Steiner-tree confinement and multiquarks

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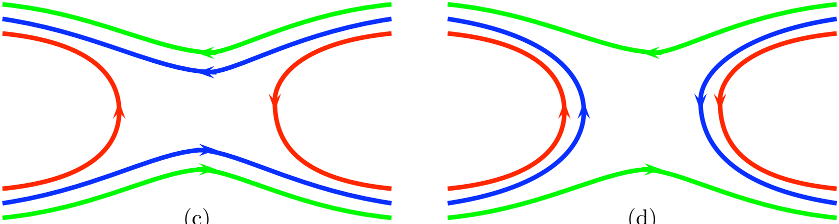
Introduction

Long and shaky history

1. Z baryons with strangeness $S = +1$ in the late 60s,
2. Baryonium in the late 70s and early 80s,
3. Dibaryon resonances?
4. H dibaryon with strangeness $S = -2$ predicted,
5. Heavy pentaquark predicted in 1987,
6. Light pentaquark predicted in 1997, base on earlier work,
7. Light pentaquark candidate in 2003,
8. Not confirmed in most other experiments
9. Etc.
10. Confusion added by theorists, jumping on a speculative idea, and producing tables and tables of multiquarks.

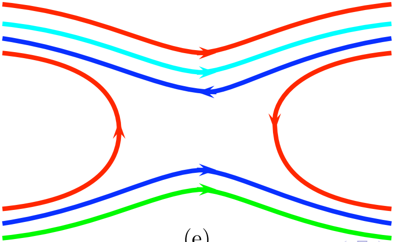
Duality

- s-channel exchanges vs. t-channel exchanges
- See Rosner, D.P. Roy, etc, baryonium partner of mesons in $\overline{N}N$,



(c) (d)

- Exotic baryon partner of mesons in baryonium–baryon, etc.



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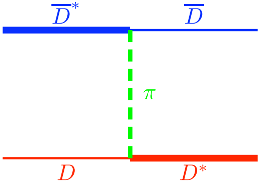
Nuclear forces: hadron molecules

- 

  Meson exchanges bind NN. Why not other hadrons containing light quarks? Non-local operator here (transfer of energy).
- In particular, the $X(3872)$ was predicted as a $D\bar{D}^*$ system,
- When the $X(3872)$ was found, greeted as a success of this approach,
- However, recent measurements also suggest a radial excitation of a $(c\bar{c})$ state,
- If the $X(3872)$ is eventually interpreted as (mainly) a molecule, other states predicted, but no proliferation (nuclear forces are spin and isospin dependent),
- In particular, the b-analogue predicted about 50 MeV below $B\bar{B}^*$,
- In the late 70s, a high-lying $(c\bar{c})$ with $J^P = 1^{--}$ state was claimed as a molecule, due to anomalous branching ratios into $D\bar{D}$, $D\bar{D}^* + \mathrm{c.c.}$ and $D^*\bar{D}^*$ (Voloshin, DeRujula and Glashow, ...)
- In fact, the branching ratios are explained by the nodal structure of the state.

Chromomagnetism

- In the 70s, the hyperfine splitting between hadrons ($J/\psi - \eta_c$, $\Delta - N$, etc.) explained à la Breit–Fermi, by a potential

$$V_{SS} = -A \sum_{i<j} \frac{\delta^{(3)}(\boldsymbol{r}_{ij})}{m_i\, m_j}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \ ,$$

  a prototype being the magnetic part of one-gluon-exchange.
- Attractive coherences in the spin-colour part: $\langle \sum \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \rangle$ sometimes larger for multiquarks than for the threshold.
- In particular $\langle \ldots \rangle$ twice larger (and attractive) in the best ($uuddss$) as compared to $\Lambda + \Lambda$.
- But $\langle \delta^{(3)}(\boldsymbol{r}_{ij}) \rangle$ much weaker for multiquarks than for ordinary hadrons, and needs to be computed. Hence uncertainties in this approach.

