# Study of the Potential Transverse Momentum and Potential Angular Momentum within the Scalar Diquark Model

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# Introduction [1]

The proton is composed of minimum three quarks (uud) in constant interaction.

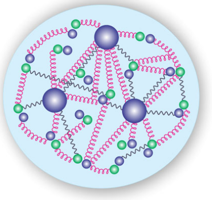

**Figure** – Sketch of the inside of a proton

Challenges :

- *Quantum* : size $\sim 10^{-15} m$
- *Relativistic* : High Energy, $m_{\text{nucleon}} \sim GeV$, $m_{\text{quarks u, d}} \sim MeV$
- *Particle number fluctuation*
- *Confinement* : QCD running coupling large at long distances → non-perturbative treatment
- *Gauge invariance* : Correct definition of observables is complicated

1. presentation based on the article : David Arturo Amor-Quiroz et al. 2020. arXiv : 2010.13451 [hep-ph]

# Nucleon Spin Decomposition

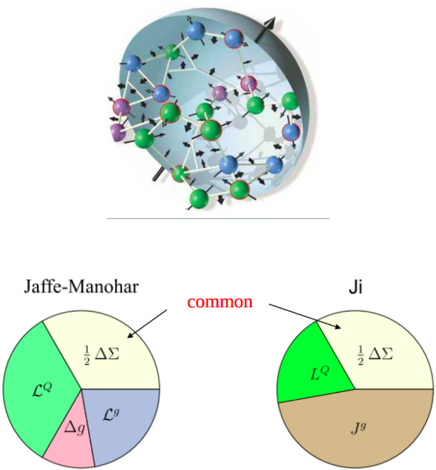


**Figure –** Different decompositions of the nucleon spin

# The Ji and JM Decomposition [2] [3],

$$\text{Ji} : J = L_q + S_q + J_g$$

- $L_q \sim \vec{r} \times i\vec{D}$
- Kinetic decomposition (corresponds to the classical notion $r \times p_{kin}$)
- $L_q, S_q, J_g$ are gauge invariant, can be measured
- Local definition of the derivative

$$\text{Jaffe-Manohar} : J = \mathcal{L}_q + S_q + \mathcal{L}_g + S_g$$

- $\mathcal{L}_q \sim \vec{r} \times i\vec{\partial}$
- Canonical decomposition in the light-cone gauge (corresponds to the Noether theorem operators)
- Total decomposition into both quark and gluon spin and OAM
- Only $\mathcal{S}_q$ and $(\mathcal{L}_q + \mathcal{L}^g_{JM} + S^g_{JM})$ are gauge invariant
- Generators of rotation
- Non-local definition of the derivative
- Gauge invariance can be restored provided one replaces $\partial$ by a non-local definition of the gauge covariant derivative

2. Elliot Leader et Cédric Lorcé. In : *Physics Reports* 541.3 (2014).
3. M. Wakamatsu. In : *Physical Review D* 81.11 (2010).

# Potential Transverse Momentum and Angular Momentum

Using the Ji and JM definitions, we investigate the following observables [4] :

$$\vec{k}_{pot} \triangleq \langle k_\perp^{JM} \rangle - \langle k_\perp^{Ji} \rangle \qquad (1)$$
$$L^z_{pot} \triangleq \langle \mathcal{L}^z_q \rangle - \langle L^z_q \rangle$$

In the light-cone gauge $A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3)$. These can be shown to be :

$$\vec{k}_{pot} = - e_q \langle \int \mathrm{d}^2 \vec{r}_\perp \bar{\psi}(\vec{r}_\perp) \gamma^+ \vec{A}_{phys,\perp}(r_\perp) \psi(\vec{r}_\perp) \rangle \qquad (2)$$
$$L^z_{pot} = - e_q \langle \int \mathrm{d}^2 \vec{r}_\perp \bar{\psi}(\vec{r}_\perp) \gamma^+ (r \times \vec{A}_{phys,\perp}) \psi(\vec{r}_\perp) \rangle$$

- $L_{pot}$ was found to vanish in an explicit one-loop perturbation theory calculation [5]
- We aim to understand if there could be a link between the two quantities

4. we use light-cone variables $[x^+, x^-, \vec{x}_\perp = (x_1, x_2)]$, $x^{+,-} = \frac{1}{\sqrt{2}}(A^0 \pm A^3)$
5. X. Ji et al. In : *Phys Rev D.* 93.5, 054013 (2016).

# Transverse Momentum Distributions

The leading-twist quark TMD correlator is defined as [6] :

$$\phi^{[\gamma^+]}(P, x, k_\perp, S; \gamma) = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} e^{-iz\cdot k} \langle P, S|\psi(-\frac{z}{2})\gamma^+ \mathcal{W}_\gamma(-\frac{z}{2}, \frac{z}{2})\psi(\frac{z}{2})|P, S\rangle_{z^+=0} \tag{3}$$

where $\mathcal{W}\gamma$ is a Wilson line (more details later).

