The deconvolution problem of deeply virtual Compton scattering

Hervé Dutrieux

herve.dutrieux@cea.fr - PhD student - supervisor: H. Moutarde
CEA Saclay - Irfu / Département de Physique Nucléaire

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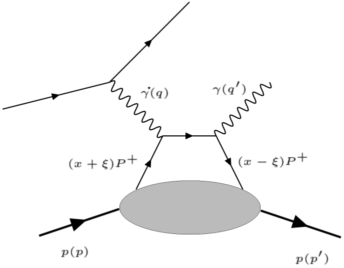
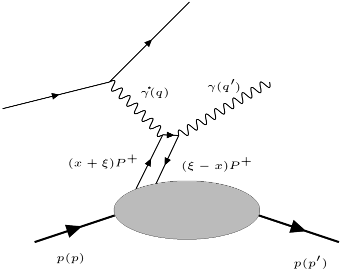
Overview

Deeply virtual Compton scattering and the structure of hadrons

Deeply virtual Compton scattering (DVCS) is the scattering of a lepton on a hadron via a photon of large virtuality, producing a real photon in the final state.

- x is the average light-front plus-momentum (longitudinal momentum in a fast moving hadron) fraction of the struck parton
- ξ describes the light-front plus-momentum transfer, linked to Björken's variable x_B
- $t = \Delta^2$ is the total four-momentum transfer squared





Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Deeply virtual Compton scattering and the structure of hadrons

Similarly to the introduction of **parton distribution functions** (PDFs) in the study of DIS,

- For a large photon virtuality $Q^2 = -q^2$, finite x_B and small total four-momentum transfer squared t, **factorisation theorems** describe DVCS in terms of a hard scattering part computable thanks to perturbative QCD, and a soft non-perturbative part described by **generalised parton distributions** (GPDs).
- The amplitude of DVCS is parametrised by **Compton form factors** (CFFs) $\mathcal{F}$, which write as convolutions of perturbative **coefficient functions** T_F^a and the **GPDs** F^a:

CFF convolution (leading twist)

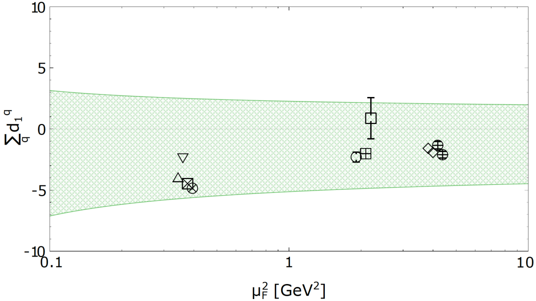
$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T_F^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) F^a(x, \xi, t, \mu^2)^a \quad (1)$$

[a] $F^g(x, \xi, t, \mu^2)/x$ for the usual definition of gluon GPD

μ is the factorisation / renormalisation scale, α_s the strong coupling.

Deeply virtual Compton scattering and the structure of hadrons

GPDs allow access to gravitational form factors of the **energy-momentum tensor** (EMT) $\rightarrow$ first data-driven extractions of **mechanical properties** of hadronic matter (*e.g.* pressure distribution) [Burkert, *et al.*, 2018], [Kumericki, 2019], [Dutrieux, *et al.*, 2021]



In green, 68% confidence interval found for $\sum_q d_1^q(t = 0, \mu^2)$, a critical parameter to evaluate pressure profiles and results obtained by other studies (black markers). The parameter is compatible with 0 with current experimental data. [Dutrieux, *et al.*, 2021]

Properties of generalised parton distributions

- Several types of GPDs: H, E, $\widetilde{H}$, $\widetilde{E}$, ... depending on helicity considerations.
- A GPD is a function of (x, ξ, t, μ^2), ξ-even, with physical region $(x, \xi) \in [-1, 1]^2$ and the dependence on μ^2 is given by renormalisation group equations.
- The **forward limit** gives back the PDF:

$$H^q(x, \xi = 0, t = 0, \mu^2) = f_q(x, \mu^2) \tag{2}$$

- **Polynomiality property**: due to Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x \, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2) \xi^k \tag{3}$$

This property implies that the GPD is the Radon transform of a **double distribution** F (DD) with an added *D*-term on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| < 1\}$:

Double distribution formalism

$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi) \left[F(\beta, \alpha, t, \mu^2) + \xi\delta(\beta) D(\alpha, t, \mu^2)\right] \tag{4}$$

Deconvoluting a Compton form factor

Position of the problem

Assuming a CFF has been extracted from experimental data with excellent precision[1], we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{5}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" eq. (5) to recover $H^q(x, \xi, t, \mu^2)$ from $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$?**

[1] and the different gluon and flavour contributions have been separated, which probably requires a global analysis with various targets and processes.

