquin 2018

Birefringence in linearized gravitational wave

$$d\tau^2 = dt^2 - (1 - \sigma \cos[\omega(t-z)])\, dx^2$$

$$- (1 + \sigma \cos[\omega(t-z)])\, dy^2 - dz^2 \; + \; O(\sigma^2)$$

LIGO/Virgo: $\omega = 2\pi/\lambda_{\text{GW}} \sim 2\pi \cdot 100$ Hz

$\sigma \sim 10^{-20}$ 'strain'

$\lambda_{\text{photon}} = 1\,\mu$m

Marsot 2019:

$$\Delta\tau_{\text{photon}}\Big(1 \qquad + \; \sigma \qquad + \sigma \, \frac{\lambda_{\text{photon}}^2}{\lambda_{\text{GW}}^2}\Big)$$

no GW	GW	GW
	no spin	spin
		$\sim 10^{-45} < \sigma^2$

3 axioms for general relativity

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\Downarrow \qquad \text{if test particle has no spin}$$

$$\not{m}\, \frac{D}{d\tau}\, \frac{d}{d\tau}\, x^\mu =: \not{m}\, \ddot{x}^\mu = 0$$

$$\Delta\tau = \oint \sqrt{g_{\mu\nu}\, \frac{dx^\mu}{dp}\, \frac{dx^\nu}{dp}}\, dp\,, \quad (+\,-\,-\,-)$$

Adding spin to the geodesic equation

Now add spin $S^{[\mu\nu]}(\tau)$.

$$
\begin{aligned}
\dot{x}^\mu &\neq P^\mu && \text{Einstein 1907 – 1915} \\
\dot{P}^\mu &= 0 - \tfrac{1}{2} R^\mu{}_{\rho\alpha\beta} S^{\alpha\beta} \dot{x}^\rho && \text{Mathisson 1937, Papapetrou 1951} \\
\dot{S}^{\mu\nu} &= P^\mu \dot{x}^\nu - P^\nu \dot{x}^\mu && \text{Dixon 1970, Souriau 1974}
\end{aligned}
$$

$\not\Rightarrow$ $P_\mu P^\mu =: m^2$ is conserved.

If $S^{\mu\nu} \doteq 0$ and if $\dot{x}^\mu$ and P^μ do not vanish, the spin evolution implies:

$$\dot{x}^\mu(\tau) = \gamma(\tau)\, P^\mu(\tau)$$

and we can reparametrize τ to achieve $\gamma(\tilde{\tau}) \doteq 1$.
However, if $S^{\mu\nu} \neq 0$ we only have $4 + 6$ equations for $4 + 4 + 6$ unknowns and we must add 4 “supplementary conditions”.

2 main chapels

Pirani 1957: $S^{\mu}{}_{\nu}\dot{x}^{\nu} = 0$

- Massless particles move on geodesics, no birefringence of light.

Tulczyjew 1959: $S^{\mu}{}_{\nu}P^{\nu} = 0$

- $P_{\mu}P^{\mu}$ is conserved unless $P_{\mu}P^{\mu} - \frac{1}{4}R_{\mu\nu\alpha\beta}S^{\mu\nu}S^{\alpha\beta} = 0$,
- $P_{\mu}\dot{x}^{\mu} = 0$,
- $\frac{1}{2}S_{\mu\nu}S^{\mu\nu} =: s^2$ is conserved.

After a $1+3$ split, the "spin scalar" s will become $s = \vec{s}\cdot\vec{p}/p =: \chi\hbar$ and $\vec{s} = s\,\vec{p}/p + \vec{s}^{\perp}$. Quantum Mechanics says: photons have "helicity" $\chi = \pm 1$ and $|\vec{s}^{\perp}| = \hbar$.

Equations of motion for massless test particles with spin

$$\dot{x}^\mu = P^\mu + 2\,\frac{S^\mu{}_\nu R^\nu{}_{\beta\rho\sigma} S^{\rho\sigma}}{R_{\alpha\beta\rho\sigma} S^{\alpha\beta} S^{\rho\sigma}}\,P^\beta$$

$$\dot{P}^\mu = -s\,\frac{\sqrt{-\det(R^\alpha{}_{\beta\rho\sigma} S^{\rho\sigma})}}{R_{\alpha\beta\rho\sigma} S^{\alpha\beta} S^{\rho\sigma}}\,P^\mu$$

$$\dot{S}^{\mu\nu} = P^\mu \dot{x}^\nu - P^\nu \dot{x}^\mu$$

Souriau 1974, Saturnini 1976

3 delicate features:

- No flat space limit
  (Non-vanishing cosmological constant Λ helps. Marsot 2021)
- No zero-spin limit
- Superluminal velocities: due to the anomalous velocity $\dot{x}^\mu$ is spacelike.