After some algebra, TMDs relate to the transverse momentum in a model-independent relation :

$$\langle \vec{k}_\perp \rangle_{JM} = \int \mathrm{d}x \; \mathrm{d}^2\vec{k}_\perp \; \vec{k}_\perp \phi^{[\gamma^+]}(P, x, \vec{k}_\perp, S) \tag{4}$$

6. D. Boer, P.J. Mulders et F. Pijlman. In : *Nuclear Physics B* 667.1-2 (2003).

# The color-gauge link

The Wilson line linking $z_1$ to $z_2$ along the path $\gamma$ can be written as :

$$\mathcal{W}_\gamma(z_1, z_2) = \mathcal{P} \exp\left( -ig \int_\gamma \mathrm{d}z \cdot A(z) \right) \quad (5)$$

It has the following properties :

- makes the correlator color-gauge invariant
- encodes Final or Initial State Interactions (FSI, ISI)

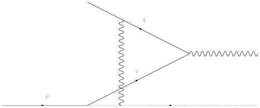

**Figure –** Example of an Initial State Interaction


**Figure –** Different path correspond to different processes

- is process dependent
- breaks the naive time reversal symmetry

J.C. et al. Collins. In : *Transversity 2005* (2006). DOI : 10.1142/9789812773272_0025

# The Sivers Shift

Generically : $\phi(P, x, k_\perp, S) = f_1(x, k_\perp) - \frac{\epsilon_\perp^{ij} S_\perp^i k_\perp^i}{M} f_{1T}(x, k_\perp)$
where $f_1$ is the unpolarized parton distributions and $f_{1T}$ is the Sivers function
Then :

$$\langle k_\perp^i \rangle_{JM} = -\,\epsilon_\perp^{ij} S_\perp^k \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \frac{k_\perp^2}{M} f_{1T}(x, k_\perp) \qquad (6)$$

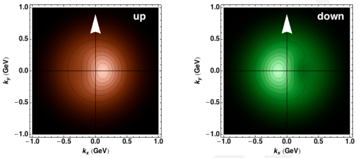


**Figure** – The up and down quark density distortion in transverse-momentum space, obtained by studies of the Sivers function

- One of two leading twist function that are odd under time reversal for a spin-1/2 target
- Can lead to a non-zero transverse momentum
- In SIDIS (Drell-Yan), the Sivers function encodes the presence of Final (Initial) State interactions through gluon exchanges

# Proposed Lensing mechanism

Burkardt[7] made the case for a "lensing mechanism" that would create a nonzero torque of the struck quark due to the Sivers Shift.

- The distribution of unpolarized quarks in a transversely polarized nucleon is shifted due to the Sivers shift before it fragments
- The attracting interactions bend the observed hadrons in the direction opposite to the struck quark
- The lensing parameter would formally be written "SSA = GPD x L(x)"

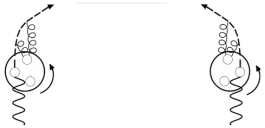

**Figure –** sketch of the proposed lensing mechanism

Pasquini, Rodini and Bacchetta showed that under restrictive conditions, such a lensing mechanism could be exhibited in the case of the pion[8].

7. Matthias Burkardt. In : *Nuclear Physics A* 735.1-2 (2004).
8. Barbara Pasquini, Simone Rodini et Alessandro Bacchetta. In : *Physical Review D* 100.5 (2019).

# The Scalar Diquark Model

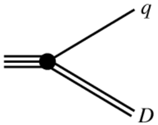


**Figure** – sketch of the scalar diquark model

- The nucleon splits into a quark and scalar diquark structure
- Lorentz covariance is maintained
- Initial/Final State interactions are considered in an Abelian (∼QED) theory
- Provides interesting calculations for analytic results that are accessible in the litterature

Without loss of generality, The perturbative calculations are all :

- computed in the light-cone gauge ($A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3) = 0$ )
- conducted for a semi-inclusive deep inelastic scattering experiment (SIDIS)
- Regularized through dimensional regularization (parameter $\varepsilon$)

# Potential momentum (1)

It can be shown that the transverse momentum following Ji and JM's decomposition write :

$$\begin{aligned} \langle \vec{k}_\perp \rangle_{Ji} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) \overleftrightarrow{D}_\perp \psi(0) | P, S \rangle \\ \langle \vec{k}_\perp \rangle_{JM} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) \overleftrightarrow{D}_{pure,\perp} \psi(0) | P, S \rangle \\ k_{pot} &= \frac{i}{4P^+} \langle P, S | \bar{\psi}(0) A_{phys,\perp} \psi(0) | P, S \rangle \end{aligned} \tag{7}$$

From symmetry :

$$\begin{aligned} \langle \vec{k}_\perp \rangle_{Ji} &= - \langle \vec{k}_\perp \rangle_{Ji} \\ \langle \vec{k}_\perp \rangle_{JM,DIS} &= - \langle \vec{k}_\perp \rangle_{JM,DY} \end{aligned} \tag{8}$$

Only the JM $k_\perp$ contributes. Also, no asymmetry can arise without a gluon or photon exchange.