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in [Freund, 1999], but the question remains essentially open.
- We show that GPDs exist which bring contributions to the LO and NLO CFF of only subleading order even under evolution. We call them **LO and NLO shadow GPDs**.

Definition of a LO shadow GPD

For a given scale μ_0^2,

$$\forall\xi, \forall t,\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{6}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{LO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s(\mu^2)) \tag{7}$$

- Let H^q be a LO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same LO CFF up to a numerically small and theoretically subleading contribution.

Shadow GPDs at leading order

- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We also omit t since it is untouched by the convolution.
- **Leading order** It is well-known that the LO CFF only probes the GPD on the $x = \xi$ line and the D-term, so a LO shadow GPD is simply given by:

$$\mathrm{Im}\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) \propto H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \tag{8}$$

$$H^q(x, \xi = 0, \mu_0^2) = 0 \tag{9}$$

where $H^{q(+)}$ denotes the singlet GPD (x-odd part of the GPD).

Shadow GPDs at leading order

- We search our DD as a polynomial of order N in (β, α), characterised by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \tag{10}$$

- The associated GPD is obtained by the linear Radon transform, given by the matrix R for $x > |\xi|$:

$$H^{q(+)}(x, \xi, \mu_0^2) = \sum_{u=1}^{N+1} \frac{1}{(1+\xi)^u} + \frac{1}{(1-\xi)^u} \sum_{v=0}^{N+1} q_{uv}\, x^v \quad \text{where} \quad q_{uv} = \sum_{m,n} R_{uv}^{mn} c_{mn} \tag{11}$$

$$R_{uv}^{mn} = \sum_{j=0}^{n} \frac{(-1)^{u+v+j}}{m+j+1} \binom{n}{j} \binom{j}{m-u+j+1} \binom{m+j+1}{v-n+j} \tag{12}$$

Shadow GPDs at leading order

- For our shadow GPD, we first want $H^{q(+)}(\xi,\xi,\mu_0^2) = 0$, so we notice that

$$H^{q(+)}(\xi,\xi,\mu_0^2) = \sum_{w=1}^{N+1} \frac{k_w}{(1+\xi)^w} \quad \text{where} \quad k_w = \sum_{u,v} C_w^{uv}\, q_{uv}\,, \quad C_w^{uv} = (-1)^{u+v+w} \binom{v}{u-w}$$

Cancelling the LO CFF

$$H^{q(+)}(\xi,\xi,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{13}$$

Shadow GPDs at leading order

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- We then want $H^{q(+)}(x,\xi=0,\mu_0^2)=0$, so we notice that

$$H^{q(+)}(x,0,\mu_0^2) = \sum_{w=0}^{N+1} q_w x^w \quad \text{where} \quad q_w = \sum_{u,v} Q_w^{uv}\, q_{uv}\ , \quad Q_w^{uv} = 2\delta_w^v$$

Cancelling the forward limit

$$H^{q(+)}(x,\xi=0,\mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{14}$$

Shadow GPDs at leading order

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$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{14}$$

- Both linear systems $C.R$ and $Q.R$ are systems of $\sim N$ equations for $\sim N^2$ variables, so the number of solutions grows quadratically with N, order of the polynomial DD.

