Weak Gravity Conjecture and Scalar Fields

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The **Weak Gravity Conjecture** (WGC) as part of the **Swampland** program constrains coupling of $U(1)$ gauge theories to gravity.

In Quantum Gravity any $U(1)$ gauge theory with gauge coupling g must contain a *superextremal* state satisfying:

$$gq \geq \frac{1}{\sqrt{2}} \frac{m}{M_P}$$

Arkani-Hamed, Motl, Nicolis, Vafa

N.B.: We restrict here to $D = 4$. In D dimensions $\frac{1}{\sqrt{2}} \to \sqrt{\frac{D-3}{D-2}}$.

Application to **magnetic monopoles** places an upper bound on the *UV cut-off* Λ of the low energy Effective Field Theory

$$\Lambda \lesssim g M_P$$

The WGC can be obtained from different requirements:

- Decay of extremal Reissner-Nordström black holes
- Overall long range repulsive force
- Subdominance of gravity
- No gravitational bound states

They are all the same in this set-up!

Natural questions to ask:

- *Is it always true?*
- How can they apply in different setups?

Let's see what happens with spin-0 force carriers

Simplest generalization: **Einstein-Maxwell-Dilaton theory**

$$\mathcal{S} = \int d^4x\, \mathcal{R} - \frac{(\partial\phi)^2}{2} - \frac{1}{4} e^{-2\alpha\phi} F^2$$

The dilaton enters the BH solution through the charge

$\Longrightarrow$ Analog physical motivation apply! Replacement:

- Reissner-Nordström $\Leftrightarrow$ GM-GHS Black Hole[1]

Dilatonic Weak Gravity Conjecture: Heidenreich, Reece, Rudelius

In any Einstein-Maxwell-Dilaton theory with gauge coupling g a *superextremal* state must exist satisfying:

$$g^2 q^2 \geq \frac{1+\alpha^2}{2} \frac{m^2}{M_P^2}$$

[1] Gibbons-Maeda Garfinkle-Horowitz-Strominger

For other massless scalar force carriers:

$$\mathcal{S} = \int d^4x\, \mathcal{R} - \frac{g_{ij}}{2}\partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{4}F^2$$

- BHs do **not** carry scalar hair

Decoupling of the BH argument

Overall long range repulsive force, subdominance of gravity and no gravitational bound states arguments are fulfilled **if** $\exists$ a state

$$g^2 q^2 \geq \frac{1}{2}\frac{m^2}{M_P^2} + g_{ij}\frac{\partial_{\phi_i} m^2 \partial_{\phi_j} m^2}{4m^2}$$

Palti

This was generalized to the **Repulsive Force Conjecture** (RFC)

> In any $U(1)$ gauge theory with massless scalar fields a *self-repulsive* state must exist
>
> Heidenreich, Reece, Rudelius

With this interpretation, **what happens if we drop the $U(1)$?**

- Relevant BHs are Schwarzschild, no bound on scalars
- *Both* scalar and gravitational forces attractive
- Cannot avoid bound states

Should $\frac{1}{2}\frac{m^2}{M_P^2} + g_{ij}\frac{\partial_{\phi_i} m^2 \partial_{\phi_j} m^2}{4m^2} \leq 0$ be satisfied? Palti

Problem: How to deal with **self-interacting** scalars?

Few attempts to define a **Scalar Weak Gravity Conjecture** (SWGC) have been made

- Palti '17 : work in the context of RFC, no clear meaning
- Gonzalo, Ibanez '19 : propose a *Strong* SWGC: scalar potentials bound by: $2\left(V'''(\phi)\right)^3 - V''(\phi)V''''(\phi) \geq \frac{\left(V''(\phi)\right)^2}{M_P^2}$.
  Extremal states verify $m^2(\phi) = \frac{m_0^2}{Ae^{-\frac{\phi}{M_P}} + Be^{\frac{\phi}{M_P}}}$
- Freivogel, Gasenzer, Hebecker, Leonhardt '19 : Counterexample to the previous, troubles with $V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$
- Benakli, C.B., Lafforgue-Marmet '20
- Gonzalo, Ibanez '20 : *Pair Production Weak Gravity Conjecture*

$$\frac{g^{ij}}{n}\,|T(ij \to \varphi\varphi^*)|^2_{th} \geq |T(gg \to \varphi\varphi^*)|^2_{th}$$

i, j run over massless scalars.

To couple self-interacting scalars to gravity, **no** black hole and **no** long range force argument can be used. **Our proposal**:

- Study possible subdominance of gravity

Start with

$$V(\phi) = \frac{1}{2} m_0^2 \phi^2 - \frac{\mu}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4$$

First question:

How should we compare self-interactions to gravity?