# Potential momentum (2)

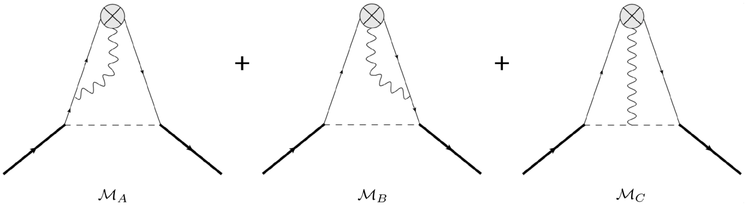


**Figure** – Diagramms contributing to $k_\perp^q$

Calculating these diagramms will give the potential OAM, keeping in mind :

- $\langle k_\perp^q \rangle_{JI}^{A+B} = \langle k_\perp^q \rangle_{JM}^{A+B} = 0$ (conservation of momentum)
- $\langle k_\perp^q \rangle_{JI}^{C} = 0$ (PT symmetry)
- $\langle k_\perp^q \rangle + \langle k_\perp^S \rangle = 0$ (Burkardt sum rule)

# Potential momentum (3)

All in all, the calculation yields[9] :

$$k^i_{pot} = -\epsilon^{ij}_{\perp} s^j_{\perp} \frac{\pi}{6}(3m_q + M)\frac{g^2 e_q e_S}{(4\pi)^2(4\pi\varepsilon)^2} + \mathcal{O}(1/\varepsilon) \quad (9)$$

Remarks :

- $k_{pot} \neq 0$ ! There is a Sivers shift in the SDM
- Crosscheck with known Sivers function[10]. Naively inputting the result from other works yields misleading results.
- The Burkardt sum rule holds $\sum_{q,S}\langle k_\perp \rangle = 0$
- If the same mechanism produces the transverse momentum and the angular momentum, we need to look at two loops.

9. $m_q$ and $M$ are the masses of the struck quark and proton respectively. $g$ is the field coupling, $e_q$ and $e_S$ are the charges of the quark and diquark repectively.

10. S. MEISSNER, A. METZ et K. GOEKE. "Relations between generalized and transverse momentum dependent parton distributions". In : *Physical Review D* 76.3 (2007).

# Potential Angular Momentum



Looking now at $L^z_{pot}$, we calculate :

$$\begin{aligned} L^z_{pot} =& \frac{-g\epsilon^{ij}_{\perp}}{2P^+}\left[-i\nabla^i_{\Delta_\perp}\langle p+\Delta, S'|\bar{\psi}(0)\gamma^+ A^j_{phys,\perp}\psi(0)|p,S\rangle\right]_{\Delta=0} \\ =& \mathcal{O}(1/\varepsilon) \end{aligned} \tag{10}$$

- Both $\mathcal{L}^z$ and $L^z$ are PT-even
- the Ji and JM definitions of OAM coïncide at two loops in the SDM to order $\mathcal{O}(1/\epsilon^2)$ unlike the transverse momentum.
- Does the two-bodied nature of the system prevent it from acquiring any Lorentz torque ? [11]

11. Cédric Lorcé. In : *The European Physical Journal C* 78.9 (2018).

# Conclusions and Outlook

Summary and Conclusions :

- Both $\vec{k}^q_{pot}$ and $L^q_{z,pot}$ were computed at two-loop in the scalar diquark model to the order $\mathcal{O}(\lambda^2 e_q e_S)$
- The difference between the two decomposition appears when interactions play a role.
- We found the surprising result $L^{z,q}_{pot} = \mathcal{O}(1/\varepsilon)$, whereas $k_{\perp,pot} = \mathcal{O}(1/\varepsilon^2)$, which puts in jeopardy the inuitive proposal of a lensing mechanism

Outlook

- Deeper perturbative calculation of $L^{q,z}_{pot}$
- Continue using the SDM as a tool to challenge our physical understanding of the problems at hand
- Adress more challenging and complex models.

# Additional frames

# The Ji and JM Decomposition (recap)

| | Ji | Jaffe-Manohar |
|---|---|---|
| $J =$ | $L_q + S_q + J_g$ | $\mathcal{L}_q + S_q + \mathcal{L}_g + S_g$ |
| decomp. interpretation | kinetic | canonical |
| local | ✓ | × |
| gauge-invariant operators | all | $S_q$, $(\mathcal{L}_q + L_g + S_g)$ |
| measurable | ✓ | ✓ |
| accessible through | Wigner functions and leading-twist GPDs | Wigner functions and higher-twist |

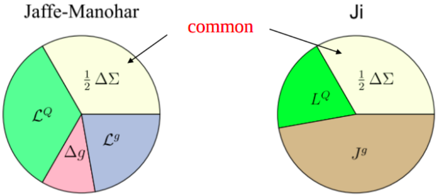


**Figure –** Different decompositions of the nucleon spin