Shadow GPDs at leading order

Cancelling the LO CFF

$$H^{q(+)}(\xi, \xi, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(C.R) \tag{13}$$

Cancelling the forward limit

$$H^{q(+)}(x, \xi = 0, \mu_0^2) = 0 \implies (c_{mn})_{m,n} \in \ker(Q.R) \tag{14}$$

LO shadow GPDs

Here is an example of an infinite family of LO shadow DDs, each being of degree $N \geq 9$ odd

$$F_N(\beta,\alpha,\mu_0^2)=\beta^{N-8}\left[\alpha^8-\frac{28}{9}\alpha^6\left(\frac{N^2-3N+20}{(N+1)N}+\beta^2\right)+\frac{10}{3}\alpha^4\left(\frac{N^2-7N+40}{(N+1)N}+\frac{2(N^2-3N+44)}{3(N+1)N}\beta^2+\beta^4\right)\right.$$
$$\left.-\frac{4}{3}\alpha^2\left(\frac{N^2-11N+60}{(N+1)N}-\frac{N-8}{N}\beta^2-\frac{N^2-3N-28}{(N+1)N}\beta^4+\beta^6\right)+\frac{1}{9}(1-\beta^2)^2\left(\frac{N^2-15N+80}{(N+1)N}-\frac{2(N-8)}{N}\beta^2+\beta^4\right)\right] \tag{15}$$

Shadow GPDs at next-to-leading order

- **Next-to-leading order** $\rightarrow$ **original result** The NLO CFF is composed of a collinear part (compensating LO evolution applied to the tree-level LO CFF) and a genuine 1-loop NLO part. An explicit calculation of each term for our polynomial double distribution gives that

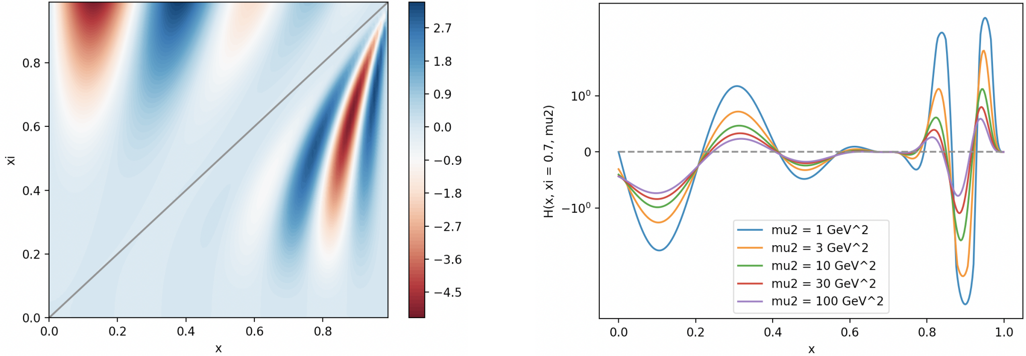
$$\mathrm{Im}\ T^q_{coll}(Q^2,\mu^2)\otimes H^q(\mu^2)\propto$$
$$\alpha_s(\mu^2)\log\left(\frac{\mu^2}{Q^2}\right)\left[\log\left(\frac{2-2\xi}{\xi}\right)\mathrm{Im}\ T^q_{LO}\otimes H^q(\mu^2)+\sum_{w=1}^{N+1}\frac{k_w^{(coll)}}{(1+\xi)^w}\right] \qquad (16)$$

and assuming $\mathrm{Im}\ T^q_{LO}\otimes H^q(\mu^2)=0$,

$$\mathrm{Im}\ T^q_1(Q^2,\mu^2)\otimes H^q(\mu^2)\propto\alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right)\mathrm{Im}\ T^q_{coll}\otimes H^q(\mu^2)+\sum_{w=1}^{N-1}\frac{k_w^{(1)}}{(1+\xi)^w}\right] \qquad (17)$$

