A two-loop study of a background gauge invariant Gribov-Zwanziger type action

In collaboration with dr. Urko Reinosa

Duifje van Egmond
Centre de Physique Théorique - École Polytechnique

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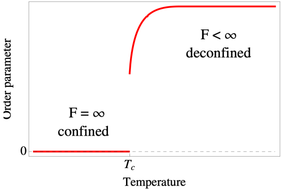
- From this, we know that:
  - At some very high temperature T_c, hadrons become free quarks and gluons $\rightarrow$ quark-gluon plasma.
  - This transition is related to the breaking of the center symmetry Z_N, for Yang-Mills theories.

Center symmetry

- The order parameter for the confinement/deconfinement transition is the Polyakov loop:

$$\mathcal{P} \propto e^{-FT} \tag{1}$$



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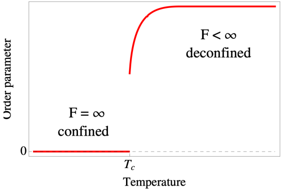

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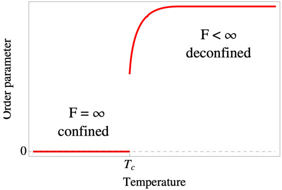

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- Under center symmetry $\mathcal{P} \rightarrow Z_N \mathcal{P}$, so breaking of the center symmetry signals deconfinement.

Analytical results

- At high energies, QCD is well described by an $SU(3)$ Yang-Mills action with a Faddeev-Popov gauge fixing:

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \overline{c}^a (D_\mu c)^a + i b^a \partial_\mu A^a_\mu \tag{2}$$

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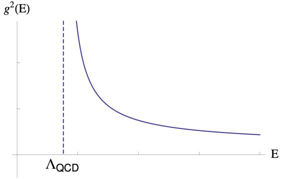

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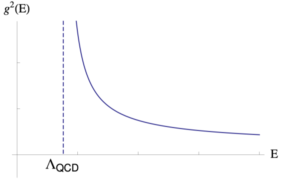

- Does this mean we have an infinite coupling at low energies? Probably not!

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- Can we use the GZ/CF model to describe the confinement/deconfinement transition? Not in the Landau gauge: no explicit center symmetry.

Background Field gauge Methods

- In the BFG formalism, an arbitrary background field $\overline{A}_\mu$ is introduced: $a^a_\mu = A^a_\mu - \overline{A}^a_\mu$.

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- The center symmetry is now governed by the background field symmetry.
- The Polyakov loop is now its functional expression at the minimum of a background field potential V.

Example: SU(2)

- $\mathcal{P} = cos(r/2)$ with $r = \beta g \overline{A}_0^a$

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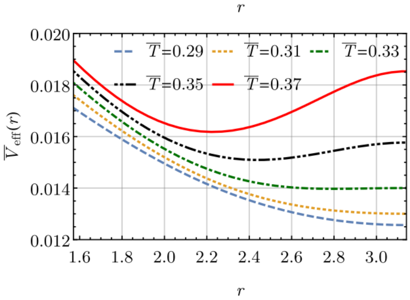

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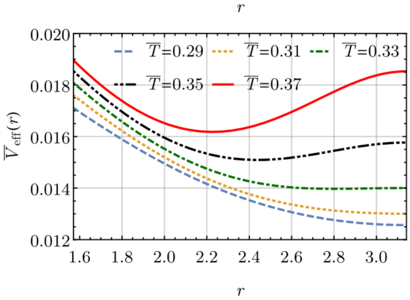

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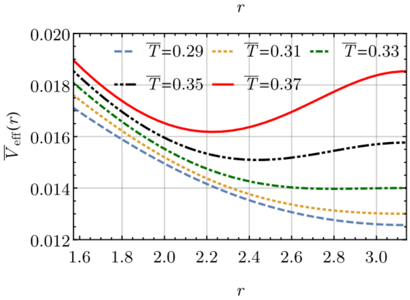

$$\frac{d^2 V(r,T)}{dr^2}\Big|_{r=\pi} = 0$$

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Other results

$T_c(MeV)$

	Lattice	CF at 1-loop [2]	CF at 2-loop [3]
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