Binding mechanisms: chromo-electricity

- What about a (confining) spin-independent confining interaction for $(q_1 q_2 \bar{q}_3 \bar{q}_4)$?
- For equal masses, found unbound.
- Symmetry breaking: if $H = H_{\rm even} + \lambda\, H_{\rm odd}$, $E(\lambda) \leq E(0)$
- But, the effect often benefits more to the threshold, and stability deteriorates,
- Exception: $(QQ\bar{q}\bar{q})$ more stable than $(qq\bar{q}\bar{q})$
- Typically $(cc\bar{u}\bar{d})$ at the edge, $(bc\bar{q}\bar{q})$ or $(bb\bar{q}\bar{q})$ required if one uses

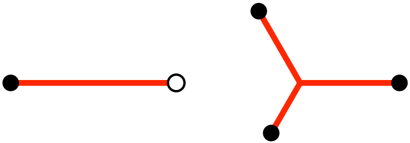
$$V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}\, v(r_{ij}) \ ,$$

- Question: what about a better model of confinement?

Steiner tree: baryons-1

- For mesons and baryons

$$v = \sigma\, r_{12}\ , \qquad V_Y = \sigma \min_J \sum_{i=1}^{3} r_{iJ}\ .$$



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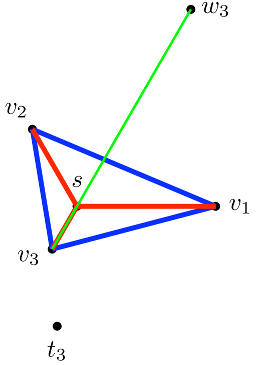
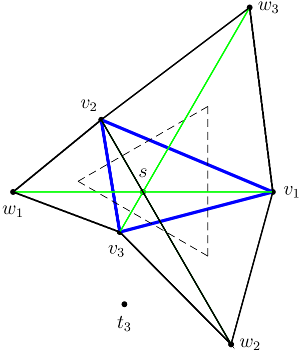
- No dramatic change for baryon spectroscopy
- As compared to the additive model, which would give

$$V_{\text{Baryon}} = \frac{\sigma}{2}\left[r_{12} + r_{23} + r_{31}\right]\ .$$

Steiner tree: baryons-2

- This baryon potential is the solution of the famous Fermat-Torricelli problem of the minimal path linking three points, with an interesting symmetry restoration, intimately related to a theorem by Napoleon.
- 



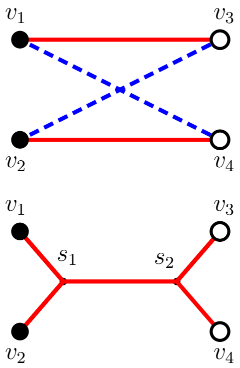
Steiner tree: tetraquarks-1

$$U = \min\{V_{\text{flip-flop}}, V_{\text{Steiner}}\}$$

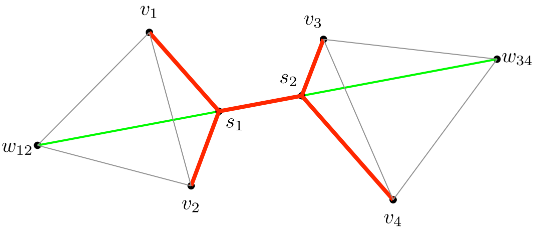
$$V_{\text{flip-flop}} = \min\{d_{13} + d_{24}, d_{14} + d_{23}\} \ ,$$

$$V_{\text{Steiner}} = \min_{s_1, s_2}\Big(\|v_1 s_1\| + \|v_2 s_1\| + \|s_1 s_2\| + \|s_2 v_3\| + \|s_2 v_4\| \Big) \ ,$$

U dominated by the flip-flop term,



Steiner tree: tetraquarks-2



In the planar case, very simple construction of the connected term of the potential (this speeds up the computation).

$$V_4 = \sigma \, \| w_{12} w_{34} \| \ ,$$

maximal distance between the two Melznak points.