Shadow GPDs at next-to-leading order

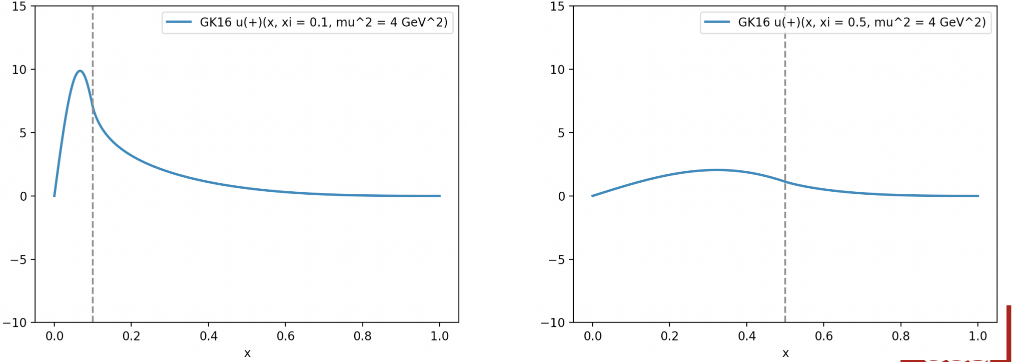
- Cancelling both terms gives rise to two additional systems with a linear number of equations. The first NLO shadow GPD is found for $N = 21$, and adding the condition that the DD vanishes at the edges of its support gives a first solution for $N = 25$ (see below).



Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.7$ up to 100 GeV^2 via APFEL++ and PARTONS [Bertone].

Shadow GPDs at next-to-leading order

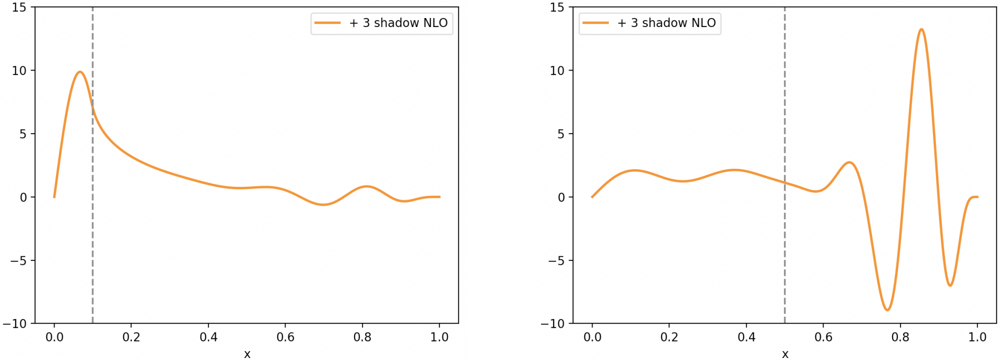
- **On a lever-arm in Q^2 of $[1, 10]$ GeV^2** (current and soon up-coming experimental data), the NLO CFF generated by the NLO shadow GPD varies as $\mathcal{O}(\alpha_s^2(Q^2))$ and its numerical value is of order 10^{-5} (although the NLO shadow GPD is itself of order 1).
- Consider this Goloskov-Kroll GPD model (via PARTONS) at scale 4 GeV^2



$\xi = 0.1$ (left) and $\xi = 0.5$ (right). The dotted line depicts $x = \xi$.

Shadow GPDs at next-to-leading order

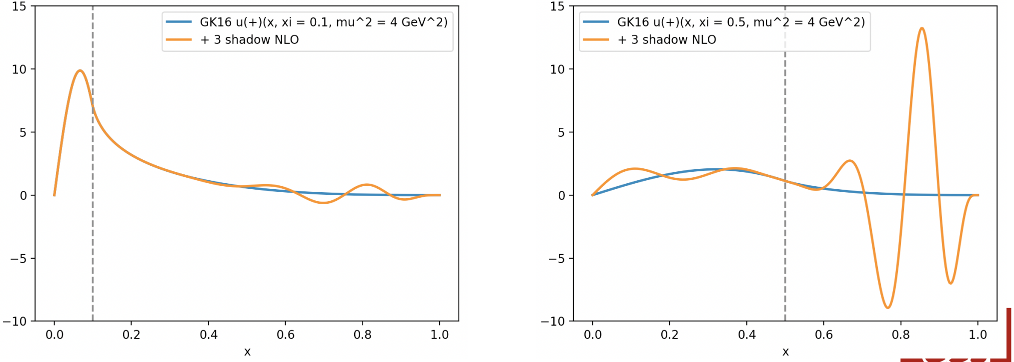
- Now consider this GPD model



$\xi = 0.1$ (left) and $\xi = 0.5$ (right). The dotted line depicts $x = \xi$.