$\Longrightarrow$Answer: At a fixed energy scale

According to the original WGC:

$$E \sim m_0$$

At these energies we are in the *non-relativistic* regime. We make the following redefinition:

$$\phi(x) = \frac{1}{\sqrt{2m_0}}\left(\psi(\vec{x},t)e^{-im_0 t} + \psi^*(\vec{x},t)e^{im_0 t}\right)$$

and match to Schrödinger's lagrangian with potential

$$V_{eff}(\psi\psi^*) = m_0\psi^*\psi + \frac{\tilde{\lambda}}{16m^2}(\psi^*\psi)^2$$

Particle's number conserved $N = \int d^3x\psi^*\psi \Rightarrow$ Only contact terms

Tree-level matching: $\tilde{\lambda} = \lambda - \frac{5}{3}\frac{\mu^2}{m_0^2}$

Two possibilities:
- $\tilde{\lambda} > 0$: Repulsive
- $\tilde{\lambda} < 0$: Attractive

Physical observable to constrain: $\phi\phi \to \phi\phi$

Graviton mediated scattering decouples into long range and short range:

$$\mathcal{A}(\phi\phi \to \phi\phi)^{NR}_{GR} \simeq 2\frac{m_0^4}{M_P^2}\left(\frac{1}{4m_0^2} + \frac{1}{t} + \frac{1}{u}\right)$$

Gravitational interactions trivially dominate at long distances . Comparison should be made between short range manifesting around the *Compton radius*

For a *self-interacting* scalar field, implementation of gravity subdominance amounts to require

$$|\tilde{\lambda}| = \left|\lambda - \frac{5}{3}\frac{\mu^2}{m_0^2}\right| \geq \frac{1}{2}\frac{m_0^2}{M_P^2}$$

Few observations

- $|\tilde{\lambda}| M_P$ could be interpreted as an **UV cut-off** imposed by coupling to gravity
- Formation of **bound states** now *critically* depends on short-range interactions $\tilde{\lambda} > 0 \Rightarrow$ No B.S.; $\tilde{\lambda} < 0 \Rightarrow$ B.S. hold up by scalar forces: formation of tower of states not guaranteed.
- Constraint obtained developing around $\phi = 0$. Around a **generic background value** ϕ_0:

$$4 m_0^2 \left| \frac{\partial^4 V_{eff}}{\partial^2 \psi \partial^2 \psi^*} \right|_{\psi=0} \geq \frac{1}{2 M_{Pl}^2} \left| \frac{\partial^2 V_{eff}}{\partial \psi \partial \psi^*} \right|_{\psi=0}^2$$

The r.h.s. represents attractive gravitational interactions only when $\frac{\partial^2 V_{eff}}{\partial \psi \partial \psi^*} \geq 0$.

Example: **SSB potential**

- $V(\phi,\phi^*) = -m_0^2\phi^*\phi + \lambda(\phi^*\phi)^2$, with $m_0^2 > 0$, $\lambda > 0$
- Use $\phi(x) = \frac{1}{\sqrt{2}}\rho(x)e^{i\pi(x)} \Longrightarrow V = V(\rho^2)$.
- Develop around *background value* $\bar{\rho}(x)$
- $\bar{\rho}^2 < \frac{m_0^2}{3\lambda} \Rightarrow$ *Tachyonic modes*. Bound still verified. $\frac{\partial^4 V_{eff}}{\partial^2\psi\partial^2\psi^*}\Big|_{\psi=0} > 0 \Rightarrow$ Repulsive contact terms
- $\bar{\rho}^2 \geq \frac{m_0^2}{3\lambda} \Rightarrow$ Bound verified up to: $\rho^2 \leqslant \frac{14}{3}\tilde{M}_{Pl}^2 + \frac{17}{21}\frac{m^2}{\lambda} + \mathcal{O}(\tilde{M}_{Pl}^{-2})$ $\frac{\partial^4 V_{eff}}{\partial^2\psi\partial^2\psi^*}\Big|_{\psi=0} > 0 \Rightarrow$ Attractive contact terms
- At the *minimum*, $\bar{\rho}^2 = \frac{m_0^2}{\lambda} \equiv v$, $\tilde{\lambda} = -24\lambda$. Gravity is *subdominant* as long as $\lambda \geq \frac{1}{48}\frac{m_0^2}{M_P^2} \simeq 10^{-16} - 10^{-17}$.
  With the EW scale in mind

$$v^2 \leq 48 M_P^2 \sim 10^{37}\, GeV^2$$

Multiple scalars and moduli

Consider a complex scalar field X and a *real* modulus ϕ with

$$\mathcal{L} \supset m_X^2(\phi)|X|^2, \;\; m_X^2(\phi) = m_X^2(\phi_0) + \partial_\phi m_X^2\big|_{\phi_0}\delta\phi + \frac{1}{2}\partial_\phi^2 m_X^2\big|_{\phi_0}(\delta\phi)^2 + ...$$

$$\delta\phi \equiv \phi - \phi_0$$

Subdominance of gravity may be inferred from $XX^* \to XX^*$ scattering computed at $E_{CM} \sim 2m_X$.