Steiner tree: tetraquarks-3

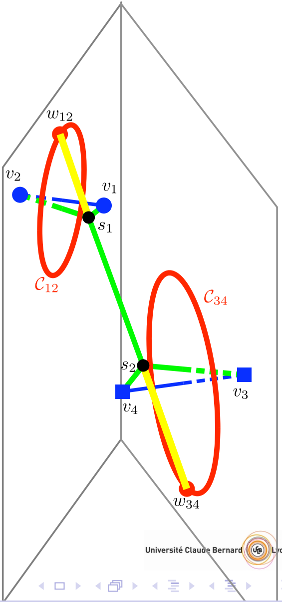
$$V_4 = \sigma \, \|w_{12} w_{34}\| \ ,$$

maximal distance between the two Melznak circles.

$$V_4 \le \sigma \left\{ \frac{\sqrt{3}}{2} \left[\|\boldsymbol{x}\| + \|\boldsymbol{y}\| \right] + \|\boldsymbol{z}\| \right\} \ ,$$

which is exactly solvable. The Jacobi var.

$$\boldsymbol{x} = v_1 v_2,$$
$$\boldsymbol{y} = v_3 v_4,$$
$$\boldsymbol{z} = (v_1 + v_2)/2 - (v_3 + v_4)/2 \ ,$$



Steiner tree: tetraquarks-4

- The crude, but rigorous, geometric considerations demonstrate stability at least for large M/m quark-to-antiquark mass ratio.
- What about an accurate numerical solution of this four-body problem?
- First estimate

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Absence of exotic hadrons in flux-tube quark models

- Second estimate (Vijande et al.)

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Stability of multiquarks in a simple string model

- However, the effect of antisymmetrisation and short-range forces not yet included.
- Tetraquarks with different flavours and large quark-to-antiquark mass ratio most likely, e.g., ($bc\bar{u}\bar{s}$).

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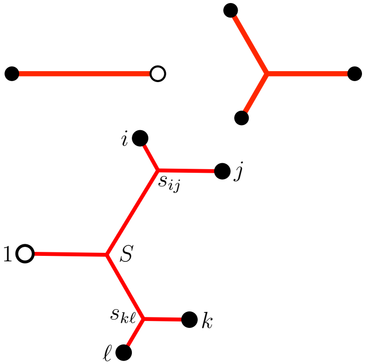
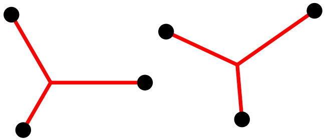
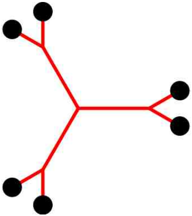
Steiner tree: pentaquark

- $U = \min\{$flip-flop, Steiner$\}$,
- Flip-flop
- Connected Steiner tree



- $(\bar{q}qqqq)$, as well as $(\overline{Q}qqqq)$, $(\bar{q}qqqQ)$ for $M \gg m$, and probably many other configurations bound vs. spontaneous dissociation. (hyperscalar approx. with flip-flop alone sufficient to prove binding)
- But short-range forces and antisymmetrisation constraints not yet included.
- Configurations such as $(\bar{c}uuds)$ should survive, as spin effects might help.

Steiner-tree: hexaquark

- Same scenario: flip-flop and connected diagrams,
- The latter, more interesting, but less important for the dynamics,
- Binding is obtained in most cases, where antisymmetrisation is neglected.
-  

Steiner-tree: baryon-antibaryon

- Again: flip-flop and connected diagrams,
- Binding obtained in most cases.
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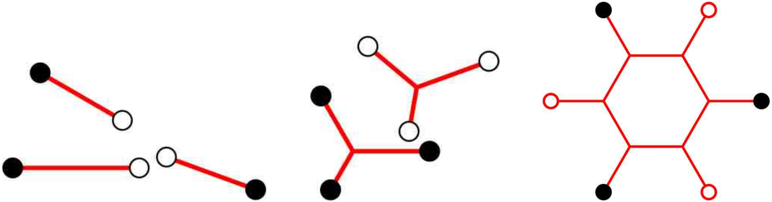

Conclusions. Multiquarks:

The stability of multiquarks remains a very important issue, with recent developments

- Lattice QCD, QCD sum rules, AdS/QCD entering the game very seriously,
- Support to the flip–flop – Steiner-tree model of confinement,
- This model turns out more attractive than the empirical colour-additive model,
- To be refined in the case of identical quarks,
- Need more relativity, short-range forces, and non-adiabatic corrections
- Stable multiquarks likely in sectors with several flavours
- In particular: $(cc\bar{u}\bar{d})$, $(bc\bar{q}\bar{q})$