Shadow GPDs at next-to-leading order

- The orange model is GK + 3 × our NLO shadow GPD. For ξ close to 0 and x close to ξ, by design, the two are very close, but vastly different otherwise. They give rise to NLO CFFs which are exactly identical at this scale, and different by a negligible amount for current Q^2 lever arm.



$\xi = 0.1$ (left) and $\xi = 0.5$ (right). The dotted line depicts $x = \xi$.

Conclusion

- We have explicitly shown that, with current lever arm in Q^2, there exist LO and NLO shadow GPDs of considerable size which present a very small and subleading contribution to CFFs. The deconvolution of DVCS seems therefore ill-posed.
- It is foreseeable that higher order DVCS, TCS or LO DVMP could present similar issues.
- On the contrary, higher order DVMP, DDVCS or Lattice QCD for instance could escape this problem or significantly constrain it $\rightarrow$ **interest of multi-channel analysis**, and the development of integrated analysis tools, like **PARTONS**.
- The increase in Q^2 lever arm promised by the EIC / EICC will be very welcomed here and for mechanical properties as well.
- **Positivity constraints** are also a significant tool to constrain the potential size of shadow GPDs.

Backup slides

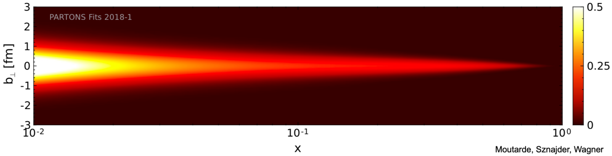
Deeply virtual Compton scattering and the structure of hadrons

Physical interest of GPDs

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \quad (18)$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework [Moutarde, Sznajder, Wagner, 2018]

Deconvoluting a Compton form factor

Remark on renormalisation dependence

- $\mathcal{H}(Q^2) = \sum_a \mathcal{H}^a(Q^2, \mu^2)$ is not r.-dependent, but the separation in quark and gluon contributions is. $\sum_a T^a(Q^2, \mu^2) \otimes H^a(\mu^2)$ has a residual r.-dependence unless T^a is summed at all orders in perturbation. $T^a(Q^2, \mu^2) \otimes H^a(\mu^2)$ has both an intrinsic and a residual r.-dependence.
- If the GPD evolution has been computed in a consistent way with coefficient functions, the residual scale dependence of $T^q_{N^nLO}(Q^2, \mu^2) \otimes H^q(\mu^2)$ will be small, in the sense that, for all Q^2 and μ^2 close to Q^2

$$T^q_{N^nLO}(Q^2, \mu^2) \otimes H^q(\mu^2) - T^q_{N^nLO}(Q^2, Q^2) \otimes H^q(Q^2) = \mathcal{O}(\alpha_s^{n+1}(Q^2)) \qquad (19)$$

- If two very different GPDs have, for all Q^2 in the experimental domain and μ^2 close to Q^2, the same N^{n}LO CFF $T^q_{N^nLO}(Q^2, \mu^2) \otimes H^q(\mu^2)$ up to $\mathcal{O}(\alpha_s^{n+1}(Q^2))$, they will be very difficult to distinguish from one another, both for **numerical reasons** ($\alpha_s^{n+1}(Q^2)$ gets very small) and **theoretical reasons** (the difference between the two is of the order of the systematic uncertainty created by varying the renormalisation scale).

Deconvoluting a Compton form factor

- This means

$$T^q_{N^nLO}(Q^2,\mu^2) \otimes H^q_1(\mu^2) - T^q_{N^nLO}(Q^2,\mu^2) \otimes H^q_2(Q^2) = \mathcal{O}(\alpha_s^{n+1}(Q^2)) \qquad (20)$$

that is, by linearity

N^{n}LO shadow GPDs

$$\begin{cases} \text{for all experimental } Q^2 \text{ and } \mu^2 \text{ close to } Q^2, \quad T^q_{N^nLO}(Q^2,\mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^{n+1}(Q^2)) \\ H^q(x,\xi=0,t=0,\mu^2) = 0 \end{cases} \qquad (21)$$

where $H^q = H^q_1 - H^q_2$. We call GPDs which satisfy eq. (21) N^{n}LO shadow GPDs. We have added the constraint that the forward limit is exactly the same since PDFs are very well known from precise DIS data.