Around a generic background value *short-range* subdominance gives:

$$(\partial_\phi m)^2 \geq \frac{1}{2}\frac{m^2}{M_P^2}$$

Extremal solution: $m^2(\phi) = m_0^2 e^{\pm 2\frac{\phi}{\tilde{M}_P}}, \quad \tilde{M}_P = \sqrt{2}M_P$
This is reminiscent of the Swampland Distance Conjecture! (SDC)

- This is the *same* result one obtains from the long range channel for either $XX \to XX$ or $XX^* \to XX^*$.
- We can consider other scatterings: $\phi\phi \to XX^*$ or $\phi X \to \phi X$

- The $\phi\phi \to XX^*$ channel gives:

$$\left| \frac{\left(\partial_\phi m^2\right)^2}{m^2} - \partial_\phi^2 m^2 \right| \geq \frac{m^2}{M_P^2}$$

This is the same inequality re-obtained in *Gonzalo, Ibanez '20*. This result is **purely** short-distance.

- The $\phi X \to \phi X$ channel is trivial!

$$\left|\partial_\phi^2 m^2\right| \geq 0$$

Gravity here is only long range, scalar forces are only short range.

Consider now a *complex* Φ with $m_X(|\Phi|)$ function only of its modulus

$$\mathcal{L} \supset m_X^2\left(|\Phi|\right)|X|^2, \;\; m_X^2\left(|\Phi|\right) = m_X^2\left(|\Phi_0|\right) + \partial_\Phi \partial_{\Phi^*} m_X^2\left(|\Phi_0|\right)|\Phi - \Phi_0|^2 + \dots$$

Comparison should now be made between contact and graviton mediated *short range* contributions to the $\Phi\Phi^* \leftrightarrow XX^*$ decay

This leads to

$$\left|\partial_\Phi \partial_{\Phi^*} m^2\right| \geq \frac{m^2}{M_P^2}$$

Again, this is intrinsically a short distance result, matching the previous $\phi\phi \rightarrow XX^*$ when $m^2(\phi)$ is developed around an extremum. Here we see it has its own importance.

Extremal states solution

$$m_X^2(|\Phi|) = Ae^{\sqrt{2}\frac{\Phi+\Phi^*}{\tilde{M}_{Pl}}} + Be^{-\sqrt{2}\frac{\Phi+\Phi^*}{\tilde{M}_{Pl}}} + Ce^{\sqrt{2}i\frac{\Phi-\Phi^*}{\tilde{M}_{Pl}}} + De^{-\sqrt{2}i\frac{\Phi-\Phi^*}{\tilde{M}_{Pl}}}$$

Using the parametrization $\Phi = \frac{\phi+i\chi}{\sqrt{2}}$ and defining $R_\phi = e^{\frac{\phi}{\tilde{M}_P}}$, $R_\chi = e^{\frac{\chi}{\tilde{M}_P}}$

$$m_X^2(R_\phi, R_\chi) = \frac{m_-^2}{R_\phi^2} + m_+^2 R_\phi^2 + \frac{n_-^2}{R_\chi^2} + n_+^2 R_\chi^2$$

reproducing the well known formula for K.K. and winding modes along two directions for the two real scalar fields.

Notice the similitude: K.K. states *saturates* the dilatonic WGC

If we allow for a more general $m_X(\Phi, \Phi^*)$ and develop it, we can have $XX^* \to XX^*$.

Both long range and short range subdominance are obtained with

$$\frac{\partial_\Phi m^2 \partial_{\Phi^*} m^2}{4m^2} \geq \frac{1}{2}\frac{m^2}{M_P^2}$$

Extremal solution: $m^2(\Phi, \Phi^*) = m_0^2 e^{2\frac{\Phi+\Phi^*}{\tilde{M}_P}}$

In general, for n real scalars the bound $\frac{\sum_{i=1}^n \left(\partial_{\phi_i} m^2\right)^2}{4m^2} \geq \frac{n}{2}\frac{m^2}{M_P^2}$ is saturated for

$$m^2(\phi_i) = m_0^2 e^{2\frac{\sum_{i=1}^n \phi_i}{\tilde{M}_P}}$$

Extremal states' formula resembles a **volume** of compactification.

Consider the case where ϕ_i, $i = 1, ..., n$ are n moduli appearing only as parameters in the couplings of X ($\langle X \rangle = 0$)

$$V(X, \phi) = m_X^2(\phi_i) X^2 + \sum_{n \geq 4} \lambda_n(\phi_i) X^n$$

- The condition on $XX \to XX$ reads

$$\left. \frac{\left| \vec{\nabla}_\Phi V(X, \Phi) \right|}{V} \right|_{X=0} \geq \frac{\sqrt{\tilde{c}}}{M_{Pl}},$$

- Alternatively, when $\vec{\nabla}_\Phi V(X, \phi)\big|_{X=0} = 0$, condition on $\phi_i \phi_i \to XX$ reads

$$\left. \frac{\left| \nabla_\Phi^2 V(X, \phi) \right|}{V} \right|_{X=0} \geq \frac{n\tilde{c}}{M_{Pl}^2}$$

where we note the similarity with the Refined de Sitter Conjecture

Conclusions

- Subdominance of gravity *can* be implemented in the presence of scalar fields
- Starting with the case of a *self-interacting* scalar we have seen how *crucial* it is to observe short-range interactions
- Provides both an alternative to RFC and *may* avoid formation of tower of gravitational bound states
- Study of cases with multiple scalar fields hints at connections to other Swampland conjectures. Extremal states *may* have interesting physical interpretations in the case of moduli fields.